# Fast Optimization of Large Antenna Arrays on Dielectric Layers using Surrogate Macro-Basis-Function Representations

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*Abstract*—**A fast full-wave simulation for the synthesis of printed antenna arrays is presented. The technique relies on a fast simulation scheme, exploiting the Macro Basis Functions (MBF) technique, to quickly analyze arrays. Mutual coupling (MC) is taken into account by exploiting the embedded element patterns in the synthesis process. The technique is hybridized with an iterative convex optimization to effectively optimize antenna excitation and locations in a double-step optimization routine. The synthesized arrays will therefore fulfill all requirements.The performance of the method is validated for arrays of printed bowtie antennas.**

## I. Introduction

Antenna array synthesis problem consists of manipulating all array parameters, i.e. antenna's position, excitation, to provide an array with required performance in terms of sidelobe level, number of required elements, as well as beampattern. Several methods have been studied to effectively control these degree of freedom [1] – [5]. Interestingly, the synthesis problem and the constraints can be formulated as a convex problem, i.e. directly (for array factor) or by relaxation, and therefore can be solved optimally using convex programming [7]–[12].

An important advantage of convex-based optimization is the optimality of the solution and the effectiveness of the method once the problem has been modeled as a convex one. While with global optimizations, one could have the power to optimize different parameters with several constraints at the cost of simulation time, which sometimes is unaffordable. Therefore, for the antenna synthesis problems, the convex optimization is effective in optimizing the antenna excitations, while their locations are mainly controlled by global optimization techniques [7]. These hybrid schemes allow the designer to exploit all degrees of freedom. However, the use of stochastic search is very time consuming. An interesting technique to optimize the position using convex optimization is proposed in [10] considering arrays with constant amplitudes.

Furthermore, in most of studies, to easy the synthesis problem, all antenna are often assumed to have identical pattern or even considered as a point. The array pattern is then converged to array factor, i.e. ignoring the mutual coupling between antenna. The main obstacle to introduce full-wave simulation in these synthesis routines is the time needed to analyze these arrays, which greatly slows down the optimization process. In some cases, for large arrays, full-wave analysis become impractical.

This paper addresses a solution for these two problems by proposing a full-wave optimization scheme, in which fast simulation techniques are combined with an iterative convex optimization to effectively synthesize arrays of printed antennas. These arrays are analyzed using the Method of Moments (MoM) combined with Macro Basis Functions (MBF) [14] (which may be regarded as a surrogate model [15] for the actual current distribution on the patch). Moreover, techniques for fast MBF interactions are implemented to quickly calculate the entries of the reduced interaction matrix. The patterns (embedded element patterns – EEPs) obtained from the fast full-wave analysis are then exploited through an optimization routine relying on convex optimization to effectively control the weights and positions of the antennas. This approach maybe viewed as an extension of the method presented in [16], [17]: now, variable weights are introduced and 2D arrays are more thoroughly analyzed using a cost function based on $l_\infty-$ norm.

The paper is formulated as: in Section II, the simulation technique is described for printed array. Then, in Section III, the synthesis problem is detailed and the optimization routine is explained. Section IV shows the optimization results for printed linear and planar bowtie antenna arrays. The paper ends with conclusions and remarks.

## II. Full-wave Simulation of Printed Antenna Arrays

To integrate a full-wave solver in the array synthesis, a fast simulation technique is needed to rapidly obtain all EEPs and array patterns. In the present work, techniques starting from Method of Moment (MoM) are implemented to quickly calculate the entries of the MoM matrix. In particular, the accelerated Macro Basis Functions (MBF) technique [14] are used to minimize the size of the problem, and to reduce solution time for the MoM system of equations. Besides, interactions between MBFs are further accelerated by two recently developed techniques, which effectively solve two parts of the layered-medium Green's function:

- Interpolatory technique [18] for the average homogeneous part,

- and Contour - FFT (C - FFT) [19] for the rest (i.e. subtracting the homogeneous part).

For the homogeneous medium, the interpolatory technique [18] is very effective in obtaining the MBFs interaction between metallic antennas. A mathematical model represented the interaction between MBFs is built from three physical transformations (farfield subtraction, phase extraction and appropriate change of variables) of MBFs interactions over a polar-radial grid of limited numbers of points. The interaction is then modeled in terms of a harmonic-polynomial (HARP) function [20], [16]. For the remained part, for each pair of MBFs, the interaction is calculated versus relative distance, thanks to the effectiveness of the C-FFT [19]. As soon as the tabulation is finished, the interaction is quickly attained by interpolating these tables.

Once the preparation is completed, the reduced MBF interaction matrix is rapidly filled by interpolating HARP and these C-FFT tables. Currents on antennas and all EEPs are then quickly calculated. These techniques allows the full-wave simulation to be integrated inside the synthesis routine, which fully takes into account the MC effects. It should be recalled that the tabulation is done only once for a given antenna independently from the array configuration; the data then can be used to analyze different arrays made of the same elements. This feature is very beneficial when array optimization with mutual coupling (full-wave) is concerned.

## III. Pencil Beam Synthesis

Assuming an array made of $N$ antennas on horizontal plan, with EEP $f_n(\theta,\phi)$ and is excited by excitation vector $a_n$, the array pattern of the array $e_{\text{array}}(\theta,\phi)$ is calculated as:

$$f_{\text{array}}(\theta,\phi) = \sum_{n=1}^{N} e_n(\theta,\phi)\; a_n\; e^{jk(u_x x_n + v_y y_n)} \qquad (1)$$

where $k$ is the wavenumber, $(u_x = \sin\theta\cos\phi, v_y = \sin\theta\sin\phi)$, and $(x_n, y_n)$ is the position of antenna $n^{th}$ . The synthesis goal considered here consists of forming a focused beam (i.e. pencil beam) at the broadside. Assuming that the sidelobe region $S$ has a maximum peak level $\rho$, the synthesis problem is formulated as:

$$\min_{w_n,x_n,y_n} \rho \qquad \text{subject to} \quad \sup_{(\theta,\phi)\in S} \mid f_{\text{array}}(\theta,\phi) \mid \leq \rho \qquad (2)$$

In Eq. (1), both the excitation $a_n$ and positions $(x_n, y_n)$ correspond to the degrees of freedom of the array. The proposed synthesis consists of a two-step scheme to independently optimize these variables, in a way similar to the hybrid approach proposed in [7]. While the technique in [7] implemented global optimization to control the positions, these positions are manipulated here using convex optimization, which is more effective. The method proposed in [10] is adopted, which transforms the problem into a convex problem. Thus, the synthesis is carried out efficiently exploiting convex optimization toolbox.

As antenna's position is involved, the EEPs are recalculated every iteration. The optimization routine is modified as follows:

- Step I: Initial design: an uniform array of equal distance is initialized, all EEPs are obtained.
- Step II: Inside the optimization loop: for each iteration, excitation vector, $a_n$, is first optimized with the current position of the antennas to minimize the SLL.
- Step III: With the optimum $a_n$, start optimizing the positions by solving another convex problem, where EEPs are momentarily assumed to be constant (see following paragraph).
- Step IV: Update the EEPs exploiting the proposed simulation technique described in the previous Section and return to Step II (for the next iteration) until requirement are fulfilled.

To incorporate the EEPs to optimize antennas positions in Step III, EEPs are considered as unchanged within a small increment of position. The upper bound is set to the movements of antennas, as described in [16], [17]. In Step IV, after finding new position, all EEPs are quickly updated thanks to the fast simulation technique. The synthesized array thus satisfies all requirements and fully takes into account the mutual coupling.

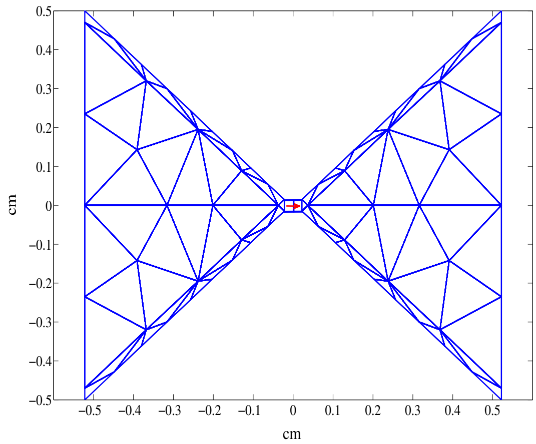


Fig. 1. Geometry of the printed bowtie antenna.

## IV. Simulation Results

To pose practical applications, bowtie antenna printed on a double substrates backed by ground plane are studied here, as shown in Fig. 1. As the actual antenna is considered, the minimum distance between antennas is set in the optimization to ensure the non-overlapping between antennas.

*1) Linear Array of 21 Printed Bowtie:* The considered example is to synthesize a linear array of maximum size of $10\lambda$, which generates an asymmetric sidelobe pattern, i.e. a 10 dB difference between sidelobe levels on each side of the main beam, as described in [6]. The optimum design in [6] is an array of 21 elements with sidelobe levels of -29.8 dB and -19.8 dB, while in [9] a better sidelobe performance is achieved, i.e. -30.3 dB and -21.3 dB, with one more element in the array.

For the proposed approach, a uniform array of 21 bowtie elements over 10 $\lambda$ length, i.e. the initial configuration as 0.5 $\lambda$ spacing, is chosen as a starting point. The minimum distance is set equal to 0.35 $\lambda$, as the size of the bowtie is 0.33×0.33 $\lambda$. Fig. 2 shows the radiation pattern of the optimized array; it clearly satisfies the constraints of asymmetric pattern with sidelobe levels of -30 dB and -20 dB. While the solution is comparable in performance to what is found in literature [9], the proposed approach has advantage of providing a feasible solution, which might not be the case in [9] as it worked on an over-sampled grid of point-like sources.

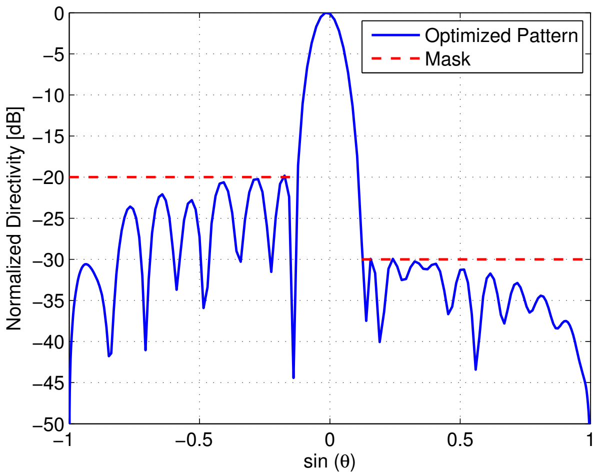


Fig. 2. A radiation pattern of the synthesized 21 antenna array in main plane cut.

*2) Sparse Planar Array:* A larger array is considered in this subsection, where elements are distributed over a square of size $5\lambda \times 5\lambda$ [7], [9]. The synthesis consists of generating a focused beam pattern with the main beamwidth of $\sin(\theta) = 0.24$ at -6 dB, and to minimize the sidelobe levels. Arrays comprising of 41 elements are both reported in [7] and [9] with sidelobe levels of -16.5 dB and -17.3 dB, respectively.

An array made of 41 elements is also implemented in this paper, where the elements initially are randomly populated over the square with minimum distance of 0.7 $\lambda$. The proposed technique is then exploited to optimize the excitation and location of antennas. Since the distance ($l_2$-norm) between antennas cannot be modeled as a constraints in a convex problem, the minimum distance requirement then is relaxed to the ($l_\infty$-norm) distance, i.e. maximum of the distance along $x$ or $y$. Although this relaxation reduces the search space of the optimization, but it enables the integration of the minimum distance in the convex optimization. Fig. 3 displays the patterns in two main planes of the optimized array, while its layout is plotted in Fig. 4. A sidelobe level below -20 dB is achieved using the proposed approach. It is interesting to see that, while including the actual array environment, the proposed technique offers better performance w.r.t. to those ignoring the mutual coupling. Finally, the synthesis time for the second example using the proposed method is reported in Table I. The convex optimization converges after about 20 iterations, which takes less than 4 minutes including the time to analyze the array every iteration, i.e. solving and calculating EEPs. It is worth noting that, while the preparation phase is the most time taking one, it was done only once and for all. The data can be re-used for analyzing different arrays made of the same type of elements. This feature allows the incorporation of full-wave solver inside the array synthesis routine.

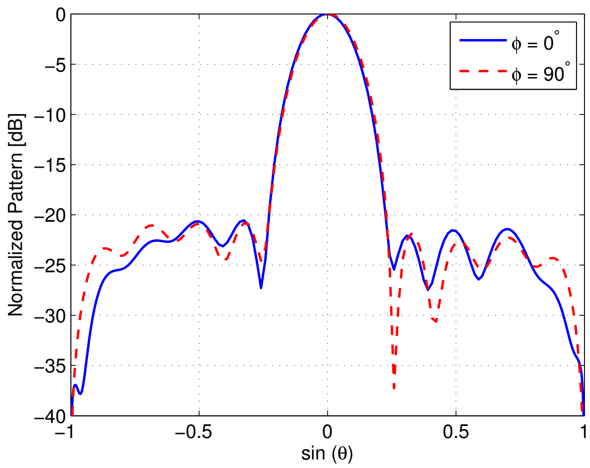


Fig. 3. Radiation pattern of sparse array of 41-element in two main planes.

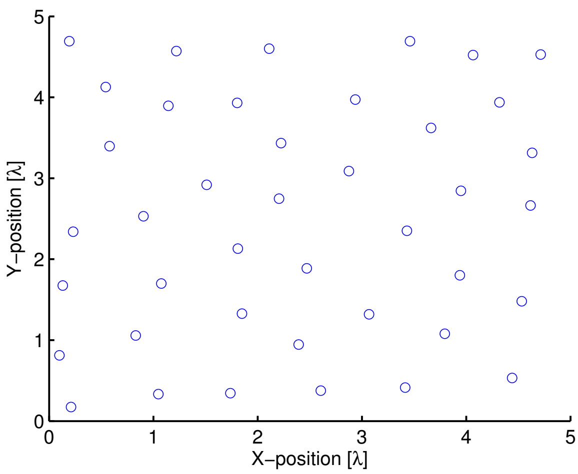


Fig. 4. Layout of sparse array of 41-bowtie element.

## V. Conclusion

A hybridized technique combining the advantage of fast simulation techniques and convex optimization is presented for the synthesis of printed antenna arrays. The MBF-based

TABLE I
SYNTHESIS TIME FOR ARRAY MADE OF 41-ELEMENTS

| **Operations** | **Required Time** (mins) |
|---|---|
| MBF Generation | 0.2 |
| Interpolatory Preparation Time | 0.56 |
| C-FFT Tabulation Time | 5.43 |
| **Total Preparation time** | **6.19** |
| EEP calculation and Array solution | 0.08 |
| **Convex Optimization Time** | **4** |

techniques are exploited to enable the integration of full-wave solver inside the optimization routine. Therefore, the optimized arrays include the actual array environment. Compared to very recent developments, the amplitudes are also modified and planar arrays are tackled based on $l_\infty$-norm cost function. Numerical results have shown the performance of the presented method.

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