Portfolio-based Contract Selection in Commodity Futures Markets

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***Abstract*—As financial future markets offer coexistent contracts that only differ in their maturities, trading mandatorily induces the task of selecting a specific contract. Assuming the objective of maximal profit, an analysis of the current market situation is inevitable. Among immediate and upcoming trading costs and market liquidity, even possible market inefficiencies might be taken into consideration. This research introduces a future selection strategy that minimizes trading costs by dynamic programming. The strategy is evaluated by Monte Carlo simulations on sets of arbitrary trading instructions on three commodity classes. The results allow the conclusion of market inefficiencies in the analyzed future markets.**

I. Introduction

Since their first launch in the 1860s at the Chicago Board of Trade, numerous future contracts have been released. Therefore, modern future markets offer a vast number of commodities and financial assets. As high liquidities and volumes yield attractive trading conditions, future markets are undoubtedly an important trading vehicle in many investment sectors.

Due to the nature of future markets, a single asset is generally represented by several future contracts with different maturities. Thus, applying conventional trading strategies to future markets yields the additional challenge of selecting specific contracts to be traded – a nontrivial task because different contracts might benefit from varying market circumstances. Figure 1 exemplarily shows quotes for different WTI future contracts during the period from June to December 2015.

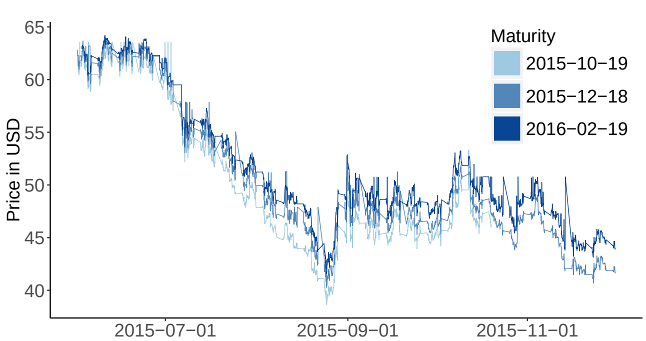

Fig. 1. Prices of three WTI future contracts with different maturitites

Considering short-term maturities, long-running positions require the expiration to be extended by closing and reopening longer-term positions for the same underlying asset – this so called future rolling result in additional trades and thus, trading costs. However, the trading volume of contracts tremendously rises with a decreasing period to maturity in the common case. Accordingly, spreads tend to be the lowest for the nearest future contracts.

That is the reason why the so called front month strategy has become an established way to open and roll future contracts [1]. It focuses on the trade of the nearest future contract whose maturity month is not reached yet. After exceeding that point in time, contracts are rolled to the subsequent maturity. Next to the advantage of most likely addressing liquid markets, it is simple to use and backtesting requires only the two nearest future contracts to be comprised.

Nevertheless, this strategy disregards several factors. Besides the evaluation of future contracts with more distant maturity dates, the current portfolio as well as statistical parameters of the trading strategy might be taken into consideration. But even if the point of interchange is reasonably adapted to the trading characteristics, the constant period may be to rigid to fit the needs of the underlying system. Especially highly varying holding times or partial reorganizations of a long-running portfolio may cause the necessity of future rolls. Correspondingly, this research introduces a procedure that optimizes the selection of futures by investigating the influence of a larger set of parameters. The described problem will be formalized in the following section.

II. Optimal future selection

The problem that is addressed in this study is the optimal selection of specific future contracts for settled trading decisions on an asset class. It is targeted at the maximization of wealth by evaluating immediate and upcoming trading costs and possible market inefficiencies.

As the selection process is intended to work with arbitrary trading strategies on a single asset class, the latter are sufficiently characterized by their trading decisions. Thus, we expect a finite sequence of trading decisions $(\mathcal{D}_t)_{t\in\mathbb{T}}$ with $\mathcal{D}_t \in \mathbb{Z}$ and time $t \in \mathbb{T}$ to be given. The value $\mathcal{D}_t$ represents the number of future contracts to be traded (bought or sold depending on sign) at time t.

Let F be the set of all future contracts of the considered asset class and $F_\tau \in F$ be a future contract with maturity $\tau \in \mathbb{T}$. Let Ω be the set of all possible market scenarios and the finite sequence $(\mathcal{F}_t)_{t\in\mathbb{T}}$ be a filtration on the space Ω, so that the element $\mathcal{F}_t$ represents the known and relevant information at time t.

The future selection strategy can then be understood as a function $q\colon \mathcal{F} \times \mathcal{D} \rightarrow F$ that returns a specific future contract F_τ for a trading decision $\mathcal{D}_t$ by interpreting the information $\mathcal{F}_t$. Let Q be the set of all possible future selection strategies.

Let $w\colon \mathcal{D} \times Q \rightarrow \mathbb{R}$ be the function that calculates the wealth after the execution of all ordered trades.

The problem is then to find a future selection strategy $\hat{q} \in Q$ at time t that maximizes the expected wealth after the execution of all trading decisions:

$$w\left(\mathcal{D}, \hat{q}\right) = \max_{q \in Q}\left(w\left(\mathcal{D}, q\right)\right) \tag{1}$$

The examination of the optimal selection strategy $\hat{q}$ consists of three steps that are introduced in the following:

1) modeling trading costs (and future roll costs) by analyzing observed spreads of different futures (section III-A)
2) introducing temporal uncertainty of trading decisions to the prediction by evaluating statistical information of the underlying trading strategy (section III-B)
3) assignment of specific future contracts to trading decisions and evaluation of all possible combinations (section III-C)

III. Minimizing trading costs

The relationship of a forward price to its underlying spot price is complex and several defining models have been proposed [2][3][4]. The cost-of-carry model explains the price $F_{t,\tau}$ of a future contract with maturity τ at time t as a function of the spot price S given by

$$F_{t,\tau} = S_t \cdot e^{(r+s-c)\cdot(\tau-t)} \tag{2}$$

with risk-free interest rate r and storage cost s. The convenience yield c is an additional parameter that includes market expectations to the formula. It allows the model to explain different market situations like contango and backwardation. It can be seen as the most volatile part of the interest rate and strongly depends on the maturity.

Assuming the future markets to be efficient in regard to the market-efficiency hypothesis [5], we do not expect over- or undervaluations of specific future contracts – in other words, we expect the interest rates of future contracts not to be cointegrated. Therefore, the maximization of wealth corresponds to the minimization of immediate and upcoming trading costs.

A. Trading cost analysis

While commissions follow a clear pattern and can be regarded as exactly predictable, spreads depend on market liquidity and may be notably fluctuating. According to this principle, a precise investigation of trading costs requires the relationship between spread and the maturity time to be explored.

Figure 2 shows average spread evolutions for six WTI contracts during the last 180 trading days (maturities from July to December 2015). Obviously, spreads and their volatilities decrease with an approaching maturity. This circumstance accords to typical future evolutions [6]. The periodic oscillation is constituted by the alternating liquidity between regular and after-hours trading. The minimum spread of about 1‰ of the price is reached with a distance of about 90 days, yet with a high volatility. However, this investigation shows that the trading costs of the nearest three monthly contracts might temporary be regarded as similar.

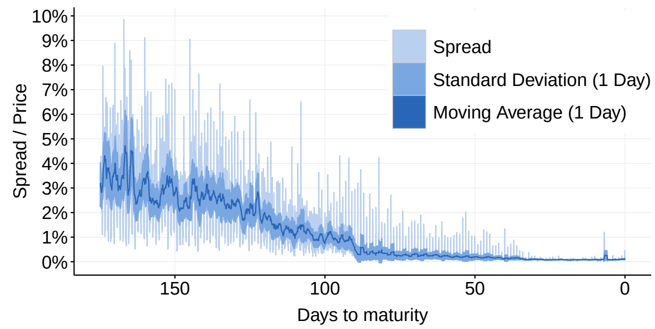

Fig. 2. Spread over the last six months to maturity for WTI Mini contracts

Obviously, the contract selection has a tremendous impact on emerging trading costs. The Figure can be understood as a clear recommendation to trade short-term future contracts as the spread commonly decreases with the remaining trading period. However, this conclusion has restricted validity. Opening long-term positions in near future contracts yields a high probability of necessary future rolls and results in additional costs.

For a more detailed view on future roll costs, let $\mathbb{D} \subset \mathbb{Z}$ be the set of time intervals. Let $d_\tau\colon \mathbb{T} \rightarrow \mathbb{D}$ with $d_\tau(t) = \tau - t$ be the time interval between a time t to the maturity date τ of a future contract F_τ.

Let $\mathrm{d}\tau \in \mathbb{D}$ be the time interval between maturities of two consecutive future contracts. It is assumed to be constant for a future class (e.g. 1 month for WTI).

Let $c\colon \mathbb{D} \rightarrow \mathbb{R}$ with $c(d), d \geq 0$ then be the average observed trading costs that incur d time steps before the maturity.

Further, let

$$c(d) = \underbrace{c(0) + c(\mathrm{d}\tau)}_{\text{Future roll cost}} + \underbrace{c(d + \mathrm{d}\tau)}_{\text{Liquidation cost}} \quad \forall\, t < 0. \tag{3}$$

be the trading costs for an exceeded maturity. The overall cost arise from three trades: firstly, the costs for closing the expiring position at the last possible point in time ($c(0)$) and re-opening one position with a $\mathrm{d}\tau$ more distant maturity ($c(\mathrm{d}\tau)$) must be considered. Secondly, the in either case upcoming liquidation cost must be payed. It is delayed by $\mathrm{d}\tau$ to $c(d+\mathrm{d}\tau)$. These additional trades result increase the expected trading costs for exceeded maturities (see Figure 3).

On the whole, decreasing spreads oppose the increasing risk of future rolls. As the probability of their occurrence

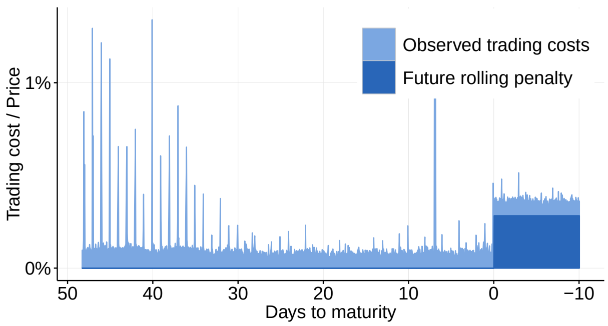

Fig. 3. The average trading costs $c(d)$ over the last 50 days for WTI Mini contracts are displayed. An exceeded maturity results in additional trades by future rolling. Therefore, the expected trading costs increase considerably for $t < 0$.

depends on the relationship between maturity and trading frequency, we include statistical information about the trading strategy to improve the rating quality of future contracts.

B. Prediction of trading costs

In this section, liquidation costs of a portfolio of future contracts are analyzed with regard to the future selection problem. A minimization procedure is proposed that requires the following information to be known:

- trading costs in relation to the distance to maturity (as described in III-A)
- future contract positions in portfolio
- statistical information about trading decisions

In the following, we suppose that statistical information of the trading decisions are available in form of a distribution function of the number of trades per time. Although this approach is not limited to special distributions, we assume the number of trades per time period to be normally distributed with an average interval between two trades μ and standard deviation σ. This model allows estimations of the trading interval of upcoming trades. Assuming t to be the time of the last trade, the time of the n-th upcoming trade is displayed as a random variable $t_n \sim \mathcal{N}(t + n\mu, n\sigma)$.

In this case, the expected number of trades from time s to time t is $\frac{t-s}{\mu}$ with a standard deviation of $\frac{t-s}{\mu} \cdot \sigma$. The method identically works on short and long positions but must be applied separately. In the following, the term liquidation is used by either meaning the liquidation of short or long positions.

Let $v : \mathbb{T} \times \mathbb{T} \times \mathbb{T} \to \mathbb{R}$ be a function so that $v(\tau, t, s)$ yields the expected trading costs for a trade of a future contract F_τ at time t estimated at time s with $s < t$. As a first approach, these costs are the average observed trading costs and actually independent from s:

$$v(\tau, t, s) = c(\tau - t)\,. \tag{4}$$

The statistical information combined with the estimate v can be used to predict future trading costs. The function $C_\tau : \mathbb{T} \to \mathbb{R}$ illustrates this relationship and enables the estimation of transaction costs of prospective trades at an approximate time t with the information of time s by

$$\mathbb{E}\left(C_\tau(t) \mid \mathcal{F}_s\right) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\left(\frac{t-s}{\mu}\cdot\sigma\right)^2}} \cdot \exp\left(-\frac{x^2}{2\left(\frac{t-s}{\mu}\cdot\sigma\right)^2}\right) \cdot v(\tau, t+x, s)\ \mathrm{d}x \tag{5}$$

The temporal uncertainty of s is then modeled by folding the given probability density function with the observed trading costs. In this manner, all observed trading costs as well as the future roll costs are weighted with their probability to assure a reasonable prediction.

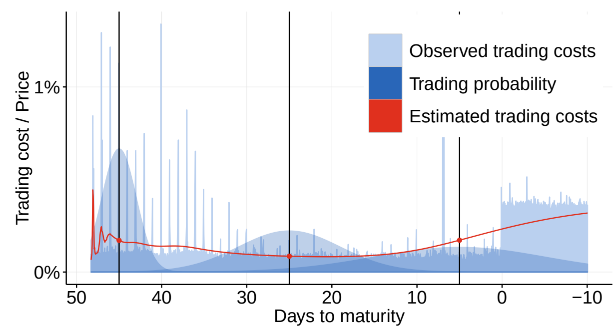

Fig. 4. Estimated trading costs $\mathbb{E}\left(C_\tau(t) \mid \mathcal{F}_s\right)$ for future F_τ 55 days before its maturity and $\frac{\sigma}{\mu} = \frac{1}{5}$. Normal distributions for three exemplary points (45, 25 and 5 days left) are schematically figured to denote the uncertainty of the effective trading time.

Figure 4 exemplarily shows a trading cost estimation for a future contract 48 days before its expiry. While short-term predictions apparently reflect temporal trading cost fluctuations, more distant predictions strongly smooth the observed trading costs. The considered quotient $\frac{\sigma}{\mu} = \frac{1}{5}$ denotes the standard deviation to rise and the minimum trading cost estimate is located approximately 20 days before the maturity. The risk of an occurring future roll increases the estimated trading costs for liquidation points near the maturity.

These predictions can be applied to future contracts with different maturities to compare the development of their trading costs. Since this method allows us to estimate these costs for all contracts In the following, we address the question, whether and how this forecast method can be used to reduce these costs. Therefore, a minimization procedure is proposed that targets that objective by finding an optimal order in which the future contracts are to be liquidated.

C. Minimizing of portfolio liquidation costs

Trading costs are evaluated by analyzing possible future selections according to the dynamic programming principle. Starting with the portfolio at the current time, trading costs of all possible trading paths are evaluated by successively removing positions up to an empty portfolio. The optimal

t	1	2	3	4	5	6
$\mathbb{E}\left(C\left(t\right) \mid \mathcal{F}_0\right)$	0.14%	0.13%	0.17%	0.13%	0.12%	0.18%

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
$L^1=0$	0.0%	0.0%	0.0%	0.0%			
$L^1=1$		0.14%	0.13%	0.13%	0.13%		
$L^1=2$			0.27%	0.27%	0.26%	0.25%	
$L^1=3$				0.44%	0.40%	0.38%	0.38%

Fig. 5. Minimization example in which all possible combinations of three liquidations during the next six trading events are examined for a single future. Trading costs differ from 0.12% to 0.18%. This trivial and conflict-free example evaluates every possible trading path according to the dynamic programming principle using Equation 9. The backtracked resulting path recommends to liquidate at times 2,4 and 5 yielding an average cost per trade of $\frac{0.38\%}{3} = 0.127\%$. Conflicts may arise in higher order tables with more than just one future.

liquidation order is then revealed by the path with minimum trading costs.

Let $L_t = \{L_t^1, L_t^2, \ldots, L_t^N\} \in \mathbb{N}_0^N$ the set of transactions that are necessary to fully liquidate the portfolio with N futures at $t \in \mathbb{T}$. Let $\mathcal{C}_s\left(L_t\right) \in \mathbb{R}$ at time $s \in \mathbb{T}, s \leq t$ be the estimate of the trading costs for all transactions in L_t. As these transactions occur at sequent points in the future, the costs can be described recursively. Obviously, no further trading costs are expected if there are no positions left:

$$\mathcal{C}_s\left(L_t\right) = 0 \text{ if } L_t^j = 0 \;\forall\; j \in [1, N]. \tag{6}$$

If the portfolio is not fully liquidated yet, each trading decision may lead to one of $N+1$ possible actions:

- liquidate an existing position of L_t of the N futures to lower the chance of future rollings (actions will be indexed by $j = [1, N]$)
- choose a transaction with minimal trading costs that is not part of the set of necessary liquidations (indexed by $j = 0$)

In the latter case, the number of necessary liquidation steps remain from t to the next trading decision at time $t + \mu$. Therefore, $L_{t+\mu} = L_t$ and accordingly

$$\mathcal{C}_s\left(L_{t+\mu}\right) = \mathcal{C}\left(L_t\right) \text{ if } j = 0. \tag{7}$$

holds. In the case that future contracts of the future with maturity τ_j are liquidated ($j = [1, N]$), the emerging costs $\mathbb{E}\left(C_{\tau_j}(t) \mid \mathcal{F}_s\right)$ must be added to the liquidation costs of the partially liquidated portfolio.

Let the function $\mathcal{L}\colon \mathbb{Z}^N \times F \to \mathbb{Z}^N$ represent contract liquidation. The number of necessary liquidations of a future F_{τ_i} in L_t is therefore decreased by the call of $\mathcal{L}\left(L_t, F_{\tau_i}\right)$. In this case, trading costs are increased by

$$\begin{aligned}\mathcal{C}_s\left(L_{t+\mu}\right) = {} & \mathcal{C}_s\left(\mathcal{L}\left(L_t, \tau_j\right)\right) \\ & + \mathbb{E}\left(C_{\tau_j}(t) \mid \mathcal{F}_s\right) \text{ if } j \in [1, N].\end{aligned} \tag{8}$$

So, the emerging costs $\mathbb{E}\left(C_{\tau_j}(t) \mid \mathcal{F}_s\right)$ are added to liquidation costs of the residual portfolio $\mathcal{L}\left(L_t, \tau_j\right)$. All in all, the optimal liquidation costs are given by the following minimization:

$$\mathcal{C}_s\left(L_{t+\mu}\right) = \min_{j \in [0,N]} \begin{cases} 0 & \text{if } L_t^j = 0 \\ & \forall\; j \in [1, N] \\ \mathcal{C}_s\left(L_t\right) & \text{if } j = 0 \\ \infty & \text{if } L_t^i = 0 \\ \mathcal{C}_s\left(\mathcal{L}\left(L_t, \tau_j\right)\right) + & \\ \mathbb{E}\left(C_\tau(t) \mid \mathcal{F}_s\right) & \text{else.} \end{cases} \tag{9}$$

The assumption of infinite trading costs in case of fully liquidated futures avoids the examination of further transactions. Every partially liquidated portfolio may be reached by a finite number of different trading combinations. The shown procedure yields the path with the minimum expected costs. The task of finding the optimal path for a single future is trivial, the complexity lies in effecting a compromise between conflicting future liquidations. As shown in Figure 5, the resulting path for one future is directly specified by the subset of trades with the minimal cumulated trading costs. However, possible collisions in which several transactions seem optimal at the time yield the problem of selecting specific ones. Therefore, the actual decision is based on the evaluation of all further trading steps as well. It is then given by the first trade of the best trading path.

As virtual transactions of futures after their maturities yield rolling costs, they are not assumed to be interesting for the minimization if there are alternative liquidation paths.

Considering k futures with ascending maturities, the recursive evaluation stops with $t_{\max} = \max(s + k \cdot \mu, \tau_k)$. The dimension of the evaluation table increases with every considered future, so that $\mathcal{O}(t_{\max} \cdot \prod_{i=1}^{|L|} L_t^i)$ table elements must be calculated. The minimum liquidation path is then evaluated by backtracking.

IV. Market inefficiencies

In the following, future prices $F_{t,\tau}$ are assumed to converge to the spot price S_t with a general interest rate $r_{t,\tau}$:

$$F_{t,\tau} = S_t \cdot e^{r_{t,\tau} \cdot (\tau - t)} \tag{10}$$

Typically, the interest rate is positive and yields higher prices for more distant futures. This contango situation is exemplarily shown in Figure 6 for different WTI futures.

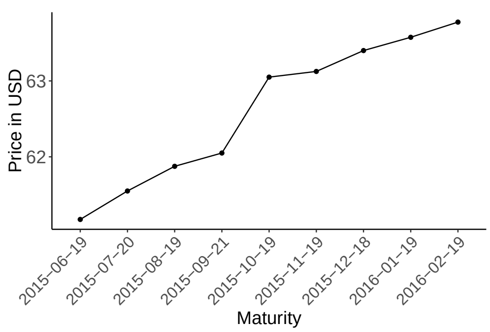

Fig. 6. Partial yield curve of the WTI Mini future at 11 June 2015.

In asset classes without spot prices, the interest rate may be approximated by analyzing the relationship between different future pairs. The following approach calculates the average differences between interest rates of futures with different maturities:

$$r_{t,\tau} = avg_{i \neq j} \frac{\ln F_{t,\tau_i} - \ln F_{t,\tau_j}}{\tau_i - \tau_j} \tag{11}$$

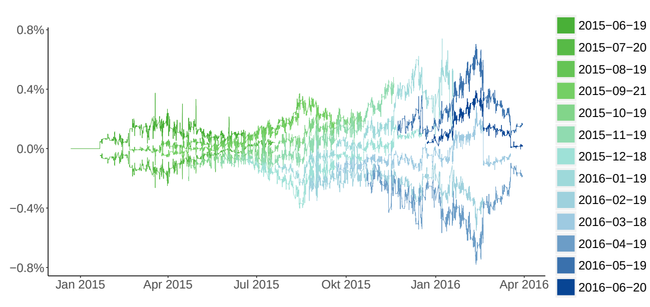

Fig. 7. Deviations of different interest rates of WTI Mini contracts to their average are displayed and reveal considerable variances.

Figure 7 shows that interest rates of futures of the same asset class with different maturities may fluctuate in an uncorrelated way. These variations represent diverse market expectations at different maturity dates and are not a sign of inefficient markets. Nevertheless, even the smallest under- and overvaluations in these price structures may be used to improve the overall performance of the future selection. There is no lower limit for their intensity as it occurs with arbitrage trading strategies because all trading decisions are already settled. Thus, the next evaluation approach considers the interest rates to be cointegrated.

Assuming the existence of inefficiencies in the term structure, we suppose the deviations from the average interest rate $\mathrm{d}r_{t,\tau}$ to be explained by over- or undervaluations. As these mispricings are characterized mean reverse, we further assume the existence of Ornstein-Uhlenbeck processes that describe the evolution of $\mathrm{d}r_{t,\tau}$ by

$$\mathrm{d}r_{t,\tau} = \theta_\tau \left(\mu_\tau - \mathrm{d}r_{t-\mu,\tau}\right) dt + \sigma_\tau dW_t \tag{12}$$

So, the mispricing $\mathrm{d}r_{t,\tau}$ converges to the value μ_τ with the mean reversion speed θ_τ. The overvaluation is expected to be counterbalanced by the market. $(W_t)_{t \in \mathbb{T}}$ is a standard Wiener process, so that σ_τ represents the influence of random noise.

The parameters of this Ornstein-Uhlenbeck process are fitted with a maximum likelihood estimation [7]. It has to be considered that input data for estimates must not cross maturity dates as the sudden disappearance of single futures may drastically change the average interest rate that affects all values $\mathrm{d}r_{t,\tau}$. These estimations can be used to improve the performance of the trading cost predictions by adding the estimated approximation $\beta_{t,\tau}$ with

$$\beta_{t,\tau} = F_{t,\tau} \cdot \mathrm{e}^{\mathrm{d}r_{t+\mu,\tau} - \mathrm{d}r_{t,\tau}} \tag{13}$$

to the average interest rate in the calculation of $v(\tau, t, x)$ (see Equation 5):

$$v(\tau, t, x) = \begin{cases} c\left((\tau - t) - x\right) + \beta_{t,\tau} & \text{for a buy} \\ c\left((\tau - t) - x\right) - \beta_{t,\tau} & \text{for a sell} \end{cases} \tag{14}$$

So, trading cost predictions may be altered by mean reverse expectations. Buying a presumably overvalued future contract is penalized in the same way in which a sell of such a future is rewarded. The success of this method does not only require the existence of mean reverse effects but also their persistence to ensure a measurable predictability.

V. Results

In this chapter, the three presented future selection strategies are analyzed and compared in regard to their trading costs and overall performance. This evaluation is based on a Monte Carlo simulation that benchmarks backtesting results on the time period from July 2015 to Januar 2016. A random set of 100 trading decisions is assigned to every simulation and accordingly creates different trading scenarios.

Figure 8 shows the evolution of the number of contracts in an exemplary test scenario. The amount changes 100 times and thus creates 100 future selection problems. Distributions of different selection strategies are shown in Figure 9 and discussed in the following.

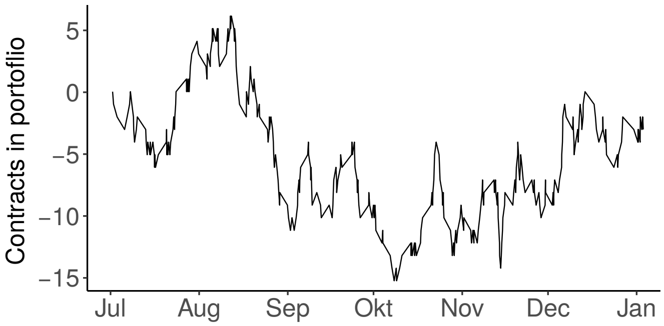

Fig. 8. Monte Carlo simulation scenario: exemplarily portfolio size evolution generated by the random aquisition of 50 long and short positions

TABLE I. PERFORMANCE OF RANDOM SETS OF TRADING DECISIONS ON THREE COMMODITIES MEASURED IN INDEX POINTS

Return	Front Month	Minimum Spread	Ornstein-Uhlenbeck
WTI	2.6695	2.5692	3.1958
Natural Gas	-0.1542	0.0245	-0.0625
Silver	0.6951	1.0625	1.4265
Costs	**Front Month**	**Minimum Spread**	**Ornstein-Uhlenbeck**
WTI	1.4896	1.4126	1.5533
Natural Gas	0.5045	0.3782	0.5913
Silver	1.2405	1.0863	1.5235

The result of the front month rolling strategy in Figure 9 shows a scenario in which the portfolio contains two different futures at maximum. One month before an upcoming maturity, all trading decisions that decrease the absolute amount of contracts are used to lower the number of expiring contracts. Other trades always enlarge the position of the subsequent future.

The future selection strategy that minimizes the expected trading costs leads to more heterogeneous combinations. The portfolio consists of up to five different futures at the same time. Therefore, futures are traded up to three months before their maturity. These results conform to the trading cost analysis in chapter III that identifies the next three upcoming WTI Mini futures to have at least temporarily similar trading costs.

The last chart of Figure 9 displays the result of the future selection strategy that assumes mean reverse compensations. Considerable is the relocation of the focus during November 2015. Unlike the previous approach, almost all short positions have the same maturity (December 2015). The reason lies in the fact that the interest rate of this future is approximately 0.5% higher than the average (see Figure 7).

Table I shows average returns and trading costs of the different future selection strategies on three commodities during June 2015 to January 2016. The values are created by a Monte Carlo simulation and show the cumulated results of 100 random and equally distributed trades in each iteration. Statistical information about the trading decisions is directly derived from the simulation input data. The minimization procedure actually reduces the average trading costs in all three cases compared to the front month strategy and consecutively increases the average return. The mean reversion approach yield a further rise for WTI and silver while lowering the quality for natural gas. However, it is noticeable that these

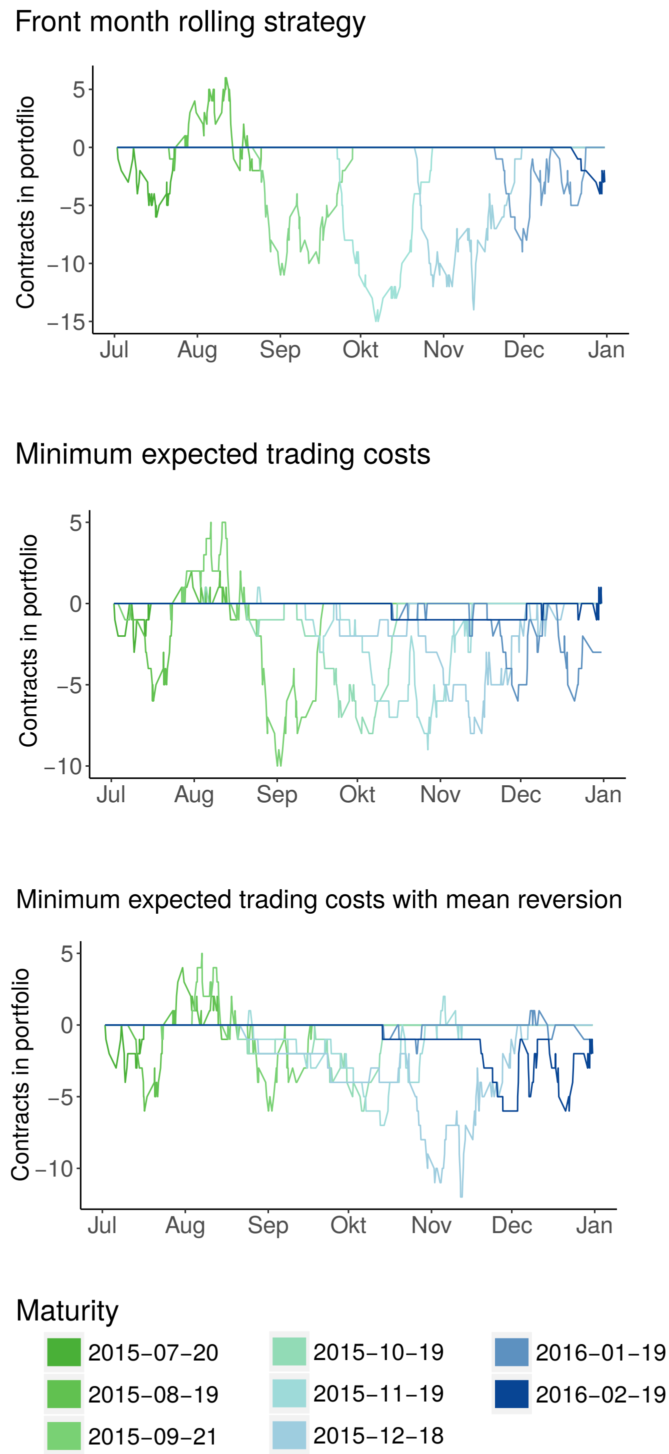

Fig. 9. The three charts show partitions of different future selection strategies for the portfolio development scenario displayed in Figure 8.

partial improvements go along with the highest average trading costs without exception. These apparently contradicting results suggest that the consideration of assumed inefficiencies in the term structure may enhance returns. In this case, this effect is able to counterbalance the higher trading costs in two of three cases.

VI. CONCLUSION

The optimal future selection is a non-trivial task that highly depends on the liquidity of different contracts as well as on the structure of the underlying trading strategy. This study indicates noticeable differences in trading costs and performance between the compared strategies.

A reasonable balance of holding times, maturities and trading costs have led to significant improvements in the Monte Carlo simulation. The consideration of mean reverse motions after over-average fluctuations interestingly yield higher trading costs as well as a higher return in the reviewed cases. A plausible interpretation of this result may be given by the fact that the cost minimization is degraded by the introduction of further indicators. The assumption of virtual trading costs composed of spread expectations as well as of additional over- or undervaluations lead to rising trading costs. However, the increasing average return suggest their existence and lead to an improved performance for all examined commodities.

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