Abstract—This paper presents the results of using experton, forgotten effects and heavy moving averages operators in three traditional models based purchasing power parity (PPP) model to forecast exchange rate. Therefore, the use of these methods is to improve the forecast error under scenarios of volatility and uncertainty, such as the financial markets and more precise in exchange rate. The heavy ordered weighted moving average weighted average (HOWMAWA) operator is introduced. This new operator includes the weighted average in the usual heavy ordered weighted moving average (HOWMA) operator, considering a degree of importance for each concept that includes the operator. The use of experton and forgotten effects methodology represents the information of the experts in the field and with that information were obtained hidden variables or second degree relations. The results show that the inclusion of the forgotten effects and heavy moving average operators improve our results and reduce the forecast error.

Keywords—Experton, forgotten effects, HOWMAWA operator, econometric forecasting.

I. INTRODUCTION

One of the most important prices in open economies is the exchange rate, however the growing volatility requires decision makers to structure models to forecast exchange rate with an acceptable forecast error [1,2].

Models based on economic fundamentals such as price index, interest rates and balance of payments, have not produced satisfactory results, having similar behavior to a random walk [3,4] The main problem of these models is that they are a simplification of the reality [5,6]. So, even though there are theoretically accepted variables, there are others whose effect within the phenomenon are forgotten, so they must be identified and then used in the models to forecast.

The forgotten effects methodology introduced by Kaufmann & Gil-Aluja [8]) allows us to identify those relationships of second degree or hidden variables, where the causal relationship is forgotten and with the use of incidence matrix we can get the degree of forgetfulness [9].

In order to make the incidence matrices is necessary to include the knowledge of the experts in the financial markets, to do this we use the experton technique [7]. The experton is a function that used a cumulative distribution and using a decimal scale from 0 to 1 and linguistic expression we can unified the information of a group of experts.

Another technique used in this paper is the heavy ordered weighted moving average (HOWMA) operator that is an extension of the ordered weighted average (OWA) operator introduced by Yager [10]. The main advantage of this operator is that unifies the moving average with a heavy weighted vector [11]. This approach is further extended by unifying the weighted average and the HOWMA operator in the same formulation, considering the degree of importance that each concept has in the aggregation. This new operator is called the heavy ordered weighted moving average weighted average (HOWMAWA) operator. Some of its main properties and particular cases are studied.

An application of these two techniques where used in an econometric forecast problem regarding the exchange rate USD/MXN for 2015. With the use of information from 1994-2014 it was generated the econometric models, then using time series, HOWMA and HOWMAWA operators we forecast the future value of the variables included in the models. After that, we use the forgotten effects methodology to identify the hidden variables. Later they were incorporated in the econometric models, obtaining new results for the future value of the exchange rate.

The remainder of the paper is organized as follows. In Section 2 we review some theories of exchange rate determination and the techniques used in this paper. Section 3 presents the HOWMAWA operator. Section 4 presents the results of the using experton and forgotten effects methodology in exchange rate. Section 5 presents the comparison of using the different techniques in exchange rate USD/MXN 2015 forecast. Finally, Section 6 presents the main conclusions of the paper.

II. PRELIMINARIES

In this section, we briefly review some basic concepts to be used in the paper. We analyze the theories of exchange rate determination, time series, expertons, forgotten effects methodology and heavy moving average operators.
2.1 Theories of exchange rate determination and times series

The theory used to forecast exchange rate are the purchasing power parity (PPP) that can be defined as follows.

**Theory 1.** PPP model postulates that variations in exchange rate in a given period must be equal to inflation differential. It is defined as follows [12]:

\[ \hat{e} = \pi_D - \pi_F, \]  

where \( \hat{e} \) is the change in exchange rate, \( \pi_D \) domestic inflation and \( \pi_F \) denotes foreign inflation.

In order to predict the future value of the independent variables in the econometric model, multiplicative decomposition time series method was used. It is formulated as follows [13]:

\[ Y_t = T_t \ast S_t \ast C_t \ast I_t, \]  

where \( Y_t \) is observed value, \( T_t \) trend, \( S_t \) seasonality, \( C_t \) cycle and \( I_t \) irregularity.

2.2 Experton

The theory of expertons [7,8] suggests that in order to obtain realistic data from a phenomenon that is not directly measurable, and aggregated set of valuation given by experts is useful. Kaufmann [7]) indicates that an experton is an measurable, and aggregated set of valuation given by experts is realized from a phenomenon that is not directly measurable.

Let assume \( E \) is a referential set, finite or not; \( r \) a certain number of experts that are asked to give their opinion about each element of \( E \). This is represented as:

\[ \forall \alpha \in [a_i(x), a_j(x)] \subset [0,1], \]

where \( \subset \) is a set of inclusion and \( j \) the expert.

A static which concerns for each \( x \in E \) is the lower and upper bound, for that a cumulative complementary law \( F_i(a, x) \) is established for the \( a_i' (x) \) and \( F^*(a, x) \) is established for the \( a_j' (x) \). So that:

\[ \forall x \in E, \forall x \in [0,1]: \tilde{A}(x) = [F_i(a, x), F^*(a, x)], \]

where \( \sim \) is the nature of the concept.

The referential set \( E \) is the following experton:

\[ \forall x \in E, \forall x \in [0,1]: [F_i(a, x), F^*(a, x)] = 1. \]

The empty experton is then given by:

\[ \forall x \in E: [F_i(a, x), F^*(a, x)] = \{ 1, \alpha = 0 \} \cup [0, \alpha \neq 0]. \]

2.3 Forgotten effects methodology

Forgotten effects methodology introduced by Kaufmann & Gil-Aluja [8] is supported on the assumption of the existence of two sets:

\[ A = \left\{ \frac{a_i}{i} = 1, 2, ..., n \right\}, \]

\[ B = \left\{ \frac{b_j}{j} = 1, 2, ..., m \right\}. \]

It is conjectured that prevails incidence of \( a_i \) on \( b_j \) if the value of the membership function characteristic of \( (a_i, b_j) \) is valued in the range \([0,1]\), i.e.:

\[ \forall (a_i, b_j) \Rightarrow \mu(a_i, b_j) \in [0,1]. \]

The set of pair of evaluated elements is known as direct impact matrix, which shows the cause-effect relationship in different degrees, caused by the corresponding assembly \( A \) (causes) and the set \( B \) (effects)

<table>
<thead>
<tr>
<th>b1</th>
<th>b2</th>
<th>b3</th>
<th>...</th>
<th>bm</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>( \mu_{a1b1} )</td>
<td>( \mu_{a1b2} )</td>
<td>( \mu_{a1b3} )</td>
<td>...</td>
</tr>
<tr>
<td>a2</td>
<td>( \mu_{a2b1} )</td>
<td>( \mu_{a2b2} )</td>
<td>( \mu_{a2b3} )</td>
<td>...</td>
</tr>
<tr>
<td>a3</td>
<td>( \mu_{a3b1} )</td>
<td>( \mu_{a3b2} )</td>
<td>( \mu_{a3b3} )</td>
<td>...</td>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>an</td>
<td>( \mu_{anb1} )</td>
<td>( \mu_{anb2} )</td>
<td>( \mu_{anb3} )</td>
<td>...</td>
</tr>
</tbody>
</table>

Now assume that there is a third set of elements, called \( C \), expressed as follows:

\[ C = \left\{ \frac{c_k}{k} = 1, 2, ..., k \right\}. \]

This new set of elements represent effects of the set \( B \), having an incidence matrix as follows:

<table>
<thead>
<tr>
<th>c1</th>
<th>c2</th>
<th>c3</th>
<th>...</th>
<th>cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>( \mu_{b1c1} )</td>
<td>( \mu_{b1c2} )</td>
<td>( \mu_{b1c3} )</td>
<td>...</td>
</tr>
<tr>
<td>b2</td>
<td>( \mu_{b2c1} )</td>
<td>( \mu_{b2c2} )</td>
<td>( \mu_{b2c3} )</td>
<td>...</td>
</tr>
<tr>
<td>b3</td>
<td>( \mu_{b3c1} )</td>
<td>( \mu_{b3c2} )</td>
<td>( \mu_{b3c3} )</td>
<td>...</td>
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<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>bn</td>
<td>( \mu_{bnc1} )</td>
<td>( \mu_{bnc2} )</td>
<td>( \mu_{bnc3} )</td>
<td>...</td>
</tr>
</tbody>
</table>

Note that we have two incidence matrixes, that have as a common element the set \( B \). So the relationship of the three sets can be expressed as:

\[ M \subset AxB \cap N \subset BxC. \]

With this information we will obtain the forgotten effects in \( A \) and \( C \), using set \( B \) as base. To do this, the max-min operator is used (expressed by symbol \( \circ \)), generating a new incidence matrix as follows:

\[ M \circ N = P, \]

\[ P \subset AxC. \]
This new relationship is formulated as:

\[ \forall (a_i, c_z \in A \times C), \]
\[ \mu(a_i, c_z) M \circ N = \forall b_j (\mu M(a_i, b_j) \wedge \mu N(b_j, c_z)). \]

\[ \text{The resulting incidence matrix of performing the operation max-min is:} \]
\[
\begin{array}{cccccc}
  & c_1 & c_2 & c_3 & \ldots & c_m \\
 a_1 & \mu a_1 c_1 & \mu a_1 c_2 & \mu a_1 c_3 & \ldots & \mu a_1 c_m \\
a_2 & \mu a_2 c_1 & \mu a_2 c_2 & \mu a_2 c_3 & \ldots & \mu a_2 c_m \\
a_3 & \mu a_3 c_1 & \mu a_3 c_2 & \mu a_3 c_3 & \ldots & \mu a_3 c_m \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
a_n & \mu a_n c_1 & \mu a_n c_2 & \mu a_n c_3 & \ldots & \mu a_n c_m \\
\end{array}
\]

The \( P \) matrix defines the casual relationships between elements of sets \( A \) and \( C \), in the intensity or degree that leads to consider those belonging to \( B \).

In order to assign values to the matrices, Kaufmann & Gil-Aluja [8] propose a decimal scale constituted by 11 values from 0 to 1, \([0,0.1,0.2,0.3,\ldots,1]\). This, point Luis & Gil-Lafuente [15], facilitates their adaptation and treatment since people are used to think and work in decimal form.

2.4 Heavy moving average operators

The use of heavy moving averages operators in forecasting can be useful because it combines the historical data through the moving averages and incorporate a weighted vector that goes from \(-\infty\) to \(\infty\). In this sense we can understand or estimate the results according to the expectations of the experts in the field. In this paper the heavy ordered weighted moving average is used. It can be defined as follows:

**Definition 1.** A HOWMA operator is defined as a given sequence \( \{a_i\}_{i=1}^{N} \), where you get a new sequence \( \{s_i\}_{i=1}^{N+n+1} \) which is multiplied by a heavy weighting vector, such that:

\[ \text{HOWMA}(s_i) = \sum_{j=1}^{m+1} w_j b_j, \tag{2} \]

where \( b_j \) is the \( j \)th largest element of the collection \( a_1, a_2, \ldots, a_n \), and \( W \) is an associated weighting vector of dimension \( m \) with \( W: 1 \leq \sum_{i=1}^{m+1} w_i \leq n \) and \( w_i \in [0,1] \). Observe that here we can also expand the weighting vector from \(-\infty\) to \(\infty\). Thus, the weighting vector \( w \) becomes \(-\infty \leq \sum_{i=1}^{m+1} w_i \leq \infty \).

If we focus in the reordering step, is possible to distinguish from the descending HOWMA (DHOWMA) operator and the ascending HOWMA (AHOWMA) operator. Also note that if the weighting vector is higher than one, is important to note that the information in some parts is independent.

Among the characteristics of the HOWMA operator we have that it is a monotonic and commutative function because:

a) It is monotonic because if \( a_i \geq d_i \), for all \( i \), then \( \text{HOWMA}(a_1, \ldots, a_n) \geq \text{HOWMA}(d_1, \ldots, d_n) \).

b) It is commutative because any permutation of the arguments has the same evaluation.

Note that we can identify two types of boundaries in the HOWMA operator depending of the weighting vector. When \( W \) ranges from \( 1 \) to \( \infty \), it is possible to distinguish from the minimum and the total operator but if \( W \) ranges from \(-\infty \) to \(\infty \) the HOWMA operator is not bounded.

The HOWMA operator follows the same parameter of beta value of the weighting vector that uses the HOWA operator [11]. This beta value normalizes the vector \( W \) and it is defined as \( \beta(W) = (|W| - 1)/(n - 1) \). Since \( |W| \in [1, n] \), then \( \beta \in [0,1] \). That is why if \( \beta = 1 \), we get the total operator and if \( \beta = 0 \), we get the usual moving average.

III. HOWMAWA OPERATOR

Another extension of the heavy moving averaging operators is presented, combining the HOWMA operator and the weighted averages, this in order to consider in the same formulation the degree of importance that each concept has in the aggregation. This new operator is called the heavy ordered weighted moving average weighted average (HOWMAWA) operator. In order to understand better this operator [16,17] initially the ordered weighted averaging-weighted average (OWAWA) operator [18] and the ordered weighted averaging weighted moving average (OWMAWA) operator are presented.

**Definition 2.** An OWA operator of dimension \( n \) is a mapping \( \text{OWA}: R^n \rightarrow R \) that has an associated weighting vector \( W \) of dimension \( n \) such that \( w_j \in [0,1] \) and \( \sum_{j=1}^{n} w_j = 1 \), according to the following formula:

\[ \text{OWA}(a_1, \ldots, a_n) = \sum_{j=1}^{n} \tilde{v}_j b_j, \tag{3} \]

where \( b_j \) is the \( j \)th largest of the \( a_i \), each argument \( a_i \) has an associated weighted (WA) \( v_i \) with \( \sum_{i=1}^{n} v_i = 1 \) and \( v_i \in [0,1] \), \( \tilde{v}_j \in [0,1] \), \( \tilde{v}_j = \beta w_j + (1 - \beta) v_j \) with \( \beta \in [0,1] \) and \( v_j \) is the weight (WA) \( v_i \) ordered according to \( b_j \), that is, according to the \( j \)th largest of the \( a_i \).

Also, Merigó [18] indicated that if \( \beta = 1 \), we get the OWA operator and if \( \beta = 0 \), the WA. The OWA operator accomplishes similar properties than the usual aggregation operators. Like in the OWA operator, we can distinguish between the descending and ascending operator.

**Definition 3.** An OWAWA operator of dimension \( m \) is a mapping \( \text{OWA}: R^m \rightarrow R \) that has an associated weighting vector \( W \) of dimension \( m \) with \( W = \sum_{j=1}^{m+1} w_j = 1 \) and \( w_j \in [0,1] \), such that:
where $b_j$ is the $j$th largest argument of the $a_t$, each argument $a_t$ has an associated weight (WA) $v_i$ with $\sum_{i=1}^{m+t} v_i = 1$ and $v_i \in [0,1]$, $\tilde{v}_j = \beta w_j + (1 - \beta)v_j$ with $\beta \in [0,1]$, $v_j$ is the weight $v_i$ ordered according to $b_j$, that is, according to the $j$th largest of the $a_t$, $m$ is the total number of arguments considered from the whole sample and $t$ indicates the movement done in the average from the initial analysis. With the ideas presented below, it is possible to define the heavy ordered weighted average weighted average (HOWMAWA) operator as follows.

**Definition 4.** An HOWMAWA operator of dimension $m$ is a mapping HOWMAWA: $R^m \rightarrow R$ that has an associated weighting vector $W$ of dimension $m$ with $W: 1 \leq \sum_{i=1}^{m+t} w_i \leq n$ and $w_i \in [0,1]$, such that:

**HOWMAWA**($a_{1+t}, ..., a_{m+t}$) = \[ \sum_{j=1+1}^{m+t} \tilde{v}_j b_j, \] (3)

where $b_j$ is the $j$th largest argument of the $a_t$, each argument $a_t$ has an associated weight (WA) $v_i$ with $\sum_{i=1}^{m+t} v_i = 1$ and $v_i \in [0,1]$, $\tilde{v}_j = \beta w_j + (1 - \beta)v_j$ with $\beta \in [0,1]$, $v_j$ is the weight $v_i$ ordered according to $b_j$, that is, according to the $j$th largest of the $a_t$, $m$ is the total number of arguments considered from the whole sample and $t$ indicates the movement done in the average from the initial analysis. Also note that the characteristics of the HOWMA operator are also applied to the HOWMAWA operator [19], such as:

a) The weighted vector can also expand from from $-\infty$ to $\infty$. Thus, the associated weight (WA) $v_i$ becomes $-\infty \leq \sum_{j=1}^{m+t} w_j \leq \infty$.

b) It is possible to distinguish between the descending (DHOWMAWA) and ascending (AHOWMAWA) operator.

c) This operator is monotonic and commutative. It will be bounded if the weight vector goes from 1 to $\infty$, but is not bounded if the weighting vector goes from $-\infty$ to $\infty$.

d) The beta value of the weighting vector applies the same way it does for the HOWMA operator, so if $\beta = 1$, we get the total operator and if $\beta = 0$, we get the usual moving average.

**IV. DETECTING EXCHANGE RATE FORGOTTEN VARIABLES**

In order to detect the hidden variables in exchange rate forecast, we integrate the opinions of five people who have knowledge and information of this topic. In this group were three people that are financial advisors in forex market and another two that are academics that has previous work in this field. With the information provide we used the expert technique and the results are presented in Tables 1-3.

<table>
<thead>
<tr>
<th>Table 1. Effect-Effect.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCI\textsubscript{EUA}</td>
</tr>
<tr>
<td>\pi\textsubscript{EUA}</td>
</tr>
<tr>
<td>\pi\textsubscript{MEX}</td>
</tr>
<tr>
<td>r\textsubscript{EUA}</td>
</tr>
<tr>
<td>r\textsubscript{MEX}</td>
</tr>
<tr>
<td>BT</td>
</tr>
<tr>
<td>FDI</td>
</tr>
<tr>
<td>FPI</td>
</tr>
<tr>
<td>R</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Cause-Cause</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Growth expectations</th>
<th>Monetary politics</th>
<th>Country risk</th>
<th>Oil price</th>
<th>Global markets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth expectations</td>
<td>1</td>
<td>0.8</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>Monetary politics</td>
<td>0.8</td>
<td>1</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Country risk</td>
<td>0.7</td>
<td>0.7</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Oil price</td>
<td>0.7</td>
<td>0.5</td>
<td>0.4</td>
<td>1</td>
</tr>
<tr>
<td>Global markets</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3. Cause-Effect</th>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Growth expectation</th>
<th>Monetary politics</th>
<th>Country risk</th>
<th>Oil price</th>
<th>Global markets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth expectation</td>
<td>0.4</td>
<td>0.6</td>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>Monetary politics</td>
<td>0.7</td>
<td>0.8</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>Country risk</td>
<td>0.3</td>
<td>0.6</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Oil price</td>
<td>0.5</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Global markets</td>
<td>0.6</td>
<td>0.4</td>
<td>0.6</td>
<td>0.4</td>
</tr>
</tbody>
</table>

where $PCI\textsubscript{USA}$ = price consumer index in USA, $PCI\textsubscript{MEX}$ = price consumer index in Mexico, $r\textsubscript{USA}$ = interest rate in USA, $r\textsubscript{MEX}$ = interest rate in Mexico, $BT$ = balance of trade in Mexico, $FDI$ = foreign direct investment in Mexico, $FPI$ = foreign portfolio investment in Mexico, $R$ = international reserves in Mexico.

With the information from table 1-3, the forgotten effects methodology developed by Kauffman & Gil-Aluja (1988) was used in order to identify the forgotten variables. The most relevant results are present in Table 7.
Table 7. Forgotten effects in determining the forward exchange rate

<table>
<thead>
<tr>
<th>Cause</th>
<th>Effect</th>
<th>M</th>
<th>M o N</th>
<th>M* = M o N o B</th>
<th>Q = M*M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth expectations</td>
<td>PCI_{USA}</td>
<td>0.4</td>
<td>0.7</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>Country risk</td>
<td>PCI_{USA}</td>
<td>0.3</td>
<td>0.7</td>
<td>0.7</td>
<td>0.4</td>
</tr>
<tr>
<td>Oil price</td>
<td>PCI_{MEX}</td>
<td>0.3</td>
<td>0.6</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Oil price</td>
<td>r_{EUA}</td>
<td>0.3</td>
<td>0.6</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Oil price</td>
<td>r_{MEX}</td>
<td>0.3</td>
<td>0.7</td>
<td>0.7</td>
<td>0.4</td>
</tr>
<tr>
<td>Oil price</td>
<td>FDI</td>
<td>0.4</td>
<td>0.7</td>
<td>0.7</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Observe that in general the variables with a higher sum of indirect effects are oil price and country risk (1.8 and 1.0 for each one). Analyzing the indirect effects of the variables, note that there are two with values of 0.4, these are: a) oil price in relation with interest rate and country risk with price consumer index in USA. The former has a second order effect via growth expectations, while the second through monetary policy.

Note. To develop these relationships, we use FuzzyLog Software. Free available in http://www.fuzzyeconomics.com/jaimegil.html

V. FORECAST OF EXCHANGE RATE USD/MXN FOR 2015

In order to forecast exchange rate USD/MXN for 2015 we use information from 1994-2014 because was in the first date when Mexico implement floating exchange rate regime. We use the following econometric model:

PPP model

\[
\begin{align*}
\text{Total} & = 4.32632314 \\
\text{Residual} & = 0.043882562 \\
\end{align*}
\]

where \( tcf \) is forward exchange rate, \( tclag \) is exchange rate with a lag, \( v \) is volatility, \( pci_{USA} \) is price consumer index in USA, \( pci_{MEX} \) is price consumer index in Mexico and \( mme \) is Mexican crude oil price. Note that all these variables are expressed in logarithm.

In order to forecast exchange rate based on the econometric equation, we used multiplicative decomposition time series method.

In the case of the HOWMA and HOWMAWA operators, a sequence of \( n = 6 \), a weighting vector \( W = (0.05, 0.15, 0.15, 0.25, 0.40) \) and a weighting vector of the weighted average \( V = (0.05, 0.05, 0.10, 0.20, 0.20, 0.40) \) were used based on the information obtained from the experts in the financial market. Also was considered a degree of importance of 70% for the OWA and 30% for the weighted average.

The results of using the different methods can be seen in Tables 4-6. (Note. FE means forgotten effects).

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 251</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>4.28253932</td>
<td>5</td>
<td>0.856518785</td>
<td>F( 5, 245) = 4798.78</td>
</tr>
<tr>
<td>Residual</td>
<td>0.043729218</td>
<td>245</td>
<td>0.00178487</td>
<td>Prob &gt; F = 0.0000</td>
</tr>
<tr>
<td>Total</td>
<td>4.32632314</td>
<td>250</td>
<td>0.017305293</td>
<td>R-squared = 0.9897</td>
</tr>
</tbody>
</table>

| tcf | Coef. | Std. Err. | t | P>|t| | (95% Conf. Interval) |
|-----|-------|-----------|---|------|----------------------|
| v   | 0.02286157 | 0.00285299 | 8.00 | 0.000 | 0.0171964-0.0284349 |
| tclag | 0.0127896 | 0.0221556 | 5.44 | 0.000 | 0.008774-0.0168012 |
| pci_{USA} | 0.0456639 | 0.0807508 | 0.57 | 0.572 | -0.0113394-0.10262 |
| pci_{MEX} | 0.0353876 | 0.0228906 | 1.55 | 0.123 | -0.0096999-0.0804751 |
| mme | -0.0098227 | 0.0105975 | -0.93 | 0.355 | -0.0306965-0.011051 |
| _cons | -0.166185 | 0.1499267 | -0.11 | 0.914 | -0.3114281-0.2791911 |

Adj R-squared = 0.9897
Root MSE = 0.01336
Table 4. USD/MXN forecast using time series

<table>
<thead>
<tr>
<th>Time</th>
<th>Spot exchange rate USD/MXN</th>
<th>Time Series</th>
<th>Error</th>
<th>Time series and FE</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>01-15</td>
<td>14.6808</td>
<td>15.6757</td>
<td>0.9949</td>
<td>15.8839</td>
<td>1.2031</td>
</tr>
<tr>
<td>02-15</td>
<td>14.9230</td>
<td>15.6272</td>
<td>0.7042</td>
<td>15.8420</td>
<td>0.9190</td>
</tr>
<tr>
<td>03-15</td>
<td>15.2136</td>
<td>15.2111</td>
<td>-0.0025</td>
<td>15.4178</td>
<td>0.2042</td>
</tr>
<tr>
<td>04-15</td>
<td>15.2208</td>
<td>15.2666</td>
<td>0.0458</td>
<td>15.4674</td>
<td>0.2466</td>
</tr>
<tr>
<td>05-15</td>
<td>15.2475</td>
<td>15.4326</td>
<td>0.1851</td>
<td>15.6321</td>
<td>0.3846</td>
</tr>
<tr>
<td>06-15</td>
<td>15.4692</td>
<td>15.7162</td>
<td>0.2470</td>
<td>15.9168</td>
<td>0.4476</td>
</tr>
<tr>
<td>07-15</td>
<td>15.9225</td>
<td>15.8889</td>
<td>-0.0336</td>
<td>16.0907</td>
<td>0.1682</td>
</tr>
<tr>
<td>08-15</td>
<td>16.5032</td>
<td>15.6731</td>
<td>-0.8301</td>
<td>15.8782</td>
<td>-0.6250</td>
</tr>
<tr>
<td>09-15</td>
<td>16.8519</td>
<td>15.6922</td>
<td>-1.1597</td>
<td>15.8910</td>
<td>-0.9609</td>
</tr>
<tr>
<td>10-15</td>
<td>16.5813</td>
<td>15.8567</td>
<td>-0.7246</td>
<td>16.0594</td>
<td>-0.5219</td>
</tr>
<tr>
<td>11-15</td>
<td>16.6325</td>
<td>15.7631</td>
<td>-0.8694</td>
<td>15.9705</td>
<td>-0.6620</td>
</tr>
<tr>
<td>12-15</td>
<td>17.0365</td>
<td>16.0779</td>
<td>-0.9586</td>
<td>16.3045</td>
<td>-0.7320</td>
</tr>
</tbody>
</table>

Average 15.8569 15.6568 -0.2001 15.8629 0.0060

Table 5. USD/MXN forecast using HOWMA operator

<table>
<thead>
<tr>
<th>Time</th>
<th>Spot exchange rate USD/MXN</th>
<th>HOWMA</th>
<th>Error</th>
<th>HOWMA and FE</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>01-15</td>
<td>14.6808</td>
<td>14.4197</td>
<td>-0.2611</td>
<td>14.5507</td>
<td>-0.1301</td>
</tr>
<tr>
<td>02-15</td>
<td>14.9230</td>
<td>14.4971</td>
<td>-0.4259</td>
<td>14.6505</td>
<td>-0.2725</td>
</tr>
<tr>
<td>03-15</td>
<td>15.2136</td>
<td>14.8056</td>
<td>-0.4080</td>
<td>14.9846</td>
<td>-0.2290</td>
</tr>
<tr>
<td>04-15</td>
<td>15.2208</td>
<td>15.1007</td>
<td>-0.1201</td>
<td>15.3050</td>
<td>0.0842</td>
</tr>
<tr>
<td>05-15</td>
<td>15.2475</td>
<td>15.4455</td>
<td>0.1980</td>
<td>15.6786</td>
<td>0.4311</td>
</tr>
<tr>
<td>06-15</td>
<td>15.4692</td>
<td>15.7565</td>
<td>0.2873</td>
<td>16.0171</td>
<td>0.5479</td>
</tr>
<tr>
<td>07-15</td>
<td>15.9225</td>
<td>16.0467</td>
<td>0.1422</td>
<td>16.3512</td>
<td>0.4287</td>
</tr>
<tr>
<td>08-15</td>
<td>16.5032</td>
<td>16.3817</td>
<td>-0.1215</td>
<td>16.6976</td>
<td>0.1944</td>
</tr>
<tr>
<td>09-15</td>
<td>16.8519</td>
<td>16.7113</td>
<td>-0.1406</td>
<td>17.0577</td>
<td>0.2058</td>
</tr>
<tr>
<td>10-15</td>
<td>16.5813</td>
<td>17.0465</td>
<td>0.4652</td>
<td>17.4243</td>
<td>0.8429</td>
</tr>
<tr>
<td>11-15</td>
<td>16.6325</td>
<td>17.3888</td>
<td>0.7563</td>
<td>17.7990</td>
<td>1.1666</td>
</tr>
<tr>
<td>12-15</td>
<td>17.0365</td>
<td>17.7368</td>
<td>0.7003</td>
<td>18.1806</td>
<td>1.1442</td>
</tr>
</tbody>
</table>

Average 15.8569 15.9462 0.0893 16.2248 0.3679

Table 6. USD/MXN forecast using HOWMAWA operator

<table>
<thead>
<tr>
<th>Time</th>
<th>Spot exchange rate USD/MXN</th>
<th>HOWMA WA</th>
<th>Error</th>
<th>HOWMA WA and FE</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>01-15</td>
<td>14.6808</td>
<td>14.2532</td>
<td>-0.4276</td>
<td>14.3696</td>
<td>-0.3112</td>
</tr>
<tr>
<td>02-15</td>
<td>14.9230</td>
<td>14.5154</td>
<td>-0.4076</td>
<td>14.6504</td>
<td>-0.2726</td>
</tr>
<tr>
<td>03-15</td>
<td>15.2136</td>
<td>14.7428</td>
<td>-0.4708</td>
<td>14.8963</td>
<td>-0.3173</td>
</tr>
<tr>
<td>04-15</td>
<td>15.2208</td>
<td>14.9880</td>
<td>-0.2328</td>
<td>15.1604</td>
<td>-0.0604</td>
</tr>
<tr>
<td>05-15</td>
<td>15.2475</td>
<td>15.1598</td>
<td>-0.0877</td>
<td>15.3517</td>
<td>0.1042</td>
</tr>
<tr>
<td>06-15</td>
<td>15.4692</td>
<td>15.4292</td>
<td>-0.0400</td>
<td>15.6407</td>
<td>0.1715</td>
</tr>
<tr>
<td>07-15</td>
<td>15.9225</td>
<td>15.6150</td>
<td>-0.3075</td>
<td>15.8422</td>
<td>-0.0803</td>
</tr>
<tr>
<td>08-15</td>
<td>16.5032</td>
<td>15.8376</td>
<td>-0.6656</td>
<td>16.0843</td>
<td>-0.4189</td>
</tr>
<tr>
<td>09-15</td>
<td>16.8519</td>
<td>16.0610</td>
<td>-0.7909</td>
<td>16.3276</td>
<td>-0.5243</td>
</tr>
<tr>
<td>10-15</td>
<td>16.5813</td>
<td>16.5040</td>
<td>-0.0773</td>
<td>16.7944</td>
<td>0.2131</td>
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<tr>
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<td>16.6325</td>
<td>16.6033</td>
<td>-0.0291</td>
<td>16.9123</td>
<td>0.2798</td>
</tr>
<tr>
<td>12-15</td>
<td>17.0365</td>
<td>16.8365</td>
<td>-0.2000</td>
<td>17.1668</td>
<td>0.1303</td>
</tr>
</tbody>
</table>

Average 15.8569 15.5455 -0.3114 15.7664 -0.0905

Note that with the information above we can make decisions based not only in one result, but now we have a range of different scenarios to analyze and improve our vision of the future. It is important to distinguish that with the HOWMA and HOWMAWA operators is easier to generate new scenarios based on changes in the weighting vector and expectations of the future of the decision makers, something that is not possible using time series.

VI. CONCLUSIONS

This paper introduced new hidden variables to the traditional PPP model with the use of two techniques: experton and forgotten effects methodology. Therefore, these techniques use the information of a group of decision makers, that are experts in the field, and with the uses of matrices we can obtained the second degree effects and with that information the forgotten effects.

Also a new extension of the OWA operator called heavy ordered weighted moving average weighted average (HOWMAWA) operator was introduced. This new operator considers a degree of importance for each concept that is used in the HOWMA operator. We have analyzed this new operator giving its definition, studying its properties and some interesting particular cases have been included.

This new operator and some others have been used in order to forecast exchange rate USD/MXN 2015. As we can note the model improve with the additional hidden variable detected through the forgotten effects methodology and HOWMA and HOWMAWA operators.

In future research, we expect to develop new extension of the OWA operator by considering intuitionistic fuzzy sets [20] or linguistic variables [21,22].
REFERENCES


