From quantum cognition to quantum agents: an agent model integrating the superposition state property

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***Abstract*—To model intelligent complex systems engineers use the techniques of distributed artificial intelligence and the agent paradigm increasingly However, the problem of decision making by components of a complex system with local, incomplete, uncertain, exchanged or observed in asynchronous manner is often present in agent models. To provide a solution to this problem, studies on quantum cognition introduce quantum properties such as superposition state and entanglement in the decision process. So how to propose quantum agents models that are capable of implementing both quantum properties of superposition state and entanglement? A case study simulating the Takuzu game illustrates our proposed quantum agent model.**

Keywords— quantum agents; complex systems modelling; quantum cognition; agent-based systems; agent-based modelling

I. Introduction

To model complex systems, and obviously intelligent complex systems, engineers use the techniques of distributed artificial intelligence and the agent paradigm increasingly. However, some issues and situations are difficult to model when agents (components of an intelligent complex system) must make decisions with uncertain, incomplete or indeterminate knowledge of their context. In the literature concerning the agent-based distributed systems with uncertain information feature, there are many approaches from the computational intelligence domains [1, 2, 3], but studies on quantum cognition seems very interesting and promising. They introduce quantum properties such as superposition state and entanglement, in the cognition and therefore in the decision process.

Quantum theories inspired a wide variety of scientific fields. Thus many quantum-like models appeared. The potential of using quantum theory to build models of cognition is one example [4]. Indeed, the open, parallel, cooperative and competitive decision processes fully benefit research on quantum probability decision models [5]. The problem of decision making with local, incomplete, uncertain, exchanged or observed in asynchronous manner is often present in agent models [6]. Also, having worked for many years on the agent-based modeling of complex systems, we are interested in modeling quantum agents [7] capable of implementing both quantum properties of superposition state and entanglement.

This paper is organized as follows: in the second section a state of the art in the fields of quantum cognition and agent modeling of complex systems is made; in the third section a quantum agent model is proposed; in the fourth section, a case study on the simulation of the Takuzu game illustrates the proposed quantum agent model; finally, conclusions of this research focused on the superposition state property are presented.

II. Review of Literature

A. Complex Systems

For minimal definition, a system is a set of elements that interact according to certain principles or rules (*i.e.* the law of system evolution). Such a system is then determined by the nature of its elements (components), all the states of its elements defining its overall state, the interactions between its elements and interactions with their environment [8].

A complex system is a system made up of a large number of components, whose behaviors are both highly variable and highly dependent on the behavior of other components [9]. From the interactions of the components of a complex system emerge collective behavior that cannot be derived as a result of the behavior of each component. The prime examples of complex systems are the human brain and human societies [10].

The many challenges of the science of complex systems include the formal definition of complex systems, the modeling and simulation of these systems with a wide variety of characteristics: many heterogeneous interacting parts, multiple scales, complicated transition laws, unpredicted emergence, sensitive dependence on initial conditions, path-dependent dynamics, ill-defined boundaries, interaction and self-organization of autonomous agents, combinatorial explosion, adaptivity to changing environments, co-evolving subsystems, and multilevel or non-equilibrium dynamics [10].

From this set of complex systems characteristics, we have retained three key properties:

- **Self-organization and complex adaptive systems** [11]. An organization is an integral arrangement according to a distribution of a set of elements in a hierarchical level; a self-organizing system has the ability to create and recreate this structure [12, 13, 14].
- **Nonlinearity**. Behavior and responses of a complex system are not deterministic and are influenced by the presence of nonlinear relationships and feedback loops. The implicit ability to exhibit linear or non-linear behaviors (order and/or chaos), is based on a response of the system to these stimuli [10].

- **Emerging behaviors**. From nonlinear interactions, self-organized or chaotic result emergent properties and complex behaviors [15, 16, 17, 18].

B. Quantum Cognition Approaches

When an human being thinks, reasons and makes a decision, he does not use the rules of classical logic, but those of quantum mechanics, where things are not well defined but intricate, ubiquitous, oscillating and superposed [19, 20]. Like photons and electrons, thoughts are superposed, interfere, and are entangled in our brain.

Using limited cognitive resources, the quantum cognition gives humans the opportunity to answer an unlimited number of questions, with limited rationality [19]. Quantum theory allows the reactions of human beings to the questions put to them or to situations in which they are placed to be evaluated [20]. Each time reasoning is applied to a decision process, human decisions are typically quantum, because opinions are not always determined [21].

Wang et al. [5] present five reasons, which become five challenges, to use quantum theory to build models of human cognition:

1) Formalizing psychological concepts of conflict, ambiguity, and uncertainty – quantum modeling allows us to formalize the state of a cognitive system moving across time in its state space until a decision is reached, at which time the state collapses to a definite value (*i.e.* indefinite state, called a superposition state at each moment in time) [6].
2) Formalizing the cognitive system's sensitivity to measurements – the quantum principles are also consistent with the idea that a choice can alter preferences (*i.e.* an intermediate choice affects the final decision) [22].
3) Formalizing order effects of cognitive measurements.
4) Understanding violations of classical probability laws in cognitive and decision studies.
5) And understanding non-decomposability of cognition.

As noted above, the quantum properties are numerous: indeterminacy, wave interference, ubiquity, oscillation, entanglement between the states, superposition principle. From this set of properties, we have retained as first studies two main properties:

- **The superposition state**. Quantum thinking is to do massively parallel calculations, reasoning operating on mental representations consisting of a superposition of states. When an observation is made, the superposition state reduces to a single classical and definite state. The superposition state provides a very good representation of conflict, ambiguity or uncertainty that we feel when we doubt.
- **Quantum entanglement**. The entanglement is the propensity that can have two (or multiple) objects, two ideas, or two arguments to form only one.

Quantum formalisms have already been proposed to develop quantum cognition models such as *SCOP* model [21]. The *SCOP* model is defined by five elements (1):

$$\left(\Sigma, M, \Lambda, \mu, \nu\right) \tag{1}$$

where Σ is the set of states, M is the set of contexts, Λ is the set of properties, μ is a probability function that describes how a state changes to another state under the influence of a context, ν is a function that describes a weight for a specific state.

In a *SCOP* model each concept is represented by defined sets of states, contexts and properties. The concepts are continuously changing under the influence of contexts. These changes are described by state changes of concepts [21].

C. Multiagent Paradigm for Complex Systems Modeling

The concept of software agent is a response to the desire to design and develop intelligent systems composed of entities which are themselves intelligent. This opens new perspectives in the research field of Distributed Artificial Intelligence [23]. The intelligence of these artificial entities is collective or individual. This allows modeling, simulating, or developing a wide variety of complex systems [24]. At least an agent is an autonomous entity that can adapt to, react to, or interact with its environment [25]. The main properties of these entities are then: autonomous, interactive and adaptive. Agents may also have excessively cognitive properties as in the BDI model (Belief, Desire, and Intention) [26]. This model is built around three concepts inspired by human behavior models: (1) beliefs, based on agent knowledge, (2) desires, corresponding to the knowledge that agent would express, and (3) intentions, or actions, that agents decide to do.

There are many definitions of the agent paradigm [27, 28, 29] and new types of agents continue to emerge [30]. So, it is difficult to establish a consensual definition. However, through these definitions we observe that three functions characterize agent activity: perceive, decide, and act. An agent has its own knowledge. It acts in autonomy to reason and decides according to its objectives, its interactions with other agents in the system, and its environment perception. By extension, considering cognitive agents, experts of this domain generally agree on the following characteristics: intentionality, rationality, commitment, adaptability, and intelligence. Agent-based systems are systems that allow distributing agents, communicating, autonomous, reactive, skillful, and finalized entities. They form intelligent solver networks, weakly bound, working together to solve problems beyond their individual capabilities and knowledge [31]. Agents may possess many other properties, like: sociable, mobile, proxy, intelligent, rational, temporally continuous, credible, transparent and accountable, coordinative, cooperative, competitive, rugged, or trustworthy [25].

Agent-based modeling and simulation is a relatively new approach to modeling complex systems made up of interactive and autonomous entities – the agents. Multi-agents modeling is a way of modeling the dynamics of complex systems and complex adaptive systems [9]. These systems are often self-organized and can create emergent orders. The behaviors of agents are described by simple rules. They interact with other agents and their environment. These interactions in turn influence their own behavior. Thus, at the system level, structures and behaviors emerge that were not explicitly programmed into the initial model, but appear through interaction of agents. This emergence in multi-agent systems is also subject to formalization proposal [32]. The focus on the modeling of heterogeneous agents in a population, the emergence of self-organization, and the self-adaptation of multi-agent systems are three distinctive features of agent-based simulation [33, 34, 35].

III. Model of Quantum Agent

A software agent, according to the well-known model of Newell and Simon [36], is an independent information processing system, which means it is made up of:

1) a receiving and a transmitting modules to exchange messages with its environment or other software agents;

2) its own execution capacity;

3) and a memory (*i.e.* a knowledge base).

Thus, an agent-based system is a society of autonomous/independant agents that work together to achieve a common goal through interaction, communication or transaction. For us, autonomy is the main differentiation of agent paradigm compared to the object paradigm. This autonomy can be achieved by:

1) an independent computer process;

2) an individual memory;

3) the ability to interact (by perception or reception, and communication or action) [24];

4) and specially for a quantum agent the ability to control superposition states.

The quantum agent-based formal approach we follow to model and design complex systems is to define the modular architecture of quantum agents (qAgents), to define their model of interaction, communication and knowledge, and to respect a rigorous methodology for acquiring expertise. Thus, a quantum agent-based system *M* is described by a 5-tuple (3):

$$\mathrm{M} = \langle \mathrm{A, I, P, O}, \Psi \rangle \qquad (3)$$

where *A* is a set of qAgents ($\alpha_i \in \mathrm{A}$, whose state vector can be written $|\Psi_i\rangle$ and read ket Ψ_i) that can superpose several states $\{\psi_{1,}, \psi_2, ..., \psi_n\} \in \Psi$; *I* is the set of interactions defined for the qAgents of *A* ($\iota_i \in I$); *P* is the set of roles to be played by the qAgents of *A* ($\rho_i \in P$), and *O* is the set of organizations of the qAgents of *A* into communities ($o_i \in O$). Moreover, considering that the multi-agent system is composed of *n* qAgents α_i , each qAgent being in state $|\Psi_i\rangle$, then the global state of the multi-agent system is $|\Psi_1\rangle \otimes |\Psi_2\rangle \otimes ... \otimes |\Psi_n\rangle$.

Many basic agent behaviors are inspired by the cycle <*perceive, decide, act*> [24]. The behavior of quantum agents is similar. They continually perform four functions: observation, interpretation according to their possible states, decision, and eventually action (Figure 1). Thus, a qAgent $\alpha_i \in \mathrm{A}$ is described by the following tuples (4):

$$\alpha_i = < \Pi(\varepsilon_j, \pi_k), \Omega^*(\pi_k, \Sigma_{\alpha_i}, \Omega_{\alpha_i}), \Delta(\Omega_{\alpha_i}, \delta_m), \Gamma(\delta_m, \gamma_n), K_{\alpha_i} > \qquad (4)$$

where $\Pi(\varepsilon_j, \pi_k)$ is the function of observations of the qAgent α_i (ε_j is an event and π_k is its observation); $\Omega^*(\pi_k, \Sigma_{\alpha_i}, \Omega_{\alpha_i})$ is the multi-function of interpretations of the qAgent α_i (Σ_{α_i} is the finite set of states of qAgent α_i , moreover, at a given time the state vector of the qAgent α_i is noted $|\psi_{\alpha_i}\rangle$), and Ω_{α_i} is the finite set of interpretations of observations made by qAgent α_i); $\Delta(\Omega_{\alpha_i}, \delta_m)$ is the function of decisions of the qAgent α_i (δ_m is a decision); $\Gamma(\delta_m, \gamma_n)$ is the function of actions of the qAgent α_i (γ_n is an action); K_{α_i} is the finite set of knowledge of the qAgent α_i (decision rules, values of the domain, acquaintances and/or networks of affinities between quantum agents, observed events, internal states, etc.).

The quantum agents communicate and influence each other (quantum property), they also have quantum states. When these states take only two values, such as cellular automata that we present below in Section 4, we can compare them to the qubit concept. A qubit (contraction of quantum and bit) represents the storage unit of quantum information [37]. A qubit is composed of a superposition of two basic states, written |0> and |1>. A qubit state consists of a linear superposition of these two states. If a classical bit is always either in the |0> state, or in the |1> state, a qubit is in a superposition of these two states. This can be described by a linear combination of two states (5):

$$|\Psi\rangle = \alpha \bullet |0\rangle + \beta \bullet |1\rangle \qquad (5)$$

where the coefficients *α* and *β* are two complex numbers satisfying the normalized relation $|\alpha|^2 + |\beta|^2 = 1$.

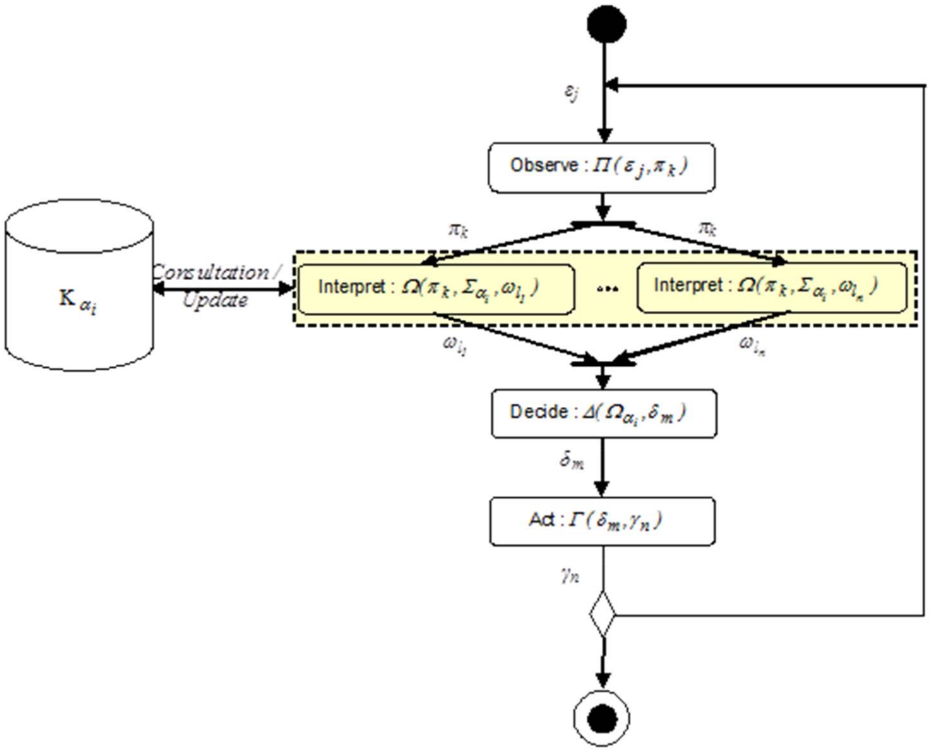

Fig. 1. Basic behavior of a quantum agent

IV. Application to the Simulation of Takuzu Game

To calculate is to observe, remember and act; thus performs a finite-state automaton. Consider a set of identical finite-state automata, with a limited number of states, placed on the squares of a chessboard (these kinds of automata are called cellular automata, or CA). Any one of these automata observes the automata around, remembers the state it is and changes state respecting the invariable rules that characterize it (rules similar to a program). This change relates all squares of the chessboard and determines a new generation of states of the squares. By applying this process again, a new generation is obtained. The most famous of cellular automaton is the Conway automaton, known as “Game of life” [38]. If we have already shown the ability to model the “Game of life” with quantum agents [39] it is a different kind of automaton that will interest us in this paper: the Takuzu game.

The popular grid games such as Sudoku or Takuzu are also other kinds of cellular automata. The cells of these games have values (states) dependent on values in other cells, neighbours, on the same column or on the same row. Takuzu game (also named binary puzzle) is a logic-based number placement puzzle. The objective for the gamer is to fill a grid with the two binary values "0" and "1", respecting two rules: 1) an equal number of “1” and “0” in each row and column, and 2) no more than two same adjacent values "0" or "1".

An illustration of a simulation based on classical agent’s model is given in Figure 3: the grid is composed of 16 cells (a 4x4 grid), each cell is modeled by a quantum agent, the initial configuration includes 16 quantum agents, 4 of which have value. Twelve steps of the evolution of this simple configuration of Takuzu game are shown in the figure 3 (3.1 to 3.12).

A. *Quantum Agents-Based Simulation*

A quantum system can be described by the sum of different superposed states (6):

$$\left|\Psi\right\rangle = \sum_{i=1}^{i=n} \alpha_i \left|\Psi_i\right\rangle \tag{6}$$

where α_i are coefficients called "probability amplitude".

In the case of a cell of a cellular automaton, without interference from its environment, the sum of different superposed states becomes (7):

$$\left|cell\right\rangle = \tfrac{1}{2}\left|0\right\rangle + \tfrac{1}{2}\left|1\right\rangle \tag{7}$$

where, for instance, |0> can represent the death state and |1> the living state. This means that before an observation (or measure) the cell is in both living and dead states, and at the next observation (or measure), the cell has the same probability of being observed alive or dead.

In the case of a 4x4 Takuzu grid, probability of a cell to be equal to 0 or 1, the coefficients α or β respectively, will depend on the states of the 3 other cells of the row and the 3 other cells of the column (8):

$$\left|cell\right\rangle = \alpha\left|0\right\rangle + \beta\left|1\right\rangle \tag{8}$$

By applying the *SCOP* model to a 4x4 Takuzu grid, where the concept is the cell (C is the set of cells and $c_i \in C$ is a given cell), we get:

Σ is a set of 2 states: {0, 1};

M is the set of cells contexts; the context e of a cell c_i is defined by the states of the 6 "neighboring cells" of c_i;

Λ is the set of cell properties: position, size, color, ...;

μ is the function of state change, *i.e.* the probability that a cell in a state p under the influence of context e changes in state q (knowing that there are 2^6=64 possible contexts for a given cell c_i) (9):

$$\mu : \Sigma \times M \times \Lambda \rightarrow [0,1]; (q,e,p) \rightarrow \mu(q,e,p) \qquad (9)$$

ν is the weight of a cell property a in a state p (10):

$$\nu : \Sigma \times \Lambda \rightarrow [0,1]; (p,a) \rightarrow \nu(p,a) \qquad (10)$$

B. Quantum Agents for the Takuzu Game

Cellular automata are synchronous massively parallel computers. Each cell is a finite state transducer which takes its inputs from neighboring cells, and determines its own output state. At every tick of the clock, cells determine their states (change of states or not). Each cell observes states of neighboring cells and, taking its own state into account, applies a defined transition rule, to decide its state at the next tick of the clock. All cells are changed at the same time. From the physical point of view, a cellular automaton belongs to the field of digital classical theories, in which space, time and states are discrete [9].

In the following we propose a quantum agents model for the Takuzu game, like the previous presentation of cellular automata, but this time, the tick of the clock is when the player writes a new value in a cell, i.e fills a cell. Quantum agents are autonomous (*i.e.* each qAgents manages its evolution cycle time), they observe the state of their neighbors (qAgents can also transmit their state by exchanging messages) and apply the two rules presented below. Indeed, when a cell becomes aware (observe) the status of a neighboring cell, the latter may be planning to change its state. Also, each cell will determine its future state by calculating the probability of its binary value "0" or "1" (α or β in eq. 8). The Figure 2 illustrates a possible evolution for a grid of 16 cells (4x4), whereas the evolution of these cells is rather synchronous. Consider the cell $Cell_{1,2}$ ('?' colored red in the figure), her probability to have the value "0" or "1" are defined by (11):

$$\alpha_{2,1}|0\rangle + \beta_{2,1}|1\rangle \Rightarrow \frac{4}{6}|0\rangle + \frac{2}{6}|1\rangle = \frac{2}{3}|0\rangle + \frac{1}{3}|1\rangle \qquad (11)$$

Figure 3 illustrates an evolution of a 4x4 Takuzu grid in twelve steps. In this illustration, the evolution of the game is not automatic, but supervised by a player. The player can fill the grid by viewing the different states of the cells (implemented by 16 quantum agents). Two situations (grids) are remarkable: 1) when cell cannot propose a defined state (0 or 1) and thus all cells have a probability amplitudes between 0 and 1 (typically, the coefficients α and β are equal to 1/3, 2 / 5, 1/2, 3/5, 2/3 in this example); 2) when a cell has determined its value, *i.e.* when its probability amplitude is either $PA = 1|0\rangle + 0|1\rangle$ or $PA = 0|0\rangle + 1|1\rangle$. In the first case, the player chooses itself a value for a cell based on the visualization of the states of all cells (choices of cells colored in blue on the figures 3.3, 3.4, 3.6, 3.7, and 3.9). In the second case, the player fills the cells according to the values proposed by the cells (choices of cells colored in red on the figures 3.5, 3.8, 3.10, 3.11, and 3.12).

The detail of the twelve steps shown in Figure 3 is as follows:

1) The initial setting of the Takuzu grid;

2) The visualization of the initial amplitude probabilities of quantum cell agents;

3) The player makes his first choice ($Cell_{1,2} = 0$);

4) The player makes his second choice ($Cell_{1,3} = 1$);

5) The player can observe that the probability values of the cell $Cell_{1,4}$ is equal to $1|0\rangle + 0|1\rangle$ (so $Cell_{1,4} = 0$);

6) The player makes his third choice ($Cell_{2,1} = 0$);

7) The player makes his fourth choice ($Cell_{3,1} = 1$);

8) The player can observe that the probability values of the cell $Cell_{1,4}$ is equal to $1|0\rangle + 0|1\rangle$ (so $Cell_{4,1} = 0$);

9) Before hesitation, because the probability values of all the cells are equal to $½|0\rangle + ½|1\rangle$, the player makes his fifth choice ($Cell_{2,3} = 1$);

10) The player can observe that the probability values of the cells $Cell_{2,4}$ and $Cell_{4,3}$ are equal to $1|0\rangle + 0|1\rangle$ (so $Cell_{2,4}$ = 0 and $Cell_{4,3}$ = 0);

11) The player can observe that the probability values of the cells $Cell_{3,4}$ and $Cell_{4,2}$ are equal to $0|0\rangle + 1|1\rangle$ (so $Cell_{3,4}$ = 1 and $Cell_{4,2}$ = 1); and finally

12) The player can observe the probability values of the last cell $Cell_{3,2}$ is equal to $1|0\rangle + 0|1\rangle$ (so $Cell_{3,2}$ = 0).

$$\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} \rightarrow \begin{pmatrix} 1&?&?&? \\ ?&1&?&? \\ ?&?&1&? \\ ?&?&?&1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \frac{2}{3}|0\rangle + \frac{1}{3}|1\rangle & \alpha_{1,3}|0\rangle + \beta_{1,3}|1\rangle & \alpha_{1,4}|0\rangle + \beta_{1,4}|1\rangle \\ \alpha_{2,1}|0\rangle + \beta_{2,1}|1\rangle & 1 & \alpha_{2,3}|0\rangle + \beta_{2,3}|1\rangle & \alpha_{2,4}|0\rangle + \beta_{2,4}|1\rangle \\ \alpha_{3,1}|0\rangle + \beta_{3,1}|1\rangle & \alpha_{3,2}|0\rangle + \beta_{3,2}|1\rangle & 1 & \alpha_{3,4}|0\rangle + \beta_{3,4}|1\rangle \\ \alpha_{4,1}|0\rangle + \beta_{4,1}|1\rangle & \alpha_{4,2}|0\rangle + \beta_{4,2}|1\rangle & \alpha_{4,3}|0\rangle + \beta_{4,3}|1\rangle & 1 \end{pmatrix}$$

Fig. 2. Illustration of the superposed state of quantum cell agents during a step of the Takuzu game

1			
	1		
		0	
			1

1) Initial setting

1	$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$
$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$	1	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$
$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	0	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$
$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$	$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	1

2) Initial probability amplitudes

1	0	$\frac{2}{5}|0\rangle$ $\frac{3}{5}|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$
$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$	1	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$
$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{2}{5}|0\rangle$ $\frac{3}{5}|1\rangle$	0	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$
$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	1

3) The player's first choice

1	0	1	$1|0\rangle$ $0|1\rangle$
$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$	1	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$
$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{2}{5}|0\rangle$ $\frac{3}{5}|1\rangle$	0	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$
$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	1

4) The player's second choice

1	0	1	0
$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$	1	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$
$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{2}{5}|0\rangle$ $\frac{3}{5}|1\rangle$	0	$\frac{2}{5}|0\rangle$ $\frac{3}{5}|1\rangle$
$\frac{2}{3}|0\rangle$ $\frac{1}{3}|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	1

5) Choice where PA=1|0> + 0|1>

1	0	1	0
0	1	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$
$\frac{2}{5}|0\rangle$ $\frac{3}{5}|1\rangle$	$\frac{2}{5}|0\rangle$ $\frac{3}{5}|1\rangle$	0	$\frac{2}{5}|0\rangle$ $\frac{3}{5}|1\rangle$
$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	1

6) The player's third choice

1	0	1	0
0	1	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$
1	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	0	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$
$1|0\rangle$ $0|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	$\frac{3}{5}|0\rangle$ $\frac{2}{5}|1\rangle$	1

7) The player's fourth choice

1	0		0
0	1	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$
1	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	0	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$
0	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	1

8) Choice where PA=1|0> + 0|1>

1	0	1	0
0	1	1	$1|0\rangle$ $0|1\rangle$
1	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	0	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$
0	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	$1|0\rangle$ $0|1\rangle$	1

9) The player's fifth choice

1	0	1	0
0	1	1	0
1	$\frac{1}{2}|0\rangle$ $\frac{1}{2}|1\rangle$	0	$0|0\rangle$ $1|1\rangle$
0	$0|0\rangle$ $1|1\rangle$	0	1

10) Choice where PA=1|0> + 0|1>

1	0	1	0
0	1	1	0
1	$1|0\rangle$ $0|1\rangle$	0	1
0	1	0	1

11) Choice where PA=0|0> + 1|1>

1	0	1	0
0	1	1	0
1	0	0	1
0	1	0	1

12) Choice where PA=1|0> + 0|1>

Fig. 3. Twelve steps of evolution of a 4X4 grid of Takuzu game: probabilities of the cells' binary values and choices of the player

The detail of the nine steps shown in Figure 4 is as follows:

1) The initial setting of the 4x4 Takuzu grid;

2) The visualization of the initial probability values of the cell $Cell_{1,2}$;

3) The player observes the light gray color of the cell $Cell_{1,2}$, which means that the probability values of this cell is equal to $\frac{2}{3}|0\rangle + \frac{1}{3}|1\rangle$ (so the player takes the decision to write 0 in this cell: $Cell_{1,2} = 0$);

4) The new setting of the 4x4 Takuzu grid;

5) The visualization of the probability values of the cell $Cell_{1,3}$;

6) The player observes the dark gray color of the cell $Cell_{1,3}$, which means that the probability values of this cell is equal to $\frac{2}{5}|0\rangle + \frac{3}{5}|1\rangle$ (so the player takes the decision to write 1 in this cell: $Cell_{1,3} = 1$);

7) The new setting of the 4x4 Takuzu grid; 8) visualization of the probability values of the cell $Cell_{1,4}$;

9) The player observes the white color of the cell $Cell_{1,4}$, which means that this cell has determinate its value, its probability values is equal to $1|0\rangle + 0|1\rangle$ (so the player decides to follow the proposition of the cell and to write 0: $Cell_{1,4} = 0$).

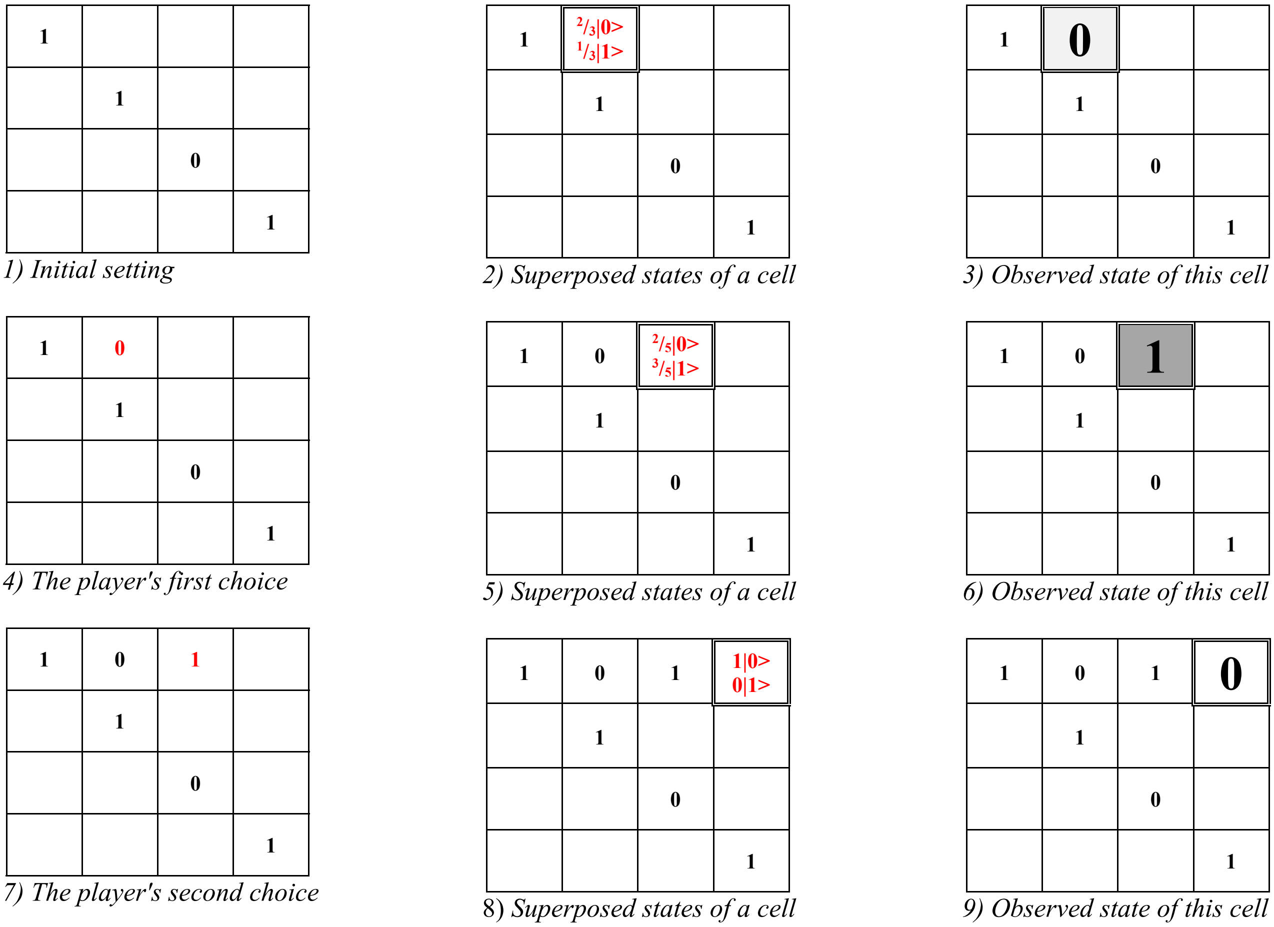

Fig. 4. Detail of the top three choices made by the player after observing the propositions made by the cells of the first row of the Takuzu grid.

V. Conclusion and Perspectives

To model complex systems, and obviously intelligent complex systems, engineers use the techniques of distributed artificial intelligence and the agent paradigm increasingly. However, some issues and situations are difficult to model when agents (components of an intelligent complex system) must make decisions with uncertain, incomplete or indeterminate knowledge of their context. To provide a solution to this problem, studies on quantum cognition introduce quantum properties such as superposition states and entanglement in the decision process. In this context, we have assumed that agent modeling can be inspired by quantum-like models. In this paper, a quantum agent's model that is capable of implementing both quantum properties of state superposition and entanglement has been proposed.

A simple case study of simulation of the Takuzu game with a grid of 4x4 cells illustrates our proposed quantum agent model. We have presented the potentiality of a quantum approach for modeling and simulate a continuous and asynchronous version of the game. From this simulation we have shown the interest of the state superposition property for the decision process of quantum agents, that this decision is made by the agents themselves or is observed by a player..

We are continuing to work on better integration of quantum properties in our quantum agent model. After the property of superposition state, we are now interested in the quantum properties of entanglement (quantum correlations between quantum agents during a collective decision), interference between states (a choice made during a decision process interferes with confidence in the final choice), and oscillation (the hesitation when quantum agents taking individual or collective decision). Agents with quantum behavior can they become agents having a quantum cognition, beyond the superposition state property that can deal with uncertainty in the decision-making?

Another perspective for our work on the treatment of uncertainty in the decision-making by intelligent agents is the comparison of the relevance of the quantum agents approach with different other approaches [1, 2, 3], mainly the fuzzy agents approach [40].

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