Comparing Selected PSO Modifications on CEC 15 Benchmark Set

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Abstract — **In this paper we choose to compare three popular modifications of Particle Swarm Optimization (PSO) using the recent CEC´15 benchmark set. Three different approaches for PSO modification are compared: the heterogeneous swarm, diversity-guided swarm and orthogonal learning based approach. The goal is to provide up-to date comparison of the performance of these methods on a state-of-art benchmark set and highlight the differences in performance on different types of fitness functions. The experiments were carried out according to CEC benchmark rules and statistically evaluated.**

Keywords— Particle swarm optimization, PSO, ARPSO, HPSO, OLPSO

I. INTRODUCTION

Despite that the original method is now over 20 years old, the Particle Swarm Optimization (PSO) remains one of the most popular and widely used metaheuristic optimizers for various problems of global optimization. Since the introduction in 1995 [1, 2] the PSO is subject of continuous research [3, 4] and performance-improving efforts. In this study we choose to compare three modifications that attracted the interest of the research community (according to the number of citations in literature) and presented promising performance on previous benchmarks.

The orthogonal learning PSO (OLPSO) was proposed in 2011 by Zhan et. al. [5]. The method utilizes a learning approach based on constructing a "guidance vector" using the OED (Orthogonal Experimental Design) method. The OLPSO has shown very promising performance [5].

From the group of PSO algorithms based on heterogeneous swarm (HPSO) [6, 7] we have selected the self-adaptive heterogeneous PSO (fk-PSO) that has competed in the CEC`13 competition with good results.

Finally, the last included algorithm in the comparison is the diversity guided PSO (ARPSO) that has been proposed in 2002 by Riget and Vesterstrom [8]. This modification proposed a simple yet effective adaptive alteration of the basic movement rule of PSO. The typical convergence behavior of the swarm is altered in such way that the particles are repulsed from the best solution in order to avoid stopping in local optima. This simple modification provided surprising boost to the performance of the algorithm and has since then been adapted by other research e.g. [10].

The research questions for this work were following:

1. Is there significant difference in performance of the compared methods on the recent CEC`15 benchmark set?
2. Is there a solo best performing algorithm among the three?
3. What conclusions can be made based on the observed performance of the methods on the CEC`15 benchmark set?

The paper is structured as follows. The original PSO algorithm is briefly described in Section 2. The compared PSO variants are described in sections 3, 4 and 5. The experiment setup is detailed in section 6. Following Section 7 contains the statistical overviews of results and performance comparisons obtained during the evaluation of CEC`15 benchmark set [15]. Following is the discussion and conclusion.

II. PARTICLE SWARM OPTIMIZATION

In the PSO each particle in the population represents a possible solution of the optimization problem, which is defined by the cost function (CF). In each iteration of the algorithm, a new location of the particle is calculated based on its previous location and velocity vector (velocity vector contains particle velocity for each dimension component).

In the original design [1 - 3], two points of interest are taken into consideration during the calculation of new velocity of the particle: the personal best solution of each particle (*pBest*) and the best solution in the whole swarm (*gBest*). The differential vectors are multiplied by acceleration constants and random dimensional weights (random numbers from interval [0, 1] different for each dimension component). The new velocity is calculated according to (1)

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$$v_{ij}^{t+1} = v_{ij}^{t} + c_1 \cdot Rand(\ ) \cdot (pBest_{ij} - x_{ij}^{t}) + c_2 \cdot Rand(\ ) \cdot (gBest_j - x_{ij}^{t}) \quad (1)$$

where:
v_{ij}^{t+1} - New velocity of the i^{th} particle for j^{th} dimension in iteration t+1.
v_{ij}^{t} - Current velocity of the i^{th} particle for j^{th} dimension in iteration t.
c_1, c_2 - Acceleration factors (set to value 2.0).
$pBest_i$ – Local (personal) best solution found by the i^{th} particle.
gBest - Best solution found in a population.
x_{ij}^{t} - Current position of the i^{th} particle for j^{th} dimension in iteration t.
Rand () – Pseudo random number, interval [0, 1].
The new position of each particle is then given by (2), where x_{ij}^{t+1} is the new particle position for dimension j:

$$x_{ij}^{t+1} = x_{ij}^{t} + v_{ij}^{t+1} \quad (2)$$

This initial design suffered from several major issues e.g. uncontrolled acceleration of particles. To address these issues the inertia weight constant w was added to the equation (3).

$$v_{ij}^{t+1} = w \cdot v_{ij}^{t} + c_1 \cdot Rand(\ ) \cdot (pBest_{ij} - x_{ij}^{t}) + c_2 \cdot Rand(\ ) \cdot (gBest_j - x_{ij}^{t}) \quad (3)$$

Usually a linear decreasing inertia weight [2] is used. In this strategy the value of inertia weight typically linearly decreases from 0.9 to 0.4.

III. Orthogonal Learning PSO (OLPSO)

The orthogonal learning method consists of following steps:

- Creating an orthogonal matrix according to rules described in [5].
- Creating trial solutions by crossover of *pBest* and *gBest* according to data in the matrix.
- Evaluating trial vectors and storing the best as x_b.
- Evaluating the most beneficial dimensional components and creating trial vector x_p according to the results.
- Comparing the fitness of x_b and x_p. The winner is used as a guidance vector P in (4).

$$v_{ij}^{t+1} = w \cdot v_{ij}^{t} + c \cdot Rand() \cdot (P_{ij} - x_{ij}) \quad (4)$$

IV. Self-adaptive heterogeneous PSO (*fk*-PSO)

The fk-PSO [5] is the self-adaptive variant of heterogeneous swarm. In general, there are three heterogeneous models: static, dynamic and self-adaptive. In static approach the roles of particles are randomly selected (from a pool) at the start of the optimization and do not change until the end. In dynamic approaches the roles of the particles may change either stochastically or deterministically. In self-adaptive approaches the behavior is changed and selected based on its success rate. In the fk-PSO the behavior pool contains following six different models:

TVAC-PSO, in this model the value of c_1 and c_2 changes linearly. The c_1 is decreasing and c_2 increasing therefore the algorithm has good exploration and exploitation ability. The values of acceleration factors are subject to (5) and (6).

$$c_1^{t+1} = (c_{1,end} - c_{1,start}) \cdot \frac{t}{t_{max}} + c_{1,start} \quad (5)$$

$$c_2^{t+1} = (c_{2,end} - c_{2,start}) \cdot \frac{t}{t_{max}} + c_{2,start} \quad (6)$$

TVIW-PSO, time variant inertia weight with constant acceleration factors as described in section 2.

sPSO, social behavior favoring model (7).

$$v_{ij}^{t+1} = v_{ij}^{t} \cdot w + c_2 \cdot Rand(\ ) \cdot (gBest_j - x_{ij}^{t}) \quad (7)$$

cPSO, cognitive behavior favoring model (8).

$$v_{ij}^{t+1} = v_{ij}^{t} \cdot w + c_1 \cdot Rand(\ ) \cdot (pBest_{ij} - x_{ij}^{t}) \quad (8)$$

modBB-PSO, is able to escape from local minima as well as explore the *pBest* areas. The probability *ep* increases linearly from 0 to 1 therefore the algorithm is more likely to select *pBest* area for exploration in the end phase.

$$\begin{cases} pBest_{ij}, if\ U(0,1) < ep \\ N\left(\frac{pBest_{ij} + gBest_j}{2}, \left|pBest_{ij} - gBest_j\right|\right), otherwise \end{cases}$$

Where N : Normal (Gaussian) distribution,
U : Uniform distribution,

QSO uses a linear decrease of "cloud size" around the *gBest*. Therefore, the particles are forced to move in shrinking space in every dimension (9).

$$x_{ij}^{t+1} \sim N(gBest_j, \sigma) \quad (9)$$

Where σ is cloud radius

For more details on the fk-PSO please refer to [8].

V. Diversity guided PSO (ARPSO)

In the ARPSO the mechanics are similar to the PSO described in the section 2. However, when the diversity of swarm collapses under a given threshold a repulsive phase is activated. In the repulsive phase the velocity calculation is adjusted to (10). Therefore, the particles are repulsed from the *gBest*. Using this simple but effective approach it is possible to improve the performance of PSO algorithm and partially avoid the premature convergence.

$$v_{ij}^{t+1} = w \cdot v_{ij}^{t} + c_1 \cdot Rand(\ ) \cdot (pBest_{ij} - x_{ij}^{t}) - c_2 \cdot Rand(\ ) \cdot (gBest_j - x_{ij}^{t}) \quad (10)$$

For more details on the ARPSO please refer to [9].

VI. Experiment Setup

The experiments were carried out for dim = 10, 30 and 50. The maximal number of explicit cost function evaluations (FEs) was set to 10000 · dim. The population size was set to 40. The control parameters of compared algorithms were set according to authors proposals [5,8 and 9]. 51 repeated runs were performed for each function.

The CEC`15 benchmark set [11] consists of 15 functions divided into 4 categories: unimodal, multimodal, hybrid and composite. In the following section the unimodal functions are denoted with ^{u}, multimodal functions are denoted with ^{m}, hybrid functions are denoted with ^{h} and the composite functions are denoted with ^{c}.

VII. Results

In this section the results of the comparative study are presented. The notation is following:

- A1 – ARPSO
- A2 – fk –PSO
- A3 – OLPSO

The Wilcoxon Signed Rank pair test was used to identify statistically significant differences in results for all pairs of algorithms. With the level of significance α = 0.05. If the *p*-value in following tables is lower than 0.05 we accept the alternative hypothesis that the results of first algorithm are lower (better) than the results of second algorithm with statistical significance. The values showing statistically significant difference are given in bold numbers.

The corresponding *p*-values are presented in Table 1 – 3 for dim = 10, 30 and 50; Furthermore, examples of the mean *gBest* value history are depicted in Figs 1 – 7.

In the following Table 4 the solo best performances are compared. The algorithm is proclaimed solo best performing if its performance is better than other two methods with statistical significance according to Table 1 – 3. For example, in Table 1 for f_3 the A3 achieved significantly lower results than both A1 and A2 and is therefore the solo best performer.

Finally, the median values are presented in Table 5 – 7 alongside with the standard deviations. The results are discussed in the following section.

TABLE I. Signed Rank Test *p*-values, α=0.05, Dim: 10

f_x	A1<A2	A1<A3	A2<A1	A2<A3	A3<A1	A3<A2
f_1^u	9.75E-01	8.45E-01	**2.53E-02**	3.01E-01	1.57E-01	7.02E-01
f_2^u	8.89E-01	7.15E-01	1.12E-01	**6.39E-02**	2.89E-01	9.37E-01
f_3^m	1.00E+00	1.00E+00	**7.71E-09**	1.00E+00	**4.97E-09**	**4.97E-09**
f_4^m	5.91E-02	3.96E-06	9.42E-01	**3.83E-03**	1.00E+00	9.96E-01
f_5^m	**6.76E-03**	**1.42E-03**	9.93E-01	1.50E-01	9.99E-01	8.52E-01
f_6^h	2.67E-01	2.76E-01	7.37E-01	2.95E-01	7.27E-01	7.08E-01
f_7^h	7.02E-01	9.74E-01	3.01E-01	8.08E-01	**2.70E-02**	1.95E-01
f_8^h	8.20E-02	8.12E-01	9.19E-01	9.87E-01	1.90E-01	**1.30E-02**
f_9^c	6.13E-01	**1.19E-08**	3.91E-01	**3.37E-09**	1.00E+00	1.00E+00
f_{10}^c	1.73E-01	1.22E-01	8.29E-01	6.02E-01	8.80E-01	4.02E-01
f_{11}^c	6.44E-01	2.03E-01	3.59E-01	**2.07E-02**	7.99E-01	9.80E-01
f_{12}^c	**1.43E-02**	**1.15E-02**	9.86E-01	2.06E-01	9.89E-01	7.97E-01
f_{13}^c	1.00E+00	1.33E-01	**9.03E-05**	**2.23E-06**	8.69E-01	1.00E+00
f_{14}^c	**4.10E-02**	1.46E-01	9.60E-01	6.91E-01	8.56E-01	3.13E-01
f_{15}^c	1.00E+00	1.00E+00	**2.43E-10**	5.00E-01	**2.43E-10**	5.00E-01

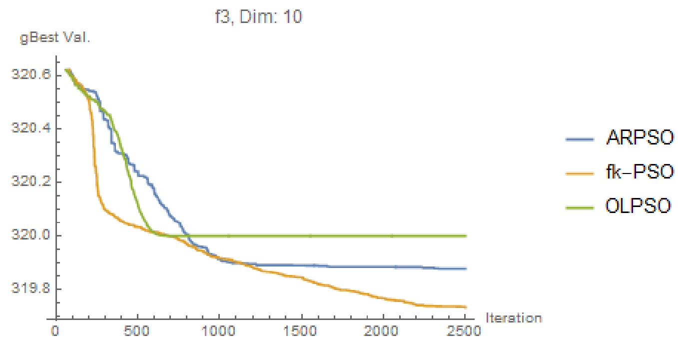

Fig 1. mean gBest history over 51 runs – comparison

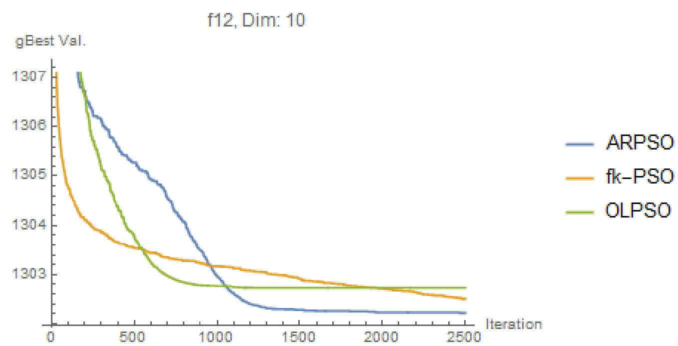

Fig 2. mean gBest history over 51 runs – comparison

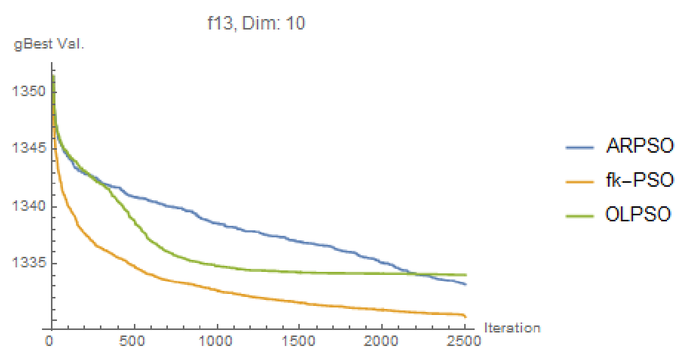

Fig 3. mean gBest history over 51 runs – comparison

TABLE II. SIGNED RANK TEST p-VALUES, α=0.05, DIM: 30

f_x	A1<A2	A1<A3	A2<A1	A2<A3	A3<A1	A3<A2
f_1^u	7.21E-01	**1.07E-02**	2.82E-01	**4.76E-04**	9.90E-01	1.00E+00
f_2^u	**2.53E-02**	7.99E-01	9.75E-01	9.99E-01	2.03E-01	**1.42E-03**
f_3^m	1.00E+00	1.00E+00	**2.65E-10**	1.00E+00	**2.65E-10**	**2.65E-10**
f_4^m	**4.23E-08**	**1.86E-03**	1.00E+00	1.00E+00	9.98E-01	**1.46E-04**
f_5^m	6.75E-01	7.63E-01	3.28E-01	7.97E-01	2.40E-01	2.06E-01
f_6^h	**1.68E-05**	1.80E-01	1.00E+00	7.08E-01	8.22E-01	2.95E-01
f_7^h	9.22E-01	9.85E-01	7.92E-02	8.12E-01	**1.54E-02**	1.90E-01
f_8^h	**2.88E-02**	1.00E+00	9.72E-01	1.00E+00	**4.21E-05**	**1.42E-05**
f_9^c	3.52E-01	**2.57E-03**	6.51E-01	**8.15E-07**	9.98E-01	1.00E+00
f_{10}^c	3.05E-01	**5.88E-06**	6.99E-01	**3.05E-06**	1.00E+00	1.00E+00
f_{11}^c	**1.30E-07**	9.11E-01	1.00E+00	1.00E+00	9.08E-02	**4.94E-08**
f_{12}^c	7.83E-01	**5.87E-07**	2.20E-01	**2.72E-07**	1.00E+00	1.00E+00
f_{13}^c	1.00E+00	**4.93E-04**	**3.26E-08**	**2.81E-10**	1.00E+00	1.00E+00
f_{14}^c	1.00E+00	8.10E-01	**1.43E-07**	**5.63E-06**	1.93E-01	1.00E+00
f_{15}^c	1.00E+00	1.00E+00	**2.63E-10**	5.00E-01	**2.63E-10**	5.00E-01

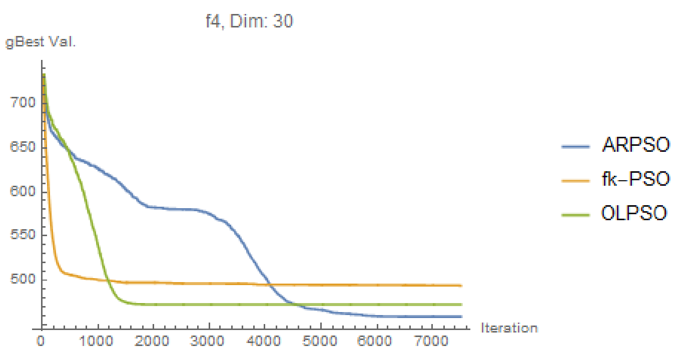

Fig 4. mean gBest history over 51 runs – comparison

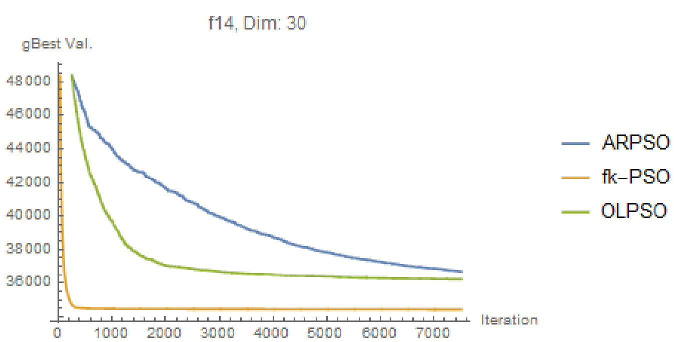

Fig 5. mean gBest history over 51 runs – comparison

TABLE III. SIGNED RANK TEST p-VALUES, α=0.05, DIM: 50

f_x	A1<A2	A1<A3	A2<A1	A2<A3	A3<A1	A3<A2
f_1^u	1.00E+00	9.08E-02	**1.24E-06**	**4.76E-04**	9.11E-01	1.00E+00
f_2^u	**2.42E-02**	6.41E-01	9.76E-01	9.96E-01	3.63E-01	**3.62E-03**
f_3^m	1.00E+00	1.00E+00	**2.65E-10**	1.00E+00	**2.65E-10**	**2.65E-10**
f_4^m	**3.36E-10**	4.61E-01	1.00E+00	1.00E+00	5.43E-01	**1.71E-09**
f_5^m	1.14E-01	1.00E+00	8.88E-01	1.00E+00	**6.68E-05**	**3.30E-07**
f_6^h	**9.20E-03**	3.98E-01	9.91E-01	8.07E-01	6.05E-01	1.96E-01
f_7^h	1.00E+00	9.82E-01	**1.84E-07**	**4.38E-05**	**1.81E-02**	1.00E+00
f_8^h	**3.23E-03**	1.88E-01	9.97E-01	3.84E-01	8.15E-01	6.20E-01
f_9^c	1.24E-01	**2.49E-04**	8.78E-01	**1.31E-05**	1.00E+00	1.00E+00
f_{10}^c	1.00E+00	**8.31E-04**	**1.93E-07**	**3.18E-09**	9.99E-01	1.00E+00
f_{11}^c	**2.51E-08**	9.99E-01	1.00E+00	1.00E+00	**7.55E-04**	**1.01E+08**
f_{12}^c	1.00E+00	**8.54E-07**	**1.73E-08**	**2.65E-10**	1.00E+00	1.00E+00
f_{13}^c	1.00E+00	9.01E-01	**3.17E-10**	**3.57E-10**	1.00E-01	1.00E+00
f_{14}^c	1.00E+00	3.63E-01	**3.17E-04**	**1.35E-04**	6.41E-01	1.00E+00
f_{15}^c	1.00E+00	1.00E+00	**3.88E-10**	5.00E-01	**7.27E-09**	9.77E-01

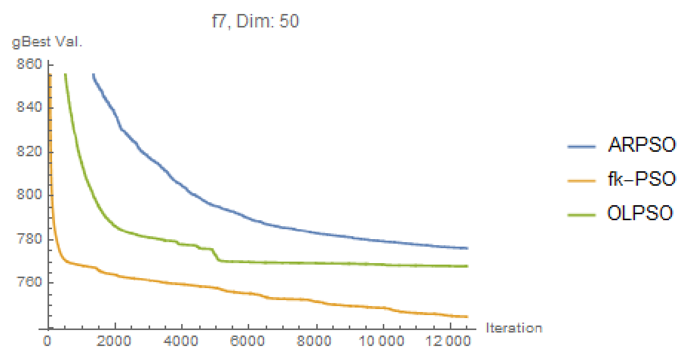

Fig 6. mean gBest history over 51 runs – comparison

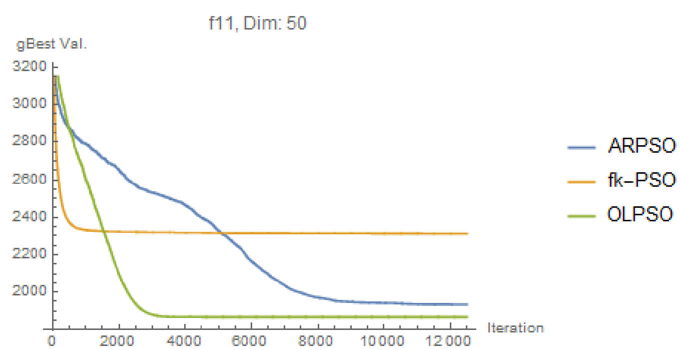

Fig 7. mean gBest history over 51 runs – comparison

TABLE IV. SOLO BEST PERFORMANCE COUNT - COMPARISON

Dimension	**ARPSO - A1**	**fk-PSO – A2**	**OLPSO – A3**
10	2	1	1
30	1	2	2
50	0	6	3

TABLE V. MEDIAN COMPARISON, DIM:10

	ARPSO		fk-PSO		OLPSO	
f_x	Median	Std. Dev.	Median	Std. Dev.	Median	Std. Dev.
f_1^u	6.85E+04	3.57E+05	5.62E+04	6.08E+04	3.25E+03	3.35E+05
f_2^u	4.66E+03	7.85E+03	2.95E+03	6.06E+03	3.67E+03	7.37E+03
f_3^m	3.20E+02	2.84E+00	3.20E+02	2.49E+00	3.203E+02	4.35E-04
f_4^m	4.08E+02	3.08E+00	4.09E+02	5.35E+00	4.11E+02	5.65E+00
f_5^m	6.89E+02	1.32E+02	7.57E+02	1.71E+02	7.59E+02	2.01E+02
f_6^h	1.46E+03	1.39E+03	1.40E+03	1.91E+03	1.51E+03	2.88E+03
f_7^h	7.01E+02	4.14E-01	7.01E+02	5.29E-01	7.01E+02	4.32E-01
f_8^h	1.36E+03	1.04E+03	1.54E+03	1.78E+03	1.26E+03	6.72E+02
f_9^c	1.00E+03	5.30E-02	1.00E+03	4.59E-02	1.00E+03	1.11E-01
f_{10}^c	1.59E+03	1.07E+03	1.68E+03	6.94E+02	1.70E+03	9.83E+02
f_{11}^c	1.40E+03	1.31E+02	1.40E+03	1.39E+02	1.40E+03	1.30E+02
f_{12}^c	1.30E+03	6.47E-01	1.30E+03	5.63E-01	1.30E+03	1.06E+00
f_{13}^c	1.33E+03	3.50E+00	1.33E+03	3.29E+00	1.33E+03	4.17E+00
f_{14}^c	4.40E+03	3.31E+03	4.38E+03	3.05E+03	4.40E+03	2.80E+03
f_{15}^c	1.60E+03	3.57E-06	1.60E+03	0.00E+00	1.60E+03	0.00E+00

TABLE VI. MEDIAN COMPARISON, DIM:30

	ARPSO		fk-PSO		OLPSO	
f_x	Median	Std. Dev.	Median	Std. Dev.	Median	Std. Dev.
f_1^u	1.07E+06	1.74E+06	1.11E+06	7.58E+05	2.10E+06	5.62E+06
f_2^u	1.70E+03	3.74E+03	3.67E+03	4.45E+03	1.42E+03	2.53E+03
f_3^m	3.21E+02	8.86E-02	3.20E+02	1.23E-01	3.20E+02	3.16E-05
f_4^m	4.59E+02	1.59E+01	4.93E+02	2.86E+01	4.72E+02	2.21E+01
f_5^m	2.95E+03	5.33E+02	2.94E+03	4.56E+02	2.98E+03	4.81E+02
f_6^h	7.52E+04	8.29E+04	1.79E+05	1.18E+05	3.67E+04	7.11E+05
f_7^h	7.08E+02	2.15E+00	7.07E+02	3.38E+00	7.06E+02	3.49E+00
f_8^h	5.58E+04	4.28E+04	6.77E+04	6.27E+04	2.44E+04	1.36E+05
f_9^c	1.00E+03	6.10E+01	1.00E+03	4.28E+01	1.00E+03	4.26E+01
f_{10}^c	1.13E+05	1.46E+05	9.96E+04	9.47E+04	3.39E+05	1.93E+06
f_{11}^c	1.64E+03	1.70E+02	1.93E+03	2.24E+02	1.54E+03	1.41E+02
f_{12}^c	1.31E+03	1.32E+00	1.31E+03	9.39E-01	1.31E+03	1.96E+00
f_{13}^c	1.42E+03	9.37E+00	1.41E+03	5.55E+00	1.42E+03	8.11E+00
f_{14}^c	3.69E+04	2.13E+03	3.44E+04	1.35E+03	3.57E+04	2.06E+03
f_{15}^c	1.60E+03	1.54E-05	1.60E+03	0.00E+00	1.60E+03	0.00E+00

TABLE VII. MEDIAN COMPARISON, DIM:50

	ARPSO		fk-PSO		OLPSO	
f_x	Median	Std. Dev.	Median	Std. Dev.	Median	Std. Dev.
f_1^u	6.86E+06	6.26E+06	3.68E+06	2.24E+06	6.65E+06	1.92E+07
f_2^u	2.71E+03	8.09E+03	5.34E+03	1.07E+04	3.23E+03	6.91E+03
f_3^m	3.21E+02	7.08E-02	3.21E+02	1.43E-01	3.20E+02	1.48E-05
f_4^m	5.38E+02	2.38E+01	6.34E+02	5.54E+01	5.39E+02	2.84E+01
f_5^m	5.28E+03	9.74E+02	5.75E+03	7.64E+02	4.64E+03	7.75E+02
f_6^h	5.43E+05	6.20E+05	8.98E+05	5.23E+05	1.74E+05	1.25E+06
f_7^h	7.81E+02	1.68E+01	7.47E+02	2.69E+01	7.75E+02	2.41E+01
f_8^h	2.63E+05	3.81E+05	4.57E+05	3.19E+05	1.17E+05	1.09E+06
f_9^c	1.01E+03	7.62E+01	1.01E+03	7.86E+01	1.01E+03	9.30E-01
f_{10}^c	2.95E+05	2.94E+05	1.16E+05	8.39E+04	4.70E+05	5.55E+05
f_{11}^c	1.95E+03	1.75E+02	2.33E+03	2.36E+02	1.87E+03	1.15E+02
f_{12}^c	1.31E+03	1.74E+00	1.31E+03	1.11E+00	1.32E+03	2.46E+00
f_{13}^c	1.52E+03	1.04E+01	1.49E+03	6.92E+00	1.52E+03	1.01E+01
f_{14}^c	7.91E+04	1.17E+04	6.06E+04	1.18E+04	7.80E+04	1.00E+04
f_{15}^c	1.60E+03	3.22E-05	1.60E+03	0.00E+00	1.60E+03	1.71E+00

VIII. RESULTS DISCUSSION

In this section we discuss the obtained results starting with a look to Table 1 (the case of dim = 10). According to the presented *p*-values, there are only two functions with all algorithms performing without significant difference (f_6 and f_{10}) therefore it is possible to answer the first research question with that the performance of compared PSO modifications is indeed significantly different on the newest Benchmark set.

However, when Table 4 is consulted it is clear that only for four functions a single algorithm outperformed other two with statistical significance. The ARPSO won on f_5 and f_{12}, the fk-PSO on f_{13} and the OLPSO on f_3. There is therefore not a single best performing algorithm among the three. The *gBest* history for f_3 is depicted in Fig. 3. It seems the performance of all three algorithms is similar but when median values (Table 5) are taken into consideration the OLPSO wins with significantly lower standard deviation of results. This can be understood as better reliability of the OLPSO in comparison to the other two algorithms on this type of multimodal fitness landscape.

Fig. 2 depicts the *gBest* history for f_{12}. The ARPSO seems to avoid premature convergence best and achieves best result despite converging slowest of the three. This is the behavior that should be expected from ARPSO and proves that the concept is feasible in such fitness landscapes as f_{12}.

In the last example for dim = 10 (Fig. 3) the f_{13} is investigated. The OLPSO seems to converge prematurely. The ARPSO dos not fail into local but the convergence speed is too slow in comparison with the fk-PSO. The fk-PSO is in average converging fastest and yet keeps improving during the whole FEs limit and achieves best overall result.

In the case of dim = 30 (Tables 2, 4 and 6) the ARPSO won f_4, the fk-PSO f_{13} and f_{14}. The OLPSO won f_3 and f_8. In the Fig. 4 it is depicted the mean *gBest* history for f4, the two phases of ARPSO are easily recognizable and show the impact on the performance that is improved by escaping the initial (premature) convergence. In Fig. 5 the superior convergence speed of fk-PSO is again highlighted.

Finally, in the case of dim = 50 (Tables 3,4,7) the ratio of won functions changes significantly. The ARPSO fails to score a single solo best performance. The fk-PSO managed to obtain six solo best results and the OLPSO has won three times. Examples of the convergence behavior are again depicted in Fig. 6 and 7.

It should be pointed out that the majority of solo best results of the fk-PSO (regardless the dimensionality) was obtained on composite functions that usually represent a very complex fitness landscape. On the other hand, the OLPSO seems to perform best on basic multimodal functions.

The performance of all three algorithms seems fairly comparable for dim = 10 and dim = 30. For dim = 50 the ARPSO is less competitive (often due to slow convergence speed not premature convergence). The fk-PSO seems to cope best with high dimensional problems.

IX. CONCLUSION

In this paper we presented results of a small-scale comparative study of three popular PSO modifications using the state of art CEC`15 benchmark set. Each represent a different basic principle (dynamic adaptation, heterogeneous swarm, learning). The results were presented and tested for statistical significance. Several trends were highlighted in figures. Based on the provided data the research questions can be addressed as follows:

1. There is statistically significant difference in the performance of the algorithms on several functions of the benchmark set. The number is highest for the highest dimension setting.
2. There is not a single best performing algorithm on the whole benchmark set.
3. The collected data support following conclusions:
    a. The fk-PSO often exhibits superior convergence speed and performs best on complex fitness landscapes in high dimension.
    b. The OL-PSO is able to solve some of the basic multimodal fitness landscapes with greater precision and reliability.
    c. The performance of ARPSO is comparable with the other two approaches in lower dimensions. In high dimension the ARPSO seems to converge too slow to obtain competitive result in the FEs limit.

The aim of this study was to show the differences in performance of well-known PSO modifications when confronted with the latest CEC benchmark set. The results of this paper will be further used in future studies to suggest possible improvements in those methods and to design new approaches for PSO based metaheuristics.

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