Interval Forecasting with Fuzzy Time Series

Petrônio C. L. Silva¹, Hossein Javedani Sadaei¹, Frederico Gadelha Guimarães²

Abstract—In recent years, the demand for developing low computational cost methods to deal with uncertainty in forecasting has been increased. Interval forecasting is a category of forecasting in which the method provides intervals as outputs of its forecasting. The initial aim of this paper is therefore proposing a new interval forecasting method based on a low cost and accurate forecasting method, namely first order Fuzzy Time Series. In this study, the results of the proposed method are compared with actual data and regular point forecasts using Fuzzy Time Series. The evaluation of the results shows the accuracy and promising performance of the proposed method.

Index Terms—Fuzzy Time Series, Uncertainty, Interval Forecasting

I. INTRODUCTION

In many practical forecasting problems, such as financial and environmental time series, one can find challenging behaviors and patterns inside the data which can turn its analysis and prediction into a very hard task. In addition to that, the volume of data is another significant challenge when accurate, reliable, and fast forecasting methods are sought for. This scenario motivated, in recent years, the development of new methods of forecasting that can deal with uncertainty by estimating distributions and low processing cost. One of these methods is Fuzzy Time Series [1], which has been drawing some attention and getting more relevance in recent years due to many studies reporting its good accuracy compared with other models [2]. Although these FTS methods have received some criticism from the literature, see for instance [3], due to methodological problems, many of these issues have been approached in more recent papers [4].

Nevertheless, all forecasting problems are subject to different types of uncertainties, either related to imprecision in the data due to measurement errors, sensor calibration and other unknown factors, or related to the forecasting activity itself, since not all characteristics of the underlying process that generated the data might be modeled perfectly, leading to errors in the forecast.

This fact led to the development of methods for Probabilistic Forecasting [5] and Interval Forecasting [6], to deal with forecasting uncertainty by estimating distributions of possible values instead of a unique point forecast. In wide sense, the Interval Forecasting can be viewed as a particular case of the Probabilistic Forecasting, if we consider an uniform distribution between the bounds of the interval forecast.

FTS models extend time series with the concept of fuzzy sets, identifying patterns and relationships between these fuzzy sets in the in-sample data. However, most papers on FTS typically produce a point forecast defuzzifying the forecast value. In the literature of FTS, the methods to produce interval forecasts are usually based on Type 2 Fuzzy Sets, as in [7] and [8]. The main drawback of these techniques is the high computational cost demanded by the Type 2 Fuzzy Sets.

The present study proposes a new approach for Interval forecasting by extending FTS point forecast. The proposed Interval Fuzzy Time Series (IFTS) is introduced to produce not only the point forecast based on the midpoints of the fuzzy sets but also an interval forecast based on the support of those fuzzy sets in the fuzzy logical relationship groups (FLRG) and using interval algebra. The proposed method is simple and computationally cheap. To explain the details, the paper is organized as follows: section II presents the related literature, section III introduces the proposed Interval Fuzzy Time Series model as an alternative method for forecasting under uncertainty, section IV provides some experiments to validate the proposed method and compare its performance with that of other methods, and finally, section V concludes the paper.

II. LITERATURE REVIEW

A. Fuzzy Time Series Models

Fuzzy Time Series (FTS) are non parametric models introduced by Song and Chissom [1] based on Fuzzy Set theory [9]. These methods are easy to implement and very flexible, affording ways to deal with numeric and non-numeric data. Some of FTS methods produce compact and human readable models of the time series behavior using fuzzy rules which can be used by business experts and researchers.

There are several categories of FTS methods, varying mainly by its order and time-variance. The order indicates how many time-delays (lags) are used in modeling the time series. Given the time series data \( F \), the First Order models use \( F(t-1) \) data to predict \( F(t) \) and the High Order models use \( F(t-1), F(t-2), \ldots, F(t-k) \) data to predict \( F(t) \). Time varying models require updates of the current model with time to produce new forecasts.

Song and Chissom [1] proposed the main steps of all FTS methods but its computation demands many matrix operations for each forecasting making the process computationally expensive. Chen [10] simplified Song and Chissom’s algorithm by creating the Fuzzy Logical Rule Groups (FLRG),

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making the forecasting process cheaper by avoiding the use of matrix manipulations. The FLRG represent the rule base of the model and are human readable and easy to interpret. Both methods are known as Conventional FTS models.

The initial step of training an FTS model is the partitioning of the Universe of Discourse $U$, that is, the data range covered by the training data that must be transformed into fuzzy sets. This is certainly one of the most crucial steps on fuzzy time series methods due to their influence on forecasting accuracy. The number of intervals, the length of these intervals and their midpoints are all parameters that should be chosen carefully or indeed optimized. The partitioning scheme initially proposed in Conventional FTS is just the division of the data range in $k$ equal length intervals, the same method adopted hereafter in this work just for simplicity. However, more accurate methods have been proposed in the literature, we refer to [11] and [12] for further details.

The generation of FLRG from the fuzzyfied data in FTS model has, at least, two drawbacks: the lost of rule’s recurrence and their chronological order. Thus at the forecasting process a very recurrent pattern of data has the same importance of a unique occurrence pattern. Moreover, newer and older patterns also have the same weight in the forecast.

To fix these drawbacks Yu [13] proposed the Weighted Fuzzy Time Series (WFTS) model by including weights on FLRG’s. These weights are monotonically increasing and have a smoothing effect, giving more importance to the most recent data on forecasting process.

The works of [14] and [15] present the Improved Weighted Fuzzy Time Series (IWFTS) model and change the way in which the weights are assigned to the RHS rules on Yu’s model. The main difference is that the weights are calculated by the recurrence of each rule, discarding the chronological order. The Exponentially Weighted Fuzzy Time Series (EWFTS) method, proposed by Sadaei [16] and [17], replaces the linear weight growth of WFTS model by an exponential growth.

There are hybrid FTS techniques such as Askari and Montazerin [18] that proposes a high-order multi-variable FTS algorithm based on fuzzy clustering, Sadaei et al. [4] combines statistical ARFIMA models with FTS for forecasting of long-memory time series.

All these models have some common drawbacks. First, in the forecasting step just one FLRG is choosen for computing the result, based on the maximum membership between the input value and all the FLRG’s. This causes the lost of “smoothing” effect of fuzzy methods, which demands mixing many sets according to their fuzzy memberships . Secondly, these models are point-based forecasters and give no uncertainty measures about their results.

**B. Interval Forecasting**

Interval Forecasts generalize point forecasts to represent and incorporate uncertainty [19], usually by estimating an inter quantile range.

Parametric approaches include Chatfield [6], Gardner and Everette [20]. These methods use strong statistical assumptions about data that can make it less useful where data is not conforming.

Some non parametric approaches were developed to bypass the normality assumptions, for instance in Koenker [21], Steinwart and Christmann [22] and Takeuchi [23]. However these methods usually involve loss functions optimizations or iterative approaches such as Markov Chain Monte Carlo or Expectation-Maximization.

Quantile and Interval Forecasts have been used in a variety of fields of forecasting including Wind Electrical Generation [24], Supermarket Sales [25] and Power Load [26], [27], [28].

Beyond the probabilistic uncertainty represented by these quantile intervals other kind of uncertainty needs to be represented, the fuzzy uncertainty. Intervals can be seen as fuzzy sets projections on real line and common arithmetic operations can be applied to them. These topological and algebraic properties of intervals are discussed in [29], which also discusses their relationships with fuzzy sets.

In the next section a new method of interval forecasting is proposed. This method represents the fuzzy uncertainty by extending the point-based FTS methods to incorporate the fuzzy bounds on prediction.

**III. THE INTERVAL FUZZY TIME SERIES - IRFTS**

Here a simple, fast and effective method is proposed to deal with uncertainty, combining the flexibility of the FTS models with the properties of Interval Forecasts without the need to resort to parametric methods or optimization techniques as in quantile estimation methods. The importance of this is, for example, when the method is used in a high sized data or on a fast streaming with concept drifts, which demands the model to be frequently updated.

The Interval Fuzzy Time Series Model (IRFTS) aims to produce a prediction interval based of fuzzy bounds but without probabilistic meaning. The method is described below in two separated procedures: the model building procedure and the forecasting procedure. The model training procedure is based on [1] and [10] and aims to construct the FLRG rule base:

**Model building procedure:**

1) Define the universe of discourse $U$ from data $D$ as $U = [D_{min} - D_1, D_{max} + D_2]$;

2) Partition the universe of discourse in $k$ even intervals $u_i$ of size $(D_{max} + D_2) - (D_{min} - D_1)$, where $D_1$ and $D_2$ are just numbers used to round $D_{max}$ and $D_{min}$ to next integer multiple of 10 ;

3) Define the fuzzy sets $A_i$ on the universe $U$. Each fuzzy set will be related to an interval $u_i$, will have a midpoint $m_{A_i}$ and will be associated with a triangular fuzzy membership function $\mu_{A_i}(x)$. The vector $\mu_{A_i} = [\mu_{A_i}(u_1), \ldots, \mu_{A_i}(u_k)]$ represents the membership values of fuzzy set $A_i$ with the midpoints of all the $u_i$ intervals;
4) Fuzzify historical data \( D \), generating a new dataset \( D_f \). Each datapoint \( d_i \in D \) will be replaced by the fuzzy set \( A_k \) which has maximum membership value \( \mu_{A_k}(d_i) \);

5) From \( D_f \) establish all the Fuzzy Logical Relationship - FLR between two following sets in the format \( A_j \rightarrow A_j \) where \( A_i \) is a fuzzified value in time \( t \) and \( A_j \) is the fuzzified value in time \( t + 1 \). After all the FLR’s are generated, eliminate the duplicate rules:

\[
\begin{align*}
A_i & \rightarrow A_j, A_k, A_l \\
A_j & \rightarrow A_j, A_l
\end{align*}
\]  

6) Generate the FLRG - Fuzzy Logic Relationship Groups by grouping the FLR’ by the left hand side (LHS) of each rule and the right hand side (RHS) sets of each rule will be grouped to form the right hand side of the FLRG. For example, the group of FLR’s on Equation 1 will generate the FLRG on Equation 2.

\[
\begin{align*}
A_i & \rightarrow A_j, A_k, A_l \\
A_j & \rightarrow A_j, A_l
\end{align*}
\]  

The FLRG make up the rule base of the model and are human readable and easy to interpret. An FLR has the form \( LHS \rightarrow RHS \) where \( LHS \) always has one fuzzy set, representing \( F(t-1) \) and the \( RHS \) has all the fuzzy sets that followed \( LHS \) in the FLR’s, representing all possible \( F(t) \) coming from \( F(t-1) \). The number and format of the FLRG rule base is closely related with the variance and stationarity of the dataset.

The forecasting procedure uses the generated model to build prediction intervals based on the mean interval of the \( RHS \) fuzzy sets on each FLRG weighted by their fuzzy membership in relation to input value:

**Forecasting procedure:**

1) For a given input value \( F(t) \), find the membership value \( \mu_i \) for all \( A_i \) fuzzy sets;

2) Choose all FLRGS which \( LHS \mu_i > 0 \);

3) Each chosen FLRG will generate a interval \( I^i = [\Pi_{min}^i, \Pi_{max}^i] \) where \( \Pi_{min}^i \) is the minimum lower bound of all \( RHS \) fuzzy sets of FLRG \( i \) and \( \Pi_{max}^i \) is the maximum upper bound of \( RHS \) fuzzy sets of FLRG \( i \);

\[
\begin{align*}
\Pi_{min}^i &= \min(A_1, \ldots, A_k) \\
\Pi_{max}^i &= \max(A_1, \ldots, A_k)
\end{align*}
\]  

4) The final forecast interval \( I_f \) is calculated as the sum of the FLRGs intervals weighted by the membership value of each FLRG, as showed in Equation 4

\[
I_f = \frac{\sum_{i \in A} \mu_i \Pi^i}{\sum_{i \in A} \mu_i} = \frac{\sum_{i \in A} (\mu_i \Pi_{min}^i, \mu_i \Pi_{max}^i)}{\sum_{i \in A} \mu_i}
\]  

The generated interval \( I_f \) is bounded by a composition of the fuzzy sets bounds on the FLRG’s which have some membership with the input value \( F(t) \) and is expected to contain the true value \( F(t+1) \). In next sections we present a demonstration of the method and propose a discussion about its main characteristics.

### A. Application example

For clarification of the method, let us take the University of Alabama Enrollments dataset, retrieved from [30]. Using a partitioning scheme with 6 equal-length intervals and triangular membership functions we get the fuzzy sets listed on table I, and the generated FLRG’s are listed in Equation 5.

<table>
<thead>
<tr>
<th>Fuzzy Set</th>
<th>Lower bound</th>
<th>Midpoint</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>10612</td>
<td>12008</td>
<td>13404</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>12008</td>
<td>13404</td>
<td>14800</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>13404</td>
<td>14800</td>
<td>16196</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>14800</td>
<td>16196</td>
<td>17592</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>16196</td>
<td>17592</td>
<td>18988</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>17592</td>
<td>18988</td>
<td>20384</td>
</tr>
</tbody>
</table>

TABLE I  
**Partition scheme for University of Alabama Enrollments dataset**

\[
\begin{align*}
A_1 & \rightarrow A_1 \\
A_2 & \rightarrow A_2, A_3 \\
A_3 & \rightarrow A_3, A_4 \\
A_4 & \rightarrow A_3, A_4, A_5 \\
A_5 & \rightarrow A_4, A_6 \\
A_6 & \rightarrow A_6
\end{align*}
\]  

Given an input value \( F(t-1) = 16,894 \), the membership values for fuzzy sets will be zero except for \( \mu_{A_1} = 0.5 \) and \( \mu_{A_5} = 0.5 \). Picking the FLRG’s \( A_1 \rightarrow A_3, A_4, A_5 \) we find the intervals \( I_1 \) and \( I_2 \) from first and second FLRGs. Following the values of the fuzzy sets the intervals will be \( I_1 = [13404, 158988] \) and \( I_2 = [14800, 20384] \). Now we can calculate the final interval \( I_f \) as showed as Equation 6 and the same process is illustrated in Figure 1. In figure 2 it is possible to check the interval forecast along the time series.

\[
I_f = [0.5 \times 13404 + 0.5 \times 158988, 0.5 \times 14800 + 0.5 \times 20384] = [14102, 19686]
\]  

### B. Method Discussion

This model has three major improvements from the models cited on section II-A: a) use of more than one FLRG on forecasting process; b) weighting the forecast output by the membership values of the input value; c) the interval forecast. The last characteristic put this model at a different category in time-invariant first-order fuzzy time series models. Other non-interval FTS methods can be compared with the \( |I|/FTS \) by comparing their point forecasts with the midpoint of the interval \( I_f \).

The prediction interval has a different meaning from intervals generated by parametric models and quantile based methods cited on section II-B. The interval produced by this model covers up the uncertainty of future values \( F(t+1) \) by their historical fuzzified values. First because the learned model, represented by the FLRG’s, is a fuzzy description of
past behaviors of data set assuming time invariance. Each FLRG represents all possibilities of change from the actual value $F(t)$ (the LHS fuzzy set) to future value $F(t+1)$ (one or more of the RHS fuzzy sets). Then, forecasting $F(t+1)$ for this model is to find the lower and upper bounds of the fuzzy uncertainty represented in the RHS fuzzy sets of all FLRG’s related with $F(t)$. The meaning of forecasted interval $I_f$ is the value range which contains the real value given the fuzzyness of data.

There is no need to know the population’s statistical parameters but the Universe of Discourse partitioning scheme affects the precision of the model by changing the length of forecast intervals. In Figure 3 and Table II, using same data set as the previous section, the effects of partitioning scheme on the intervals can be seen. It shows that the accuracy of the intervals is adjustable by changing the partitioning scheme of the model. This is indeed the unique required parameter for this method.

In next sections we compare the effectiveness of this proposed model with common FTS point forecasts methods and study the properties of the forecast intervals.

### IV. Benchmarks

Comparing point forecasts with interval forecasts is just possible if we consider the midpoint of the interval, which is the transformation of the interval forecast to a point forecast. This is clearly not a good way to evaluate interval forecasts but it makes possible to compare these two different approaches. Further ahead in this section we describe the properties of the proposed method results using interval metrics as coverage, calibration and sharpness.

To measure the performance of the proposed model, two well known financial time series data (the TAIEX and NASDAQ data sets) were selected, where a cross-validation method was applied for training and forecasting on test data. The results were then compared with the methods of Conventional FTS from Chen [10], Weighted FTS of Yu [13], Improved Weighted FTS of Ismail and Efendi [14] and Exponentially Weighted FTS of Sadaei [3] models, all of them trained with the same methods and data.

The Taiwan Stock Exchange Capitalization Weighted
Stock Index (TAIEX)\(^1\), a well known economic time series data commonly used in FTS literature (\([13, 31, 7, 32, 4]\), etc) was sampled on 2005 to 2013 time window, and has the averaged daily index by business day, totalizing 2000 instances where 1400 were used in training and the remaining 600 in validation. At Figure 4 the sample time series is shown, and the vertical line separates training and validation sets. The National Association of Securities Dealers Automated Quotations - Composite Index (NASDAQ IXIC)\(^2\) was sampled on 2001 to 2008 time window, and has the averaged daily index by business day, also totalizing 2000 instances where 1400 were used in training and the remaining 600 in validation. At Figure 5 the sample data is shown with its train and test sub samples.

The accuracy metrics used to evaluate models are the Mean Average Percent Error (MAPE), described in Equation 7, and Root Mean Squared Error (RMSE), described in Equation 8, where \(Y\) means the real data and \(\hat{Y}\) the forecasted values. The universe of discourse was partitioned in a grid scheme, where all partitions have the same length. Each model was trained and tested for 15, 20, 25, 30, 35 and 40 partitions and the results are shown in Table III.

\[
MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Y_i - \hat{Y}_i}{Y_i} \right| 
\tag{7}
\]

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{n}} 
\tag{8}
\]

The results make clear the strong impact that the number of partition intervals has on the accuracy of point forecasts. The proposed model has a bit lower performance when compared with the other models, but it is necessary to remind that this comparisons is made over the midpoint of the interval which is not the focus of the interval forecast. When comparing the results in Figures 6 and 8 with all test data, and Figures 7 and 9 with a smaller sample, the desired effect of the interval forecast becomes more visible. The produced intervals were capable of encapsulating the predictions of all other models and delimiting the boundaries of models uncertainty.

![Fig. 5. NASDAQ dataset sample used on benchmarks](image)

\begin{table}[h]
\centering
\caption{Model error comparisons}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Model & Partitions & TAIEX & & & NASDAQ & \\
\hline
 & RMSE & MAPE & RMSE & MAPE & & \\
\hline
Chen & 15 & 122.819 & 0.0136 & 36.024 & 0.0133 & \\
 & 20 & 121.830 & 0.0128 & 36.144 & 0.0125 & \\
 & 25 & 86.919 & 0.0091 & 40.993 & 0.0134 & \\
 & 30 & 81.442 & 0.0083 & 38.666 & 0.0127 & \\
 & 35 & 85.403 & 0.0088 & 46.687 & 0.0151 & \\
 & 40 & 62.913 & 0.0066 & 44.855 & 0.0152 & \\
\hline
Yu & 15 & 122.078 & 0.0135 & 37.625 & 0.0137 & \\
 & 20 & 93.133 & 0.0104 & 35.670 & 0.0124 & \\
 & 25 & 72.081 & 0.0079 & 38.877 & 0.0126 & \\
 & 30 & 57.412 & 0.0062 & 34.853 & 0.0118 & \\
 & 35 & 51.746 & 0.0065 & 42.749 & 0.0131 & \\
 & 40 & 40.009 & 0.0050 & 47.745 & 0.0151 & \\
\hline
Ismail & 15 & 123.844 & 0.0136 & 39.301 & 0.0144 & \\
 & 20 & 93.049 & 0.0104 & 35.085 & 0.0122 & \\
 & 25 & 72.036 & 0.0079 & 36.833 & 0.0124 & \\
 & 30 & 56.935 & 0.0061 & 39.078 & 0.0125 & \\
 & 35 & 57.412 & 0.0055 & 47.795 & 0.0146 & \\
 & 40 & 45.794 & 0.0050 & 49.429 & 0.0155 & \\
\hline
Efendi & 15 & 143.302 & 0.0146 & 42.840 & 0.0150 & \\
 & 20 & 110.091 & 0.0116 & 36.678 & 0.0124 & \\
 & 25 & 77.507 & 0.0080 & 35.252 & 0.0120 & \\
 & 30 & 70.400 & 0.0073 & 36.120 & 0.0120 & \\
 & 35 & 63.462 & 0.0065 & 45.432 & 0.0144 & \\
 & 40 & 52.064 & 0.0053 & 48.615 & 0.0156 & \\
\hline
Sadaei & 15 & 77.684 & 0.0072 & 7.722 & 0.0000 & \\
 & 20 & 99.734 & 0.0087 & 17.213 & 0.0035 & \\
 & 25 & 86.351 & 0.0081 & 29.061 & 0.0087 & \\
 & 30 & 93.559 & 0.0093 & 26.268 & 0.0071 & \\
 & 35 & 97.293 & 0.0097 & 36.609 & 0.0104 & \\
 & 40 & 84.125 & 0.0080 & 31.743 & 0.0107 & \\
\hline
\end{tabular}
\end{table}

Now three main properties for interval forecasts must be evaluated: coverage rate, calibration and sharpness, as proposed in [33] and [24]. The coverage refers to the statistical consistency between the forecasts and the observations, and measures which proportion of the observations are inside the interval. This can be done by an Indicator Function, developed by [34], as shown in Equation 9. Given a forecasting interval \(I = [\alpha, \beta]\), and the real value \(y\), the indicator function \(I\) verifies if \(y\) is covered by \(I\) or not.

\[
I(y, I) = \begin{cases} 
1 & \text{if } y \in I \\
0 & \text{if } y \not\in I 
\end{cases} 
\tag{9}
\]

The coverage rate is the average value of indicator function between forecasted intervals and the real values, in which the ideal value is 1. The coverage rate is shown at Equation 10 where \(y_i \in Y\) are the real values and \(I_i \in I\) are the predicted intervals for these values.

\[
C(Y, I) = \frac{\sum_{i=1}^{n} I(y_i, I_i)}{|Y|} 
\tag{10}
\]

The property of sharpness and resolution refers to the concentration of the predictive distribution, or how wide and variable are the intervals and refers uniquely to the forecasts. Sharpness, presented in Equation 11, is the average size of


the intervals and resolution, presented in the equation 12, is the variability of the intervals.

\[ \delta_T = \sum_{i=1}^{|I|} \delta_{I_i} = \frac{\sum_{i=1}^{|I|} \beta_i - \alpha_i}{|I|} \]  

(11)
\[
\sigma_I = \sum_{i=1}^{\lfloor I \rfloor} |\delta_{I_i} - \bar{\delta_I}| \tag{12}
\]

While small values of \(\delta_I\) are desirable, meaning a compact interval, wide values of \(\sigma_I\) are best, meaning the capability of the model to adapt the interval with the increase of uncertainty. There’s no absolute reference values for sharpness and resolution which depends on the statistical properties of the data.

At Table IV it is possible to compare the coverage rate, sharpness and resolution of the proposed model for each number of partitions and each data set. This table shows that in the worst case the real value is covered by the interval in 94\% of time even with thin intervals, by augmenting the number of partitions. The effects of the number of partitions in the length of intervals is also displayed at Figure 10 for TAIEX data set. It is important to observe that the intervals are not always evenly distributed around the real value, as effect to the weighting of the fuzzy membership of input value and the distribution of the intervals along the Universe of Discourse, but will be inside the interval almost all time.

<table>
<thead>
<tr>
<th>Partitions</th>
<th>TAIEX</th>
<th>NASDAQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1</td>
<td>1320.27</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>1333.61</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
<td>1078.45</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>954.16</td>
</tr>
<tr>
<td>35</td>
<td>1</td>
<td>827.28</td>
</tr>
<tr>
<td>40</td>
<td>1</td>
<td>787.29</td>
</tr>
</tbody>
</table>

**TABLE IV**

**COVERAGE, SHARPNESS AND RESOLUTION BY NUMBER OF PARTITIONS**

These results show that the proposed method has similar accuracy to the FTS methods with point forecasts and, in reference of its interval properties, has good reliability (or calibration) and adjustable sharpness according to the number of the partitions on the Universe of Discourse.

**V. CONCLUSION**

Point forecasting methods has as its main general drawback the inability to measure the uncertainty of their results. Depending on the field of application, knowing the uncertainty associated with forecast is indispensable. A way to introduce, and bound, the forecasting uncertainty is using the Interval Forecasting methods such as Quantile Regression and Estimation. In the other hand, Fuzzy Time Series methods have been gaining attention in recent years by their simplicity and accuracy.

In this work a new method for interval forecasting is proposed, using First Order Time Invariant Fuzzy Time Series. This method extends classical model of [10] to produce intervals and include two others innovations: new weighting method based of membership value and the use of multiple FLRG’s on the forecasting procedure.

The experiments with benchmarks, comparing some literature FTS models and the proposed method, shows its effectiveness, with accuracy values near the point forecasters, 94\% of coverage rate and adjustable sharpness.

**A. Method Limitations**

It has been observed on the benchmarks that this method has a low resolution, therefore the forecasted intervals have low variation independent of input. It is desirable that the
interval length has variations measuring the confidence of that interval due to input value.

B. Future works

For the next works some improvements are planned:

1) Extending the proposed interval forecast to a probabilistic forecast, producing a probability mass distribution as result;
2) Introducing a conditional probability distribution (or Bayesian stochastic model) for calculating the transitions of FLRG’s with time;
3) Weight the FLRG rules by their confidence of result based on input values, thus correcting the resolution issue.

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