Iterated Problem Reformulation for Evolutionary Large-Scale Multiobjective Optimization

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***Abstract*—Due to the curse of dimensionality, two main issues remain challenging for applying evolutionary algorithms (EAs) to large-scale multiobjective optimization. The first issue is how to improve the efficiency of EAs for reducing computation cost. The second one is how to improve the diversity maintenance of EAs to avoid local optima. Nevertheless, these two issues are somehow conflicting with each other, and thus it is crucial to strike a balance between them in practice. Thereby, we propose an iterated problem reformulation based EA for large-scale multiobjective optimization, where the problem reformulation based method and the decomposition based method are used iteratively to address the aforementioned issues. The proposed method is compared with several state-of-the-art EAs on a variety of large-scale multiobjective optimization problems. Experimental results demonstrate the effectiveness of our proposed iterated method in large-scale multiobjective optimization.**

***Index Terms*—Evolutionary algorithm, multiobjective optimization, large-scale optimization, problem reformulation**

I. Introduction

Multiobjective optimization problems (MOPs) are widely seen in real-world applications. In an MOP, there are two or more conflicting objectives which should be optimized simultaneously [1]. Due to the conflicting property of the multiple objectives, the Pareto dominance relationship is usually adopted to distinguish the qualities of two candidate solutions of an MOP. For two candidate solutions $\mathbf{x}_1$ and $\mathbf{x}_2$ in an MOP, if all the objective values of $\mathbf{x}_1$ are not worse than those of $\mathbf{x}_2$, and at least one objective value of $\mathbf{x}_1$ is strictly better than that of $\mathbf{x}_2$, $\mathbf{x}_1$ Pareto dominates $\mathbf{x}_1$ [2]. Consequently, if a candidate solution is not dominated by any solution of an MOP, it is called a Pareto optimal solution; the collection of all the Pareto optimal solutions is called the Pareto optimal set (PS); the projection of the PS in the objective space is called the Pareto optimal front (PF).

To solve MOPs, a variety of evolutionary algorithms (EAs) have been proposed in the past decades. Thanks to the population based property of EAs, multiobjective EAs (MOEAs) are capable of obtaining a set of representative solutions in a single run for solving MOPs effectively. Existing MOEAs can be roughly divided into three categories [3], i.e., the dominance based MOEAs, the decomposition based MOEAs, and the indicator based MOEAs. The first category adopts the dominance based relationships, e.g., the ε-dominance [4] and the strength Pareto dominance [5], [6], to distinguish the qualities of different solutions. The second category first decomposes an MOP into several subproblems by some weight (reference) points (vectors), and then optimize the subproblems simultaneously. Typical algorithms include the decomposition (DE) based MOEA (MOEA/D) [7] and its variations, such as the differential evolution based MOEA/D (MOEA/D-DE) [8]. As for the third category, it adopts the performance indicators to assess the contribution of a candidate solution to the quality of the population, such as the indicator based EA (IBEA) [9], the dominated hypervolume based MOEA (SMS-EMOA) [10], and the enhanced inverted generational distance based MOEA (MOEA/IGD-NS) [11]. There are still some MOEAs that do not fall into the above three categories, e.g., the local search based MOEA (NSLS) [12] and the collaborative neurodynamic based method (MONO) [13].

Despite the promising performance in solving MOPs, conventional MOEAs fail to handle MOPs with a large number of decision variables, a.k.a., large-scale MOPs (LSMOPs) [14]. On one hand, LSMOPs are challenging to conventional MOEAs due to the curse of dimensionality [15]. As the increase in the number of decision variables, the volume and complexity of the search space increase exponentially [16]. Consequently, conventional MOEAs will have to use a huge number of function evaluations for obtaining a set of acceptable solutions. On the other hand, LSMOPs widely exist in real-world applications, such as the optimization of power dispatch in the power system [17] and the critical point detection in complex network [18]. Thus, there is a growing demand for effective large-scale MOEAs for handling real-world LSMOPs.

Some large-scale MOEAs have been proposed in recent

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years, which can be divided into two different types [19]. The first type is known as the decision variable analysis based approaches [20], which classify the decision variables into different groups and then the original LSMOP is decomposed into several simpler sub-MOPs. Typical algorithms include the decision variable analysis based MOEA (MOEA/DVA) [21] and the decision variable clustering-based large-scale EA (LMEA) [16]. The second type consists of the problem transformation/reformulation based approaches, which transform/reformulate the original LSMOP into a simpler problem, and then optimize the transformed/reformulated problem to obtain some promising candidate solutions of the original problem. The weighted optimization framework (WOF) [22] and the problem reformulation based large-scale framework (LSMOF) [19] are two representative algorithms. Moreover, some MOEAs also show promising scalability due to their special reproduction operators, despite that they are not tailored for solving LSMOPs. These MOEAs use some efficient/effective offspring generation methods, e.g., the particle swarm optimization based method. For instance, the competition mechanism proposed in single-objective optimization [23] is used in solving LSMOPs. Typical algorithms include the competition mechanism based multi-objective particle swarm algorithm (CMOPSO) [24] and the efficient large-scale competitive swarm optimizer (LMOCSO) [25]. Despite the different mechanisms, the two types of large-scale MOEAs intend to maintain the convergence and diversity independently. As a consequence, MOEAs of the first type could be inefficient in terms of computation time or function evaluations, while those of the second type could be ineffective due to the greedy convergence enhancement.

Despite that most existing large-scale MOEAs have achieved remarkable results in handling LSMOPs, the balance between effectiveness and efficiency should be further improved. To better strike a balance between the effectiveness and efficiency, we propose an iterated problem reformulation based algorithm, termed iLSMOA, for large-scale multiobjective optimization. To be more specific, we use the problem reformulation method to enhance the convergence and the decomposition based method for diversity maintenance in an iterated manner. Different from conventional two-stage based strategies, the iterated strategy is effective in balancing the convergence and diversity dynamically. The proposed algorithm is compared with different categories of MOEAs on a variety of LSMOPs, and the experimental results have indicated its encouraging performance.

The rest of this paper is organized as follows. In Section II, we briefly recall some background of related works, including the problem reformulation, MOEA/D-DE, and the improved strength Pareto based selection. The details of the proposed algorithms are presented in Section III. Experimental comparisons between our proposed method and the state-of-the-art algorithms on the benchmark problems are presented in Section IV. Finally, we draw the conclusions in Section V.

II. Background

A. Problem Reformulation

The problem reformulation is proposed to reformulate the LSMOP into a low-dimensional single-objective optimization problem for accelerating the convergence rate of existing MOEAs [19]. To begin with, it selects some reference solutions in the decision space to construct several bi-directional reference vectors. Note that this selection is usually achieved by some strategies with strong capabilities in selecting well-distributed solutions, e.g., the improved strength Pareto based selection [5]. Then the step sizes from the ideal/nadir points in the decision space to the PS are defined as the decision variables of the reformulated problem. Finally, the objective space is reformulated by a performance indicator, e.g., the hypervolume (HV) indicator [26], where the obtained value is used as the fitness of the reformulated problem. Assuming that $\mathbf{s_1} = (x_1, \dots, x_D)$ is a reference solution in the D-dimensional decision space; $\mathbf{o}$ and $\mathbf{t}$ are the ideal and nadir boundary points of the decision space; $\mathbf{v}_l$ and $\mathbf{v}_u$ are vectors starting from $\mathbf{o}$ and $\mathbf{t}$ and pointing to $\mathbf{s}_1$ respectively:

$$\mathbf{v}_l = \mathbf{s}_1 - \mathbf{o}, \tag{1}$$

$$\mathbf{v}_u = \mathbf{t} - \mathbf{s}_1, \tag{2}$$

where l_{max}=$||\mathbf{t} - \mathbf{o}||$ is the maximum diagram length in the decision space. Supposing that $\mathbf{p}_1, \mathbf{p}_2$ are two solutions on vectors $\mathbf{v}_l$ and $\mathbf{v}_u$ respectively, and the distances from $\mathbf{o}$ to $\mathbf{p}_1$ and $\mathbf{t}$ to $\mathbf{p}_2$ are $\lambda_{11} \times \frac{\mathbf{v}_l}{||\mathbf{v}_l||} l_{max}$ and $\lambda_{12} \times \frac{\mathbf{v}_u}{||\mathbf{v}_u||} l_{max}$, respectively. Thus, two solutions $\mathbf{p}_1$ and $\mathbf{p}_2$ can be obtained:

$$\mathbf{p}_1 = \mathbf{o} + \lambda_{11} \times \frac{\mathbf{v}_l}{||\mathbf{v}_l||} l_{max}, \tag{3}$$

$$\mathbf{p}_2 = \mathbf{t} - \lambda_{12} \times \frac{\mathbf{v}_u}{||\mathbf{v}_u||} l_{max}, \tag{4}$$

where λ_{11} and λ_{11} are two weight variables. Afterwards, r reference solutions are used to construct $2r$ subproblems. For example, two subproblems constructed on the basis of $\mathbf{s_1}$ are

$$z_{11}(\lambda_{11}) = F(\mathbf{o} + \lambda_{11} \times \frac{\mathbf{v}_l}{||\mathbf{v}_l||} l_{max}), \tag{5}$$

$$z_{12}(\lambda_{12}) = F(\mathbf{t} - \lambda_{12} \times \frac{\mathbf{v}_u}{||\mathbf{v}_u||} l_{max}). \tag{6}$$

The constructed objective functions are $Z'(\mathbf{\Lambda}) = \{z_{11}(\lambda_{11})$, $z_{12}(\lambda_{12})$, $\dots$,$z_{r1}(\lambda_{r1})$, $z_{r2}(\lambda_{r2})\}$, and the decision space is $\mathbf{\Lambda}$=$\{\lambda_{11}, \lambda_{12}, \dots, \lambda_{r1}, \lambda_{r2}\}$. Once the subproblems are reconstructed, the optimization of the decision vector $\mathbf{x}$ in the original decision space is transformed to the optimization of $\mathbf{\Lambda}$ in the reconstructed decision space. Correspondingly, the new optimization problem can be reformulated as

$$\begin{array}{ll} \text{Maximize} & G(\mathbf{\Lambda}) = H(Z'(\mathbf{\Lambda})) \\ \text{subject to} & \mathbf{\Lambda} \in \Re^{2r}, \end{array} \tag{7}$$

where H is the HV indicator.

B. MOEA/D-DE

MOEA/D-DE is a variation of MOEA/D, which uses the differential evolution (DE) operator for offspring generation [8], as presented in Algorithm 1. In this algorithm, the parent solutions are selected from the neighborhood with a probability of δ, and then a DE operator is adopted for offspring generation. Specifically, three parent solutions are used for offspring generation. Assuming that $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ are three parent solutions, an offspring solution $\mathbf{x}'$ can be generated by

$$\mathbf{x}' = \mathbf{x}_1 + \mathrm{F} \times (\mathbf{x}_2 - \mathbf{x}_3), \tag{8}$$

where F is a control vector, and the polynomial mutation operation [27] is used after this crossover. Here, another control parameter (CR) for determining the use of DE is not mentioned as it is set to one as recommended in [8]. Next, the Tchebycheff aggression method is used to construct subproblems for assessing the quality of each solution, which can be formulated as (9).

$$\text{minimize } g(\mathbf{x}|\gamma, \mathbf{z}^*) = \max_{1 \le i \le M} \{\gamma_i |f_i(\mathbf{x}) - z_i^*|\}, \tag{9}$$

where $\gamma = (\gamma_1, \dots, \gamma_M)$ is a weight vector (M is the number of objectives), f_i is the ith objective value of solution $\mathbf{x}$, and $\mathbf{z}^* = (z_1^*, \dots, z_M^*)$ is the reference point with $z_i^* = \min\{f_i(\mathbf{x})|\mathbf{x} \in \Omega\}$ (Ω is the decision space).

Algorithm 1 The main framework of MOEA/D-DE.

Input: N (population size), δ (probability of choosing parents locally).
Output: P (final population).

```
P, W, E ← Initialization(N) /*P is the initial population, W is the weight vector set, and E is the neighborhood index set*/
while termination criterion is not fulfilled do
  for i ← 1 : N do
    if rand < δ then
      Randomly choose two solutions x2, x3 from the neighborhood E_i /*E_i denotes the neighborhood index set of the ith solution in P*/
    else
      Randomly choose x2, x3 from P
    end
    x' ← Offspring_Generation (P_i, x2, x3) /*P_i denotes the ith solution in P*/
    g1, g2, g3 ← Tchebycheff_Aggression(x', x2, x3)
    for j ← 2 : 3 do
      P_i ← x' if g_j > g1
    end
  end
end
```

C. Improved Strength Pareto Based Selection

The improved strength Pareto based EA (SPEA2) [5] incorporated a tailored fitness assignment strategy, a density estimation technique, and an enhanced truncation method. In the tailored fitness assignment strategy, the dominance relationship between the pairwise candidate solutions is first detected, and then a strength value is assigned to each candidate solution. It can be formulated as

$$Str(\mathbf{x}_i) = |\{j|\mathbf{x}_j \in P \wedge \mathbf{x}_i \prec \mathbf{x}_j\}|, \tag{10}$$

where P is the population and $\mathbf{x}_i, \mathbf{x}_j$ are the candidate solutions in it. Then, the raw fitness can be calculated by

$$Raw(\mathbf{x}_i) = \sum_{\mathbf{x}_j \in P \wedge \mathbf{x}_j \prec \mathbf{x}_i}^{N} Str(\mathbf{x}_j). \tag{11}$$

Besides, the additional density information, termed Den, is used to discriminate the candidate solutions having identical raw fitness values as

$$Den(\mathbf{x}_i) = \frac{1}{\sigma_i^k + 2}, \tag{12}$$

where k is the square root of the population size, and σ_i^k denotes the kth nearest Euclidean distance from $\mathbf{x}_i$ to the candidate solutions in the population. Finally, the fitness function is

$$Fit(\mathbf{x}_i) = Raw(\mathbf{x}_i) + Den(\mathbf{x}_i). \tag{13}$$

This selection strategy is capable of obtaining a set of representative solutions from a population even on MOPs with complex PFs.

III. The Proposed Algorithm

The main scheme of the proposed iterated problem reformulation based large-scale multiobjective algorithm, termed iLSMOA, is presented in Algorithm 2. Generally, the proposed algorithm uses the problem reformulation based single-objective optimization (Steps 3–4) and the decomposition based multiobjective optimization (Steps 5–6) in an iterated manner. To begin with, a population of size N is randomly generated from the original LSMOP (Step 1). Then we select r solutions as the reference solutions by the environmental selection strategy in SPEA (as given in Section II-C), and the problem reformulation based single-objective optimization (as given in Section II-A) is conducted to obtain a set of solutions A (Step 4). Afterwards, the same environmental selection strategy is adopted to select N well-distributed solutions from A (Step 5), where the selected solutions are used as the initial population of MOEA/D-DE. Note that our empirical results on a variety of LSMOPs indicate that it is essential to select a set of evenly distributed solutions as the initial population of MOEA/D-DE; if only the convergence criterion is considered for obtaining the initial population, MOEA/D-DE could trap in local optima easily. Finally, the above procedures are repeated in an iterated manner until the termination criterion is fulfilled.

Notably, it is out of several considerations that we choose MOEA/D-DE as the optimizer to spread the quasi-optimal solutions over the approximate PF. First, due to the explicit directional guidance of the weight vectors, MOEA/D-DE will stick to some subspaces/subproblems even if the current candidate solutions are not promising. It substantially maintains

Algorithm 2 The proposed iLSMOA.

Input: Z (original LSMOP), N (population size), r (number of reference solutions), g_{max} (maximum iteration).

Output: P (final population).

```
P ← Initialization(N, Z)
while termination criterion is not fulfilled do
    Z' ← Problem_Reformulation(P, r, Z)
    A ← Single_Objective_Optimization(Z', g_max)
    P ← Environmental_Selection(A, N)
    P ← Evolve_by_MOEA/D-DE(P, N, g_max)
end
```

the diversity of the population and will also enhance the global search ability of the problem reformulation based optimization. Second, since the decomposition based method generates offspring solutions from neighborhood solutions, it enhances the local search ability of the algorithm for convergence enhancement. Third, the DE operation enables the proposed algorithm to handle problems with complex PSs as suggested in [8].

Moreover, there are two motivations for adopting the iterated framework instead of the two-stage strategy in LSMOF [19]. First, the quash-optimal solutions obtained by the problem reformulation based optimization can be local optima, while the iterated strategy enables the population to escape from them. Second, the iterated strategy provides the problem reformulation optimization some dynamic reference solutions, which also enhances the global search ability of the proposed algorithm.

IV. Empirical Studies

To empirically investigate the performance of the proposed iLSMOA, six representative MOEAs, namely, IBEA [9], CMOPSO [24], GDE3 [28], LMEA [16], MOEA/D-DE [8], and LSMOF [19], are compared on eight test problems from the LSMOP test suite [14]. Each algorithm is run for 30 times on each test problem independently, and the Wilcoxon rank-sum test [29] is used to compare the results obtained by the proposed iLSMOA and the compared algorithms at a significance level of 0.05. Symbols '+', '−', and '=' indicate that the compared algorithm is significantly better than, significantly worse than, and statistically tied by iLSMOA.

A. Experimental Settings

For fair comparisons, we adopt the recommended parameter settings for the compared algorithms that have achieved the best performance as reported in the literature. All the compared algorithms are implemented in PlatEMO [30].

1) Reproduction Operators. In this work, the simulated binary crossover (SBX) [31] and the polynomial mutation (PM) [27] are adopted in the compared algorithms for offspring generation in IBEA and LSMOF. The distribution index of crossover is set to n_c=20 and that of mutation is set to n_m=20, as recommended in [31]. The crossover probability p_c is set to 1.0 and the mutation probability p_m is set to $1/D$, where D is the number of decision variables. In MOEA/D-DE and GDE3, the DE operator [32] and PM are used for offspring generation, where the control parameters are set to CR=1, F=0.5, p_m=$1/D$, and η=20 as recommended in [8]. As for CMOPSO, the particle swarm operator [33] and PM are used, where parameters R_1 and R_2 are randomly selected from $[0, 1]$ with γ set to 10 as recommended in [24].

2) Population Size. The population size is set to 100 for test instances with two objectives and 105 for test instances with three objectives.

3) Specific Parameter Settings in Each Algorithm. In MOEA/D-DE, the neighborhood size T is set to 20, the probability of choosing parents locally δ is set to 0.9, and the maximum number of solutions replaced by each offspring n_r is set to 2. Meanwhile, NSGA-II is embedded in LSMOF, the number of reference solutions r is set to 10, the population size for the single-objective optimization is set to 30, and the mutation factor F in DE is set to 0.8. In iLSMOA, the number of reference solutions and the population size of the reformulated problems are set to 10, and g_{max} is set to 150.

4) Termination Condition. A total number of 500000 FEs is adopted as the termination condition for all the test instances.

5) Performance Indicator. In the experiments, two widely used performance indicators, the inverted generational distance (IGD) [34] indicator and the HV [26] indicator, are adopted for evaluating the performance of the compared algorithms.

Both the IGD and HV indicators can assess both the convergence and diversity of the solution set, the size of reference points is set to 10000 (or a close number) for the IGD calculations. Note that, a smaller value of IGD will indicate better performance of the algorithm; in contrast, a greater value of HV will indicate better performance of the algorithm.

B. Results on Bi-Objective LSMOPs

Here, we present the experimental results achieved by the seven compared algorithms on 24 bi-objective LSMOPs. The statistics of IGD and HV values are given in Table I and Table II, respectively. It can be observed from these two tables that iLSMOA has achieved the most best results (10 and 11 of 24 respectively), followed by MOEA/D-DE, LSMOF, GDE3, and CMOPSO. Moreover, iLSMOA has outperformed LSMOF on most of the test instances, indicating the effectiveness of the proposed iterated problem reformulation method in large-scale multiobjective optimization.

The final non-dominated solutions obtained by the compared algorithms on bi-objective LSMOP2 and LSMOP9 with 1000 decision variables in the run associated with the median IGD value are displayed in Fig. 1 and Fig. 2, respectively. It can be observed that iLSMOA has obtained the most evenly distributed solutions converged to the PF, indicating the superiority of our proposed iLSMOA in handling LSMOPs in terms of both convergence enhancement and diversity maintenance.

C. Results on Tri-Objective LSMOPs

The experimental results achieved by the seven compared algorithms on 24 tri-objective LSMOPs are presented. The

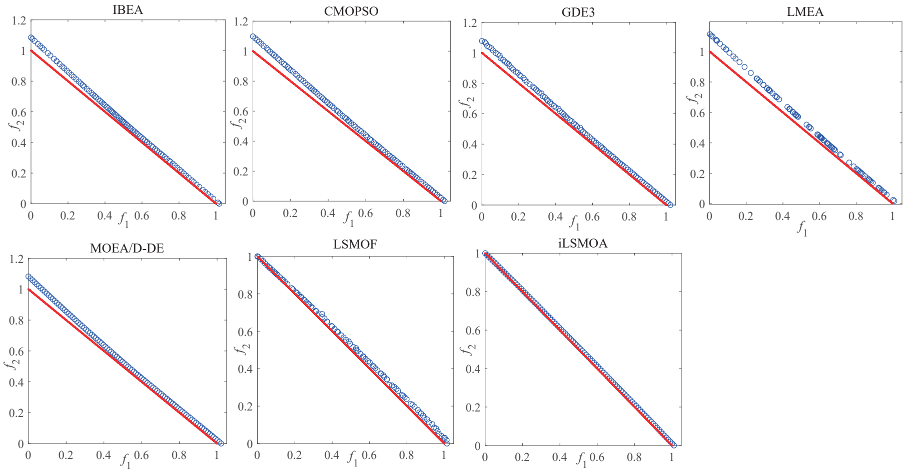

Fig. 1. The final non-dominated solutions obtained by the compared algorithms on bi-objective LSMOP2 with 1000 decision variables in the run associated with the median IGD value.

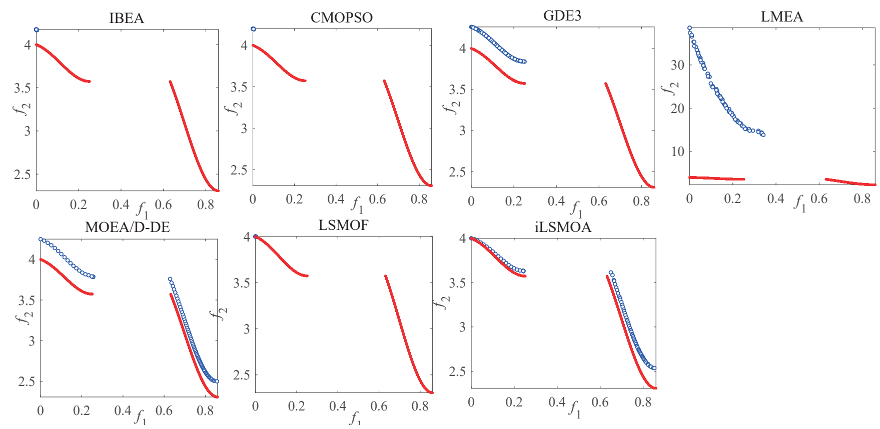

Fig. 2. The final non-dominated solutions obtained by the compared algorithms on bi-objective LSMOP9 with 1000 decision variables in the run associated with the median IGD value.

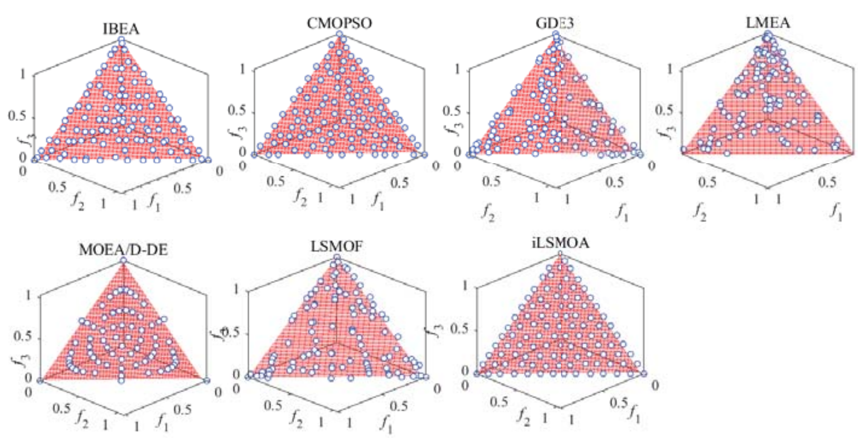

Fig. 3. The final non-dominated solutions obtained by the compared algorithms on tri-objective LSMOP2 with 1000 decision variables in the run associated with the median IGD value.

TABLE I
THE STATICS OF IGD RESULTS ACHIEVED BY SEVEN COMPARED ALGORITHMS ON 24 BI-OBJECTIVE TEST INSTANCES FROM LSMOP TEST SUITE.

Problem	D	IBEA	CMOPSO	GDE3	LMEA	MOEA/D-DE	LSMOF	iLSMOA
LSMOP1	200	2.5198e-1 (7.03e-2) -	2.1175e-1 (4.06e-2)-	9.6936e-3 (8.50e-4) =	5.0654e-1 (4.89e-1)-	4.2729e-3 (1.50e-4) +	5.1178e-1 (5.06e-2)-	9.4559e-3 (5.85e-4)
	500	2.7042e-1 (6.29e-2)-	2.4426e-1 (2.39e-3)-	3.5996e-1 (1.47e-2)-	8.1454e+0 (3.81e+0)-	2.4239e-1 (5.63e-3)-	5.8458e-1 (3.67e-2)-	9.0907e-2 (1.16e-2)
	1000	5.6997e-1 (2.96e-2)-	2.6044e-1 (2.99e-3)-	1.0485e+0 (6.19e-2)-	9.5576e+0 (2.76e+0)-	4.7337e-1 (3.09e-2)-	6.2352e-1 (2.32e-2)-	1.5589e-1 (4.67e-3)
LSMOP2	200	5.9069e-2 (2.98e-3)-	7.1472e-2 (3.40e-3)-	7.6401e-2 (4.68e-3)-	1.2715e-1 (1.57e-2)-	6.7485e-2 (4.62e-3)-	3.6640e-2 (4.84e-3)-	1.2989e-2 (1.19e-3)
	500	4.4650e-2 (1.45e-3)-	5.3040e-2 (1.09e-3)-	4.6716e-2 (7.12e-4)-	7.3208e-2 (2.44e-3)-	4.2944e-2 (1.09e-3)-	2.4521e-2 (1.86e-3)-	1.2148e-2 (2.98e-4)
	1000	2.9728e-2 (5.32e-4)-	3.6781e-2 (1.01e-3)-	2.8326e-2 (4.26e-4)-	4.0621e-2 (3.42e-4)-	2.5279e-2 (6.78e-4)-	1.9026e-2 (3.42e-4)-	8.3608e-3 (1.94e-4)
LSMOP4	200	8.0944e-2 (3.69e-3)-	9.0600e-2 (6.05e-3)-	2.1677e-2 (4.89e-4)-	1.2916e-1 (2.15e-2)-	1.6077e-2 (7.93e-4) =	9.6697e-2 (8.63e-3)-	1.5735e-2 (8.11e-4)
	500	6.0860e-2 (1.02e-3)-	7.3844e-2 (2.39e-3)-	5.6064e-2 (5.70e-3)-	1.2929e-1 (1.38e-2)-	6.8900e-2 (5.45e-3)-	5.7244e-2 (5.31e-3)-	2.0903e-2 (1.86e-3)
	1000	3.9911e-2 (6.37e-4)-	6.7472e-2 (9.93e-4)-	5.2558e-2 (7.59e-4)-	7.6263e-2 (3.78e-3)-	5.1241e-2 (8.41e-4)-	3.9428e-2 (2.27e-3)-	2.1165e-2 (3.11e-4)
LSMOP5	200	3.4242e-1 (3.12e-7) =	3.4112e-1 (3.63e-4) =	8.4394e-3 (9.53e-4)+	6.7184e-1 (1.06e+0)-	3.0886e-2 (1.34e-1)+	7.4209e-1 (1.13e-16)-	3.3779e-1 (8.79e-3)
	500	3.4149e-1 (7.63e-4)-	3.3522e-1 (7.71e-4)+	7.1452e-1 (9.41e-2)-	2.3140e+1 (6.56e-1)-	3.1759e-1 (5.48e-3)+	7.4209e-1 (1.13e-16)-	3.3979e-1 (1.25e-2)
	1000	4.6862e-1 (3.00e-2)-	8.6448e-1 (6.88e-1)-	2.3129e+0 (1.86e-1)-	2.4012e+1 (5.26e-1)-	8.1039e-1 (5.22e-2)-	7.4209e-1 (1.13e-16)-	3.3977e-1 (8.84e-3)
LSMOP6	200	1.1790e+0 (1.39e-1)-	5.9087e-1 (2.73e-1)+	5.9651e-1 (1.01e-1) =	3.4361e+0 (6.59e+0)-	7.4240e-1 (2.22e-4)-	3.5761e-1 (3.74e-3)+	6.1155e-1 (1.46e-2)
	500	1.0323e+0 (5.64e-2)-	4.5314e-1 (6.55e-2)+	5.5133e-1 (1.25e-1)+	1.9444e+3 (1.51e+3)-	7.3382e-1 (5.13e-2)-	3.1987e-1 (8.56e-4)+	6.5781e-1 (1.14e-2)
	1000	2.0751e+0 (5.96e-1)-	4.8573e-1 (1.29e-1)+	5.9818e-1 (1.27e-1) =	2.7388e+3 (2.37e+3)-	7.3992e-1 (4.45e-2)-	3.1267e-1 (1.11e-3)+	6.7023e-1 (1.88e-2)
LSMOP7	200	1.8708e+0 (5.44e-1)-	1.2786e+0 (3.65e-1)+	1.4243e+0 (6.90e-1) =	8.5527e+0 (1.11e+1)-	1.2929e+0 (3.24e-1)+	1.3388e+0 (1.54e-1) =	1.4133e+0 (1.85e-2)
	500	2.8632e+0 (5.94e-1)-	1.9421e+0 (2.39e-1)-	2.6717e+0 (6.64e-1)-	8.2086e+4 (3.96e+3)-	1.6908e+0 (2.17e-1)-	1.4977e+0 (1.91e-3)-	1.4917e+0 (7.89e-3)
	1000	4.6098e+0 (6.91e-1)-	3.0587e+0 (2.03e+0)-	4.1317e+1 (4.60e+0)-	8.6965e+4 (3.59e+3)-	2.3039e+0 (1.77e-1)-	1.5078e+0 (6.81e-4) =	1.5099e+0 (4.92e-3)
LSMOP8	200	3.4548e-1 (9.79e-4)-	2.8622e-1 (8.70e-2)-	5.3672e-2 (2.73e-3) =	1.6457e-1 (1.03e-1)-	4.0598e-2 (4.68e-3)+	7.4209e-1 (1.13e-16)-	4.7619e-2 (1.26e-2)
	500	3.4720e-1 (8.60e-4)-	3.1401e-1 (3.71e-2)-	6.3451e-2 (6.95e-3)-	1.9658e+1 (6.19e-1)-	2.7007e-2 (1.03e-3)+	7.4209e-1 (1.13e-16)-	3.6966e-2 (4.02e-3)
	1000	3.3120e-1 (2.83e-3)-	9.6596e-1 (1.40e-1)-	1.6454e+0 (1.27e-1)-	2.0165e+1 (3.66e-1)-	2.5620e-2 (8.99e-4)+	7.4209e-1 (1.13e-16)-	2.6138e-1 (4.20e-2)
LSMOP9	200	8.1004e-1 (8.82e-16)-	8.1004e-1 (7.04e-16)-	3.4718e-1 (1.00e-1)+	9.5738e-1 (4.14e-1) =	6.2908e-2 (6.14e-2)+	8.1004e-1 (3.39e-16) =	8.1004e-1 (3.39e-16)
	500	8.1121e-1 (2.43e-3)-	8.1880e-1 (2.07e-2) =	2.4712e-1 (9.44e-2)+	4.5385e+1 (2.07e+1)-	1.7551e-1 (8.04e-3)+	8.0922e-1 (5.21e-4)-	7.5028e-1 (1.98e-1)
	1000	9.5720e-1 (5.66e-3)-	9.9859e-1 (7.19e-3)-	2.7692e-1 (1.85e-1)+	5.2523e+1 (1.65e+1)-	1.0076e-1 (1.03e-3)+	8.0692e-1 (7.32e-4)-	6.3344e-1 (3.24e-1)
+/-/=		0/23/1	5/19/2	5/14/5	0/23/1	10/13/1	3/18/3	

'+', '-' and '=' indicate that the result is significantly better, significantly worse and statistically similar to that obtained by iLSMOA, respectively. The best result in each row is highlighted.

TABLE II
THE STATICS OF HV RESULTS ACHIEVED BY SEVEN COMPARED ALGORITHMS ON 24 BI-OBJECTIVE TEST INSTANCES FROM LSMOP TEST SUITE.

Problem	D	IBEA	CMOPSO	GDE3	LMEA	MOEA/D-DE	LSMOF	iLSMOA
LSMOP1	200	2.9149e-1 (8.17e-2)-	2.9453e-1 (4.56e-2)-	5.7237e-1 (1.11e-3) =	2.1791e-1 (1.45e-1)-	5.8040e-1 (3.09e-4)+	1.1153e-1 (5.79e-3)-	5.7289e-1 (7.59e-4)
	500	2.6114e-1 (6.74e-2)-	1.9960e-1 (3.02e-3)-	2.0722e-1 (1.19e-2)-	0.0000e+0 (0.00e+0)-	3.1233e-1 (5.27e-3)-	1.0923e-1 (6.29e-3)-	4.6290e-1 (1.75e-2)
	1000	6.6189e-2 (1.59e-2)-	1.7366e-1 (2.95e-3)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	1.3485e-1 (2.00e-2)-	1.0490e-1 (5.27e-3)-	3.5600e-1 (8.28e-3)
LSMOP2	200	5.0631e-1 (3.64e-3)-	4.9301e-1 (4.58e-3)-	4.8645e-1 (5.80e-3)-	4.2546e-1 (1.72e-2)-	4.9845e-1 (5.63e-3)-	5.3752e-1 (5.75e-3)-	5.6827e-1 (1.47e-3)
	500	5.2617e-1 (1.84e-3)-	5.1605e-1 (1.38e-3)-	5.2348e-1 (8.87e-4)-	4.8762e-1 (3.57e-3)-	5.2846e-1 (1.36e-3)-	5.5273e-1 (2.33e-3)-	5.6928e-1 (3.77e-4)
	1000	5.4562e-1 (7.06e-4)-	5.3689e-1 (1.31e-3)-	5.4768e-1 (5.74e-4)-	5.2873e-1 (9.50e-4)-	5.5170e-1 (8.89e-4)-	5.5999e-1 (4.46e-4)-	5.7416e-1 (2.57e-4)
LSMOP4	200	4.6602e-1 (3.91e-3)-	4.6758e-1 (6.94e-3)-	5.5672e-1 (6.17e-4)-	4.2325e-1 (2.27e-2)-	5.6400e-1 (1.03e-3) =	4.5675e-1 (1.14e-2)-	5.6445e-1 (1.10e-3)
	500	5.0359e-1 (1.12e-3)-	4.9013e-1 (2.97e-3)-	5.1167e-1 (7.21e-3)-	4.2331e-1 (1.69e-2)-	4.9665e-1 (6.52e-3)-	5.0863e-1 (6.32e-3)-	5.5766e-1 (2.42e-3)
	1000	5.3155e-1 (8.35e-4)-	4.9955e-1 (1.22e-3)-	5.1583e-1 (9.41e-4)-	4.8440e-1 (6.28e-3)-	5.1784e-1 (1.01e-3)-	5.3305e-1 (2.75e-3)-	5.5755e-1 (4.08e-4)
LSMOP5	200	1.7354e-1 (3.00e-6)+	1.7354e-1 (2.18e-5)+	3.3891e-1 (1.48e-3)+	3.0585e-2 (2.40e-2)-	3.3435e-1 (4.60e-2)+	9.0909e-2 (7.06e-17)-	1.6971e-1 (2.71e-3)
	500	1.6856e-1 (9.49e-4)+	1.7215e-1 (3.48e-4)+	2.0860e-6 (1.14e-5)-	0.0000e+0 (0.00e+0)-	1.3779e-1 (3.42e-3)-	9.0909e-2 (7.06e-17)-	1.6391e-1 (1.16e-2)
	1000	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	9.0909e-2 (7.06e-17)-	1.6232e-1 (7.25e-3)
LSMOP6	200	0.0000e+0 (0.00e+0)-	2.9099e-2 (2.49e-2)-	2.1392e-3 (7.14e-3)-	0.0000e+0 (0.00e+0)-	9.0057e-2 (6.34e-5)+	3.1489e-2 (7.77e-3)-	8.1316e-2 (2.61e-3)
	500	0.0000e+0 (0.00e+0)-	2.8729e-2 (9.60e-3)-	3.0488e-2 (1.06e-2)-	0.0000e+0 (0.00e+0)-	8.4753e-2 (1.05e-3)+	3.5892e-2 (2.85e-3)-	7.9234e-2 (3.21e-3)
	1000	0.0000e+0 (0.00e+0)-	4.1955e-2 (4.62e-3)-	5.5831e-2 (2.04e-3)-	0.0000e+0 (0.00e+0)-	7.2626e-2 (2.12e-3)-	6.9943e-2 (1.14e-3)-	7.4072e-2 (2.31e-3)
LSMOP7	200	0.0000e+0 (0.00e+0) =	7.5507e-4 (3.11e-3) =	8.9751e-4 (3.07e-3)+	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0)
	500	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0)
	1000	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0)
LSMOP8	200	1.5421e-1 (5.01e-3)-	1.5375e-1 (3.81e-2)-	2.7710e-1 (3.25e-3)-	1.7090e-1 (6.19e-2)-	2.9727e-1 (4.53e-3)-	9.0909e-2 (7.06e-17)-	3.0594e-1 (6.59e-3)
	500	1.4311e-1 (4.47e-3)-	9.9071e-2 (4.47e-2)-	2.6492e-1 (8.82e-3)-	0.0000e+0 (0.00e+0)-	3.1307e-1 (1.41e-3)+	9.0909e-2 (7.06e-17)-	3.0528e-1 (2.89e-3)
	1000	2.2380e-2 (1.06e-2)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	3.1560e-1 (1.18e-3)+	9.0909e-2 (7.06e-17)-	1.5370e-1 (1.94e-2)
LSMOP9	200	9.0909e-2 (2.75e-16)-	9.0909e-2 (1.37e-16)-	1.2117e-1 (1.54e-2)+	5.0302e-2 (2.47e-2)-	2.1379e-1 (2.70e-2)+	9.0909e-2 (7.06e-17) =	9.0909e-2 (7.06e-17)
	500	9.0540e-2 (6.73e-4)-	8.8550e-2 (5.42e-3) =	1.4451e-1 (1.19e-2)+	0.0000e+0 (0.00e+0)-	1.6402e-1 (2.88e-3)+	9.0993e-2 (5.23e-5)-	1.0135e-1 (3.40e-2)
	1000	5.2897e-2 (1.41e-3)-	4.2505e-2 (1.79e-3)-	1.5783e-1 (2.26e-2)+	0.0000e+0 (0.00e+0)-	1.9473e-1 (4.24e-4)+	9.1291e-2 (9.39e-5)-	1.2066e-1 (5.48e-2)
+/-/=		2/19/3	2/18/4	5/16/3	0/21/3	10/10/4	0/20/4	

'+', '-' and '=' indicate that the result is significantly better, significantly worse and statistically similar to that obtained by iLSMOA, respectively.

statistics of IGD and HV values are given in Table III and Table IV, respectively. It can be observed that iLSMOA has achieved the most best results (9 and 14 out of 24 respectively), followed by LSMOF, IBEA, and MOEA/D-DE. Thus, iLSMOA has shown its competitive performance in comparison with state-of-the-arts.

The final non-dominated solutions obtained by the compared algorithms on tri-objective LSMOP2 with 1000 decision variables in the run associated with the median IGD value are displayed in Fig. 3. It can be observed that iLSMOA has obtained the most evenly distributed solutions converged to the PF, demonstrating the effectiveness of our proposed iLSMOA in handling LSMOPs.

V. CONCLUSION

In this work, we have proposed an iterated problem reformulated based EA for large-scale multiobjective optimization. The basic idea is to adopt the reformulation based single-objective optimization and the decomposition based

TABLE III
THE STATICS OF IGD RESULTS ACHIEVED BY SEVEN COMPARED ALGORITHMS ON 24 TRI-OBJECTIVE TEST INSTANCES FROM LSMOP TEST SUITE.

Problem	D	IBEA	CMOPSO	GDE3	LMEA	MOEA/D-DE	LSMOF	iLSMOA
LSMOP1	200	2.8015e-1 (8.35e-2)+	5.6450e-1 (1.42e-1)-	4.5507e-1 (1.15e-1)-	2.3396e-1 (1.56e-1)+	3.8721e-1 (2.55e-2)=	2.2044e-1 (1.98e-3)+	3.6433e-1 (5.84e-2)
	500	3.3940e-1 (8.39e-2)+	7.1589e-1 (3.05e-1)-	2.2043e+0 (2.04e-1)-	1.0272e+1 (2.93e+0)-	1.2956e+0 (1.22e-1)-	2.4511e-1 (6.47e-3)+	3.8725e-1 (4.12e-2)
	1000	9.7374e-1 (1.76e-1)-	1.0664e+0 (1.20e-1)-	2.9007e+0 (2.47e-1)-	1.0985e+1 (1.63e+0)-	1.6348e+0 (1.52e-1)-	4.6671e-1 (1.57e-2)-	3.9434e-1 (3.73e-2)
LSMOP2	200	6.9216e-2 (1.76e-3)+	1.2829e-1 (1.35e-3)-	1.3519e-1 (1.39e-3)-	1.0861e-1 (1.30e-2)-	1.0226e-1 (2.15e-3)-	1.4419e-1 (3.37e-3)-	8.2122e-2 (3.81e-3)
	500	5.8267e-2 (8.67e-4)-	6.9321e-2 (5.19e-4)-	7.7044e-2 (1.48e-3)-	7.6191e-2 (5.82e-3)-	7.1028e-2 (5.99e-4)-	8.8067e-2 (3.55e-3)-	5.4155e-2 (1.07e-3)
	1000	5.0042e-2 (6.66e-4)-	5.1527e-2 (3.80e-4)-	6.0000e-2 (1.51e-3)-	6.2756e-2 (2.67e-3)-	6.2599e-2 (4.61e-4)-	7.1666e-2 (3.87e-3)-	4.5826e-2 (7.18e-4)
LSMOP4	200	1.0821e-1 (8.62e-3)+	3.1461e-1 (1.50e-2)-	3.7425e-1 (9.66e-3)-	1.9185e-1 (7.24e-2)+	2.8530e-1 (9.48e-3)-	2.7341e-1 (8.05e-3)-	2.3526e-1 (1.26e-2)
	500	9.1916e-2 (1.92e-3)+	2.0022e-1 (3.70e-3)-	2.0717e-1 (3.52e-3)-	2.1371e-1 (8.60e-3)-	1.6393e-1 (1.87e-3)-	2.2041e-1 (5.27e-3)-	1.2076e-1 (4.76e-3)
	1000	7.3730e-2 (1.08e-3)+	1.2171e-1 (1.37e-3)-	1.2988e-1 (1.51e-3)-	1.3119e-1 (5.87e-3)-	1.0563e-1 (1.17e-3)-	1.4357e-1 (5.11e-3)-	8.0868e-2 (2.25e-3)
LSMOP5	200	4.8029e-1 (1.43e-5)-	3.7630e-1 (1.09e-1)=	2.4584e+0 (1.44e+0)-	5.5177e+0 (3.82e+0)-	5.3134e-1 (1.24e-2)-	4.8445e-1 (4.46e-2)-	3.5042e-1 (2.35e-2)
	500	4.5555e-1 (3.85e-2)-	8.5919e-1 (6.80e-2)-	6.2034e+0 (6.29e-1)-	1.8764e+1 (3.47e+0)-	1.2976e+0 (4.88e-1)-	5.2359e-1 (2.46e-2)-	3.5819e-1 (2.78e-2)
	1000	8.3798e-1 (5.99e-2)-	5.0869e+0 (4.04e+0)-	6.1954e+0 (8.59e-1)-	1.9979e+1 (7.13e-1)-	2.8023e+0 (2.24e-1)-	5.3908e-1 (5.78e-3)-	3.6679e-1 (3.59e-2)
LSMOP6	200	1.1726e+0 (3.20e-1)=	2.0063e+0 (5.99e-1)-	1.5981e+1 (1.36e+1)-	7.0272e+2 (2.44e+3)-	1.0922e+0 (1.59e-1)+	8.0008e-1 (2.21e-1)+	1.1598e+0 (1.01e-1)
	500	2.1183e+0 (6.34e-1)-	4.8347e+0 (2.40e+0)-	9.7084e+1 (1.52e+2)-	2.7444e+4 (1.75e+4)-	1.6705e+0 (3.01e-1)-	7.1883e-1 (1.06e-1)+	1.2837e+0 (1.47e-1)
	1000	6.3596e+0 (7.48e+0)-	7.0079e+1 (6.15e+1)-	2.6498e+3 (6.82e+2)-	3.8392e+4 (5.48e+3)-	2.1176e+0 (2.95e-1)-	7.7849e-1 (1.51e-1)+	1.3575e+0 (1.32e-1)
LSMOP7	200	1.1232e+0 (1.07e-1)-	1.3901e+0 (2.85e-1)-	1.4584e+0 (2.14e-1)-	3.3842e+0 (4.98e+0)-	9.3049e-1 (7.50e-2)-	9.4541e-1 (3.10e-2)-	8.2773e-1 (1.06e-2)
	500	1.4500e+0 (5.74e-2)-	1.3305e+0 (1.07e-2)-	1.2558e+0 (8.46e-3)-	1.3391e+3 (1.04e+3)-	9.4609e-1 (3.97e-5)-	8.8794e-1 (7.07e-3)-	8.3832e-1 (4.99e-3)
	1000	4.1276e+0 (7.09e-1)-	1.1213e+0 (3.01e-3)-	1.0821e+0 (1.98e-3)-	1.6670e+3 (1.81e+3)-	9.6206e-1 (5.86e-3)-	8.6210e-1 (2.69e-3)-	8.3855e-1 (2.50e-3)
LSMOP8	200	4.4399e-1 (5.35e-2)-	3.2610e-1 (4.77e-2)-	1.7919e-1 (4.86e-3)+	2.0521e-1 (5.75e-2)+	2.8105e-1 (6.08e-2)=	3.5433e-1 (2.13e-2)-	2.7723e-1 (4.67e-2)
	500	4.8821e-1 (1.51e-3)-	3.3712e-1 (4.78e-2)+	3.1323e-1 (1.62e-1)+	7.7210e-1 (1.71e-1)-	5.4179e-1 (1.06e-2)-	3.4357e-1 (2.36e-2)=	3.3907e-1 (3.97e-2)
	1000	4.8935e-1 (4.23e-4)-	2.8960e-1 (3.48e-2)+	9.3916e-1 (4.78e-2)-	7.3129e-1 (8.99e-2)-	5.2533e-1 (7.33e-3)-	3.4560e-1 (1.80e-2)=	3.4162e-1 (3.96e-2)
LSMOP9	200	1.1101e+0 (1.42e-1)-	6.5832e-1 (1.68e-1)-	1.8979e+0 (1.66e-1)-	1.5160e+0 (1.12e+0)-	3.8831e-1 (1.39e-2)+	1.4941e+0 (1.21e-1)-	6.0080e-1 (6.54e-3)
	500	1.2093e+0 (4.27e-2)-	1.0625e+0 (5.01e-2)-	4.4051e+0 (2.21e+0)-	1.0897e+2 (5.52e+1)-	4.0453e-1 (3.98e-2)+	1.4463e+0 (1.69e-1)-	5.9954e-1 (6.80e-3)
	1000	1.5400e+0 (8.91e-3)-	9.1503e-1 (6.35e-3)-	2.0675e+1 (1.57e+0)-	1.2147e+2 (4.87e+1)-	5.5751e-1 (8.41e-2)+	1.1711e+0 (9.97e-2)-	6.0855e-1 (1.99e-2)
+/-/=		6/17/1	2/21/1	2/22/0	3/21/0	4/18/2	5/17/2	

'+', '-' and '=' indicate that the result is significantly better, significantly worse and statistically similar to that obtained by iLSMOA, respectively.

TABLE IV
THE STATICS OF HV RESULTS ACHIEVED BY SEVEN COMPARED ALGORITHMS ON 24 TRI-OBJECTIVE TEST INSTANCES FROM LSMOP TEST SUITE.

Problem	D	IBEA	CMOPSO	GDE3	LMEA	MOEA/D-DE	LSMOF	iLSMOA
LSMOP1	200	5.3504e-1 (9.97e-2)+	1.7229e-1 (1.08e-1)-	2.8966e-1 (7.21e-2)-	5.4541e-1 (1.52e-1)+	3.3301e-1 (4.21e-2)-	6.0196e-1 (4.38e-3)+	4.2314e-1 (5.74e-2)
	500	4.5195e-1 (9.34e-2)+	7.0115e-2 (6.01e-2)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	4.8436e-5 (1.77e-4)-	5.7763e-1 (9.81e-3)+	3.8404e-1 (2.57e-2)
	1000	1.2532e-2 (3.23e-2)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	3.1644e-1 (2.85e-2)-	3.5781e-1 (2.18e-2)
LSMOP2	200	7.9339e-1 (5.66e-3)+	7.2646e-1 (1.79e-3)-	7.0807e-1 (3.41e-3)-	7.4130e-1 (1.06e-2)-	7.2766e-1 (2.85e-3)-	6.9845e-1 (5.01e-3)-	7.8136e-1 (4.13e-3)
	500	8.0749e-1 (1.28e-3)-	7.9704e-1 (3.21e-4)-	7.8207e-1 (3.06e-3)-	7.6918e-1 (1.37e-2)-	7.6665e-1 (9.37e-4)-	7.7118e-1 (3.94e-3)-	8.1610e-1 (1.42e-3)
	1000	8.1938e-1 (1.47e-3)-	8.1943e-1 (3.56e-4)-	8.0596e-1 (2.83e-3)-	7.8396e-1 (7.60e-3)-	7.8262e-1 (1.18e-3)-	7.9514e-1 (5.10e-3)-	8.2797e-1 (1.11e-3)
LSMOP4	200	7.4058e-1 (1.61e-2)+	4.8981e-1 (2.82e-2)-	3.9535e-1 (9.49e-3)-	6.2354e-1 (6.96e-2)+	5.0335e-1 (1.08e-2)-	5.2197e-1 (1.08e-2)-	5.9470e-1 (1.74e-2)
	500	7.6036e-1 (6.28e-3)+	6.3412e-1 (4.64e-3)-	6.1308e-1 (3.98e-3)-	5.8486e-1 (2.19e-2)-	6.5314e-1 (2.48e-3)-	6.0328e-1 (7.35e-3)-	7.3918e-1 (5.77e-3)
	1000	7.7837e-1 (4.30e-3)-	7.3479e-1 (1.45e-3)-	7.1367e-1 (3.67e-3)-	6.9882e-1 (1.51e-2)-	7.2351e-1 (1.52e-3)-	7.0373e-1 (6.51e-3)-	7.8375e-1 (3.04e-3)
LSMOP5	200	2.4855e-1 (1.35e-4)-	1.6381e-1 (5.45e-2)-	7.2946e-2 (1.42e-1)-	1.2598e-2 (2.13e-2)-	9.4708e-2 (6.40e-3)-	3.4636e-1 (2.11e-4)-	3.9438e-1 (8.27e-3)
	500	1.4869e-1 (4.79e-2)-	2.8324e-3 (5.49e-3)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	7.9513e-3 (2.30e-2)-	3.4635e-1 (2.53e-4)-	3.8433e-1 (3.20e-2)
	1000	1.6769e-4 (4.82e-4)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	3.4637e-1 (2.31e-4)-	3.5761e-1 (4.55e-2)
LSMOP6	200	1.1738e-2 (1.84e-2)+	0.0000e+0 (0.00e+0)=	0.0000e+0 (0.00e+0)=	0.0000e+0 (0.00e+0)=	1.1951e-4 (5.39e-4)=	2.2056e-2 (1.72e-2)+	6.6222e-6 (3.63e-5)
	500	0.0000e+0 (0.00e+0)=	0.0000e+0 (0.00e+0)=	0.0000e+0 (0.00e+0)=	0.0000e+0 (0.00e+0)=	0.0000e+0 (0.00e+0)=	1.7361e-2 (9.76e-3)+	1.2754e-4 (5.99e-4)
	1000	0.0000e+0 (0.00e+0)=	0.0000e+0 (0.00e+0)=	0.0000e+0 (0.00e+0)=	0.0000e+0 (0.00e+0)=	0.0000e+0 (0.00e+0)=	9.6670e-3 (6.66e-3)+	1.1670e-4 (6.39e-4)
LSMOP7	200	1.9447e-3 (1.07e-2)-	6.6533e-4 (3.34e-3)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	9.0818e-2 (3.30e-5)+	0.0000e+0 (0.00e+0)-	4.4418e-2 (2.73e-2)
	500	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	9.0581e-2 (7.75e-5)+	0.0000e+0 (0.00e+0)-	6.2597e-2 (1.86e-2)
	1000	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	6.1338e-2 (1.06e-2)-	0.0000e+0 (0.00e+0)-	7.5048e-2 (9.18e-3)
LSMOP8	200	2.5008e-1 (5.62e-2)-	2.6301e-1 (5.91e-2)-	3.0269e-1 (9.12e-3)-	3.0145e-1 (4.58e-2)-	2.8941e-1 (2.15e-2)-	3.6832e-1 (1.52e-2)-	3.8939e-1 (6.53e-3)
	500	1.8762e-1 (1.04e-2)-	2.8316e-1 (3.81e-2)-	2.5561e-1 (7.77e-2)-	3.7624e-2 (1.32e-2)-	8.4670e-2 (4.73e-3)-	3.1664e-1 (6.88e-2)-	3.9545e-1 (3.91e-3)
	1000	1.7715e-1 (2.74e-3)-	2.0457e-1 (3.74e-2)-	6.9951e-2 (4.27e-4)-	4.0528e-2 (2.30e-2)-	9.3089e-2 (4.44e-3)-	2.9546e-1 (7.27e-2)-	3.9711e-1 (2.58e-3)
LSMOP9	200	1.5042e-1 (1.12e-2)-	1.8068e-1 (1.20e-2)-	4.0445e-3 (2.92e-3)-	2.7140e-2 (1.49e-2)-	8.5088e-2 (4.58e-3)-	9.6974e-2 (1.73e-2)-	1.8707e-1 (1.30e-3)
	500	1.3524e-1 (8.15e-3)-	6.0020e-2 (1.07e-2)-	8.5330e-3 (1.60e-2)-	0.0000e+0 (0.00e+0)-	9.6036e-2 (1.85e-2)-	1.0414e-1 (2.44e-2)-	1.8701e-1 (2.15e-3)
	1000	7.6488e-2 (1.49e-3)-	9.3864e-2 (1.51e-3)-	0.0000e+0 (0.00e+0)-	0.0000e+0 (0.00e+0)-	6.3387e-2 (3.29e-3)-	1.4383e-1 (1.44e-2)-	1.8409e-1 (1.11e-2)
+/-/=		6/16/2	0/21/3	0/21/3	2/19/3	2/19/3	5/19/0	

'+', '-' and '=' indicate that the result is significantly better, significantly worse and statistically similar to that obtained by iLSMOA, respectively.

multiobjective optimization in an iterated manner. Despite its simplicity, the proposed algorithm has revealed the superiority of the iterated strategy over the static two-stage based strategy in LSMOF for large-scale multiobjective optimization. It could be attributed to the fact that the reformulation based optimization is greedy in the original LSMOF, thus leading to local optima. By contrast, in our proposed iLSMOA, the reference solutions for problem reformulation are dynamically updated, and thus the global search ability of the problem reformulation based optimization is enhanced. Moreover, since the computational complexity of iLSMOA is the same as that of LSMOF, the computation cost of iLSMOA is comparable to LSMOF. As indicated by the experimental results, the effectiveness and efficiency of iLSMOA are well balanced for solving large-scale multiobjective optimization problems.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (Grant No. 61903178, 61906081, 61906001, and U1804262), the Program for Guangdong Introducing Innovative and Entrepreneurial Teams (Grant No. 2017ZT07X386), the Shenzhen Peacock Plan (Grant No. KQTD2016112514355531), and the Program for University Key Laboratory of Guangdong Province (Grant No. 2017KSYS008).

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