A Multi-Objective Approach for 3D Airspace Sectorization:
A Study on Singapore Regional Airspace

Cheryl Wong Sze Yin
School of Computer Science and Engineering
Nanyang Technological University
Singapore 639798
Email: cwong019@e.ntu.edu.sg

T.K. Venugopalan
School of Computer Science and Engineering
Nanyang Technological University
Singapore 639798
Email: tvenugopalan@ntu.edu.sg

S. Suresh
School of Computer Science and Engineering
Nanyang Technological University
Singapore 639798
Email: ssundaram@ntu.edu.sg

Abstract—This paper presents an automatic airspace sectorization method for air traffic management in presence of many climbing trajectories. For this purpose, we have modified the formulation, and solved it using Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Voronoi diagrams. The air traffic controller’s monitoring workload is formulated with the use of modified dynamic density, which is a more rigorous and accurate formulation as compared to the simplistic metrics used in other works. The multi-objective formulation is implemented using historical flight data within Singapore regional airspace. The results clearly indicate that the en-route 3D sectorization using dynamic density in monitoring workload formulation provides better workload balance in ATC compared to 2D sectorization.

I. INTRODUCTION

Airspace sectorization is defined as the partitioning of a given airspace into smaller areas to ensure safety and increase the capacity of airspace. Many different methodologies have been used to model this as an optimization problem. The sectorization approach may be classified into two categories: clustering, and free-form sectorization. Clustering based sectorization involves combining smaller units of airspace into sectors, and has been observed to yield better solutions but may require post-processing steps to ensure that the convexity and connectedness of the shape of sectors are being satisfied [1], [2]. On the other hand, free-from sectorization that does not break down the airspace into fundamental blocks is found to be more computationally faster [3]. Voronoi diagram [4] is a popular free-form sectorization technique that guarantees both the convexity and connectedness of sectors [5]. Voronoi diagram is predominantly used with Genetic Algorithms (GA) [5], [6] because there is currently no known correlation between the selected sites and the objective functions. Hence, in this paper, we present a multi-objective formulation of 3D sectorization and for easy implementation, solving it using Voronoi diagram and NSGA-II [7].

Another important component in airspace sectorization, which is the focus of this paper, is to formulate the objective functions. The key determinant of airspace capacity is the air traffic controllers’ workload [8]. Therefore, many objective functions are based on the air traffic controllers’ monitoring and coordination workload such as minimizing average/max number of aircrafts in the sector, and number of sector crossings. However, these metrics are too simplistic to realistically portray the traffic controller’s workload. This is especially so when there are many climbing flights in the airspace, a situation that is being focused on in this paper and evident in the Singapore Regional Airspace.

Dynamic density is an concept that has been proposed to measure the workload of air traffic and has a variety of definitions [9]. In addition to number of flights monitored, it incorporates heading, speed, altitude changes and the number of close proximity flights, to evaluate the workload. This metric has previously been introduced in air traffic sectorization in [10] in a clustering based sectorization. The authors concluded that the dynamic density metrics that they used did not have the required sensitivity for airspace design. Therefore, in this paper, we propose a formulation of dynamic density which has been modified from [11] instead, as one of the objective functions. The problem of 3D airspace sectorization has been formulated as a constrained bi-objective optimization formulation which minimize both dynamic density based monitoring workload and the coordination workload of ATC. We first present the formulation of air-space sectorization using Voronoi with objective function that closely represent actual workload of the air-traffic controller. Then, we investigate the benefits of 3D sectorization for Singapore’s air traffic that contains a large percentage of climbing flights within en-route airspace using the proposed formulation.

The optimization is cast in a Multi-Objective Optimization (MOO) framework, resolved using a combination of Voronoi diagram [4] and NSGA-II [7]. The results were analysed and insights on the proposed methodology and Singapore regional airspace are provided. Our findings reflect greater efficiency in using 3D sectorization as compared to 2D in the given airspace.

Section II provides an overview of the framework, motivation and details on the formulation of the model. Section
III provides an analysis on the proposed methodology on the Singapore regional Airspace. Lastly, Section IV provides a summary of the results and the direction of our future work.

II. METHODOLOGY: MULTI-OBJECTIVE OPTIMIZATION FRAMEWORK

The problem of 3D airspace sectorization has been modeled as constrained multi-objective optimization. For this purpose, this paper uses Voronoi diagram to realize the sectors and NSGA-II [7] evolutionary algorithm to tackle the multi-objective optimization problem. First, we present the Voronoi based sector representation in Section II-A, followed by the modified conflicting objectives and constraints formulation in sectorization in Section II-B and brief summary of NSGA-II in Section II-C.

A. Voronoi Diagrams

Voronoi diagram is a partitioning technique of subdividing a given region into sub-regions. A set of points known as sites, are used to generate these partitions satisfying that, any point within each partition is closest to its corresponding site. Such diagrams have an interesting property that the resulting shape of the partitions are strictly convex (internal angles are less than 180°). This is a widely used technique in automatic sectorization literature, since convex sector shapes are preferred than 180° diagrams have an interesting property that the resulting shape within each partition is closest to its corresponding site. Such are used to generate these partitions satisfying that, any point within a given region into sub-regions. A set of points known as sites, are used to generate these partitions satisfying that, any point within each partition is closest to its corresponding site. Such diagrams have an interesting property that the resulting shape of the partitions are strictly convex (internal angles are less than 180°). This is a widely used technique in automatic sectorization literature, since convex sector shapes are preferred than 180° diagrams have an interesting property that the resulting shape within each partition is closest to its corresponding site. Such are used to generate these partitions satisfying that, any point within a given region into sub-regions. A set of points known as sites, are used to generate these partitions satisfying that, any point within each partition is closest to its corresponding site. Such diagrams have an interesting property that the resulting shape of the partitions are strictly convex (internal angles are less than 180°). This is a widely used technique in automatic sectorization literature, since convex sector shapes are preferred than 180° diagrams have an interesting property that the resulting shape within each partition is closest to its corresponding site. Such are used to generate these partitions satisfying that, any point within a given region into sub-regions. A set of points known as sites, are used to generate these partitions satisfying that, any point within each partition is closest to its corresponding site. Such diagrams have an interesting property that the resulting shape of the partitions are strictly convex (internal angles are less than 180°). This is a widely used technique in automatic sectorization literature, since convex sector shapes are preferred than 180° diagrams have an interesting property that the resulting shape within each partition is closest to its corresponding site. Such are used to generate these partitions satisfying that, any point within a given region into sub-regions. A set of points known as sites, are used to generate these partitions satisfying that, any point within each partition is closest to its corresponding site. Such diagrams have an interesting property that the resulting shape of the partitions are strictly convex (internal angles are less than 180°). This is a widely used technique in automatic sectorization literature, since convex sector shapes are preferred than 180°

\[ v_i = \left( \frac{v_i^\lambda}{v_i^\phi} \right) \text{ for } i = 1 \ldots \eta \]  

(1)

Here, \( \eta \) is the number of sectors being designed. Superscript \( \lambda \) and \( \phi \) denote the longitude and latitude of the sites location respectively. The sector boundaries are determined through the voronoi decomposition of \( V \).

B. Motivation: Conflicting Objectives

For every given sector in the airspace, there are usually 2 air traffic controllers, a radar controller and a planning controller. The radar controller is in charge of transmitting all necessary information for a safe execution of flight while the planning controller coordinates transition information of inbound and outbound within the sector. Simplifying this idea, we could classify the air traffic controllers workload into monitoring (radar) and coordination (planning) workload.

Figure 1 illustrates a situation where reducing coordination workload increases monitoring workload. In Figure 1a, the plane re-enters the same sector, causing extra coordination workload for the planning controller. In order to reduce this coordination workload without changing the trajectory of the plane, the triangular sector in the middle has to be grouped into the same sector as illustrated in Figure 1b. In this situation (Figure 1b), the reduction of coordination workload actually lead to an increase in monitoring workload. Depending on how experienced the air traffic controllers are, the balance between monitoring and coordination workload required for each sector could be different. Therefore, in this paper, we aim to search for all the Pareto optimal solutions, displaying all the possible combination of workload for the user to choose from.

1) Objective Function: Monitoring workload: The monitoring workload in previous literature [5], [12], [13] usually account for monitoring workload only using number of traffic hit counts indicating the amount of traffic in the sector. However, not all flights carry the same workload. Flights that requires a change in altitude, speed or heading would require greater effort from the air traffic controllers than those just without such changes. Hence, in this paper, we propose the use of a modified dynamic density from [11], which accounts for these changes mentioned previously. Additionally, the number of crossing points are also included as part of the monitoring workload as it is a crucial aspect to be monitored by the radar controller.

The monitoring workload within a sector has been formulated as

\[ \text{Monitoring}_{\text{sector}} = (W1 * N + W2 * NH + W3 * NS + W4 * NA + W5 * CP) / (W1 + W2 + W3 + W4 + W5) \]  

(2)

Where \( N \) is the number of flights, \( NH \) is the number of heading changes, \( NS \) is the number of speed changes, \( NA \) the number of altitude changes and \( CP \) is the number of crossing points, within the sector in the given time window. The \( W1, W2, W3, W4 \) are the subjective weights as proposed in [11] and \( W5 \) is a subjective weight of 8 proposed by us.

Definitions from [11]:

\( \text{NH} \) = Number of aircraft with Heading Change greater than 15°,

\( \text{NS} \) = Number of aircraft with Speed Change greater than 10 knots or 0.02 Mach,

\( \text{NA} \) = Number of aircraft with Altitude Change greater than 750 feet,

and defined by us,

\( \text{CP} \) = Number of aircraft with lateral distance between 0-5 nautical miles and vertical separation less than 2000/1000 feet above/below 29000 ft,

where each of these parameters are measured during a sample interval of one minute.

In order to balance the monitoring workload among all sectors, the standard deviation of the monitoring workload is
minimized. The objective function for monitoring workload is formally defined as

\[ \text{Obj}_{\text{Monitoring}} = \min \{ \text{SD}(\text{Monitoring}) \} \] (3)

where \( \text{Monitoring} \) is the vector consisting of \( \text{Monitoring}_{\text{sector no.}} \) of all sectors, defined in Equation 2 previously.

2) Objective Function: Coordination workload: Coordination workload is the number of flights leaving a particular sector within the time window.

The coordination workload is directly affected by how the sectors are formed and shaped, hence the coordination workload is minimized here. The objective function for coordination workload is formally defined as

\[ \text{Obj}_{\text{Coordination}} = \min \{ \text{Coordination} \} \] (4)

where \( \text{Coordination} \) is the vector consisting of the number of flights leaving each sector for all sectors.

3) Constraint: Minimum distance of CP to boundaries: This measure is the minimum distance from the CP within the sector to the boundaries of the sector.

\[ \text{Distance}_{\text{min}} = \min \{ \text{CP} - \text{sector boundaries} \} \] (5)

In previous literature, it is usually formulated as an objective function to maximize this distance. However, we believe that this is a safety issue and should be satisfied as a constraint. In our case, we set the CP to be at least 10 nautical miles (NM) away from the boundaries,

\[ \text{Min}_D > 10 \text{NM} \] (6)

where \( \text{Min}_D \) is a vector consisting of \( \text{Distance}_{\text{min}} \) of all sectors, defined in Equation 5 above.

4) Constraint: Deviation from Average Monitoring Workload (AMW): Minimizing the standard deviation of the workload may not necessary ensure a balance of monitoring workload among sectors. Therefore, a constraint that limits the deviation from AMW workload could aid in balancing monitoring workload among sectors. The constraint is defined as,

\[ \text{Monitoring} - \text{Monitoring}_{\text{average}} < k \times \text{Monitoring}_{\text{average}} \] (7)

where \( \text{Monitoring} \) is the vector consisting of the monitoring workload of all sectors, \( \text{Monitoring}_{\text{average}} \) is the AMW of all sectors and \( k \) is a user-defined value that determines the amount of deviation allowed.

Model Formulation:

**Objectives:**
1. Minimize standard deviation of monitoring workload between sectors
2. Minimize total coordination workload across all sectors

**Constraints:**
1. Distance between crossing points and sector boundaries > 10 NM
2. Deviation from Average Monitoring Workload is kept within user-defined range

C. NSGA-II formulation

The Voronoi sites points are decision variables stored in the chromosome of NSGA-II. In 2D sectorization, there are a total of \( 2 \times \text{no of sectors} \) variables. In 3D sectorization, there would be extra variables depending on the number of vertical splits. The Voronoi site points are randomly generation during initialization. Using the generated site points, the required parameters are created for the calculation of objectives and constraints.

In each iteration, new population of chromosomes (solutions) are selected using binary tournament selection, modified with the use of crossover and mutation operators. The solutions from the old and new populations would then be ranked based on violation of constraints and then objective values. The best set of solutions would then be kept as the population for the next iteration using rank and crowding distance as measures. More details on NSGA-II can be found in [7].

III. EXPERIMENTAL EVALUATION BASED ON SINGAPORE REGIONAL AIRSPACE

The proposed MOO formulation of airspace sectorization is applied to an en-route airspace in the Singapore region. For this purpose, we consider all flight trajectories above 24,500 ft, which are shown in Figure 2. The airspace has many flights have trajectories that climb/descend to different altitude. Hence, it will be interesting to study the 2D and 3D sectorization on the airspace using the proposed formulation, which takes into account altitude changes in the monitoring workload. The data input into the model besides flight trajectories includes crossing points and altitude, heading and speed changes which are derived from the flight trajectories using the definitions previously mentioned in Section II-B1. First, we present the results for 2D sectorization in Section III-A, followed by 3D sectorization in Section III-B.

![Flight Trajectories](image-url)
A. Results for 2D Sectorization

For the study, number of sectors is fixed as 5 and number of generation in NSGA-II is kept as 2000. The simulation study has been conducted to understand the effect of variation in AMW constraint in sectorization. First, the simulation results without the AMW constraint is presented, followed by the results for with AMW constraint.

1) 2D Sectorization Results without AMW Constraint: In this case, multi-objective formulation consider both the objectives defined in Section II-B1 and II-B2 and only the safety constraint in Section II-B3.

Figure 3 illustrates 4 airspace sectorization which has the highest hypervolumes. The blue lines represents trajectories and the red dots represents crossing points. Figure 5 illustrates monitoring workload and coordination workload of each airspace sectorization (from left to right).

2) 2D Sectorization Results with AMW Constraint: In this case, multi-objective formulation consider both objectives defined in Section II-B1 and II-B2 and both constraints in Section II-B3 and II-B4, with the k value for AMW constraint to be 0.2.

Figure 4 illustrates top 4 results of airspace sectorization which has the highest hypervolumes. The blue lines represents trajectories and the red dots represents crossing points. Figure 6 illustrates monitoring workload and coordination workload of each airspace sectorization (from left to right).

3) Analysis: From Figure 3, it can be observed that the top 4 airspace sectorization are very diverse. On the other hand, with the use of the deviation from monitoring workload constraint, the shape of the sectors in the top 4 airspace sectorization becomes very similar, as shown in Figure 4.

In Airspace Sectorisation (c) ranked 3 of Figure 5, it is shown that the total coordination workload is only 10, while the monitoring workload is extremely unbalanced, with the highest workload of a sector being almost 18 and Sector 3 having no monitoring and coordination workload at all. This is also reflected in Figure 3 where sector 3 is empty without any flight trajectories. Thus, the effective number of sector in this solution is 4. On the other hand, in Airspace Sectorisation (d) ranked 4, a more balanced workload is shared among the sectors, ranging from 7 to 10. However, the coordination workload of the sectors increased to a total of 43. This in turn supports the theory that was proposed in Section II-B where monitoring and coordination workload are conflicting objectives.

With the implementation of another constraint to restrict the amount of deviation from the average workload, the monitoring workload is more balanced between the sectors while the coordination workload ranges between 7 to 10. With this constraint, the number of solutions that are optimal became a subset of the previous set of solutions on the Pareto-optimal front (without the constraint). This in turn leads to a faster convergence in finding the optimal solution that satisfies the constraint.

For practical uses, when the number of sectors are defined, the constraint is useful for a faster convergence to an optimal airspace sectorization. However, using this framework without such a constraint could aid in analyzing the airspace, such as finding an optimal number of sectors that could give the best balance between the monitoring and coordination workload.

B. Results for 3D Sectorisation

The airspace sectors in this section are split both laterally and vertically. However, the vertical split only happens after the lateral split to ensure the convexity, connectivity and right prism satisfaction of the sector shapes. In this section, similar to the Section III-A, 3D sectorization will be conducted with and without the deviation of monitoring workload constraint.

The number of sectors is set to 5, with a single vertical split. Hence there are a total of 4 areas created using Voronoi Diagrams, in which 1 area will be split vertically. The number of iterations is set to 2000 and the value of k for the monitoring workload deviation constraint is set to 0.1.

Figure 7 and Figure 9 displays the airspace sectorization and the monitoring and coordination workload for the 3D sectorization without the use of monitoring workload deviation constraint respectively. Figure 8 and Figure 10 displays the airspace sectorization and the monitoring and coordination workload for the 3D sectorization with the use of monitoring workload deviation constraint respectively.

Similar to the 2D sectors, the sectorizations formed without the constraint differs from one another as shown in Figure 7, while sectorizations formed with the monitoring workload deviation constraint are similar to one another as shown in Figure 8. By comparing the monitoring workload of the 3D airspace sectorization in Figure 9 and 10, the vast difference between the absence and presence of the monitoring workload deviation constraint is again exemplified.

As the number of areas and vertical splits are user-defined functions in this 3D sectorization model, it is unable to determine the optimal number of areas or vertical splits that is ideal for the airspace yet. One need to vary the number of sectors and vertical splits based on the required ATC availability.

C. 2D vs 3D Sectorization

Since the traffic data and the number of sectors are the same for the 2D and 3D sectorization, we may compare the results directly with each other under similar conditions. One method of comparison is through the average values of the two objectives of the whole population after 2000 iterations. Table 1 illustrates the average values of results in the case where both constraints are used and the value of k for both 2D and 3D sectorization is 0.2. From Table 1, it can be observed that the average values for both objectives, the standard deviation

<table>
<thead>
<tr>
<th></th>
<th>Obj 1 II-B1</th>
<th>Obj 2 II-B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>0.605587</td>
<td>54.72</td>
</tr>
<tr>
<td>3D</td>
<td>0.542563</td>
<td>43.5</td>
</tr>
</tbody>
</table>
of monitoring workload and the total coordination workload is lower for 3D sectorization when compared to 2D. This in turn allows us to conclude that 3D sectorization would indeed be a better choice in the given airspace.

The value of $k$ used for the monitoring workload deviation constraint for the 2D and 3D sectors (0.2 and 0.1 respectively) may be used as another comparison basis. This means that the workload of each sector is able to deviate $0.2 \times \text{average workload}/0.1 \times \text{average workload}$ from the average workload. The value of 0.2 was used for 2D sectors is because the value of 0.1 would restrict it from fulfilling the crossing point constraint (Section II-B3). From this, we can deduce that 3D sectors, that has vertical splits, are better suited for the given airspace. This is probably because the airspace is located on top of multiple airports and there are a significant number of climbing aircrafts above the 24500 ft, as illustrated in Figure 2.

**IV. CONCLUSION**

This paper proposed a 3D airspace sectorization formulation based on dynamic density workload metric. The optimization was cast as multi-objective instead of single objective, to get a range of solutions with a varying tradeoff among the objectives. This allows the ATC to use their subjective preferences to choose the best suited sectorization.

The proposed algorithm was applied on historical flight data in the Singapore region. The results support the use of vertical splits in the airspace allows for a better balance of ATC workload. Further, a workload deviation constraint, when imposed, led to a faster convergence in the results. One can improve the sector shape further by incorporating flow conformance or geometric constraints.

Although there are various proposed methods for 3D airspace sectorization, the current approach to airspace sectorization is to have permanent sectors and supply ATCs...
Fig. 4: Top 4 results based on Hypervolume indicator in 2D Airspace Sectorisation with 2 constraints

Fig. 5: Monitoring and Coordination Workload for Top 4 results in 2D Airspace Sectorisation with 1 constraint

Fig. 6: Monitoring and Coordination Workload for Top 4 results in 2D Airspace Sectorisation with 2 constraints
Fig. 7: Top 4 results based on Hypervolume indicator in 3D Airspace Sectorisation with 1 constraint

according to workload due to safety issues. However, we hope that our formulation would provide a realistic reflection of ATC workload and aid in improving efficiency in ATM.

ACKNOWLEDGMENT

The authors wish to extend their thanks to the ATMRI:2014-R8, Singapore, for providing financial support to conduct this study.

REFERENCES

Fig. 8: Top 4 results based on Hypervolume indicator in 3D Airspace Sectorisation with 2 constraint

Fig. 9: Monitoring and Coordination Workload for Top 4 results in 3D Airspace Sectorisation with 1 constraint

Fig. 10: Monitoring and Coordination Workload for Top 4 results in 3D Airspace Sectorisation with 2 constraint