Forgotten effects and heavy moving averages in exchange rate forecasting

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Abstract— **This paper presents the results of using experton, forgotten effects and heavy moving averages operators in three traditional models based purchasing power parity (PPP) model to forecast exchange rate. Therefore, the use of these methods is to improve the forecast error under scenarios of volatility and uncertainty, such as the financial markets and more precise in exchange rate. The heavy ordered weighted moving average weighted average (HOWMAWA) operator is introduced. This new operator includes the weighted average in the usual heavy ordered weighted moving average (HOWMA) operator, considering a degree of importance for each concept that includes the operator. The use of experton and forgotten effects methodology represents the information of the experts in the field and with that information were obtained hidden variables or second degree relations. The results show that the inclusion of the forgotten effects and heavy moving average operators improve our results and reduce the forecast error.**

Keywords—Experton, forgotten effects, HOWMAWA operator, econometric forecasting.

I. Introduction

One of the most important prices in open economies is the exchange rate, however the growing volatility requires decision makers to structure models to forecast exchange rate with an acceptable forecast error [1,2].

Models based on economic fundamentals such as price index, interest rates and balance of payments, have not produced satisfactory results, having similar behavior to a random walk [3,4] The main problem of these models is that they are a simplification of the reality [5,6]. So, even though there are theoretically accepted variables, there are others whose effect within the phenomenon are forgotten, so they must be identified and then used in the models to forecast.

The forgotten effects methodology introduced by Kaufmann & Gil-Aluja [8]) allows us to identify those relationships of second degree or hidden variables, where the causal relationship is forgotten and with the use of incidence matrix we can get the degree of forgetfulness [9].

In order to make the incidence matrices is necessary to include the knowledge of the experts in the financial markets, to do this we use the experton technique [7]. The experton is a function that used a cumulative distribution and using a decimal scale from 0 to 1 and linguistic expression we can unified the information of a group of experts.

Another technique used in this paper is the heavy ordered weighted moving average (HOWMA) operator that is an extension of the ordered weighted average (OWA) operator introduced by Yager [10]. The main advantage of this operator is that unifies the moving average with a heavy weighted vector [11]. This approach is further extended by unifying the weighted average and the HOWMA operator in the same formulation, considering the degree of importance that each concept has in the aggregation. This new operator is called the heavy ordered weighted moving average weighted average (HOWMAWA) operator. Some of its main properties and particular cases are studied.

An application of these two techniques where used in an econometric forecast problem regarding the exchange rate USD/MXN for 2015. With the use of information from 1994-2014 it was generated the econometric models, then using time series, HOWMA and HOWMAWA operators we forecast the future value of the variables included in the models. After that, we use the forgotten effects methodology to identify the hidden variables. Later they were incorporated in the econometric models, obtaining new results for the future value of the exchange rate.

The remainder of the paper is organized as follows. In Section 2 we review some theories of exchange rate determination and the techniques used in this paper. Section 3 presents the HOWMAWA operator. Section 4 presents the results of the using experton and forgotten effects methodology in exchange rate. Section 5 presents the comparison of using the different techniques in exchange rate USD/MXN 2015 forecast. Finally, Section 6 presents the main conclusions of the paper.

II. Preliminaries

In this section, we briefly review some basic concepts to be used in the paper. We analyze the theories of exchange rate determination, time series, expertons, forgotten effects methodology and heavy moving average operators.

2.1 Theories of exchange rate determination and times series

The theory used to forecast exchange rate are the purchasing power parity (PPP) that can be defined as follows.

Theory 1. PPP model postulates that variations in exchange rate in a given period must be equal to inflation differential. It is defined as follows [12]:

$$\widehat{e} = \pi_D - \pi_F, \quad (1)$$

where $\hat{e}$ is the change in exchange rate, π_D domestic inflation and π_F denotes foreign inflation.

In order to predict the future value of the independent variables in the econometric model, multiplicative decomposition time series method was used. It is formulated as follows [13]:

$$Y_t = T_t * S_t * C_t * I_t, \quad (2)$$

where Y_t is observed value, T_t trend, S_t seasonality, C_t cycle and I_t irregularity.

2.2 Experton

The theory of expertons [7,8] suggests that in order to obtain realistic data from a phenomenon that is not directly measurable, and aggregated set of valuation given by experts is useful. Kaufmann [7]) indicates that an experton is an extension of probabilistic set, where each $\alpha - cut$, is replaced by a range of probabilities collected by expert opinion [14].

Let assume *E* is a referential set, finite or not; *r* a certain number of experts that are asked to give their opinion about each element of E. This is represented as:

$$\forall a \in: \left[a_*^j(x), \alpha_j^*(x)\right] \subset [0,1],$$

where ⊂ is a set of inclusion and *j* the *j*th expert.

A static which concerns for each $x \in E$ is the lower and upper bound, for that a cumulative complementary law $F_*(a,x)$ is established for the $a_*^j(x)$ and $F^*(a,x)$ is established for the $\alpha_j^*(x)$. So that:

$$\forall x \in E, \forall x \in [0,1]: \tilde{A}(x) = [F_*(a,x), F^*(a,x)],$$

where ~ is the nature of the concept.

The referential set *E* is the following experton:

$$\forall x \in E, \forall \alpha \in E[0,1]: [F_*(a,x), F^*(a,x)] = 1.$$

The empty experton is then given by:

$$\forall x \in E: [F_*(a,x), F^*(a,x)] = \begin{cases} 1, \alpha = 0 \\ 0, \alpha \neq 0. \end{cases}$$

2.3 Forgotten effects methodology

Forgotten effects methodology introduced by Kaufmann & Gil-Aluja [8] is supported on the assumption of the existence of two sets:

$$A = \left\{\frac{a_i}{i} = 1,2, \dots, n\right\},$$

$$B = \left\{\frac{b_j}{j} = 1,2, \dots, m\right\}.$$

It is conjectured that prevails incidence of a_i on b_j if the value of the membership function characteristic of (a_i, b_j) is valued in the range [0,1], i.e.:

$$\forall(a_i, b_j) \Longrightarrow \mu(a_i, b_j) \in [0,1].$$

The set of pair of evaluated elements is known as direct impact matrix, which shows the cause-effect relationship in different degrees, caused by the corresponding assembly A (causes) and the set B (effects)

M=

	b_1	b_2	b_3	…	b_m
a_1	$\mu_{a_1b_1}$	$\mu_{a_1b_2}$	$\mu_{a_1b_3}$	…	$\mu_{a_1b_m}$
a_2	$\mu_{a_2b_1}$	$\mu_{a_2b_2}$	$\mu_{a_2b_3}$	…	$\mu_{a_2b_m}$
a_3	$\mu_{a_3b_1}$	$\mu_{a_3b_2}$	$\mu_{a_3b_3}$	…	$\mu_{a_3b_m}$
:	…	…	…	…	…
a_n	$\mu_{a_nb_1}$	$\mu_{a_nb_2}$	$\mu_{a_nb_3}$	…	$\mu_{a_nb_m}$

Now assume that there is a third set of elements, called *C*, expressed as follows:

$$C = \left\{\frac{C_k}{k} = 1,2, \dots, k\right\}.$$

This new set of elements represent effects of the set *B*, having an incidence matrix as follows:

N=

	c_1	c_2	c_3	…	c_m
b_1	$\mu_{b_1c_1}$	$\mu_{b_1c_2}$	$\mu_{b_1c_3}$	…	$\mu_{b_1c_m}$
b_2	$\mu_{b_2c_1}$	$\mu_{b_2c_2}$	$\mu_{b_2c_3}$	…	$\mu_{b_2c_m}$
b_3	$\mu_{b_3c_1}$	$\mu_{b_3c_2}$	$\mu_{b_3c_3}$	…	$\mu_{b_3c_m}$
:	…	…	…	…	…
b_n	$\mu_{b_nc_1}$	$\mu_{b_nc_2}$	$\mu_{b_nc_3}$	…	$\mu_{b_nc_m}$

Note that we have two incidence matrixes, that have as a common element the set *B*. So the relationship of the three sets can be expressed as:

$$M \subset A\text{x}B \text{ y } N \subset B\text{x}C.$$

With this information we will obtain the forgotten effects in *A* and *C*, using set *B* as base. To do this, the max-min operator is used (expressed by symbol ∘), generating a new incidence matrix as follows:

$$M \circ N = P,$$

$$P \subset A\text{x}C.$$

This new relationship is formulated as:

$$\forall(a_i, c_z \in A\text{x}C),$$

$$\mu(a_i, c_z)M \circ N = \forall_{bj}(\mu M(a_i, b_j) \wedge \mu N(b_j, c_z)).$$

The resulting incidence matrix of performing the operation max-min is:

		c_1	c_2	c_3	…	c_m
	a_1	$\mu_{a_1c_1}$	$\mu_{a_1c_2}$	$\mu_{a_1c_3}$	…	$\mu_{a_1c_m}$
	a_2	$\mu_{a_2c_1}$	$\mu_{a_2c_2}$	$\mu_{a_2c_3}$	…	$\mu_{a_2c_m}$
P=	a_3	$\mu_{a_3c_1}$	$\mu_{a_3c_2}$	$\mu_{a_3c_3}$	…	$\mu_{a_3c_m}$
	:	…	…	…	…	…
	a_n	$\mu_{a_nc_1}$	$\mu_{a_nc_2}$	$\mu_{a_nc_3}$	…	$\mu_{a_nc_m}$

The *P* matrix defines the casual relationships between elements of sets *A* and *C*, in the intensity or degree that leads to consider those belonging to *B*.

In order to assign values to the matrixes, Kaufmann & Gil-Aluja [8] propose a decimal scale constituted by 11 values from 0 to 1, $[0{,}0.1{,}0.2{,}0.3, \ldots ,1]$. This, point Luis & Gil-Lafuente [15], facilitates their adaptation and treatment since people are used to think and work in decimal form.

2.4 Heavy moving average operators

The use of heavy moving averages operators in forecasting can be useful because it combines the historical data through the moving averages and incorporate a weighted vector that goes from $-\infty$ to ∞. In this sense we can under or over estimate the results according to the expectations of the experts in the field. In this paper the heavy ordered weighted moving average is used. It can be defined as follows:

Definition 1. A HOWMA operator is defined as a given sequence $\{a_i\}_{i=1}^{N}$, where you get a new sequence $\{s_i\}_{i=1}^{N-n+1}$ which is multiplied by a heavy weighting vector, such that:

$$HOWMA(s_i) = \sum_{j=1+t}^{m+t} w_j b_j, \quad (2)$$

where b_j is the *j*th largest element of the collection $a_1, a_2, \ldots, a_n$, and W is an associated weighting vector of dimension m with $W: 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ and $w_i \in [0,1]$. Observe that here we can also expand the weighting vector from $-\infty$ to ∞. Thus, the weighting vector *w* becomes $-\infty \leq \sum_{i=1+t}^{m+t} w_i \leq \infty$.

If we focus in the reordering step, is possible to distinguish from the descending HOWMA (DHOWMA) operator and the ascending HOWMA (AHOWMA) operator. Also note that if the weighting vector is higher than one, is important to note that the information in some parts is independent.

Among the characteristics of the HOWMA operator we have that it is a monotonic and commutative function because:

a) It is monotonic because if $a_i \geq d_i$, for all *i*, then $HOWMA(a_1, \ldots, a_n) \geq HOWMA(d_1, \ldots, d_n)$.

b) It is commutative because any permutation of the arguments has the same evaluation.

Note that we can identify two types of boundaries in the HOWMA operator depending of the weighting vector. When W ranges from 1 to ∞, it is possible to distinguish from the minimum and the total operator but if W ranges from $-\infty$ to ∞ the HOWMA operator is not bounded.

The HOWMA operator follows the same parameter of beta value of the weighting vector that uses the HOWA operator [11]. This beta value normalizes the vector W and it is defined as $\beta(W) = (|W| - 1)/(n - 1)$. Since $|W| \in [1, n]$, then $\beta \in [0,1]$. That is why if $\beta = 1$, we get the total operator and if $\beta = 0$, we get the usual moving average.

III. HOWMAWA Operator

Another extension of the heavy moving averaging operators is presented, combining the HOWMA operator and the weighted averages, this in order to consider in the same formulation the degree of importance that each concept has in the aggregation. This new operator is called the heavy ordered weighted moving average weighted average (HOWMAWA) operator. In order to understand better this operator [16,17] initially the ordered weighted averaging-weighted average (OWAWA) operator [18] and the ordered weighted averaging weighted moving averaging (OWAWMA) operator are presented.

Definition 2. An OWAWA operator of dimension *n* is a mapping $OWAWA: R^n \rightarrow R$ that has an associated weighting vector *W* of dimension *n* such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$OWAWA(a_1, .., a_n) = \sum_{j=1}^{n} \hat{v}_j b_j, \quad (3)$$

where b_j is the *j*th largest of the a_i, each argument a_i has an associated weight (WA) v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, $\hat{v}_j \in [0,1]$, $\hat{v}_j = \beta w_j + (1 - \beta) v_j$ with $\beta \in [0,1]$ and v_j is the weight (WA) v_i ordered according to b_j, that is, according to the *j*th largest of the a_i.

Also, Merigó [18] indicated that if $\beta = 1$, we get the OWA operator and if $\beta = 0$, the WA. The OWAWA operator accomplishes similar properties than the usual aggregation operators. Like in the OWA operator, we can distinguish between the descending and ascending operator.

Definition 3. An OWAWMA operator of dimension *m* is a mapping $OWAWMA: R^m \rightarrow R$ that has an associated weighting vector W of dimension *m* with $W = \sum_{j=1+t}^{m+t} w_j = 1$ and $w_j \in [0,1]$, such that:

$$OWAWMA\ (a_{1+t}, \dots, a_{m+t}) = \sum_{j=1+t}^{m+t} \hat{v}_j\, b_j, \qquad (3)$$

where b_j is the *j*th largest argument of the a_i, each argument a_i has an associated weight (WA) v_i with $\sum_{i=1+t}^{m+t} v_i = 1$ and $v_i \in [0,1], \hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0,1]$, v_j is the weight v_i ordered according to b_j, that is, according to the *j*th largest of the a_i, m is the total number of arguments considered from the whole sample and *t* indicates the movement done in the average from the initial analysis. With the ideas presented below, it is possible to define the heavy ordered weighted moving average weighted average (HOWMAWA) operator as follows.

Definition 4. An HOWMAWA operator of dimension m is a mapping $HOWMAWA: R^m \to R$ that has an associated weighting vector W of dimension m with $W: 1 \le \sum_{i=1+t}^{m+t} w_i \le n$ and $w_j \in [0,1]$, such that:

$$HOWMAWA\ (a_{1+t}, \dots, a_{m+t}) = \sum_{j=1+t}^{m+t} \hat{v}_j\, b_j, \qquad (4)$$

where b_j is the *j*th largest argument of the a_i, each argument a_i has an associated weight (WA) v_i with $v_i : 1 \le \sum_{i=1+t}^{m+t} w_i \le n$ and $v_i \in [0,1], \hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0,1]$, v_j is the weight v_i ordered according to b_j, that is, according to the *j*th largest of the a_i, m is the total number of arguments considered from the whole sample and *t* indicates the movement done in the average from the initial analysis. Also note that the characteristics of the HOWMA operator are also applied to the HOWMAWA operator [19], such as:

a) The weighted vector can also expand from from $-\infty$ to ∞. Thus, the associated weight (WA) v_i becomes $-\infty \le \sum_{j=1}^{n} w_j \le \infty$.

b) It is possible to distinguish between the descending (DHOWMAWA) and ascending (AHOWMAWA) operator

c) This operator is monotonic and commutative. It will be bounded if the weight vector goes from 1 to ∞, but is not bounded if the weighting vector goes from $-\infty$ to ∞.

d) The beta value of the weighting vector applies the same way it does for the HOWMA operator, so if $\beta = 1$, we get the total operator and if $\beta = 0$, we get the usual moving average.

IV. Detecting exchange rate forgotten variables

In order to detect the hidden variables in exchange rate forecast, we integrate the opinions of five people who have knowledge and information of this topic. In this group were three people that are financial advisors in forex market and another two that are academics that has previous work in this field. With the information provide we used the experton technique and the results are presented in Tables 1-3.

Table 1. Effect-Effect.

	PCI_{EUA}	PCI_{MEX}	r_{EUA}	r_{MEX}	*BT*	*FDI*	*FPI*	*R*
π_{EUA}	1	0.4	0.6	0.3	0.3	0.3	0.3	0.3
π_{MEX}	0.4	1	0.6	0.6	0.4	0.4	0.4	0.4
r_{EUA}	0.8	0.3	1	0.6	0.2	0.4	0.5	0.3
r_{MEX}	0.4	0.7	0.7	1	0.4	0.5	0.5	0.4
BT	0.3	0.5	0.3	0.4	1	0.6	0.6	0.5
FDI	0.2	0.4	0.5	0.4	0.5	1	0.6	0.5
FPI	0.3	0.3	0.4	0.5	0.5	0.6	1	0.4
R	0.3	0.4	0.4	0.5	0.4	0.6	0.6	1

Table 2. Cause-Cause

	Growth expectations	Monetary politics	Country risk	Oil price	Global markets
Growth expectations	1	0.8	0.5	0.7	0.7
Monetary politics	0.8	1	0.7	0.6	0.6
Country risk	0.7	0.7	1	0.5	0.7
Oil price	0.7	0.5	0.4	1	0.8
Global markets	0.5	0.4	0.3	0.7	1

Table 3. Cause-Effect

	PCI_{EUA}	PCI_{MEX}	r_{EUA}	r_{MEX}	*BT*	*FDI*	*FPI*	*R*
Growth expectation	0.4	0.6	0.6	0.7	0.8	0.7	0.7	0.5
Monetary politics	0.7	0.8	0.7	0.8	0.7	0.6	0.6	0.5
Country risk	0.3	0.6	0.5	0.8	0.5	0.7	0.7	0.5
Oil price	0.5	0.3	0.3	0.3	0.6	0.4	0.5	0.5
Global markets	0.6	0.4	0.6	0.4	0.4	0.6	0.6	0.4

where PCI_{USA}= price consumer index in USA, PCI_{MEX}= price consumer index in Mexico, r_{USA}= interest rate in USA, r_{MEX}= interest rate in Mexico, *BT*= balance of trade in Mexico, *FDI* = foreign direct investment in Mexico, *FPI*= foreign portfolio investment in Mexico, *R*= international reserves in Mexico.

With the information from table 1-3, the forgotten effects methodology developed by Kauffman & Gil-Aluja (1988) was used in order to identify the forgotten variables. The most relevant results are present in Table 7.

Table 7. Forgotten effects in determining the forward exchange rate

Cause	**Effect**	**M**	$M \circ N$	**M*** = $M \circ N \circ B$	**Q = M*-M**
Growth expectations	PCI_{USA}	0.4	0.7	0.7	0.3
Country risk	PCI_{USA}	0.3	0.7	0.7	0.4
Oil price	PCI_{MEX}	0.3	0.6	0.6	0.3
Oil price	r_{EUA}	0.3	0.6	0.6	0.3
Oil price	r_{MEX}	0.3	0.7	0.7	0.4
Oil price	*FDI*	0.4	0.7	0.7	0.3

Observe that in general the variables with a higher sum of indirect effects are oil price and country risk (1.8 and 1.0 for each one). Analyzing the indirect effects of the variables, note that there are two with values of 0.4, these are: a) oil price in relation with interest rate and country risk with price consumer index in USA. The former has a second order effect via growth expectations, while the second through monetary policy.

Note. To develop these relationships, we use FuzzyLog Software. Free available in http://www.fuzzyeconomics.com/jaimegil.html

V. Forecast of Exchange Rate USD/MXN for 2015

In order to forecast exchange rate USD/MXN for 2015 we use information from 1994-2014 because was in the first date when Mexico implement floating exchange rate regime. We use the following econometric model:

PPP model

```
      Source |       SS       df       MS              Number of obs =     251
-------------+------------------------------           F(  4,   246) = 6001.70
       Model |  4.28244058     4  1.07061014           Prob > F      =  0.0000
    Residual |  .043882562   246  .000178384           R-squared     =  0.9899
-------------+------------------------------           Adj R-squared =  0.9897
       Total |  4.32632314   250  .017305293           Root MSE      =  .01336

------------------------------------------------------------------------------
         tcf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           v |   .0234641    .002765     8.49   0.000      .018018    .0289101
       tclag |   .9141334   .0220617    41.44   0.000     .8706795    .9575873
      pciUSA |  -.0194534   .0398029    -0.49   0.625    -.0978513    .0589445
      pciMEX |   .0407992   .0221272     1.84   0.066    -.0027837    .0843821
       _cons |   .1066104    .070307     1.52   0.131    -.0318702     .245091
------------------------------------------------------------------------------
```

PPP model with forgotten effects

```
      Source |       SS       df       MS              Number of obs =     251
-------------+------------------------------           F(  5,   245) = 4798.78
       Model |  4.28259392     5  .856518785           Prob > F      =  0.0000
    Residual |  .043729218   245  .000178487           R-squared     =  0.9899
-------------+------------------------------           Adj R-squared =  0.9897
       Total |  4.32632314   250  .017305293           Root MSE      =  .01336

------------------------------------------------------------------------------
         tcf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           v |   .0228157   .0028529     8.00   0.000     .0171964    .0284349
       tclag |   .9127896   .0221156    41.27   0.000     .8692286    .9563505
      pciUSA |   .0456639   .0807508     0.57   0.572    -.1133904    .2047182
      pciMEX |   .0353876   .0228906     1.55   0.123    -.0096999    .0804751
         mme |  -.0098227   .0105975    -0.93   0.355    -.0306965     .011051
       _cons |  -.0161185   .1499267    -0.11   0.914    -.3114281    .2791911
------------------------------------------------------------------------------
```

where tc_F is forward exchange rate, tc_{lag} is exchange rate with a lag, v is volatility, pci_{USA} is price consumer index in USA, pci_{MEX} is price consumer index in Mexico and *mme* is Mexican crude oil price. Note that all these variables are expressed in logarithm.

In order to forecast exchange rate based on the econometric equation, we used multiplicative decomposition time series method.

In the case of the HOWMA and HOWMAWA operators, a sequence of $n = 6$, a weighting vector $W = (0.05, 0.15, 0.15, 0.25, 0.40)$ and a weighting vector of the weighted average $V = (0.05, 0.05, 0.10, 0.20, 0.20, 0.40)$ were used based on the information obtained from the experts in the financial market. Also was considered a degree of importance of 70% for the OWA and 30% for the weighted average.

The results of using the different methods can be seen in Tables 4-6. (Note. FE means forgotten effects).

Table 4. USD/MXN forecast using time series

Time	Spot exchange rate USD/MXN	Time Series	Error	Time series and FE	Error
01-15	14.6808	15.6757	0.9949	15.8839	1.2031
02-15	14.9230	15.6272	0.7042	15.8420	0.9190
03-15	15.2136	15.2111	-0.0025	15.4178	0.2042
04-15	15.2208	15.2666	0.0458	15.4674	0.2466
05-15	15.2475	15.4326	0.1851	15.6321	0.3846
06-15	15.4692	15.7162	0.2470	15.9168	0.4476
07-15	15.9225	15.8889	-0.0336	16.0907	0.1682
08-15	16.5032	15.6731	-0.8301	15.8782	-0.6250
09-15	16.8519	15.6922	-1.1597	15.8910	-0.9609
10-15	16.5813	15.8567	-0.7246	16.0594	-0.5219
11-15	16.6325	15.7631	-0.8694	15.9705	-0.6620
12-15	17.0365	16.0779	-0.9586	16.3045	-0.7320
Average	15.8569	15.6568	-0.2001	15.8629	0.0060

Table 5. USD/MXN forecast using HOWMA operator

Time	Spot exchange rate USD/MXN	HOWMA	Error	HOWMA and FE	Error
01-15	14.6808	14.4197	-0.2611	14.5507	-0.1301
02-15	14.9230	14.4971	-0.4259	14.6505	-0.2725
03-15	15.2136	14.8056	-0.4080	14.9846	-0.2290
04-15	15.2208	15.1007	-0.1201	15.3050	0.0842
05-15	15.2475	15.4455	0.1980	15.6786	0.4311
06-15	15.4692	15.7565	0.2873	16.0171	0.5479
07-15	15.9225	16.0647	0.1422	16.3512	0.4287
08-15	16.5032	16.3817	-0.1215	16.6976	0.1944
09-15	16.8519	16.7113	-0.1406	17.0577	0.2058
10-15	16.5813	17.0465	0.4652	17.4243	0.8429
11-15	16.6325	17.3888	0.7563	17.7990	1.1666
12-15	17.0365	17.7368	0.7003	18.1806	1.1442
Average	15.8569	15.9462	0.0893	16.2248	0.3679

Table 6. USD/MXN forecast using HOWMAWA operator

Time	Spot exchange rate USD/MXN	HOWMA WA	Error	HOWMA WA and FE	Error
01-15	14.6808	14.2532	-0.4276	14.3696	-0.3112
02-15	14.9230	14.5154	-0.4076	14.6504	-0.2726
03-15	15.2136	14.7428	-0.4708	14.8963	-0.3173
04-15	15.2208	14.9880	-0.2328	15.1604	-0.0604
05-15	15.2475	15.1598	-0.0877	15.3517	0.1042
06-15	15.4692	15.4292	-0.0400	15.6407	0.1715
07-15	15.9225	15.6150	-0.3075	15.8422	-0.0803
08-15	16.5032	15.8376	-0.6656	16.0843	-0.4189
09-15	16.8519	16.0610	-0.7909	16.3276	-0.5243
10-15	16.5813	16.5040	-0.0773	16.7944	0.2131
11-15	16.6325	16.6033	-0.0291	16.9123	0.2798
12-15	17.0365	16.8365	-0.2000	17.1668	0.1303
Average	15.8569	15.5455	-0.3114	15.7664	-0.0905

Note that with the information above we can make decisions based not only in one result, but now we have a range of different scenarios to analyze and improve our vision of the future. It is important to distinguish that with the HOWMA and HOWMAWA operators is easier to generate new scenarios based on changes in the weighting vector and expectations of the future of the decision makers, something that is not possible using time series.

VI. CONCLUSIONS

This paper introduced new hidden variables to the traditional PPP model with the use of two techniques: experton and forgotten effects methodology. Therefore, these techniques use the information of a group of decision makers, that are experts in the field, and with the uses of matrixes we can obtained the second degree effects and with that information the forgotten effects.

Also a new extension of the OWA operator called heavy ordered weighted moving average weighted average (HOWMAWA) operator was introduced. This new operator considers a degree of importance for each concept that is used in the HOWMA operator. We have analyzed this new operator giving its definition, studying its properties and some interesting particular cases have been included.

This new operator and some others have been used in order to forecast exchange rate USD/MXN 2015. As we can note the model improve with the additional hidden variable detected through the forgotten effects methodology and HOWMA and HOWMAWA operators.

In future research, we expect to develop new extension of the OWA operator by considering intuitionistic fuzzy sets [20] or linguistic variables [21,22].

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