Parallel Multi-Strategy Evolutionary Algorithm Using Massage Passing Interface for Many-Objective Optimization

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***Abstract*—Evolutionary multi-objective optimization (EMO) algorithms have become prevalent and obtained a great success for solving two- or three-objective problems. However, with the number of objectives increases, most of the algorithms cannot perform well due to the expansion of the objective space. Therefore, there is an urgent need for improving EMO algorithms to handle many-objective (four or more objectives) optimization problems (MaOPs). To this end, this paper proposes a parallel multi-strategy evolutionary algorithm (PMEA) to make full use of the advantages of different selection strategies. Specially, PMEA maintains three populations in parallel to select individuals based on three strategies as decomposition-based approach, indicator-based approach, and shift-based density estimation approach. PMEA uses message passing interface (MPI) to share the information after the selection of the three strategies, so that the advantages of diverse approaches can be utilized. In this way, PMEA can explore the objective space more thoroughly and thus achieve more promising performance. We evaluated PMEA on two frequently used MaOP suites and compared the results with several state-of-the-art many-objective peer algorithms. Numerical results demonstrate that PMEA can achieve a statistically superior performance, or at least highly competitive performance on most of the problems instances.**

Keywords—Parallel, Multi-Strategy, Message Passing Interface (MPI), PMEA, Many-Objective Optimization

I. INTRODUCTION

Many real-world problems can be formulated as multi-objective problems (MOPs), since they naturally have two or more conflict objectives that must be optimized simultaneously. Research on MOPs has drawn a considerable attention over the past several decades [1]-[4]. As evolutionary computation (EC) algorithms have obtained a great success in single-objective optimization [5]-[10], many researchers try to extend EC algorithms to MOPs [11]-[16], which has gradually formed the current evolutionary multi-objective optimization (EMO) methodologies. However, since an MOP has multiple objectives that often contradict with each other, there does not exist a single solution that can optimize all the objectives. Besides, it is hard to determine whether one individual is better than another if it is better on one objective but worse on another objective. As a result, many researchers encounter the problem on how to select good individuals for the next generation [13]. This problem will be aggravated when dealing with four or more objectives. Hence, optimizers are required to find a set of trade-off solutions among the conflicting objectives. This set of solutions is called Pareto optimal solutions. Generally, there are two ultimate goals when solving MaOPs. One is the convergence, which aims to minimize the distance to the Pareto front. The other one is the diversity, which aims to maximize the distribution of solutions. A number of EMO algorithms have been proposed to achieve these two goals simultaneously, mainly being classified into the following four major categories.

(1) Pareto dominance based methods: This method uses Pareto dominance relation as the selection criterion. Individuals are ranked based on their dominance relations with other individuals. Besides, this method incorporates a density selection criterion to keep the population diversity, such as the crowding distance in NSGA-II [13] and k-th nearest distance in SPEA2 and PESAII [14]-[16].

(2) Decomposition-based methods: This method decomposes an MOP into several single objective subproblems. By solving each subproblem, we can get the optimum of each subproblem, which approaches the Pareto front. The most representative algorithm is MOEA/D [17].

(3) Indicator-based methods: This method constructs a fitness evaluation function as a performance indicator according to the distribution of population. Three famous exemplars are IBEA [18], SMS-EMOA [19] and HypE [20].

(4) Multiple populations coevolutionary technique: Zhan et al. [11] proposed a novel coevolutionary framework named multiple populations for multiple objectives (MPMO), which let each population correspond with only one objective. Different populations exchange search information by using an external shared archive.

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The above EMO algorithms work well on two- or three-objective problems. However, their performances are not entirely satisfying when dealing with many-objective optimization problems (MaOPs). First and foremost, the proportion of non-dominated solutions becomes exponentially large when the number of objectives increases. This poses great challenges to Pareto dominance based algorithms as their selection operators cannot well distinguish the qualities of solutions in high-dimensional objective space. Second, the implementation of diversity-preservation operators [14]-[16] becomes computationally expensive, which slows down the search process considerably. Therefore, the performance of Pareto dominance based algorithms is seriously damaged. Nevertheless, in recent years, research into exploiting the potential of Pareto dominance based algorithms in handling MaOPs are gaining popularity and have made some achievements. Some new methods, such as ε-dominance [21], fuzzy Pareto dominance [22], and grid dominance [23], concentrate on relaxing the conditions of the traditional dominance relation and propose new dominance relations. Besides, there are also some studies, like the shifted-based density estimation proposed in [24] and the niche-preservation operation in NSGA-III [25], dedicated to the improvement of the diversity maintenance mechanism.

In the literatures, some enhanced Pareto dominance based algorithms such as shifted-based density estimation [24], decomposition-based methods [17], indicator-based methods [18], and MPMO based methods [26] have shown certain promising performance in dealing with MaOPs. However, there is still much room for improvement. To do so, a promising way is to combine the advantages of different computation paradigms to obtain promising solution. Besides, parallel computation has also been found a straightforward way to accelerate algorithms through distributed processing. Therefore, this paper is devoted to the design of a parallel many-objective algorithm which can combine the advantages among decomposition-based method, indicator-based method, and shifted-based density estimation method using a MASTER-SLAVE model. Based on the consideration, a parallel multi-strategy evolutionary algorithm (PMEA) is proposed. During the evolutionary process, we maintain three parallel populations on three SLAVEs. Each population utilizes a different selection mechanism. The first population is updated according to the aggregation function values of individuals. The selection operator of the second population is based on the binary additive ε-indicator [27]. While the shifted-based density estimation is adopted in the third population. The information sharing among all the three populations are implemented using a mating selection strategy that picks parents from the three populations on MASTER, with the help of message passing interface (MPI). Experiments have been carried out on two commonly used many-objective test suites to examine the performance of PMEA. Experimental results show that the proposed algorithm is compared favorably with several state-of-the-art many-objective algorithms.

The rest of the paper is organized as follows. Section II introduces the some basic definitions of MaOPs and their developments briefly. PMEA method is proposed in Section III. Section IV compares PMEA with other state-of-the-art peer algorithms on two commonly used many-objective test suites and makes discussions. Conclusions are given in Section V.

II. Related Works

In this section, some basic definitions in MaOPs are first given. Then, we will briefly introduce three famous many-objective optimization algorithms, named SDE, MOEA/D and IBEA, which are the basis of our proposed algorithm.

A. Basic Definitions

Generally speaking, MaOPs are defined as problems with four or more objectives. A MaOP with minimization objectives can be formulated as:

$$\begin{aligned} &MIN \quad F(X) = (f_1(X), f_2(X), ..., f_m(X))^{\mathrm{T}} \\ &\text{subject to} \quad X \in \Omega \subseteq \mathbb{R}^n \end{aligned} \tag{1}$$

where X is a n-dimensional decision variable vector from the decision space Ω. $m \geq 4$ is the number of objectives and f_i represents the i-th objective. More definitions are given below.

Definition 1: Given two decision vectors X, $Y \in \Omega$, X is said to **Pareto dominate** Y, denoted by $X \prec Y$, iff for all $i \in \{1,2,\ldots,m\}$, $f_i(X) \leq f_i(Y)$ and there exists at least one index j such that $f_j(X) < f_j(Y)$.

Definition 2: A solution $X^* \in \Omega$ is **Pareto optimal** iff no solution in the decision space Ω can Pareto dominate it.

Definition 3: The **Pareto set** (*PS*) is defined as the set of all Pareto optimal solutions.

Definition 4: The **Pareto front** (*PF*) is the set of the corresponding objective vectors of solutions in *PS*.

Definition 5: The **ideal point** $Z^*=(z_1^*, z_2^*,\ldots,z_m^*)^{\mathrm{T}}$ and the **nadir point** $Z^{\mathrm{nad}}=(z_1, z_2, \ldots, z_m)^{\mathrm{T}}$ are two points in the objective space, where z_i^* and z_i^{nad} are the infimum and supremum of the i-th objective (subject to *PS*) respectively.

The goal of many-objective optimization is to find a set of approximate true Pareto front. The quality of the solution set is measured from two aspects: convergence, which is the distance of the solutions to the Pareto front; and diversity, which is the distribution of the solutions.

B. Shift-Based Density Estimation (SDE)

Generally, density estimation techniques estimate the density of an individual by considering the mutual position relation between it and other individuals in the population. And the density of an individual X in the population P can be presented as:

$$D(X,P) = D(d(X,Y_1), d(X,Y_2), ..., d(X,Y_{N-1})) \tag{2}$$

where $Y_i \in P$ and $Y_i \neq X$; N is the size of P and $d(X,Y)$ is the similarity degree between X and Y, often measured by their Euclidean distance. $D()$ is the function of the similarity degree between the interested individual and other individuals in the population.

The main idea in SDE is to give the individuals which have no clear advantage over other individuals in the population high density value, so as to filter the poorly converged individuals by the density-based second selection criterion [24]. Specifically, if an individual performs better than X for an objective, it will be shifted to the same position of X on this objective; otherwise, it remains unchanged. As for a minimization MOP, the shifted density $SD(X, P)$ of individual X in the population P can be described as follows:

$$SD(X,P)=D(d(X,Y_1'),d(X,Y_2'),...,d(X,Y_{N-1}')) \quad (3)$$

where Y' is the shifted position of individual Y, which can be formulated as:

$$Y'_{i(j)}=\begin{cases}X_{(m)}, & if\ Y_{i(m)}<X_{(m)}\\ Y_{i(m)}, & otherwise\end{cases} \quad (4)$$

where $X_{(m)}$, $Y_{i(m)}$ and $Y'_{i(j)}$ denote the mth objective value of individuals $X_{(m)}$, $Y_{i(m)}$ and $Y_{i(j)}$, respectively.

C. Decomposition-based Algorithms

The key idea of decomposition-based algorithm is to divide an MOP into a number of single-objective optimization subproblems by using aggregation functions. The most representative technique is MOEA/D, which was proposed by Zhang and Li [17]. In MOEA/D, a subproblem can be defined as follows:

$$\begin{aligned}&MIN\ \ G(F(X),W)\\ &\text{subject to}\ \ X\in\Omega\subseteq\mathbb{R}^n\end{aligned} \quad (5)$$

where W is a set of evenly distributed weight vectors $\{W^1, W^2, \ldots, W^N\}$, G is an aggregation function that maps an objective vector $F(X)$ and a weight vector W into a single real value. Generally, three aggregation functions, named weighted sum functions, Tchebycheff functions, and penalty-based boundary intersection (PBI) functions can well serve the purpose in MOEA/D. Just take the weighted sum function as an example, the aggregation function value is obtained by calculating the inner product of the two input vectors, namely, $G(F(X), W)=F(X)^{\mathrm{T}}W$. The weight vector $W=(w_1, w_2, \ldots, w_m)^{\mathrm{T}}$ is an m-dimensional vector that satisfies two constraints: (1) $w_i\geq 0$ for any $i\in\{1, 2, \ldots ,m\}$, where m is the number of objectives, (2) $\sum_{i=1}^{m} w_i=1$. The i-th element w_i represents the relative importance of the i-th objective. Multiple subproblems are obtained by multiple weight vectors. MOEA/D aims to optimize these subproblems simultaneously by evolving a population of N individuals $\{X_1, X_2, \ldots, X_N\}$, where X_i represents the current solution to the i-th subproblem. When generating the i-th offspring, two parents are chosen from the predefined neighborhood of W^i. Then, an offspring Y_i is produced by performing SBX and polynomial mutation operators on these two parents. Once Y_i is obtained, it is compared with its neighborhood solutions. Suppose X_j is a neighbor of Y_i, X_j will be replaced by Y_i if the condition $G(F(Y_i),W^j)<G(F(X_j),W^j)$ is satisfied. A new population is formed after N offspring have been produced in this manner one after another. The procedures are repeated until the termination criterion is met. Further details of MOEA/D can be referred in [17].

Experimental results reported in [17] show that the performance of MOEA/D is quite promising on some groups of multi-objective problems, as a result, growing attentions such as self-adaptation mechanism [28], local search strategy [29] have been paid on improving the performance of the decomposition-based algorithms.

D. Indicator-based Algorithms

The basic framework of indicator-based algorithms is to construct a fitness evaluation function as a performance indicator to guide the search of individuals. And this performance indicator transfers the convergence and diversity to a single value. Zitzler and Kunzli [18] proposed the indicator-based EA (IBEA), which is a pioneer study and makes the predominant contribution in this group of algorithms. The main idea is based on a dominance preserving quality indicator, which amplifies the influence of non-dominated individuals over the dominated ones. It proposed two quality indicators called the binary additive ε-indicator and the hypervolume indicator. The experimental results in [18] show that IBEA is superior to several famous EMO algorithms. Based on this thought, numerous new algorithms emerged. Beume *et al.* [19] proposed an S-metric selection operator which is based on hypervolume and incorporated it into EMO algorithm (SMS-EMOA). Individuals with little hypervolume will be removed from the population. However, it is not appropriate for MaOPs since its computational time is unacceptable when the number of objectives increases to four or more. Therefore, Bader and Zitzler proposed a hypervolume estimation algorithm (HypE) by using the Monte Carlo simulation to reduce the computational time. Since the exact calculation of hypervolume is avoided, the algorithm can be used to solve many-objective problems. Moreover, there are also several other performance indicators, such as the $R2$ indicator [30], which have been proved to be quite effective in tackling many-objective problems.

From the mentioned above, it is clear that the shift-based density estimation methods, decomposition-based methods and indicator-based methods can produce satisfying results on many-objective problems. However, their performance may be further improved when combining the advantages of all of the three methods. Note that the major difference among the three groups of algorithms is the selection mechanism, it is possible to design a framework that can exploit the advantages of all the three methods. To this end, a parallel multi-strategy evolutionary algorithm (PMEA) is proposed in this paper. The detailed procedures of PMEA are presented in the next section.

III. PMEA Approach

A. Parallel Framework

To combine the above three algorithms together, a parallel strategy is needed. The message passing interface (MPI) is adopted herein. In this strategy, four virtual processors are used. The processor, whose role is to control the whole evolution process, is called MASTER. The other three processors, each responsible for one of the three aforementioned algorithms, act as SLAVEs.

To give an overview of the PMEA, the parallel structure and the complete flowchart are depicted in Fig. 1 and Fig. 2. At

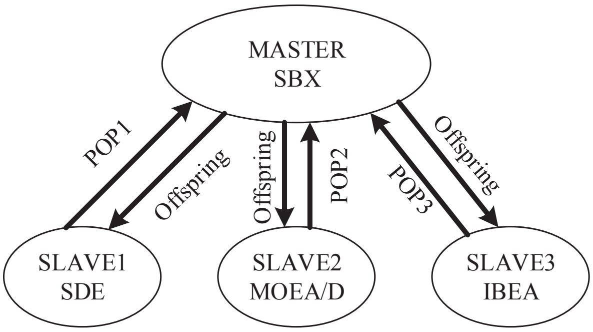

Fig. 1 Parallel structure in PMEA

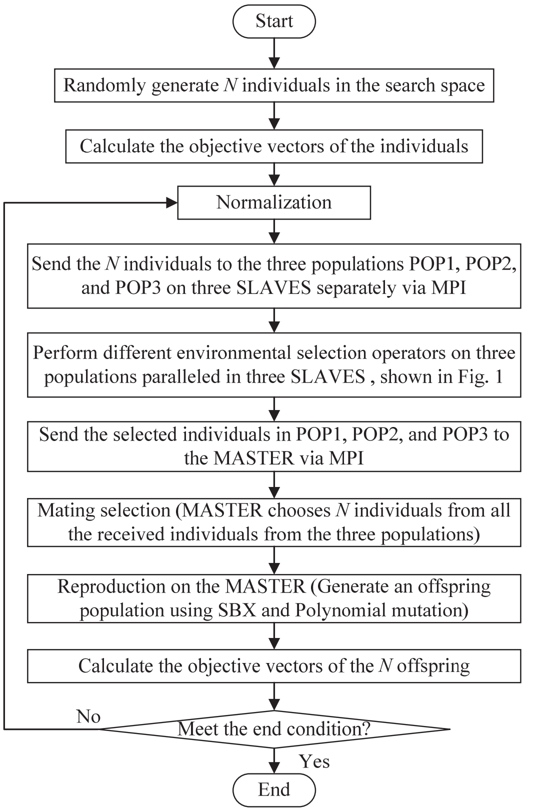

Fig. 2 The flowchart of PMEA

first, we will generate N evenly distributed weight vectors $\{W^1, W^2, \ldots, W^N\}$ using a systematic approach and a two layer generation scheme with inner and outer divisions [25], [31]. At each generation, we keep three parallel populations POP1, POP2, and POP3 with size N on three SLAVEs (each of them has N population slots). Each population slot is associated with a weight vector W^i and is used to store the current best solution. All the population slots are initialized as empty. After that, we randomly generate N individuals $\{X_1, X_2, \ldots, X_N\}$ in the decision space and calculate their objective vectors. And it is needed to normalize them to ensure that they are in the same space as the weight vectors. Then, we put the individuals to the population slots of POP1, POP2, and POP3 separately according to their perpendicular distances to the weight vectors. After the assignment, we perform environmental selection on each population in parallel using three distinct operators based on MOEA/D, IBEA, and SDE respectively. Note that the individuals assigned to the same population slot will have to compete for the living space since it can only accommodate one individual. In contrast, there is no competitive relation between individuals in different slots. To have a more intuitive look, this procedure is illustrated in Fig. 3. The detailed steps of performing environmental selection on the three populations will be shown in the following subsections. After the environmental selection, we choose parents from all the three populations and the offspring population of size N is produced using the SBX and polynomial mutation operators on the MASTER. The offspring are evaluated and normalized, and continued to be assigned to the three populations on three SLAVEs for selection. Up to now, we have finished a loop sequent. And the procedures are repeated until the termination criterion is met. For completeness, some important steps are further described as follows.

1) Normalization. Normalization is a significant step for handling MaOPs whose objectives are of different scales. It is implemented using two important points, the ideal point Z* and the nadir point Z^{nad}, shown below:

$$f_i = \frac{f_i - z_i^*}{z_i^{\mathrm{nad}} - z_i^*},\ i = 1, 2, \ldots, m. \tag{6}$$

Note that Z^* and Z^{nad} are unknown before solving a MaOP. They are estimated by the non-dominated individuals in the population. Z^* is estimated using the minimum objective values found so far. While the Z^{nad} is approximated according to m extreme points from the non-dominated solutions. For more details of estimating Z^* and Z^{nad}, please refer to [32].

2) Individual assignment. For each individual X, we first calculate its perpendicular distances to all the weight vectors $\{W^1, W^2, \ldots, W^N\}$ in the normalized objective space, shown in Fig. 4. X will be assigned to the population slots POP1[i], POP2[i] and POP3[i] respectively if W^i is the nearest weight vector to X.

3) Mating selection. After the environmental selection, we picks parents from all the three populations randomly. Specifically, for each $i \in \{1,2,\ldots,N\}$, we select a parent from {POP1[i], POP2[i], POP3[i]} with equal probability.

B. Environmental Selection of the First Population

The selection operator in the first population is based on MOEA/D, which is determined by an aggregation function. The smaller the function value, the better the individual. Zhang and Li proposed three aggregation functions to solve MOPs [17]. However, according to the results in [25], it indicates that the penalty-based boundary intersection (PBI) approach is more suitable for MaOPs than other approaches. So we also use the aggregation function defined by the PBI approach here in POP1. The aggregation function is defined as follows:

$$G^{\mathrm{PBI}}(F(X), W) = d_1 + \theta d_2 \tag{7}$$

where d_1 is the distance between the origin and the projection point of $F(X)$, while d_2 is the perpendicular distance to the weight vector W, shown in Fig. 4. As we can see, d_1 is used to measure the closeness to the Pareto front, while d_2 is used to maintain the diversity of individuals. θ is a predefined penalty parameter that combines d_1 and d_2 into a single value. The individuals with small distances to the weight vectors and the Pareto front is preferred. Besides, different objectives may

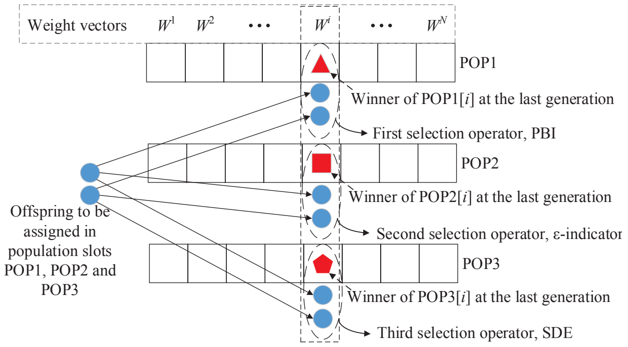

Fig.3 Parallel muti-strategy in PMEA

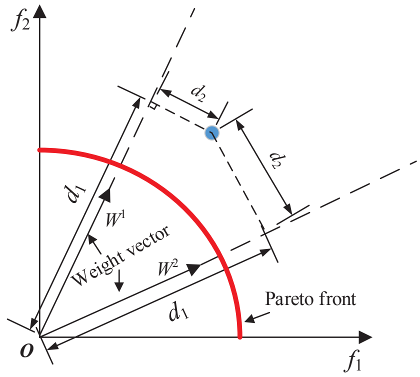

Fig.4 Illustration of the distance d_1 and d_2.

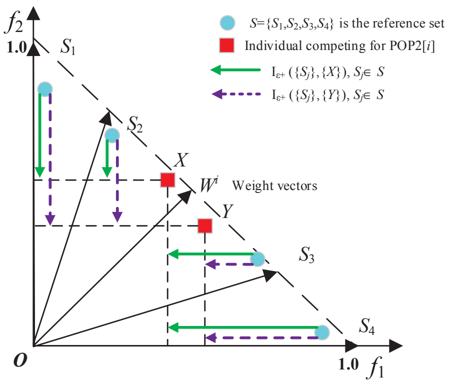

Fig.5 Illustration of binary additive ε-indicator

have different ranges of values, the two distances are also need to be normalized, given as:

$$d_1 = F(X)^{\mathrm{T}} W / \| W \| \tag{8}$$

$$d_2 = \| F(X) - d_1 (W / \| W \|) \| \tag{9}$$

Suppose there are k individuals $\{X_{p1}, X_p, .., X_{pk}\}$ assigned to the population slot POP1[i] and X_{p0} is the winner of the last generation. We choose the best individual from $\{X_{p0}, X_{p1}, X_{p2}, .., X_{pk}\}$ with the smallest function values of $G^{PBI}(F(X), W^i)$ and remove other individuals.

C. Environmental Selection of the Second Population

The selection operator in the second population is based on IBEA, which is determined by a performance indicator. There are several indicators such as binary additive ε-indicator, the hypervolume indicator, the $R2$ indicator and hypervolume estimation algorithm (HypE). However, as we have mentioned, hypervolume indicator is not computationally efficient when the number of objectives increases, while the implement of $R2$ indicator and HypE is conceptually complicated. As a result, in our method, the binary additive ε-indicator ($I_{\varepsilon+}$) is adopted as the basis of the second selection operator. And its property of dominance preserving and the effectiveness of solving MaOPs have been demonstrated in [18]. Given two solution sets A and B, the indicator is defined as follows:

$$I_{\varepsilon+}(A,B) = \min_{\varepsilon}\{\forall Y \in B\ \exists X \in A : f_i(X) - \varepsilon \le f_i(Y)\ \text{ for } i \in \{1,2,...,m\}\} \tag{10}$$

The main idea is to get the minimum shift required by each dimension of the objectives so that A can weakly dominate B. To see the idea more apparently, Fig. 5 illustrates our idea. In our method, for each individual X that in the population slot POP2[j], we construct a reference set S by choosing all the other winning individuals of POP2[i] ($i\in\{1,2,\ldots,N\}\backslash\{j\}$) in the previous generation. And the fitness of X is calculated as:

$$Fit(X) = \sum_{Y\in S} -e^{-I_{\varepsilon+}(\{Y\},\{X\})/0.05} \tag{11}$$

The larger the function value, the better the individual. Therefore, after the fitness assignment, we choose the best individual with the largest fitness value and remove all other individuals in each population slot of POP2.

D. Environmental Selection of the Third Population

In this population, we use shift-based density estimation (SDE) to evaluate the density of individuals. In [24], SDE are applied into NSGA-II, SPEA2, and PESA-II to prove its effectiveness. And experimental results show that SPEA2-SDE is more competitive and has a clear advantage over the other Pareto dominance based algorithms, which is more suitable for MaOPs. So we also use this mechanism in our method, which is used in each population slot and choose the best individual. Specifically, for each individual X that in the population slot POP3[j], we also construct a reference set S by choosing all the other winning individuals of POP3[i] ($i\in\{1,2,\ldots,N\}\backslash\{j\}$) in the previous generation, similar to POP2. And the shifted density of X is calculated using (3)-(4) and the kth nearest neighbor density estimator in [15]. In this way, these poorly converged individuals will be assigned a high density value, thus being eliminated easily during the evolutionary process. Therefore, after the fitness assignment, we choose the best individual with the least density value and remove all other individuals in each population slot of POP3.

To summarize, the fitness of individuals in the first population in PMEA is updated according to the individual itself, which is a local view. While in the other two populations, the fitness is updated from a global view, which is calculated according to the distribution of the entire population. Therefore, PMEA can achieve a good balance between convergence and diversity. Moreover, with the help of three populations, we can increase the algorithm's exploration ability and avoid getting trapped in local Pareto fronts. Although we use three populations, only N offspring are produced at each iteration. As a result, PMEA does not increase the number of fitness evaluations.

IV. Experiment Testing And Results

A. Experimental Setup

1) Test Functions: In this paper, two commonly used many-objective problem suites DTLZ [33] and WFG [34] are adopted to test the performance of PMEA. The problems in this two test suites have different characteristics, which make our experiments more comprehensive and convincing. Note that DTLZ5 and DTLZ6 are unincluded in our experiments because their Pareto fronts are unclear when the number of objectives is larger than three, as mentioned in [33].

2) Algorithms for Comparison: We compare PMEA with eight state-of-the-art many-objective algorithms, MOEA/D-PBI [17], NSGA-III [25], GrEA [23], SDE-SPEA2 [24], IBEA [18], θ-DEA [36], KnEA [37] and EPSMOEA [38]. To have a reliable and fair comparison, the recommended parameter configurations are set for all competitor algorithms as suggested in their original papers.

3) Parameter Settings: Five different settings of the number of objectives, i.e., 3, 5, 8, 10, and 15, are used in our experiments. The weight vectors and the settings of the outer and inner divisions (H_1 an H_2) used in PMEA are provided in Table I. The population sizes of all the algorithms are equal to the number of weight vectors. Besides, the crossover and mutation probabilities (P_c and P_m) are set at 0.9 and $1/n$, respectively, where n is the number of decision variables. In SBX and polynomial mutation operators, the distribution indexes (η_c and η_m) are both set to 20. And the penalty factor θ in PMEA is set to 5 as recommended in [17]. Algorithms are executed until the maximum number of generations is met. Table II shows the settings of MaxGen for all the MaOPs. Each algorithm runs 25 times for each problem instance. For all algorithms, the non-dominated individuals in the final generation are used for the performance evaluation.

TABLE I. SETTINGS OF DIVISIONS AND WEIGHT VECTORS

Objectives	Divisions(H1, H2)	Weight Vectors
3	12, 0	91
5	6, 0	210
8	3, 2	156
10	3, 2	275
15	2, 1	135

TABLE II. MAXIMUM GENERATIONS FOR DIFFERENT MaOPs

Problem	m	Max_Gen	Problem	m	Max_Gen
DTLZ1	3	400	DTLZ4	3	600
	5	600		5	1000
	8	750		8	1250
	10	1000		10	2000
	15	1500		15	3000
DTLZ2	3	250	DTLZ7	3	1000
	5	350		5	1000
	8	500		8	1000
	10	750		10	1500
	15	1500		15	2000
DTLZ3	3	1000	WFG	3	400
	5	1000		5	750
	8	1000		8	1500
	10	1500		10	2000
	15	2000		15	3000

4) Performance Measure: Hypervolume indicator is adopted here to evaluate the performance of many-objective algorithms, which is currently the only single set quality measure that known to be strictly monotonic with respect to Pareto dominance. Given a reference point r, the hypervolume (HV) of a solution set A is calculated as:

$$HV(A,r) = volume(\bigcup_{X\in A} Cube(X,r)) \quad (12)$$

where $Cube(X,r)$ represents the hypercube constructed by X and r. The points in A which dominates r are used for calculation. Note that the hypervolume should be normalized first and the reference point r is set to $(1.1, \ldots, 1.1)^T$.

B. Results and Discussion

The mean HV values of different algorithms on DTLZ and WFG are presented in Table III. For clarity, the results of the best algorithms are marked in **boldface**. In addition, Wilcoxon's rank sum test [35] at α=0.05 is tested to evaluate the statistical significance of the results between different algorithms. The symbols +, ≈, and - indicates PMEA performs significantly better (+), similarly (≈), or worse (-) when compared with the corresponding algorithm. From the Table III, we can see that PMEA can achieve the largest HV values on most of DTLZ problems, especially on DTLZ1-DTLZ4. Note that the DTLZ1 and DTLZ3 are two multimodal problems with many local Pareto fronts, however, PMEA is superior to other algorithms and can converge to the real Pareto front. GrEA and IBEA are able to achieve better results on DTLZ5. As for WFG problems, GrEA and SPEA2-SDE show great performance on WFG1. While IBEA performs relatively well on WFG3 and WFG9. But PMEA performs the best in most of the remaining problems. Overall, PMEA outperforms NSGA-III, MOEA/D-PBI, GrEA, IBEA, KnEA, EPSMOEA, SPEA2-SDE and θ-DEA on 57, 69, 61, 45, 65, 70, 63 and 61 problem instances respectively, in other words, PMEA achieves better results on a large proportion of the problem instances. And the better performance is due to the well combination of advantages among three different selection operator, which can achieve a good balance between exploration and exploitation.

V. Conclusion

In this paper, a parallel multi-strategy evolutionary algorithm (PMEA) is proposed for solving MaOPs. The algorithm makes use of the potential of decomposition-based method, indicator-based methods and shift-based density estimation by maintaining three populations in parallel. The individuals in the first population are evaluated and selected from a local view, while in the other two populations, the individuals are chosen from a global view according to the distribution of population. Furthermore, with the support of three populations, we can improve the population diversity and increase the algorithm's exploration ability. Experimental results also illustrate that PMEA is very competitive and performs better than some other peer algorithms on several benchmark functions frequently used in a statistically meaningful way. In the future, the proposed PMEA is expected to be applied to more complex real-world MaOPs like power systems or cloud computing resources scheduling.

TABLE III. EXPERIMENTAL RESULTS OF PMEA AND OTHER STATE-OF-THE-ART MaOPs

Problem	m	PMEA	NSGA-III	MOEA/D-PBI	GrEA	IBEA	KnEA	EPSMOEA	SPEA2-SDE	θ-DEA
DTLZ1	3	**1.1231**	1.1184(+)	1.1172(+)	1.0717(+)	0.5964(+)	1.0972(+)	1.0080(+)	1.1003(+)	1.1183(+)
	5	**1.5834**	1.5776(+)	1.5777(+)	1.4725(+)	1.2123(+)	1.3889(+)	1.3260(+)	1.5505(+)	1.5779(+)
	8	**2.1417**	2.1378(+)	2.1358(+)	2.1135(+)	1.7129(+)	1.3821(+)	1.9093(+)	2.0947(+)	2.1380(+)
	10	**2.5937**	2.5928(≈)	2.5924(≈)	2.5691(+)	2.4243(+)	1.7101(+)	2.5693(+)	2.5663(+)	2.5927(≈)
	15	**4.1752**	4.1751(≈)	4.1678(+)	3.9382(+)	3.0039(+)	0(+)	4.0875(+)	4.0971(+)	4.1745(≈)
DTLZ2	3	**0.7638**	0.7439(+)	0.7440(+)	0.7405(+)	0.7479(+)	0.7253(+)	0.5350(+)	0.7430(+)	0.7443(+)
	5	**1.3336**	1.3042(+)	1.3070(+)	1.3042(+)	1.3223(+)	1.2740(+)	1.1883(+)	1.3042(+)	1.3073(+)
	8	**2.0117**	1.9689(+)	1.9778(+)	1.9904(+)	1.9972(+)	1.6263(+)	1.6463(+)	1.9875(+)	1.9786(+)
	10	**2.5418**	2.5088(+)	2.5150(+)	2.5158(+)	2.5222(+)	2.1349(+)	2.2249(+)	2.5195(+)	2.5145(+)
	15	**4.1477**	4.1301(+)	4.1363(+)	4.0773(+)	4.1144(+)	3.4854(+)	3.0998(+)	4.1022(+)	4.1362(+)
DTLZ3	3	**0.7576**	0.7370(+)	0.7359(+)	0.7009(+)	0.3307(+)	0.6892(+)	0.5928(+)	0.7406(+)	0.7369(+)
	5	**1.3311**	1.3012(+)	1.3040(+)	1.1229(+)	0.6156(+)	0.4483(+)	0.7962(+)	1.3049(+)	1.3058(+)
	8	**2.0208**	1.9462(+)	1.7484(+)	1.6574(+)	1.1292(+)	0.5142(+)	0(+)	1.9770(+)	1.9718(+)
	10	**2.5612**	2.5087(+)	2.5139(+)	2.1785(+)	1.5998(+)	0.2461(+)	0.8443(+)	2.5185(+)	2.5141(+)
	15	**4.1417**	3.9494(+)	3.6948(+)	3.2441(+)	2.0544(+)	0(+)	0(+)	4.0757(+)	4.1345(+)
DTLZ4	3	**0.7518**	0.7212(+)	0.6866(+)	0.5836(+)	0.7021(+)	0.7164(+)	0.6744(+)	0.6878(+)	0.7070(+)
	5	**1.3322**	1.3015(+)	1.2960(+)	1.3077(+)	1.3233(+)	1.2791(+)	1.2238(+)	1.3067(+)	1.3087(+)
	8	**2.0327**	1.9805(+)	1.9672(+)	1.9901(+)	1.9884(+)	1.8227(+)	1.7968(+)	1.9867(+)	1.9808(+)
	10	**2.5654**	2.5152(+)	2.5154(+)	2.5129(+)	2.5251(+)	0.4361(+)	2.4155(+)	2.5146(+)	2.5154(+)
	15	**4.1456**	4.1364(+)	4.1209(+)	4.1292(+)	4.1140(+)	4.0976(+)	3.7494(+)	4.1150(+)	4.1358(+)
DTLZ7	3	1.1722	1.1739(-)	0.8760(+)	1.1571(+)	**1.1914(-)**	1.1623(+)	1.0459(+)	1.1492(+)	1.1453(+)
	5	1.1431	1.1356(+)	0.7344(+)	1.2381(-)	**1.2764(-)**	1.1634(-)	0.9914(+)	1.2720(-)	1.0274(+)
	8	1.0237	1.1176(-)	0.7212(+)	**1.2278(-)**	1.1619(-)	0.9672(+)	0.6844(+)	0.6728(+)	1.0111(+)
	10	1.0772	1.2005(-)	0.9066(+)	**1.3142(-)**	1.2800(-)	0.0045(+)	0.9632(+)	0.8140(+)	1.1953(-)
	15	1.1007	1.2903(-)	0.4038(+)	**1.3653(-)**	1.2852(-)	0(+)	0.8431(+)	0.0782(+)	1.3197(-)
WFG1	3	0.5711	0.5120(+)	0.5475(+)	0.5990(-)	0.5800(-)	0.5584(+)	0.3671(+)	**0.6569(-)**	0.5346(+)
	5	0.7022	0.6465(+)	0.6307(+)	0.7506(-)	0.7034(-)	0.6808(+)	0.3633(+)	**1.0306(-)**	0.6708(+)
	8	0.9887	0.7925(+)	0.5444(+)	1.5326(-)	0.8341(+)	0.8315(+)	0.4782(+)	**1.5536(-)**	0.8306(+)
	10	1.1357	0.9578(+)	0.6039(+)	1.0740(+)	0.9879(+)	0.9645(+)	0.6810(+)	**2.2261(-)**	0.9808(+)
	15	1.5705	1.4283(+)	0.4107(+)	**3.4222(-)**	1.4033(+)	1.3588(+)	1.0715(+)	2.8975(-)	1.4533(+)
WFG2	3	1.2283	1.2266(≈)	1.1900(+)	1.2281(≈)	**1.2334(-)**	1.2258(≈)	1.1564(+)	1.2268(≈)	1.2310(-)
	5	**1.5996**	1.5981(≈)	1.5812(+)	1.5704(+)	1.5845(+)	1.5968(≈)	1.4934(+)	1.5812(+)	1.5901(+)
	8	**2.1366**	2.1349(≈)	0.7176(+)	2.0979(+)	2.1088(+)	2.1241(+)	1.9549(+)	2.1070(+)	2.0892(+)
	10	**2.5878**	2.5876(≈)	1.1461(+)	2.5669(+)	2.5715(+)	2.5819(≈)	2.4452(+)	2.5727(+)	2.5346(+)
	15	4.1696	**4.1701(-)**	0.5361(+)	4.0941(+)	4.1217(+)	4.1070(+)	3.9153(+)	4.1459(+)	3.8475(+)
WFG3	3	0.8511	0.8198(+)	0.7805(+)	0.8337(+)	**0.8532(-)**	0.8181(+)	0.5413(+)	0.8439(+)	0.8185(+)
	5	**1.1125**	1.0167(+)	0.9270(+)	1.0157(+)	1.1040(+)	1.0169(+)	0.7626(+)	0.9872(+)	1.0274(+)
	8	1.2931	1.2665(+)	0.2179(+)	1.2667(+)	**1.4892(-)**	1.3351(-)	1.0982(+)	1.1471(+)	1.1162(+)
	10	**1.8455**	1.5867(+)	0.2692(+)	1.5905(+)	1.8450(≈)	1.6622(+)	1.4351(+)	1.4399(+)	1.5463(+)
	15	2.8176	2.4341(+)	0.3858(+)	2.7079(+)	**2.9146(-)**	1.5118(+)	2.1678(+)	1.7674(+)	2.5293(+)
WFG4	3	0.7399	0.7292(+)	0.6904(+)	0.7176(+)	**0.7436(-)**	0.7120(+)	0.5726(+)	0.7256(+)	0.7295(+)
	5	**1.3199**	1.2854(+)	1.2656(+)	1.2710(+)	1.3125(≈)	1.2610(+)	0.8474(+)	1.2387(+)	1.2873(+)
	8	**2.0074**	1.9621(+)	1.9577(+)	1.9332(+)	1.9853(+)	1.9223(+)	0.9797(+)	1.8346(+)	1.9649(+)
	10	**2.5146**	2.5029(+)	2.5099(+)	2.4794(+)	2.5105(≈)	2.4883(+)	1.3430(+)	2.3638(+)	2.5037(+)
	15	**4.1359**	4.1351(≈)	4.1160(+)	3.8933(+)	4.1143(+)	3.9054(+)	1.8238(+)	3.7139(+)	4.1354(≈)
WFG5	3	**0.7002**	0.6864(+)	0.6578(+)	0.6693(+)	0.6934(+)	0.6728(+)	0.5373(+)	0.6847(+)	0.6873(+)
	5	**1.2488**	1.2216(+)	1.2085(+)	1.2194(+)	1.2401(≈)	1.1922(+)	0.9204(+)	1.1912(+)	1.2227(+)
	8	**1.8866**	1.8500(+)	1.8431(+)	1.8620(+)	1.8681(+)	1.8262(+)	0.9487(+)	1.7510(+)	1.8501(+)
	10	**2.3771**	2.3462(+)	2.3459(+)	2.3347(+)	2.3525(+)	2.3324(+)	1.3314(+)	2.2441(+)	2.3465(+)
	15	**3.8447**	3.8315(+)	3.1930(+)	3.4544(+)	3.8160(+)	3.8131(+)	1.6755(+)	3.1853(+)	3.8316(+)
WFG6	3	0.6992	0.6867(+)	0.6475(+)	0.6760(+)	**0.6994(≈)**	0.6697(+)	0.5292(+)	0.6890(+)	0.6899(+)
	5	**1.2522**	1.2188(+)	1.2104(+)	1.2258(+)	1.2499(≈)	1.1945(+)	0.8924(+)	1.1884(+)	1.2238(+)
	8	**1.8819**	1.8444(+)	1.8400(+)	1.8587(+)	1.8641(+)	1.8024(+)	0.8728(+)	1.7304(+)	1.8434(+)
	10	**2.3501**	2.3230(+)	2.3408(+)	2.3233(+)	2.3332(+)	2.3089(+)	1.2795(+)	2.2196(+)	2.3307(+)
	15	**3.7200**	3.7064(+)	0.5036(+)	3.5043(+)	3.7096(+)	3.5525(+)	1.5877(+)	3.1997(+)	3.7148(+)
WFG7	3	**0.7545**	0.7295(+)	0.6710(+)	0.7218(+)	0.7470(+)	0.7258(+)	0.5920(+)	0.7354(+)	0.7313(+)
	5	**1.3233**	1.2924(+)	1.2866(+)	1.2983(+)	1.3197(≈)	1.2736(+)	1.0166(+)	1.2592(+)	1.2956(+)
	8	**2.0025**	1.9717(+)	1.9665(+)	1.9925(+)	1.9928(+)	1.9385(+)	1.0541(+)	1.8438(+)	1.9740(+)
	10	**2.5256**	2.5074(+)	2.5093(+)	2.5008(+)	2.5124(+)	2.5053(+)	1.4704(+)	2.3874(+)	2.5086(+)
	15	**4.1396**	4.1339(+)	1.0186(+)	3.8402(+)	4.1145(+)	3.5187(+)	1.9407(+)	3.9081(+)	4.1349(≈)
WFG8	3	**0.6829**	0.6664(+)	0.6396(+)	0.6571(+)	0.6804(≈)	0.6375(+)	0.4963(+)	0.6643(+)	0.6689(+)
	5	1.1962	1.1820(+)	1.1860(+)	1.1738(+)	**1.2144(-)**	1.1289(+)	0.8356(+)	1.1453(+)	1.1837(+)
	8	1.7725	1.7642(+)	1.7686(+)	1.7327(+)	**1.7873(-)**	1.6954(+)	0.9625(+)	1.7046(+)	1.7685(+)
	10	**2.3551**	2.2910(+)	2.3082(+)	2.2511(+)	2.3039(+)	2.2533(+)	1.4460(+)	2.2138(+)	2.2874(+)
	15	3.7805	**3.9032(-)**	0.3799(+)	3.6693(+)	3.8388(-)	2.6674(+)	1.8559(+)	3.7565(+)	3.8639(-)
WFG9	3	**0.7123**	0.6782(+)	0.6124(+)	0.6730(+)	0.7112(≈)	0.6854(+)	0.5683(+)	0.6949(+)	0.6698(+)
	5	**1.2877**	1.2161(+)	1.1959(+)	1.2386(+)	1.2871(≈)	1.2231(+)	0.9359(+)	1.1944(+)	1.2198(+)
	8	**1.8891**	1.8214(+)	1.8076(+)	1.8728(+)	1.8517(+)	1.8591(+)	0.9922(+)	1.7231(+)	1.8411(+)
	10	**2.3690**	2.3020(+)	2.2292(+)	2.3413(+)	2.3587(+)	2.3076(+)	1.2755(+)	2.2197(+)	2.3689(≈)
	15	**3.8903**	3.8172(+)	0.3880(+)	3.6751(+)	3.7398(+)	2.0019(+)	1.6700(+)	3.6339(+)	3.8720(+)
+(PMEA significantly better)			57	69	61	45	65	70	63	61
-(PMEA significantly worse)			6	0	8	15	2	0	6	4
≈			7	1	1	10	3	0	1	5

+, ≈, - indicates PMEA performs significantly better (+), similar (≈), and worse (-) compared the corresponding algorithm according the Wilcoxon rank-sum test.

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