A multi-swarm approach to multiobjective synthesis of linear antenna array design

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***Abstract*—Research in the synthesis of antenna is starting to pay attention to optimize the multiobjective problems. While evolutionary algorithms has obtained some satisfactory performance, there still be necessary to make further improvements especially on the suppression of premature. In this paper, we introduce the multi-swarm technique into the multiobjective particle swarm optimization and present a MOP approach to linear array antenna design. Two objective functions concerning sidelobe level and nulls control is considered in the optimization. The result of comparison experiments proves that the proposed method can achieve better performance in multiobjective antenna synthesis, meanwhile, indicates the advantages and applicability in practical MOP applications.**

I. Introduction

With the increasing demands of antenna engineering, including the electromagnetic-based geometry design and unconventional antenna array design, there has been a growing interest in solving the synthesis optimization problems. Antenna synthesis is introduced in determining the parameters in antenna engineering for a desired shape in radiation pattern, which corresponds to the antenna application. Many synthesis standards are concerned with suppressing the sidelobe level(SLL) while maintaining the main beam, meanwhile setting up deep or broad null areas as targeted anti-interference measures. As a result, antenna synthesis become a multiobjective optimization problem(MOP). In recent years, with more efficient computational algorithms and faster processors, these complex nonlinear synthesis problems can be solved. Among the optimization techniques, evolutionary algorithms(EAs) becomes widely used in antenna design.

EAs are famous for the simplicity, versatility and robustness, and the introduction of genetic algorithm(GA), particle swarm optimization(PSO), differential evolution(DE) and other EAs have achieved positive results in antenna synthesis. However, the premature in convergence and parameter tuning in initialization are still drawbacks especially in some sensitive situations. Moreover, due to the high nonlinearness, large-scale or medium-scale decision variables and multi-demand during synthesis designs, the traditional and introduced EA approaches always start from the simplified model or combined fitness evaluations, which leads to a lower precision and a incomplete solution space. Most of the multi-demand synthesis problem are transformed into single-objective optimization using a set of weight coefficients. These techniques has been proved to achieve satisfied compromise solutions [1]–[3]. The combination of objective functions leads to the miss of most feasible solutions, and can not give various options for decision maker. Aim at this problem, the multiobjective evolutionary algorithms(MOEAs) such as NSGA-II, MOEA/D and MOPSO, are introduced to solve these multiple criteria decision making in antenna design [4]–[7]. As the number of decision parameters becomes large, the nonlinear becomes more complex. In these situations, the traditional MOEAs gets into the local extremum points easily and the convergence and diversity become poor in the evening of evolution.

Compared with other EAs, PSO choose developing directions based on similar reference solutions and group optima. Benefit from the group intelligence, PSO has faster searching speed and relatively accurate intuition, especially for solutions with non-uniform distribution. However, the possibility of premature also increases because of the directional learning. Antenna synthesizes are complex non-uniform problems which fit the PSO procedure. Aim at antenna synthesis MOPs and solving premature phenomenon, a multi-swarm multiobjective particle swarm optimization(MSMOPSO) is proposed in this paper. This work use the radiator parameters (amplitudes and phase) as the decision variables and pattern performance(SLL and null control) to constitute the objective vector. According to the experimental results, MSMOPSO can achieve better or competitive performance in both convergence and diversity than the other comparison algorithms.

The rest part of this paper is presented as the following. Section II give the brief overview of multi-objective problem and the particle swarm optimization in MOPs. The proposed algorithm MSMOPSO is detailed in Section III. The formulation of the array synthesis as the optimization task is discussed in Section IV. The comparative simulations and results are provided in Section V. Finally, the conclusions are in Section VI.

II. Multiobjective evolutionary algorithms

Multiobjective optimization, also known as an area of multiple criteria decision making, is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously. A multiobjective optimization problem can be stated as follow:

$$\begin{aligned} &Min \quad F(X) = (F_1(X), F_2(X), \ldots, F_m(X)) \\ &Subject\ to \quad X \in \Omega \end{aligned} \tag{1}$$

Here $\Omega = \prod_{i=1}^{n}[a_i, b_i] \subseteq \mathbb{R}^n$ is the decision space, $X = (x_1, x_2, \ldots, x_n)^T \in \Omega$ is the candidate solution, and $F \in \mathbb{R}^m$ is the objective space, which constitutes m objective functions. For most MOPs, the objectives are conflicting, and there does not exist a single solution that simultaneously minimize each objective. A Pareto optimal set is exist contains all nondominated solutions which have considered equally good performance. The goal of MOP methods is to find the representative set of Pareto optimal solutions.

The recent years saw the development of many different evolutionary algorithms working on the MOPs [8]–[10], and these evolutionary technique based MOP algorithms are called multi-objective evolutionary algorithms(MOEAs). Among the evolutionary techniques, the population-based metaheuristic methods with logical simplicity are proved to be efficient and feasible. Particle swarm optimization(PSO) is one of the outstanding and popular swarm intelligence, which has been transformed into multiobjective usage. If in the d dimensional search space, the velocity and position of particle i are $X_i = (x_{i1}, x_{i2}, \ldots, x_{id})$ and $V_i = (v_{i1}, v_{i2}, \ldots, v_{id})$, in each iteration, record the particle i's best position it has ever been ($pBest$), and the global optimum position all particles have ever been ($gBest$). Based on the following equations updating the particles velocity and position.

$$\begin{aligned} v_i(k+1) =& \omega v_i(k) + c_1 r_1 (pBest_i - x_i(k)) \\ &+ c_2 r_2 (gBest - x_i(k)) \\ x_i(k+1) =& x_i(k) + v_i(k+1) \end{aligned} \tag{2}$$

Although the past decades has witnessed several efficient approaches to enhance the performance of PSO [11], the study of PSO applied in MOPs are relatively few. MOPSO [12] proposed by Coello is the most famous PSO emerged into MOPs, which proposed a secondary repository(REP) for elite collecting and global best selection, and introduced adaptive hypercubes in repository maintenance as well as diversity maintaining. The brief outline of a traditional MOPSO is provided in Algorithm 1.

Algorithm 1 MOPSO

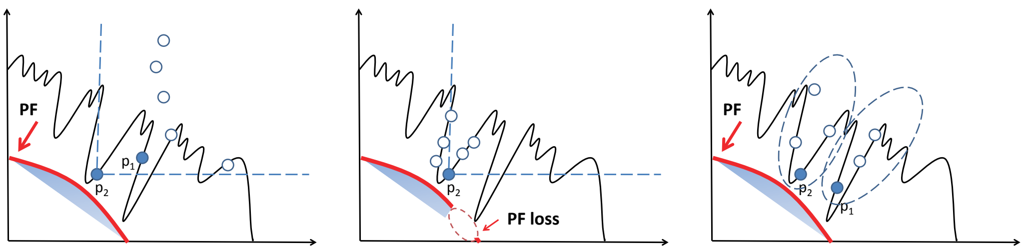
```
Initialization: Generate and evaluate a population POP of design vectors,
  select the nondominated ones into REP and generate grids for each elite.
while computational budget is not exhausted do
    for each individual p in POP do
        Choose gBest from REP according to the distribution grid.
        Update the velocity and position according to Equation (2).
        Update pBest
    end for
    Update REP and the grids.
end while
```

However, when the dimension of decision space increases, the neighborhood relations among particles become various. Since in higher dimensional space each particles have more choices to settle down, many feasible positions are neglected and never been visit through the whole evolution. Under this environment, the earliest elites in repository, which depend on the initialization position, has more change to form the direction of the following evolutions. As a serious result, the algorithm will trapped in local extremums. Furthermore, at late stage of the algorithm, particles gather around the considered optimums, the performance of diversity is limited. The premature phenomenon appears in MOPSO involved array antenna application, especially when the number of elements increases or taking more radiator parameters into consideration.

III. The Multi-Swarm Multiobjective Particle Swarm Optimization

MOPSO needs to be improved to against premature and poor diversity in special cases. First of all, we should analyse the the cause of this drawback. We can illustrate the premature phenomenon with the help of the diagrams in Figure 1. The black solid lines in the figures represent the possible solutions between the found area and the undiscovered optima. Assume that at iteration k, p_1 is an nondominated solution and collected into the repository; at iteration $k+1$, another nondominated solution is found, we call it p_2, and p_2 dominates p_1 (denoted as $p_2 \succ p_1$, Figure 1(a)). According to the archive selection strategy, only the nondominated ones go to the repository, p_1 is removed and the solution in repository decreases. The elite replace behaviors happen frequently at early iterations with the number of found elite remains the same or decreases. Because of the delete of some dominated solutions, the corresponding discovery areas are locked and limited. After several iterations, particles gathered around p_2 and the corresponding local area, and many new nondominated solutions are emerging, accelerate the speed of elite discovery. Back to iteration $k+1$, suppose p_1 is the only particle in the corresponding local optimal area, affected by this adjustment, p_1 will turn to learn from p_2, and leave the local optimal area empty. After sever iterations, the PF which connected to the skipped local area has great change be uncultivated because of the remove of solution p_1, which leads to the loss of diversity(Figure 1(b)).

To solve the drawbacks above, the multi-swarm strategy is introduced in antenna synthesis. In previous studies, multi-swarm strategy has been used to maintain diversity [13], [14] and improve the PSO performance [15]. Assume that under the whole population, sub-swarms containing parts of particles exist, and the solution p_1 and p_2 happen to be assigned into different swarms(Figure 1(c)). The sub-swarm works as a semi isolated local neighborhood, where an exclusive repository is built to keep it own elite solutions. Under this structure, p_1 won't be deleted and has chances to make better evolution. A rectification is proposed and carried out every defined period. If the repository of sub-swarm calculated have no contribution to the global REP, the sub-swarm repository will be replaced by the leaders in other sub-swarms randomly; otherwise, the sub-swarm will be maintained unchange until the next rectification. A brief outline of the generalized multi-swarm multiobjective particle swarm optimization algorithm(MSMOPSO) is presented in Algorithm 2.



(a) (b) (c)

Fig. 1. The diagram of premature in MOP optimization

Algorithm 2 MSMOPSO

Initialization: Generate and evaluate a population POP of design vectors, average distribute particle into sub-swarms, select the nondominated ones into repositories.
while computational budget is not exhausted **do**
  /****************** The rectification ******************/
  if rectification condition is met **then**
    Combine all sub-swarm repository into REP, remove the dominated ones.
    for each swarm$_i$ **do**
      if no solution in REP belongs to swarm$_i$ **then**
        Replace the repository in swarm$_i$ with the elite solutions in other swarms.
      end if
    end for
  end if
  /****************** End of rectification ****************/
  for each individual p **do**
    Choose $gBest$ from the corresponding repository.
    Update the velocity and position according to Equation (2).
    Update $pBest$
  end for
  Update repositories and the grids.
end while

IV. Formulation of the Antenna Array

An antenna array is consist of some antenna elements with a geometrical arrangement of a deliberate relationship between their currents, and always can achieve a desired radiation pattern. Phased array is a kind of antenna array to gain a directional pattern, in which the antenna elements have modified phases, amplitudes and distributions.

For a linear phased antenna array, assume that we have N isotropic radiators placed uniformly along the x-axis, whose geometry is shown in Fig.2. Each radiator element has independent current amplitude and phase. The array factor in x-y plane can be expressed as Equ.3 [2].

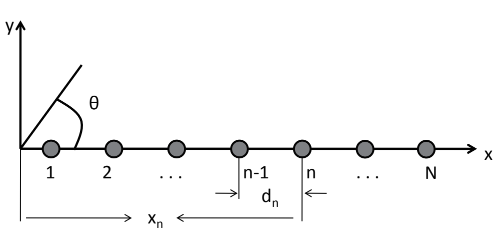


Fig. 2. Geometry of 2N-element linear array

$$AF(\theta, \vec{I}, \vec{\phi}) = 2\sum_{n=1}^{N} I_n \cos(k\frac{2n-1}{2} d \sin\theta + \phi_n) \quad (3)$$

where I_n and ϕ_n are, respectively, the excitation amplitude and phase of the nth element, and $\vec{I}$, $\vec{\phi}$ are the corresponding vectors to be determined. $k = \frac{2\pi}{\lambda}$, k is the wavenumber, and λ is the signal wavelength. d is the distance between two neighbouring elements. The array factor expressed in dB can be written as

$$AF_{dB}(\theta, \vec{I}, \vec{\phi}) = 20 \log \left|\frac{AF(\theta, \vec{I}, \vec{\phi})}{AF(\theta_0, \vec{I}, \vec{\phi})}\right| \quad (4)$$

where θ_0 is the direction of the maximum.

For sidelobe level(SLL) suppression, the objective functions are defined to be minimized and can be written as

$$Min \qquad f_1 = \max_{\theta \in S} AF_{dB}(\theta) \quad (5)$$

where S is the region that the sidelobes are suppressed, which has obvious impact on the optimization. For nulls control, we use the average broad null level (ABNL) minimization, and the fitness can be written as

$$Min \qquad f_2 = \frac{1}{B} \sum_{\theta \in S_B} AF_{dB}(\theta) \quad (6)$$

where S_B is the required broad null region and B is the length of S_B.

V. Experimental Studies

We use MSMOPSO, comparing with MOPSO and NSGA-II in the array antenna design. The two PSO-based algorithms share the same parameters: $c_1 = 1.2, c_2 = 1.2, \omega = 0.2$, and ω is multiplied by a damping of 0.99 during every iteration. To all three comparison algorithms, the population size is set to be 100. The computational budget is 500 iterations. As a preliminary investigation, the number of sub-swarm K is set to be 5 For MSMOPSO in this paper. The rectification periods is 50 iterations.

Settings of antenna: 2N-element uniform linear array, $N = 16$, $d = 0.7\lambda$. Objective f_1 is corresponding to the sidelobe level, where the mainlobe range is $\theta \in [-7, 7]$. Objective f_2 is set to control the broad nulls level, and the broad null area is $[-60, -40] \cup [40, 60]$. The decision vectors are $\vec{I} = [I_1, ..., I_N]$ and $\vec{\phi} = [\phi_1, ..., \phi_N]$, amplitudes and phases respectively.

The optimal fronts found by the three algorithms are shown in Figure 3, each point of PF represents a potential array design case, the optimal sets provide flexible choice for different situations. The solutions found by MSMOPSO range from -25.2 to -19.3 on sidelobe level, and from -59.1 to -44.8 on average broad null level. It can be seen that NSGA-II has a better diversity performance than MSMOPSO, but the weakness in convergence indicates the occurrence of premature. The curve of MOPSO is almost overlapped by NSGA-II but has much small range, which indicates that the particles are crowed and trapped in one local optimal. Moreover, the parallel coordinate plots of the related REPs' decision spaces are shown in Figure 4, among which the curves of MSMOPSO is colour-coded according to the contributions of each swarm. In the parallel coordinate plots, the lines located above are the optimal amplitude values of antenna elements, and the lines located below are the optimal phase values. It can be seen from (a), the result of MSMOPSO, that the traces of lines are distinguished by the colors related to different swarms, which indicated the diversity progress made by the multi-swarm. As contract, in MOPSO(b) and NSGA-II(c), all the lines gathered together and the solutions are limited in single discovery direction, which lead to premature and poor diversity.

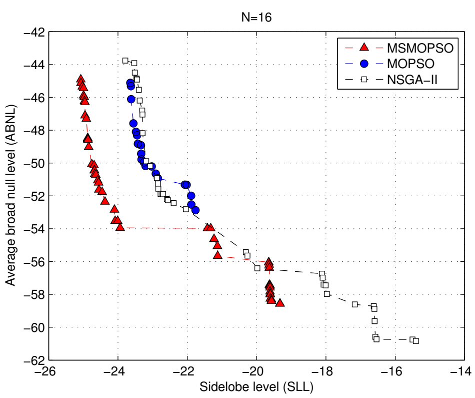
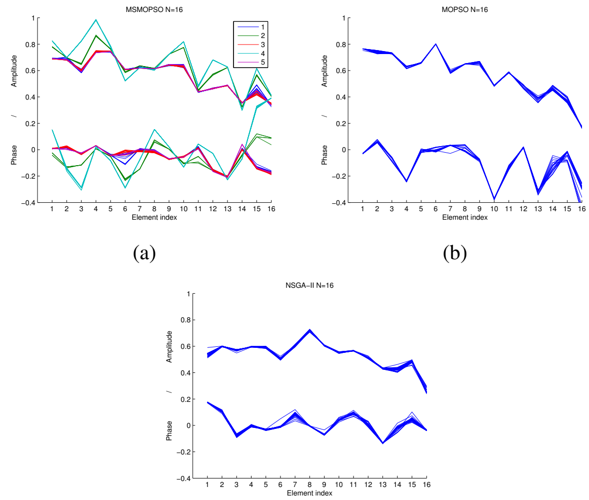


Fig. 3. The distribution of solutions optimized by MSMOPSO, MOPSO and NSGA-II

Figure 5 presents the radiation patterns of three example cases found by MSMOPSO, and Table I presents the corresponding optimized array parameters of the three cases. The first design case present a lower SLL value than case 3 and a better broad null control than case 2; case 2 has best SLL value among the three cases, but the ABNL performance is comparatively poor; case 3 presents the best ABNL value and a poorest SLL. PF found by NSGA-II range from -23.9 to -15.2 on SLL, and -60.9 to -43.9 on ABNL.

In our experiments, we select HV metric and nRep value as the performance measurements of the MOP algorithm. The HV metric can measure diversity as well as convergence performance, and it is particularly recommended for practical implications without knowing the true Pareto front. The HV we used is calculated based on the percentage of the area between the PF known and reference point in the whole



(a) (b) (c)

Fig. 4. The parallel coordinate plots in decision space

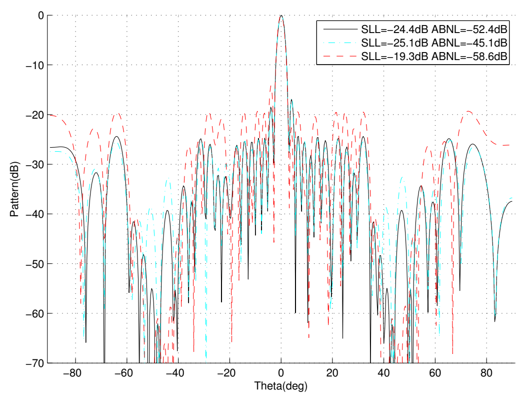


Fig. 5. Optimized radiation patterns by MSMOPSO

TABLE I
OPTIMIZED PARAMETERS BY MSMOPSO

	Case1		Case2		Case2	
n	I_n	ϕ_n	I_n	ϕ_n	I_n	ϕ_n
1	0.70	0.38	0.69	0.26	0.83	8.68
2	0.69	0.57	0.68	1.73	0.70	-9.34
3	0.60	-1.68	0.60	-1.84	0.83	-17.59
4	0.74	1.76	0.75	1.83	0.99	1.39
5	0.74	-2.58	0.75	-2.76	0.78	-4.78
6	0.61	-3.90	0.61	-0.27	0.52	-16.74
7	0.62	0.33	0.62	-0.90	0.62	-4.41
8	0.61	-0.50	0.61	-0.99	0.60	8.79
9	0.64	-3.83	0.64	-3.94	0.72	1.22
10	0.63	-3.12	0.62	-2.94	0.82	-7.52
11	0.44	0.48	0.44	0.90	0.47	2.46
12	0.46	-8.44	0.46	-8.57	0.68	-1.62
13	0.49	-11.49	0.49	-11.50	0.63	-13.02
14	0.35	0.40	0.36	-0.04	0.30	-3.96
15	0.46	-7.34	0.42	-8.12	0.62	18.99
16	0.34	-10.00	0.35	-10.51	0.41	22.44

hypercube area, and multiplied by a distance factor. The HV is defined as equation 7.

$$HV = \prod_{i=1}^{m} (\overrightarrow{r} - \overrightarrow{\min PF}) \times \frac{v}{V} \tag{7}$$

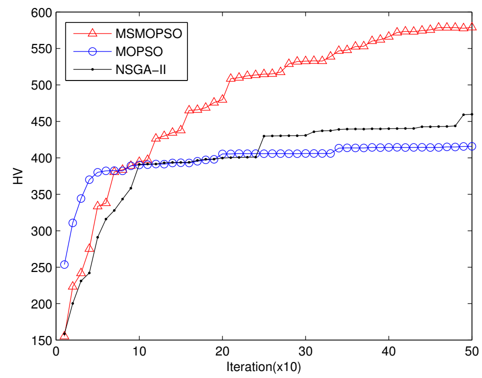
The number of solutions in REP is denoted as nRep, which can reflect the plumpness and diversity of the feasible solutions.

The performance results, including the mean and standard deviation of the metric HV and nRep generated by 20 independent simulations performed on the antenna instance, are summarized in Tab.II, where the best results are highlighted. MSMOPSO has the best HV score and the nRep value, and highest stability which inspired by standard deviation values among the three algorithms. The shortest running time is expressed by MOPSO, followed by MSMOPSO. NSGA-II has comparatively long runtime.

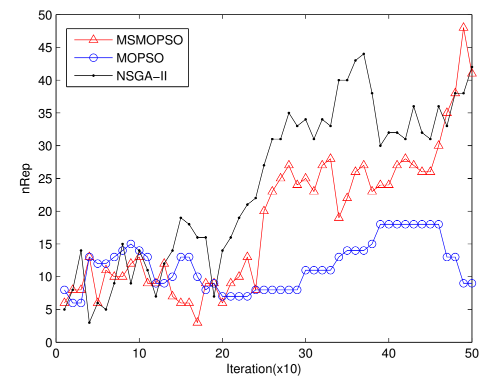
TABLE II
THE PERFORMANCES AMONG THE COMPARISON ALGORITHMS

Algorithm	HV		nRep		toc	
	mean	std.	mean	std.	mean	std.
MSMOPSO	**1.14E+02**	2.21E+01	**42**	8	6.75E+02	1.34E+02
MOPSO	6.23E+01	3.22E+01	20	12	**3.88E+02**	2.37E+01
NSGA-II	9.83E+01	3.76E+01	33	13	1.97E+03	1.29E+02

The performance curves consisted of performances of every 10 iterations. The randomicity dominates the performances at early iterations. During the middle evaluation period, the convergence advances stage by stage, where MSMOPSO presents best speed and potential. MOPSO and NSGA-II ends at local optima in most simulations, while MSMOPSO achieves satisfied solutions. The nRep values show large fluctuate during the procedure. MOPSO has poor performance in nRep value, which indicates the particles gathered together and new elite solutions are difficult to be found. NSGA-II presents a better performance in REP maintenance which benefits from the hybrid process, yet a slower convergence speed as a sacrifice.



(a)



(b)

Fig. 6. The performance curves of the comparison algorithms

VI. Conclusions

In this paper, a multi-swarm multiobjective particle swarm optimization is proposed. The MSMOPSO algorithm is used in the synthesis of linear array antenna with the goals of minimize sidelobe level and broad null level in a multiobjective approach. Comparing with MOPSO and NSGA-II, MSMOPSO obtains satisfied solutions to the antenna parameters in amplitudes and phases simultaneously, greatly improves premature and the trapping in local optima. Future studies should include the analysis of the swarm number K and the interactions between different swarms. With the low complexity and less timeconsuming, the extension of MSMOPSO can be indicated in other practical MOP applications.

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