An illustration of different concepts of solutions in semivectorial bilevel programming

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Abstract— **Bilevel programming deals with hierarchical decision processes with two decision levels, in which the upper level (leader) and the lower level (follower) decision makers control different sets of variables and pursue different objective functions. The problem is even more complicated when multiple objective functions are considered in the lower level (semivectorial bilevel problem) since a set of efficient solutions for each upper level decision exists. This paper presents and illustrates two novel types of solutions (*deceiving* and *rewarding*) in addition to the *optimistic* and *pessimistic* solutions to the semivectorial bilevel problem. These four solutions represent possible "extreme" outcomes of the decision process, capturing distinct leader's stances and follower's reactions.**

Keywords— semivectorial bilevel programming; optimistic solution; pessimistic solution; deceiving solution; rewarding solution

I. INTRODUCTION

Bilevel programming problems enable to model hierarchical decision processes with two decision levels. The *leader* (upper level decision maker) and the *follower* (lower level decision maker) control different sets of variables and pursue different objective functions in a non-cooperative manner subject to constraints involving both sets of variables. The leader makes his decision first by setting the values of his variables, but he must integrate into his optimization problem the reaction of the follower because it affects the leader´s objective value and even the feasibility of the solution. For a given leader's decision, the follower chooses an optimal candidate for his objective function within the feasible choices restricted by the leader.

A *semivectorial bilevel problem* (SVBP) is a bilevel problem with a single objective at the upper level and multiple objectives at the lower level. The existence of multiple objective functions at the lower level gives rise to a set of lower level efficient solutions for each leader's decision, which causes additional difficulties for the leader in anticipating the follower's reaction. Therefore, to provide decision aid to the leader in SVBP, different types of solutions should be computed that offer further information about possible outcomes and ranges of objective values resulting from different decisions.

This paper presents and illustrates using graphical examples four types of solutions: *optimistic*, *pessimistic*, *deceiving* and *rewarding*. These solutions represent possible "extreme" outcomes of the decision process based on SVBP, capturing distinct leader's stances and follower's reactions. This information offers the leader relevant insights about the ranges of possible values for his objective function hedging against the follower's decision resulting from trading-off his objective function values.

In section II the bilevel programming problem is presented. Semivectorial bilevel programming is introduced in section III discussing why the *optimistic* solution may not be representative of most practical decision situations and defining the *optimistic*, *pessimistic*, *deceiving* and *rewarding* solutions. These solutions are illustrated using graphical displays in section IV. Concluding remarks are presented in section V.

II. BILEVEL PROGRAMMING

A general bilevel programming problem with a single objective function at each decision level can be defined as follows:

$$\begin{aligned} &\underset{x}{\text{"max"}} \quad F(x,y) \\ &s.t. \quad G(x,y) \le 0 \\ &\qquad\quad y \in \arg\max_{y} \left\{ f(x,y) : g(x,y) \le 0 \right\} \end{aligned}$$

where $x \in \Re^{n_1}$ is the vector of the upper level decision variables and $y \in \Re^{n_2}$ is the vector of the lower level decision variables; $F(x, y)$ and $f(x, y)$ represent the leader's and the follower's objective functions, respectively.

The follower's rational reaction set to a given x' is:

$$\Psi(x') = \left\{ y' \in \Re^{n_2} : y' \in \arg\max_{y} \left\{ f(x',y) : g(x',y) \le 0 \right\} \right\}.$$

The feasible region for the bilevel problem (BP) is called the *induced region* (*IR*):

$$IR = \left\{ (x,y) \in \Re^{n_1+n_2} : G(x,y) \le 0,\ y \in \Psi(x) \right\}.$$

The bilevel problem is the problem seen by the leader. Quotation marks in $\text{"max"}\, F(x,y)$ express the undecided definition of the objective function value $F(x,y)$ from the leader's perspective (since he has control only over x) if the set of optimal solutions to the lower level problem is not singleton [1]. Even in the case where the lower level problem is a scalar optimization problem, more than one possible response of the follower may exist resulting from alternative optimal solutions to the follower's objective function. This poses a problem to the leader because the follower's choice may affect significantly the leader's decision. Most of the work on bilevel programming circumvents this difficulty by supposing that there is a single optimal solution to the lower level problem or adopting an optimistic approach. The optimistic approach presumes that the follower's response is always the one most convenient for the leader. Under this assumption, the upper level optimization is executed with respect to x and y, which means that the leader can influence the decision of the follower [1]. But, if the leader is risk-averse and wishes to limit the harm resulting from an undesirable option of the follower, a pessimistic approach may be considered. In this case the leader hedges against the worst case. Therefore, he chooses values for his variables that perform 'best' in view of the 'worst' follower's response for the leader [2]. The pessimistic bilevel problem is even more difficult to solve than the optimistic one.

III. Semivectorial Bilevel Programming

A general semivectorial bilevel problem (SVBP) with m objective functions at the lower level can be formulated as follows:

$$\begin{aligned} &\text{"}\max_{x}\text{"} \quad F(x,y) \\ &s.t. \quad G(x,y) \le 0 \\ &\qquad y \in \arg\max_{y} \left\{ \left(f_1(x,y),\ldots,f_m(x,y)\right) : g(x,y) \le 0 \right\} \end{aligned}$$

Without loss of generality, the SVBP has been defined with maximizing upper level and lower level objective functions. Below we present examples with maximizing and minimizing functions.

Since multiple objective functions are considered at the lower level, a set of efficient solutions for each leader's decision exists. This creates further difficulties for the leader regarding the anticipation of the follower's reaction within his efficient solutions.

Only *efficient* (Pareto optimal or nondominated) solutions to the lower level problem for each x-vector are feasible to the SVBP. Let $Y(x) = \left\{ y \in \Re^{n_2} : g(x,y) \le 0 \right\}$. For a given x', a solution $y' \in Y(x')$ is *efficient to the lower level problem* if and only if there is no other $y \in Y(x')$ that dominates y', i.e. such that $f_j(x',y) \ge f_j(x',y')$ for all $j=1,\ldots,m$, and $f_j(x',y) > f_j(x',y')$ for at least one j.

Therefore, the set of efficient solutions to the lower level problem of the SVBP for a given x' can be defined as: $\Psi_{\mathrm{Ef}}(x') = \{ y' \in Y(x')$: there is no $y \in Y(x')$ such that $f(x',y) \succ f(x',y') \}$, where $\succ$ denotes the dominance relation.

The *induced region* of the SVBP is

$$IR = \left\{ (x,y) \in \Re^{n_1+n_2} : G(x,y) \le 0,\ y \in \Psi_{\mathrm{Ef}}(x) \right\}$$

The SVBP was firstly addressed by Bonnel [3] who developed first-order necessary optimality conditions for the solution of an optimistic formulation. The optimistic formulation of a SVBP assumes that the solution to the lower level problem is the efficient solution with highest value according to the upper level objective function $F(x,y)$. Bonnel and Morgan [4] proposed an approach based on a penalty method dealing with weakly efficient solutions to the lower level problem.

Ankhili and Mansouri [5] developed an exact penalty method for the SVBP with a linear multiobjective lower level problem. Following the work in [5], Zheng and Wan [6] proposed a new penalty function method with two penalty parameters for the same problem. Calvete and Galé [7] also focused on bilevel problems with a linear multiobjective lower level problem (with all constraints linear and the upper level objective function being quasiconcave). The problem is reformulated as an optimization model with a nonconvex feasible region given by the union of faces of the polyhedron defined by all constraints, so that an extreme point of the polyhedron solves the problem. An enumerative exact algorithm and a genetic-based algorithm are proposed.

More recently, and still considering the optimistic formulation of a SVBP (like all the studies mentioned above), Dempe et al. [8] derived necessary optimality conditions for the problem using the classical scalarization technique to convert the lower level multiobjective problem into a parameterized single objective program. Following a related approach but considering the pessimistic formulation, Liu et al. [9] developed necessary optimality conditions for the pessimistic SVBP by transforming the problem into a single-level generalized minimax optimization problem with constraints.

As can be seen in the previous references, theoretical and algorithmic contributions made thus far to solve the SVBP have generally adopted an optimistic approach. The optimistic approach disregards the follower's preferences, assuming that the leader is able to freely select the more convenient solution among the set of efficient solutions to the follower. However, this may not be a reasonable assumption because it barely occurs in most practical decision-making problems with multiple objective functions. Alves et al. [10] pointed out and discussed the issues associated with assuming an optimistic approach in actual decision situations. These authors also introduced a new solution concept called *deceiving* solution (the worst outcome of a failed optimistic approach) and developed an algorithm based on particle swarm optimization to approximate the *optimistic*, *pessimistic*, and *deceiving* solutions to general SVBP.

To provide decision aid to the leader in SVBP, different types of solutions could be computed to give further information about possible outcomes and ranges of objective values resulting from different decisions.

The *optimistic* solution indicates the leader his maximum objective value when the follower's decision for each x setting is the best for the leader. Associated with the optimistic approach the *deceiving* solution can also be defined, which is obtained if the leader makes an optimistic decision and the follower's reaction is against the interests of the leader, i.e., a solution resulting from a failed optimistic approach. The *deceiving* solution can be worse (and is never better) than the *pessimistic* solution.

On the other hand, the *pessimistic* solution is the one that gives the maximum objective value for the leader when the follower's decision for each x setting is the worst for the leader. In this paper we further introduce a new type of solution in SVBP. It represents the best outcome of a pessimistic approach, which we call the *rewarding* solution. This solution is obtained whenever the leader takes a pessimistic approach and the follower's reaction is the most favorable to the leader.

Regarding the formulation above of the SVBP (where the upper level objective function is to be maximized), the four different types of solutions can be defined as:

- the *optimistic* solution, $O = (x^o, y^o)$, is given by

$$\max_{x,y} \left\{F(x,y) : y \in \Psi_{\mathrm{Ef}}(x), G(x,y) \leq 0\right\}$$

- given the optimistic upper level decision x^o, the *deceiving* solution is $D = (x^d, y^d) = (x^o, y^d)$ where y^d is given by $\min_y \left\{F(x^o, y) : y \in \Psi_{\mathrm{Ef}}(x^o)\right\}$.

- the *pessimistic* solution, $P = (x^p, y^p)$, is given by

$$\max_x \left\{\min_y \left\{F(x,y) : y \in \Psi_{\mathrm{Ef}}(x)\right\} : G(x,y) \leq 0\right\}$$

- given the pessimistic upper level decision x^p, the *rewarding* solution is $R = (x^r, y^r) = (x^p, y^r)$ where y^r is given by $\max_y \left\{F(x^p, y) : y \in \Psi_{\mathrm{Ef}}(x^p)\right\}$.

These four solutions represent "extreme" outcomes that can provide the leader important insights about the ranges of possible values for his objective function. This information may be particularly interesting if the leader has poor information about the tradeoffs the follower is willing to make concerning his multiple objective functions after knowing the decision made by the leader.

IV. Illustration of the Optimistic, Pessimistic, Deceiving and Rewarding Solutions in SVBP

In this section we present three examples each one illustrating these four "extreme" solutions.

A. Example 1

Consider the following SVBP in which all objective functions are to be minimized. This problem is an adaptation of Problem 3 in [11] and was presented in [10].

$$\begin{aligned} \min \quad & F(x,y) = (y_1 - 1)^2 + y_2^2 + x^2 \\ s.t. \quad & \min_y f_1(x,y) = y_1^2 + y_2^2 \\ & \min_y f_2(x,y) = (y_1 - x)^2 + y_2^2 \\ & s.t. \quad -1 \leq y_1, y_2, x \leq 2 \end{aligned}$$

where x is the only upper level variable and $y = (y_1, y_2)$ is the vector of the lower level variables.

For a given value of x, the efficient solutions to the lower level problem are given as follows: $\left\{(y_1, y_2) \in \Re^2 \middle| y_1 \in [0, x], y_2 = 0\right\}$, for $0 \leq x \leq 2$ and $\left\{(y_1, y_2) \in \Re^2 \middle| y_1 \in [x, 0], y_2 = 0\right\}$, for $-1 \leq x \leq 0$.

$\Psi_{\mathrm{Ef}}(x)$ denotes the set of efficient solutions to the lower level problem for a given x and let Ψ_{Ef} denote the union of $\Psi_{\mathrm{Ef}}(x)$ for all x. $\Psi_{\mathrm{Ef}} \equiv IR$ (induced region) is represented by the shaded area in Fig. 1, which also shows the level curves of $F(x,y)$ – circles centered at the point $(y_1, x) = (1,0)$. In Fig. 1 only y_1 and x are represented because y_2 is 0 in all efficient solutions to the lower level problem.

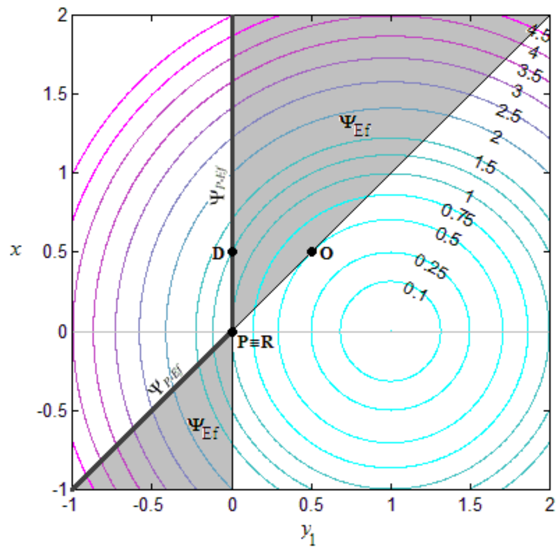
For the optimistic approach, the solution to the SVBP is the one that optimizes the leader's objective function over the set of efficient solutions to the follower, i.e., Ψ_{Ef}. The *optimistic* solution is the point O in Fig. 1, where $(x, y_1, y_2) = (0.5, 0.5, 0)$, $(f_1, f_2) = (0.25, 0)$ and $F = 0.5$.

For the pessimistic approach, the solution to the SVBP is the one that optimizes the leader's objective function within the subset of the follower's efficient solutions that are "worst for the leader", which is denoted by $\Psi_{P\text{-}Ef}$ in Fig. 1 and depicted by a thick line (left boundary of Ψ_{Ef}). The solution minimizing F over $\Psi_{P\text{-}Ef}$, i.e. the *pessimistic* solution, is point P in Fig. 1, where $(x, y_1, y_2) = (0, 0, 0)$, $(f_1, f_2) = (0, 0)$ and $F = 1$. Whenever the leader cannot anticipate the follower's reaction, the leader's pessimistic decision is the most conservative decision, whilst the optimistic decision could offer a better outcome but has a higher risk.

Another solution that can be interesting to analyze is the worst outcome of a "failed optimistic" approach, i.e. the *deceiving* solution. This solution indicates the maximum risk the leader incurs if he adopts an optimistic approach. The *deceiving* solution to this problem is the point D with $(x, y_1, y_2) = (0.5, 0, 0)$, $(f_1, f_2) = (0, 0.25)$ and $F = 1.25$.

On the other hand, if the leader takes a pessimistic approach (i.e. he chooses x=0), the only efficient solution to the follower is solution P. Therefore, in this example the leader has no chance of obtaining a better value of F if a pessimistic approach is adopted, which means that the *rewarding* solution coincides with the *pessimistic* solution. Table 1 summarizes the F values obtained in these four solutions for Example 1.

Fig. 1. Example 1 - Efficient solutions to the lower level problem for each x and level curves of F.



In this problem, adopting an optimistic approach may be considered an interesting option if the leader is willing to take some risk because the worst outcome of the optimistic approach, given by the *deceiving* solution (F=1.25), is not far from the *pessimistic* solution (F=1.0); on the other hand, the *optimistic* solution provides a considerable improvement to the leader's objective with respect to the *pessimistic* one (F=0.5 vs. 1.0 – recall that F is a minimizing function). Moreover, if the leader adopts a pessimistic approach, there will be no opportunity in this problem to obtain a solution better than the *pessimistic* one because there is only one efficient solution to the follower (P≡R).

B. Example 2

Consider the following semivectorial linear bilevel problem with maximizing objective functions at both levels.

$$\begin{aligned}
&\max\ F = x + 2y_1 - y_2\\
&s.t.\\
&\left.\begin{aligned}
&2 \le x \le 5\\
&\max\ f_1 = y_1 + 2y_2\\
&\max\ f_2 = y_1 - y_2\\
&s.t.\ y_1 \le 6\\
&\quad y_1 + y_2 \le 10 - x\\
&\quad y_2 \le x\\
&\quad y_1, y_2 \ge 0
\end{aligned}\right\} IR\ (\text{induced region})
\end{aligned}$$

x is the upper level variable and y_1, y_2 are the lower level variables.

TABLE I. LEADER'S OBJECTIVE VALUES FOR EXAMPLE 1

F (min)	**Optimistic approach**	**Pessimistic approach**
optimistic / *rewarding* solutions	$\mathbf{F^O = 0.5}$	$F^R = 1.0$
deceiving / ***pessimistic*** solutions	$F^D = 1.25$	$\mathbf{F^P = 1.0}$

Fig. 2 (a-d) illustrates the set of efficient solutions to the lower level problem for different values of x: integer x values from 2 to 5, respectively in (a) to (d). In each graph for a given x, F^p represents the worst value and F^o represents the best value for F within the lower level efficient solution set for that value of x.

Analytically, the lower level efficient solutions for each x can be defined as follows:

$$\Psi_{\text{Ef}}(x) = \begin{cases} (y_1 = 6,\ 0 \le y_2 \le 4 - x) & \\ \vee\ (y_1 + y_2 = 10 - x, y_1 < 6,\ 0 \le y_2 \le x) & \text{for } 2 \le x \le 4 \\ & \\ y_1 + y_2 = 10 - x,\ 0 \le y_2 \le x & \text{for } 4 \le x \le 5 \end{cases}$$

The *optimistic* solution is (x, y_1, y_2) = (4, 6, 0), (f_1, f_2) = (6, 6) and F = 16 – Fig. 2(c).

The *deceiving* solution is the worst solution for the leader among $\Psi_{\text{Ef}}(x = 4)$, i.e., (x, y_1, y_2) = (4, 2, 4), (f_1, f_2) = (10, -2) and F = 4.

The *pessimistic* solution is (x, y_1, y_2) = (2, 6, 2), (f_1, f_2) = (10, 4) and F = 12 – Fig. 2(a).

The *rewarding* solution is the best solution for the leader among $\Psi_{\text{Ef}}(x = 2)$, i.e., (x, y_1, y_2) = (2, 6, 0), (f_1, f_2) = (6, 6) and F = 14.

Fig. 3 highlights the complete induced region IR of the problem (in dark gray), the subset of efficient solutions "best for the leader", where the *optimistic* and *rewarding* solutions are located, and the subset of efficient solutions "worst for the leader", where the *pessimistic* and *deceiving* solutions are located (in solid thick lines).

Table 2 summarizes the F values obtained in these four solutions for Example 2.

In this example we observe a large difference in the leader's outcome between the *optimistic* and the *deceiving* solutions, which means that the leader may take a high risk if he adopts an optimistic approach. On the other hand, the leader's objective value in the *rewarding* solution is not very far from the *optimistic* solution (F=14 vs. 16) and the pessimistic F value is significantly higher than the *deceiving* one (F=12 vs. 4).

TABLE II. LEADER'S OBJECTIVE VALUES FOR EXAMPLE 2

F (min)	**Optimistic approach**	**Pessimistic approach**
optimistic / *rewarding* solutions	$\mathbf{F^O = 16}$	$F^R = 14$
deceiving / ***pessimistic*** solutions	$F^D = 4$	$\mathbf{F^P = 12}$

Fig. 2. Example 2 - Efficient solutions to the lower level problem for different values of *x*.

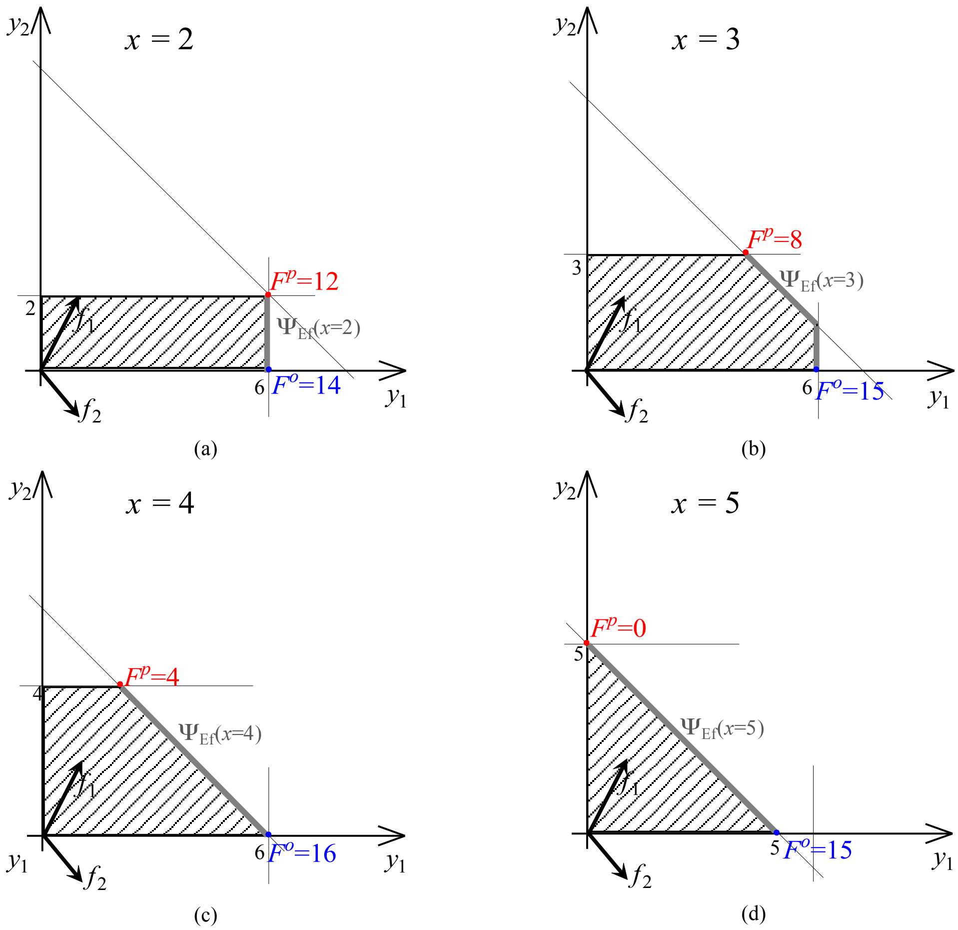
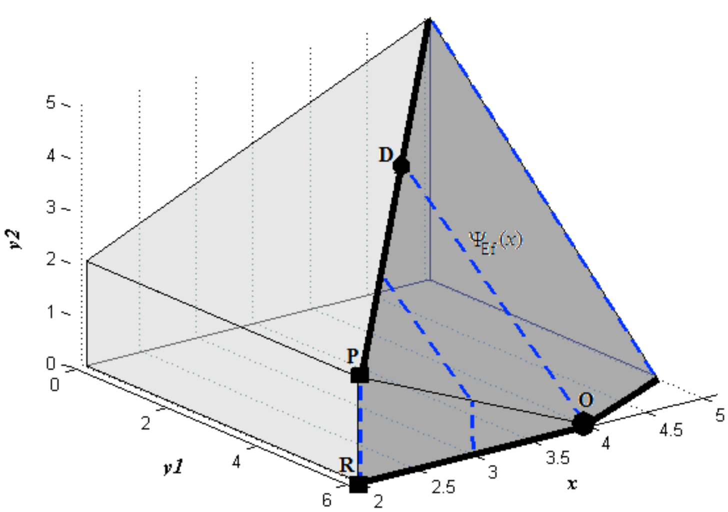


Fig. 3. Example 2 – Induced Region and the *optimistic* (O), *deceiving* (D), *pessimistic* (P) and *rewarding* (R) solutions.



C. Example 3

Consider the problem of Example 2 where only the upper level objective function changes:

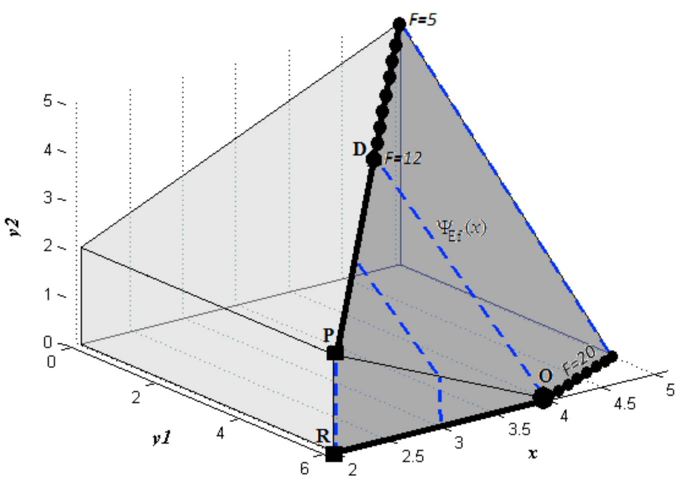
$$\max F = 2x + 2y_1 - y_2$$

The induced region and the *pessimistic* and *rewarding* solutions are the same as in Example 2 (shown in Fig. 3 and Fig. 4). The *pessimistic* solution is $(x, y_1, y_2) = (2, 6, 2)$, $(f_1, f_2) = (10, 4)$, $F = 14$ and the *rewarding* solution is $(x, y_1, y_2) = (2, 6, 0)$, $(f_1, f_2) = (6, 6)$, $F = 16$.

However, this problem admits alternative *optimistic* solutions. The vertices $(x, y_1, y_2) = (4, 6, 0)$ and $(5, 5, 0)$ and all solutions in the edge between these two vertices present the maximum leader's objective value, $F = 20$. But, if the optimistic approach fails due to a follower's reaction different from the one expected in the optimistic approach, the worst outcomes for the leader are different for the x values from 4 to 5 leading to different *'deceiving'* solutions: $F(4, 6, 4)=12$ and $F(5,0,5)=5$. These solutions are shown in Fig. 4.

This example brings attention to the importance of exploring alternative optimal solutions for the optimistic approach (and similar cases may occur for the pessimistic approach). Although all $4 \leq x \leq 5$ lead to an equal value (maximum) for the leader in an optimistic assumption, the choice x=4 is less risky as it leads to a better *deceiving* solution.

Fig. 4. Example 3 – Illustrating alternative *optimistic* solutions with different corresponding deceiving outcomes.



V. Conclusion

The existence of multiple objective functions at the lower level of a SVBP leads to a set of efficient solutions for the follower for each leader's decision. This poses additional difficulties for the leader to anticipate the follower's reaction to the underlying trade-offs between the lower level competing objectives. Therefore, different types of solutions should be computed, which provide broader information about the ranges of objective function values resulting from decisions associated with different trade-offs.

In addition to the *optimistic* and *pessimistic* solutions to the SVBP, this paper presents and illustrates two novel concepts: *deceiving* and *rewarding* solutions. The *deceiving* solution results from a failed optimistic approach, in the sense that the leader makes an optimistic decision and the follower's reaction is against the interests of the leader. The *rewarding* solution results if the follower's reaction is the most favorable to the leader when the leader takes a pessimistic approach. These four solutions characterize distinct attitudes from the leader and the follower, delimiting the ranges of possible optimal values for the leader taking into account the follower's decision.

The computation and exploration of these solutions is underway in a SVBP to model the interaction between electricity retailers and consumers. Dynamic tariffs, i.e. electricity prices varying in short periods of time, are expected to offer consumers the incentives to adopt different consumption patterns by using the flexibility in the operation of some end-use loads. The retailer (upper level decision maker) establishes dynamic electricity prices to maximize profits. The consumer (lower level decision maker) responds by selecting, under that price setting, an efficient solution establishing a trade-off between minimizing the electricity bill and the dissatisfaction associated with the corresponding load scheduling in face of his preferences and comfort requirements. The lower level optimization problem is formulated as a bi-objective mixed-integer linear programming (MILP) problem. An evolutionary algorithm approach and a hybrid approach, which consists of an evolutionary algorithm for the upper level problem and an exact MILP solver to solve scalarization problems at the lower level, are being developed to compute the four types of solutions illustrated in this paper, which are relevant for decision support purposes.

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References

[1] S. Dempe, "Bilevel programming: implicit function approach," in Encyclopedia of Optimization, vol. 2, Floudas and Pardalos, Eds., Springer US, pp. 260-266, 2009.

[2] A. Tsoukalas, W. Wiesemann, and B. Rustem, B, "Global optimisation of pessimistic bi-level problems," Lectures on Global Otimization, vol. 55, pp. 215-243, 2009.

[3] H. Bonnel, "Optimality conditions for the semivectorial bilevel optimization problem," Pac. J. Optim., vol. 2(3), pp. 447-468, 2006.

[4] H. Bonne and J. Morgan, "Semivectorial bilevel optimization problem: penalty approach," J. Optimiz. Theory App., vol. 131(3), pp.365-382, 2006.

[5] Z. Ankhili and A. Mansouri, "An exact penalty on bilevel programs with linear vector optimization lower level", Eur. J. Oper. Res.,vol. 197(1), pp. 36-41, 2009.

[6] Y. Zheng and Z. Wan, "A solution method for semivectorial bilevel programming problem via penalty method," J. Appl. Math. Comput., vol. 37(1-2), pp. 207-219, 2011.

[7] H. I. Calvete and C. Galé, "On linear bilevel problems with multiple objectives at the lower level," Omega, vol. 39(1), pp. 33-40, 2011.

[8] S. Dempe, N. Gadhi and A. B. Zemkoho, "New optimality conditions for the semivectorial bilevel optimization problem," J. Optimiz. Theory App., vol. 157(1), pp. 54-74, 2013.

[9] B. Liu, Z. Wan, J. Chen and G. Wang, "Optimality conditions for pessimistic semivectorial bilevel programming problems," J. Inequal. Appl., vol. 2014-41, pp. 1-26, 2014.

[10] M.J. Alves, C. H. Antunes and P. Carrasqueira, "A PSO approach to semivectorial bilevel programming: pessimistic, optimistic and deceiving Solutions," in Proc. of the Genetic and Evolutionary Computation Conference (GECCO 2015), pp. 599–606, 2015.

[11] K. Deb and A. Sinha, "Solving bilevel multi-objective optimization problems using evolutionary algorithms," in Proc. of EMO 2009, LNCS, vol. 5467, Springer, pp. 110-124. 2009.