Balancing the Influence of Evolutionary Operators for Global Optimization

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***Abstract*—The proper use of evolutionary operators is crucial to find optimal solutions in a search space. Moreover, the diversity of the population affects the performance of Evolutionary Algorithms (EAs). This article introduces an EA called BWEAD which balances the influence of the operators. The proposal also performs a statistical analysis of the population when the diversity is low and decides which solutions might be replaced. Then BWEAD is able to explore the search space and exploit the prominent regions. The BWEAD has been tested over the CEC2014 set of benchmark functions. The experiments provide competitive results showing an improvement of 30% in 30-dimensional and 50-dimensional functions in comparison with state-of-the-art algorithms, overcoming some addressed instances and providing evidence of its capabilities on complex optimization problems.**

***Index Terms*—Balancing operators, Diversity, Evolutionary Algorithms, Crossover, Mutation**

I. Introduction

Evolutionary algorithms (EAs) are widely used to solve complex optimization problems. The idea behind EA is to use operators inspired in an evolutionary process to explore the search space. No matter the operators, the EA have two main phases, exploration and exploitation [1, 2, 3]. In exploration the algorithm verifies the search space trying to detect the areas with more probabilities to find the optimal solution; meanwhile, the exploitation phase permits to locally analyze the specific regions of the search space. Some examples of evolutionary optimization methods are the Genetic Algorithms (GA) [4] and the Differential Evolution (DE) [5].

The GA and DE have two groups of operators, mutation and crossover. Such operators permit to explore and exploit the search spaces in optimization. Since they were proposed, GA and DE have attracted the attention of researchers from different fields and have been used in several applications such as optimal power flow solutions [6], Economic and Emission Dispatch [7], and prediction of continuous blood glucose [8].

Both DE and GA have shown their superior performance among other EAs in terms of robustness. However, the optimization process in GA and DE (as other EA) depends on different factors, such as random variables that affect their performance, and in some cases, the algorithms fall into suboptimal solutions. To overcome such problems, several enhanced versions of the EA have been proposed. For example, the GA has been combined with several methods to enhance their ability. Elaziz et al. [9] combined the Salp Swarm Algorithm with GA and used the modified algorithm to find the parameters of Adaptive Neuro-Fuzzy Inference System to improve the forecasting of the Crude Oil Price. In [10], GA is combined with neural networks to enhance the Airblast prediction. Meanwhile, Wodecki et al. [11] proposed a method to determine the local damage in a rolling bearing by using GA as a filter.

Regarding the use and modification of DE there exist several applications. For example, Hancer et al. [12] applied it to improve the performance of a dataset classification by using DE as a feature selection method. DE is also combined with a fuzzy wrapper–filter approach to form a feature selection method, as proposed in [13]. Tey et al. [14] applied DE to improve the accuracy of the photovoltaic arrays under partial shading conditions which have a different number of maximum power point. Since most of the conventional algorithms can not track the global maximum power point.

DE is also applied as a local search method to improve the performance of the moth-flame optimization (MFO) as in [15], and the developed method is used as a feature selection method providing competitive results in comparison with other methods.

One of the main problems in EA is to find the proper balance between the operators. In most of the cases, the balance depends on internal parameters (or probabilities) that need to be manually tuned. Another important situation that affects the EA is the diversity that is a property desired in the population of solutions. A low diversity means that the solutions are in the same region of the search space, and high diversity mean the solutions are dispersed along the search

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space.

Considering the above, different metrics to evaluate the diversity of the population in EA have been proposed. In terms of EA such metrics are called Genotypic Diversity Measures and most of them are based on distances [16]. On the other hand, entropy as a tool for information analysis is also used as a diversity metric. Here the entropy defines the degree of disorder in the population [17]. Both the balance between operators and the diversity are drawbacks whose in most of the cases are not commonly considered in the design and implementation of EAs.

This article proposes an evolutionary algorithm that balances the influence of operators and includes diversity to enhance the performance. The approach is called Balanced Weights Evolutionary Algorithm with Diversity (BWEAD) and it has been used to solve the CEC2014 benchmark functions. The BWEAD is designed over a GA background structure and considers the blend crossover (better known as BLX-α) and the mutation operation of DE. A solution is created by a weighted combination of the operators and the weights are balanced using trigonometric functions. Besides, diversity is verified using statistical analysis to decide which solutions can be used to compose the population.

The remainder of this paper is organized as follows. Section II briefly presented some related work related to diversity in EA. Section III describes in detail the proposed approach. Section IV describes the conducted experiments and discusses the results. Finally, conclusions and future directions are presented in Section V.

II. Backgorund

In this section, some related works which address the balance between convergence and diversity in EA are presented. Moreover, it is explored the diversity related to the use of the EA operators.

A. Balancing the operators in evolutionary algorithms

Several approaches have been developed to address exploration and exploitation in the search process [18]. The concept of diversity is extremely ligated to the proper use of the operators in EA. The convergence of an algorithm is then depended on how the solutions are moved in the search space; if all of them are together the algorithm starts to lose information and the convergence becomes premature [19].

To overcome the problems some authors have proposed different techniques to find the equilibrium in the EA optimization process. In [18] is proposed a memetic GA that finds the stability between exploration and exploitation by using probabilities and a penalty function evaluated by a learning automata. Meanwhile, in [20] is introduced an algorithm that balances the exploration and exploitation in DE. This method uses a linear equation that combines a different version of the DE mutation. The method also included a self-tuning mechanism based on the simulated annealing.

Another interesting modifications of DE are presented by Sharma et al. [21]. The authors proposed an enhanced version of DE with two modifications: one including a cognitive learning factor that permits to change the behavior of the algorithm depending on some rules; and other incorporating a dynamic parameter that permits to choose the rules according to the iterations.

Such approaches do not consider the influence of the operators individually because they try to find an equilibrium in the optimization phases. Meanwhile, in the proposed BWEAD, the weights represent the amount of contribution of each operator to create a new solution.

B. Diversity in evolutionary algorithms

The diversity of the population can affect the performance of EA [22]. In general, high diversity values are required to improve the ability of the EA to avoid local optima and premature convergence - enhancing exploration.

Some authors propose different diversity measures that have been applied to guide the EA. The author in [23] developed a diversity guided evolutionary algorithm (DGEA) which applies a measure called disruption operator (DO) to switch between the exploration and exploitation phases. Ginley et al. [24] proposed an improved GA, called ACROMUSE, by adapting the crossover, mutation, and selection parameters by introducing two diversity measures: the standard population diversity (SPD) which controls the crossover and mutation parameters; and the healthy population diversity (HPD) which guides the selection pressure.

Regarding DE approaches, Coelho et al. [25] improved the performance of the DE algorithm using the diversity measure in combination with a cultural algorithm. This method is applied to the economic load dispatch problems of thermal generators.

More attractive diversity measures have been developed such as the metric that depends on the diameter of the population (the distance between the average solution and the solution farthest away from this measure), called the radius of the population [16]. However, most of them suffer from the computational time since it depends on pairwise measurements (i.e., $O(n.N^2)$, with n represents the dimension and N the population size). To address this issue, Wineberg and Oppacher [26] developed the true diversity measure which reduces the complexity (i.e., $O(n.N)$). The true diversity measure refers to the mean of the standard deviation of each gene. This paper employs the true diversity to decide whether is necessary to include exploitation in the search process. However, the creation of new solutions is not arbitrary and different from the other EA, the BWEAD performs a statistical analysis of the population to decide which solutions need to be replaced.

III. Proposed Approach

In this work, an algorithm called Balanced Weights Evolutionary Algorithm with Diversity (BWEAD) is proposed, which is based on the standard GA and aims to obtain better results for the global optimization on benchmark functions. Mainly, the modifications made are found in the evolutionary operators (crossover and mutation) and is added a stage of a balance of the operators' influence weights using a defined equation with a memory of the best weights. Also, a diversity stage is added to verify whether the algorithm does not get stuck in local optima by eliminating members of duplicate populations.

Figure 1 illustrates the flowchart of BWEAD. First, an initial population is generated, and the fitness value is calculated for each individual in the population. Then, parents are chosen at random, to whom the mutation and crossover operations are applied by recombining them to have better solutions. Next, the equation's weights of the operators' sum are adjusted to have better fitness than the worst fitness value stored through a weights selection method. Finally, the diversity stage is carried out to assess the diversity of the positions of all individuals in the population. If this diversity of the population (D_{TD}^{N}) is less than a defined percentage of diversity, only the positions of individuals with lower fitness values than the first quartile will be maintained. BWEAD stops when the stop criterion is satisfied.

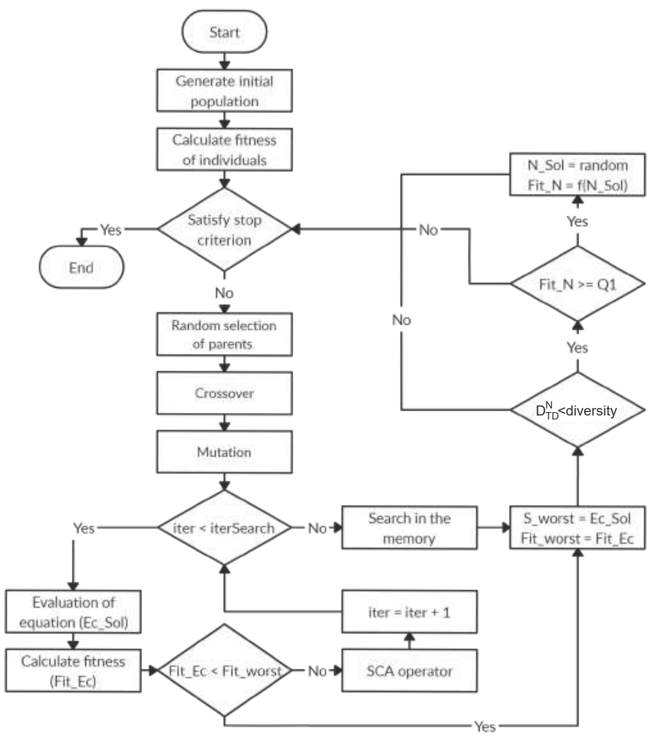


Fig. 1: The flowchart of BWEAD.

A. Initialization

BWEAD uses a random initialization of the population, and each individual is evaluated according to the specific objective functions to calculate the fitness values. The first weights (w_1, w_2) are randomly initialized with numbers between 0 and 1. Subsequently, three randomly parents are chosen to undergo to the evolutionary operators.

B. Evolutionary Operators

Once the three parents were randomly selected, the crossover is applied over two parents and it results in a first candidate solution, while the mutation is applied to the three parents resulting in a second candidate solution. In the BWEAD are used the operators following explained.

The blend crossover operator (BLX-α) performs the recombination of the two randomly parents from the population $(\mathbf{X^1}, \mathbf{X^2})$ generating one child individual $(\mathbf{x^c})$ [27]. A value of each element x_i^c, $i = 1, 2, \ldots, D$ of the offspring vector $\mathbf{x^c}$ with the D being the number of dimensions, is defined by the following uniform distribution:

$$x_i^c = (1 - \gamma_i)X_i^1 + \gamma_i X_i^2 \tag{1}$$

where

$$\gamma_i = (1 + 2\alpha)u_i - \alpha \tag{2}$$

Thus, u_i is a random number between 0 and 1, and α is a positive parameter to control the search domain, and it is usually set to 0.5 [28].

The DE mutation operator (DE/rand/1) creates a mutant offspring vector $\mathbf{x^m}$ from the three selected parents $(\mathbf{X^1}, \mathbf{X^2}, \mathbf{X^3})$ [5]. This vector is formed by perturbing a randomly selected parent vector $\mathbf{Xr^1}$ with the difference of two other randomly selected parent vectors $\mathbf{Xr^2}$ and $\mathbf{Xr^3}$ by:

$$\mathbf{x^m} = \mathbf{Xr^1} + F(\mathbf{Xr^2} - \mathbf{Xr^3}) \tag{3}$$

where the F is randomly a scaling factor between 0.2 and 0.8.

C. Balance Influence

The balance weights is defined by the following equation:

$$Ec\_Sol = w_1\mathbf{x^c} + w_2\mathbf{x^m} \tag{4}$$

where w_1 and w_2 are the weights of the operators for the proportions of mutation and crossover to obtain the new candidate solution (Ec_Sol).

This candidate's fitness value is obtained (Fit_Ec) and compared to the worst fitness value (Fit_worst) of the initial population. If the Fit_Ec is smaller than the Fit_worst then the solution of the worst fitness value (S_worst) will be equal to Ec_Sol and Fit_worst will be equal to Fit_Ec. On the other hand, if the Fit_Ec is higher than the Fit_worst then, the weights (w_1,w_2) are adjusted through the weight selection method.

This method aims to modify the weights by using an operator based on trigonometric functions taken from [29]. This process is performed until the candidate solution whose fitness is better than Fit_worst is obtained or until the maximum number of search iterations ($iter\_Search$) is achieved. If the fitness of the candidate solution is not enhanced, the last five weights in the memory are tested in Ec_Sol to search for a fitness better than Fit_worst.

D. Diversity

True diversity normalized (D_{TD}^{N}) is used to measure the diversity of the population [16, 26]. This diversity represents the average standard deviation of each individual and is given by the expression:

$$D_{TD}^{N} = \frac{\frac{1}{n}\sqrt{\sum_{k=1}^{n}(\overline{x_k^2} - (\overline{x_k})^2)}}{NMDF} \tag{5}$$

where

$$\overline{x_k^2} = \frac{1}{N}\sum_{i=1}^{N} x_{i,k}^2 \tag{6}$$

In Equation 5, $\overline{x_k}$ is the average individual value with n being the dimensionality of each individual, while $\overline{x_k^2}$ is the average of the individual value squared with N being the number of the individuals of the population. $NMDF$ is normalization with maximum diversity so far and it represents the maximum diversity obtained in the initialization of the population.

If the D_{TD}^{N} is smaller or equal to the 25% of the diversity in the population, the positions of individuals with a fitness value less than the first quartile (Q_1) will be replaced by random positions within the search space. In other words, only the 75% of the population is randomly modified. With the use of diversity the BWEAD reforces the interchange between exploration and exploitation. It means that when the diversity is low the algorithm was exploiting and needs more exploration.

IV. Experiments and Results

This section aims to compare the results of BWEAD with Differential Evolution (DE) [5], Particle Swarm Optimization (PSO) [30], Genetic Algorithms (GA) [4], Simulated Annealing (SA) [31], and Stochastic Fractal Search (SFS) [32] to solve CEC2014 benchmark functions in 30 and 50 dimensions (D). It is made up of 30 minimization problems with an established search range of $[-100, 100]^D$, which comprise three unimodal functions (F1-F3), thirteen simple multimodal functions (F4-F16), six hybrid functions (F17-F22), and eight composition functions (F23-F30) [33]. These benchmark problems are specially designed with novel basic problems, the composition of problems through the extraction of characteristics, problems of rotated and scaled trap, etc.

The parameters configured for the proposed algorithm BWEAD are for the $iter\_Search$ variable of 100 iterations and the minimum diversity percentage of 25%. Also, the stop criteria established for all the algorithms to be compared are 50,000 function accesses for 30-dimensional tests, and 80,000 for 50-dimensional tests.

The Shapiro-Wilk normality test [34] is applied to verify whether results are normally distributed. All instances show a non-normal distribution, therefore the Kruskal-Wallis [35] and Dunn-Sidak's post-hoc tests are applied for statistical analysis, considering all approaches being compared using their objectives values from all the 35 runs. All tests have been executed with a confidence level of 95% ($\alpha = 5\%$).

Tables I and II show the average and the corresponding standard deviation subscribed of the objective values over 35 executions for instances with 30 and 50 dimension, respectively. The best values are highlighted in bold and results with no statistically significant differences with the best values are emphasized in light blue for each instance.

From Tables I and II it can be seen that BWEAD are always similar or better than the other approaches. The results for BWEAD are better in 30% of the instances with 30-dimension and also 30% of the instances with 50-dimension (in bold), and, for the remaining instances, there are no statistically significant differences among the approaches, especially among DE and PSO.

Figures 2 and 3 show, for a particular run considering instances with 50 dimension[1], the best score of objective values along the evolutionary process until the maximum number of function evaluation is achieved. One curve for each algorithm is considered. It is possible to observe that the BWEAD (green triangle) can reach the minimum values using a small number of function evaluations. These results corroborate with the comparison obtained in Table II.

V. Conclusions

This paper proposed an evolutionary algorithm based on diversity to improve the performance of the search process. The approach, called Balanced Weights Evolutionary Algorithm with Diversity (BWEAD), is modeled using a GA-based framework and it balances the use of crossover and mutation operators by using trigonometric functions. The diversity is also verified using the true diversity metric to decide whether to replace individuals.

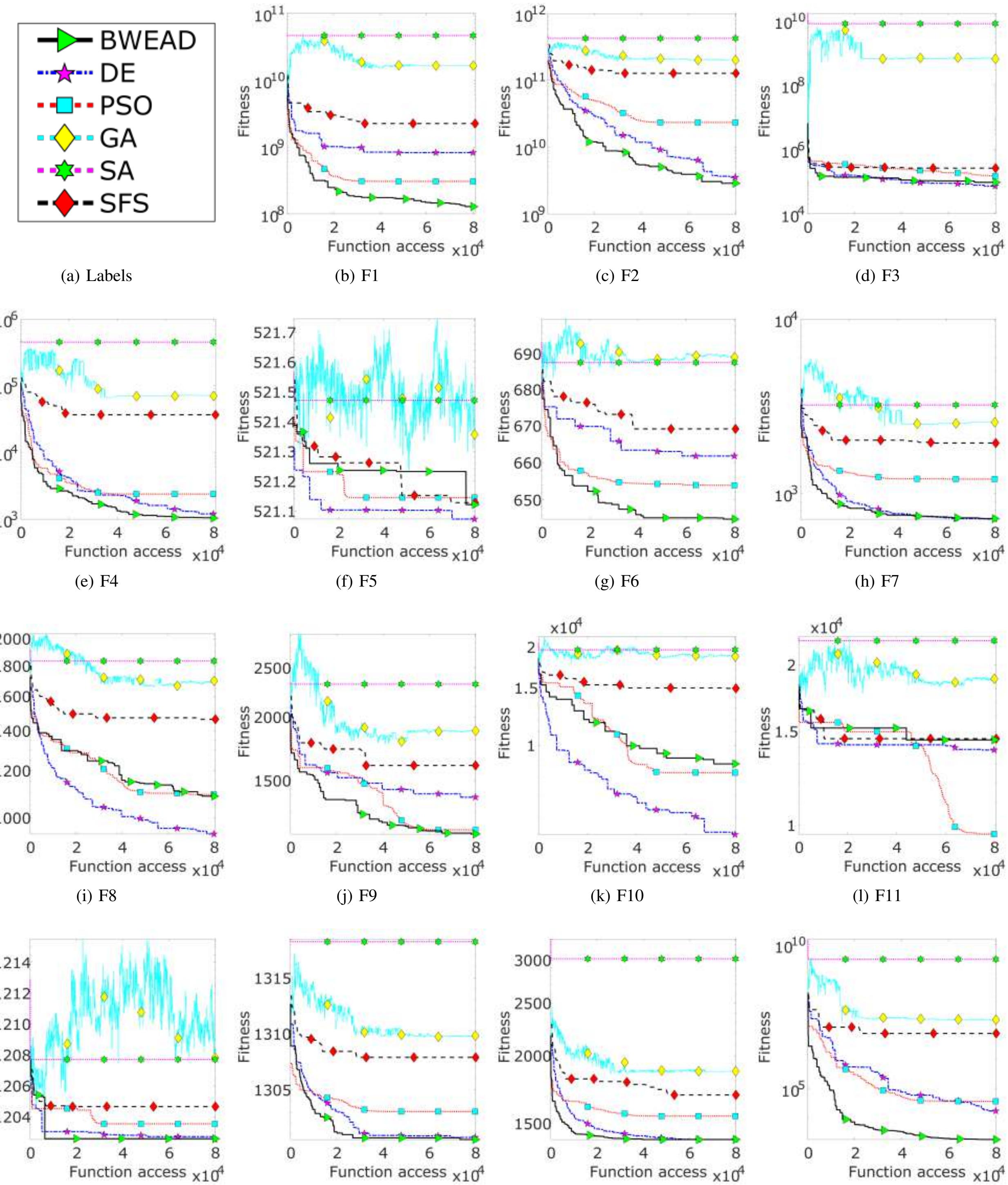
The BWEAD approach is compared using statistical tests with DE, PSO, GA, SA, and SFS to solve CEC2014 benchmark functions. BWEAD showed better results in 30% of the instances with 30-dimension and 50-dimension, presenting, for the remaining instances, no statistically significant differences among the approaches, specially among DE and PSO.

Besides the statistical tests the convergence curves were plot, showing that BWEAD can reach the minimum values using a few amount of function evaluations.

There are some limitations in the proposed algorithm, one is that only two operators are used, and the other is that one of them contains a fixed hyperparameter (α).

There are some limitations in the proposed algorithm, one is that only two evolutionary operators are used, and the other is that one of them contains a fixed hyperparameter (α). In future directions, we are planning to explore the use of reinforcement learning strategies to modify operator weights and also add more evolutionary operators by selecting them through hyperheuristics. Finally, we are going to test the approaches to more complex and real optimization problems.

[1]The convergence curves for the 30 dimension present some advantages for BWEAD as well, however they were suppressed due to the space limitation.



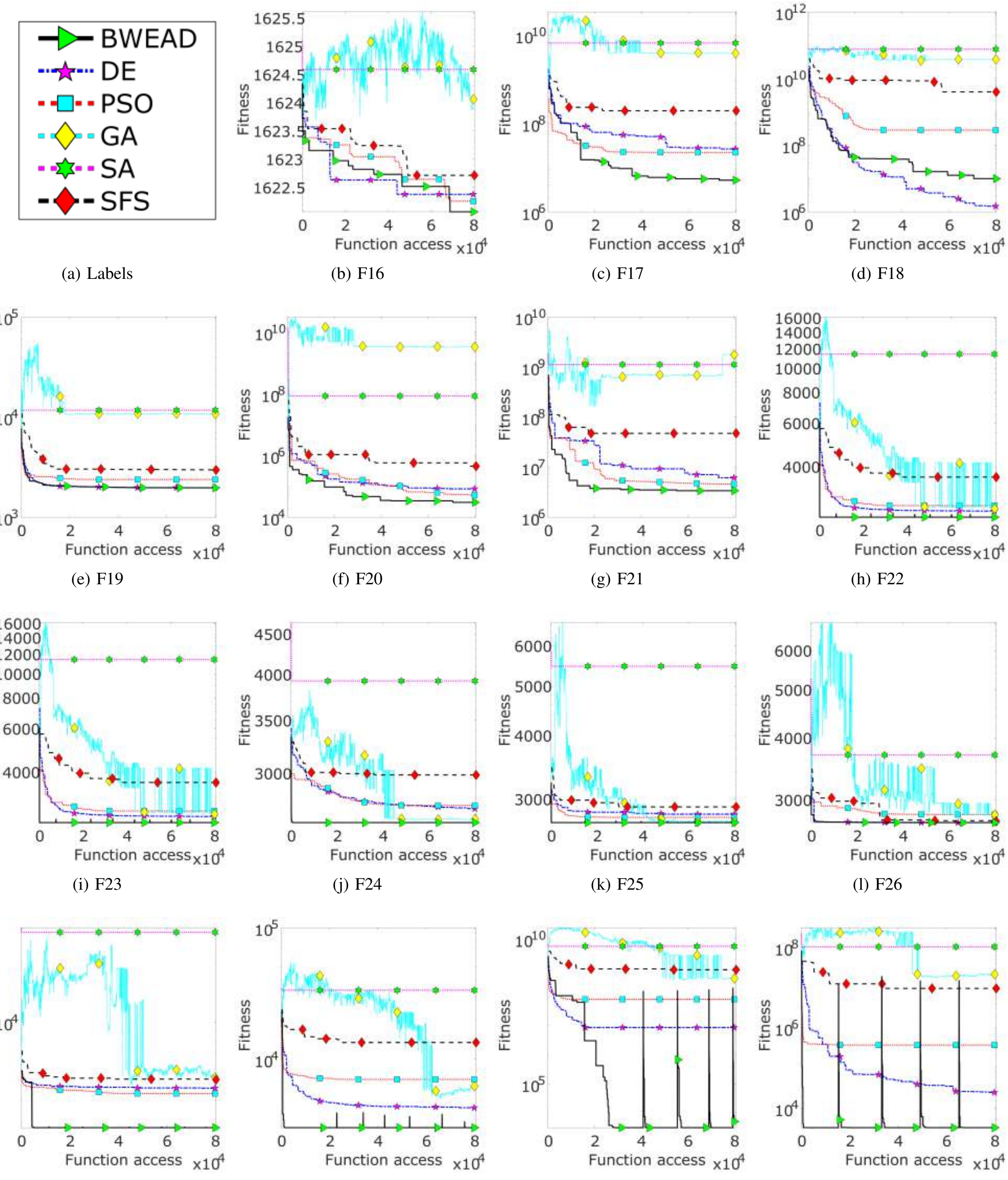
(a) Labels (b) F1 (c) F2 (d) F3

(e) F4 (f) F5 (g) F6 (h) F7

(i) F8 (j) F9 (k) F10 (l) F11

(m) F12 (n) F13 (o) F14 (p) F15

Fig. 2: Convergence



(a) Labels (b) F16 (c) F17 (d) F18

(e) F19 (f) F20 (g) F21 (h) F22

(i) F23 (j) F24 (k) F25 (l) F26

(m) F27 (n) F28 (o) F29 (p) F30

Fig. 3: Convergence

TABLE I: Average and standard deviation results for 30-dimension instances.

Instance	BWEAD	DE	PSO	GA	SFS	SA
F1	$8.17E+07_{3.06E+07}$	$1.60E+08_{3.40E+07}$	$\mathbf{6.19E+07_{4.15E+07}}$	$1.96E+09_{3.90E+08}$	$7.68E+08_{1.58E+08}$	$2.00E+10_{1.02E+10}$
F2	$7.98E+08_{3.65E+08}$	$\mathbf{4.67E+07_{9.82E+06}}$	$7.75E+09_{3.41E+09}$	$9.09E+10_{9.73E+09}$	$5.16E+10_{5.91E+09}$	$2.65E+11_{7.12E+10}$
F3	$4.21E+04_{7.81E+03}$	$\mathbf{3.09E+03_{8.24E+02}}$	$2.70E+04_{2.12E+04}$	$1.41E+06_{1.87E+06}$	$1.32E+05_{1.55E+04}$	$1.23E+09_{2.13E+09}$
F4	$\mathbf{7.02E+02_{5.81E+01}}$	$7.11E+02_{4.10E+01}$	$1.05E+03_{5.99E+02}$	$2.05E+04_{4.16E+03}$	$7.72E+03_{1.31E+03}$	$1.24E+05_{4.69E+04}$
F5	$5.21E+02_{6.09E-02}$	$\mathbf{5.21E+02_{5.70E-02}}$	$5.21E+02_{8.21E-02}$	$5.21E+02_{5.79E-02}$	$5.21E+02_{5.79E-02}$	$5.21E+02_{8.58E-02}$
F6	$6.22E+02_{2.62E+00}$	$6.32E+02_{1.30E+00}$	$\mathbf{6.19E+02_{3.17E+00}}$	$6.44E+02_{1.93E+00}$	$6.40E+02_{1.02E+00}$	$6.56E+02_{4.47E+00}$
F7	$7.10E+02_{3.83E+00}$	$7.01E+02_{3.64E-02}$	$7.84E+02_{2.80E+01}$	$1.61E+03_{1.22E+02}$	$1.16E+03_{6.12E+01}$	$2.90E+03_{7.24E+02}$
F8	$9.86E+02_{3.24E+01}$	$\mathbf{8.26E+02_{2.42E+00}}$	$8.82E+02_{2.45E+01}$	$1.26E+03_{4.25E+01}$	$1.13E+03_{1.38E+01}$	$1.50E+03_{1.43E+02}$
F9	$1.14E+03_{2.96E+01}$	$1.13E+03_{1.40E+01}$	$\mathbf{1.02E+03_{2.66E+01}}$	$1.33E+03_{2.58E+01}$	$1.28E+03_{1.98E+01}$	$1.81E+03_{1.67E+02}$
F10	$5.87E+03_{1.08E+03}$	$\mathbf{1.68E+03_{1.11E+02}}$	$3.97E+03_{7.98E+02}$	$9.83E+03_{5.10E+02}$	$8.24E+03_{2.80E+02}$	$1.17E+04_{7.91E+02}$
F11	$8.34E+03_{2.96E+02}$	$7.11E+03_{2.59E+02}$	$\mathbf{4.81E+03_{5.68E+02}}$	$9.49E+03_{5.59E+02}$	$8.72E+03_{3.38E+02}$	$1.23E+04_{9.78E+02}$
F12	$1.20E+03_{3.80E-01}$	$1.20E+03_{1.88E-01}$	$1.20E+03_{8.79E-01}$	$1.20E+03_{7.73E-01}$	$1.20E+03_{3.84E-01}$	$1.21E+03_{1.98E+00}$
F13	$1.30E+03_{1.25E-01}$	$1.30E+03_{6.09E-02}$	$1.30E+03_{9.33E-01}$	$1.31E+03_{7.92E-01}$	$1.31E+03_{4.49E-01}$	$1.32E+03_{3.12E+00}$
F14	$1.40E+03_{1.13E+00}$	$1.40E+03_{8.65E-02}$	$1.43E+03_{1.97E+01}$	$1.71E+03_{5.82E+01}$	$1.56E+03_{1.82E+01}$	$2.13E+03_{1.64E+02}$
F15	$\mathbf{1.54E+03_{1.24E+01}}$	$1.56E+03_{1.61E+01}$	$2.13E+03_{1.05E+03}$	$7.52E+05_{1.24E+05}$	$5.83E+05_{2.27E+05}$	$3.71E+08_{3.84E+08}$
F16	$1.61E+03_{3.66E-01}$	$1.61E+03_{3.38E-01}$	$1.61E+03_{5.92E-01}$	$1.61E+03_{2.58E-01}$	$1.61E+03_{1.91E-01}$	$1.61E+03_{2.22E-01}$
F17	$\mathbf{2.64E+06_{2.53E+06}}$	$6.94E+06_{2.85E+06}$	$3.48E+06_{3.53E+06}$	$2.15E+08_{8.92E+07}$	$2.85E+07_{1.06E+07}$	$2.91E+09_{2.46E+09}$
F18	$3.43E+06_{3.40E+06}$	$\mathbf{1.07E+06_{5.29E+05}}$	$3.19E+07_{1.13E+08}$	$7.95E+09_{2.73E+09}$	$1.06E+09_{3.36E+08}$	$3.92E+10_{1.80E+10}$
F19	$1.94E+03_{2.02E+01}$	$\mathbf{1.92E+03_{3.50E+00}}$	$1.93E+03_{3.04E+01}$	$2.64E+03_{1.82E+02}$	$2.16E+03_{3.48E+01}$	$8.75E+03_{3.46E+03}$
F20	$3.78E+04_{1.38E+04}$	$\mathbf{1.40E+04_{4.95E+03}}$	$2.42E+04_{2.11E+04}$	$1.34E+07_{2.12E+07}$	$1.67E+05_{7.11E+04}$	$1.95E+09_{4.34E+09}$
F21	$7.67E+05_{6.77E+05}$	$1.53E+06_{7.42E+05}$	$\mathbf{7.12E+05_{4.09E+05}}$	$1.11E+08_{6.22E+07}$	$1.01E+07_{4.60E+06}$	$2.60E+09_{3.06E+09}$
F22	$2.89E+03_{2.05E+02}$	$\mathbf{2.68E+03_{1.01E+02}}$	$2.71E+03_{2.49E+02}$	$7.37E+04_{1.01E+05}$	$3.50E+03_{1.96E+02}$	$5.00E+07_{1.19E+08}$
F23	$\mathbf{2.50E+03_{0.00E+00}}$	$2.62E+03_{8.30E-01}$	$2.65E+03_{2.00E+01}$	$2.62E+03_{1.01E+01}$	$2.93E+03_{5.84E+01}$	$9.18E+03_{3.56E+03}$
F24	$\mathbf{2.60E+03_{4.13E-07}}$	$2.64E+03_{2.61E+00}$	$2.64E+03_{5.29E+00}$	$2.61E+03_{1.20E+00}$	$2.76E+03_{1.11E+01}$	$3.51E+03_{1.19E+03}$
F25	$\mathbf{2.70E+03_{0.00E+00}}$	$2.73E+03_{4.33E+00}$	$2.71E+03_{4.00E+00}$	$2.70E+03_{1.66E-01}$	$2.77E+03_{8.44E+00}$	$3.54E+03_{3.89E+02}$
F26	$\mathbf{2.70E+03_{9.95E-02}}$	$2.70E+03_{7.57E-02}$	$2.71E+03_{2.74E+01}$	$2.77E+03_{3.73E+01}$	$2.71E+03_{7.27E-01}$	$3.85E+03_{4.42E+02}$
F27	$\mathbf{2.90E+03_{0.00E+00}}$	$3.54E+03_{1.39E+02}$	$3.55E+03_{2.95E+02}$	$3.82E+03_{1.96E+02}$	$3.80E+03_{1.62E+02}$	$1.09E+04_{7.00E+03}$
F28	$\mathbf{3.00E+03_{9.48E-14}}$	$3.76E+03_{2.81E+01}$	$4.66E+03_{4.14E+02}$	$3.89E+03_{9.91E+01}$	$8.06E+03_{3.53E+02}$	$1.94E+04_{7.29E+03}$
F29	$\mathbf{3.12E+03_{1.06E+02}}$	$4.37E+04_{2.40E+04}$	$1.30E+07_{4.76E+06}$	$1.43E+08_{8.60E+06}$	$1.47E+08_{5.15E+07}$	$2.66E+09_{1.95E+09}$
F30	$7.39E+04_{6.60E+04}$	$\mathbf{1.56E+04_{1.72E+03}}$	$1.85E+05_{1.63E+05}$	$7.91E+06_{2.53E+06}$	$1.08E+06_{2.82E+05}$	$1.32E+08_{2.14E+08}$

TABLE II: Average and standard deviation results for 50-dimension instances.

Instance	BWEAD	DE	PSO	GA	SFS	SA
F1	$\mathbf{1.79E+08_{5.16E+07}}$	$6.96E+08_{7.17E+07}$	$3.64E+08_{3.05E+08}$	$7.60E+09_{2.24E+09}$	$3.20E+10_{1.31E+10}$	$2.24E+09_{2.73E+08}$
F2	$3.43E+09_{1.26E+09}$	$\mathbf{3.25E+09_{4.55E+08}}$	$3.23E+10_{1.27E+10}$	$1.88E+11_{9.37E+09}$	$4.62E+11_{1.11E+11}$	$1.25E+11_{9.71E+09}$
F3	$1.01E+05_{8.32E+03}$	$\mathbf{6.14E+04_{7.32E+03}}$	$8.24E+04_{3.57E+04}$	$7.36E+05_{7.23E+05}$	$3.30E+09_{4.86E+09}$	$2.27E+05_{3.38E+04}$
F4	$1.13E+03_{1.40E+02}$	$\mathbf{1.05E+03_{8.39E+01}}$	$3.07E+03_{1.42E+03}$	$6.44E+04_{6.52E+03}$	$3.16E+05_{1.72E+05}$	$3.16E+04_{3.71E+03}$
F5	$5.21E+02_{4.71E-02}$	$5.21E+02_{4.01E-02}$	$5.21E+02_{5.76E-02}$	$5.21E+02_{4.96E-02}$	$5.21E+02_{1.04E-01}$	$5.21E+02_{3.47E-02}$
F6	$6.46E+02_{4.76E+00}$	$6.64E+02_{1.44E+00}$	$\mathbf{6.43E+02_{5.46E+00}}$	$6.80E+02_{2.72E+00}$	$6.95E+02_{3.73E+00}$	$6.72E+02_{1.43E+00}$
F7	$7.37E+02_{1.69E+01}$	$\mathbf{7.11E+02_{1.24E+00}}$	$9.72E+02_{9.62E+01}$	$2.46E+03_{7.19E+01}$	$4.74E+03_{9.34E+02}$	$1.94E+03_{8.99E+01}$
F8	$1.15E+03_{6.65E+01}$	$\mathbf{9.34E+02_{6.77E+00}}$	$1.01E+03_{3.90E+01}$	$1.64E+03_{2.39E+01}$	$1.98E+03_{1.29E+02}$	$1.46E+03_{1.83E+01}$
F9	$1.31E+03_{5.57E+01}$	$1.39E+03_{1.39E+01}$	$\mathbf{1.19E+03_{5.93E+01}}$	$1.78E+03_{6.12E+01}$	$2.60E+03_{3.05E+02}$	$1.68E+03_{2.74E+01}$
F10	$1.04E+04_{1.12E+03}$	$\mathbf{5.07E+03_{4.21E+02}}$	$7.74E+03_{1.10E+03}$	$1.69E+04_{6.75E+02}$	$1.94E+04_{1.12E+03}$	$1.46E+04_{3.43E+02}$
F11	$1.49E+04_{3.58E+02}$	$1.32E+04_{5.47E+02}$	$\mathbf{8.32E+03_{1.07E+03}}$	$1.64E+04_{5.86E+02}$	$2.00E+04_{1.20E+03}$	$1.53E+04_{3.57E+02}$
F12	$1.20E+03_{4.27E-01}$	$1.20E+03_{2.68E-01}$	$1.20E+03_{1.23E+00}$	$1.20E+03_{7.84E-01}$	$1.21E+03_{1.43E+00}$	$1.20E+03_{4.16E-01}$
F13	$1.30E+03_{1.44E-01}$	$1.30E+03_{7.08E-02}$	$1.30E+03_{9.53E-01}$	$1.31E+03_{3.73E-01}$	$1.32E+03_{2.02E+00}$	$1.31E+03_{3.34E-01}$
F14	$1.40E+03_{4.21E+00}$	$1.40E+03_{2.41E-01}$	$1.48E+03_{3.18E+01}$	$1.84E+03_{2.53E+01}$	$2.49E+03_{2.19E+02}$	$1.72E+03_{2.37E+01}$
F15	$\mathbf{2.52E+03_{9.72E+02}}$	$9.89E+03_{3.27E+03}$	$5.83E+04_{9.26E+04}$	$1.64E+07_{5.78E+06}$	$3.42E+09_{4.55E+09}$	$6.28E+06_{1.83E+06}$
F16	$1.62E+03_{3.78E-01}$	$1.62E+03_{2.32E-01}$	$1.62E+03_{5.06E-01}$	$1.62E+03_{2.04E-01}$	$1.62E+03_{3.02E-01}$	$1.62E+03_{1.53E-01}$
F17	$\mathbf{1.34E+07_{7.75E+06}}$	$3.32E+07_{6.66E+06}$	$1.89E+07_{1.71E+07}$	$1.01E+09_{4.24E+08}$	$6.89E+09_{4.72E+09}$	$1.68E+08_{5.20E+07}$
F18	$2.73E+07_{1.82E+07}$	$\mathbf{9.31E+05_{3.58E+05}}$	$6.05E+08_{4.65E+08}$	$2.67E+10_{6.17E+09}$	$7.75E+10_{2.71E+10}$	$5.88E+09_{1.05E+09}$
F19	$1.99E+03_{2.36E+01}$	$\mathbf{1.96E+03_{5.87E+00}}$	$2.11E+03_{1.26E+02}$	$6.56E+03_{1.56E+03}$	$3.49E+04_{2.17E+04}$	$2.79E+03_{1.25E+02}$
F20	$\mathbf{4.06E+04_{1.13E+04}}$	$4.21E+04_{1.36E+04}$	$6.01E+04_{3.02E+04}$	$2.47E+07_{4.62E+07}$	$2.18E+09_{5.15E+09}$	$3.20E+05_{1.24E+05}$
F21	$\mathbf{3.64E+06_{1.71E+06}}$	$1.15E+07_{4.59E+06}$	$6.59E+06_{8.55E+06}$	$3.51E+08_{2.13E+08}$	$4.49E+09_{3.62E+09}$	$3.88E+07_{1.29E+07}$
F22	$4.01E+03_{3.09E+02}$	$\mathbf{3.59E+03_{1.71E+02}}$	$3.87E+03_{3.90E+02}$	$4.52E+05_{4.80E+05}$	$1.04E+08_{1.03E+08}$	$6.42E+03_{6.33E+02}$
F23	$\mathbf{2.50E+03_{0.00E+00}}$	$2.65E+03_{5.25E-01}$	$2.79E+03_{7.52E+01}$	$2.71E+03_{1.65E+01}$	$1.33E+04_{4.84E+03}$	$3.57E+03_{1.28E+02}$
F24	$\mathbf{2.60E+03_{1.39E-06}}$	$2.71E+03_{2.57E+00}$	$2.72E+03_{2.68E+01}$	$2.62E+03_{2.01E+00}$	$4.18E+03_{6.48E+02}$	$2.97E+03_{1.78E+01}$
F25	$2.70E+03_{0.00E+00}$	$2.80E+03_{9.28E+00}$	$2.73E+03_{1.18E+01}$	$2.70E+03_{4.26E-01}$	$4.30E+03_{8.58E+02}$	$2.89E+03_{2.24E+01}$
F26	$2.70E+03_{6.39E-01}$	$2.70E+03_{4.77E-01}$	$2.76E+03_{5.57E+01}$	$2.80E+03_{1.61E+01}$	$5.35E+03_{1.68E+03}$	$2.81E+03_{7.78E+01}$
F27	$\mathbf{2.90E+03_{1.39E-12}}$	$4.54E+03_{3.98E+01}$	$4.44E+03_{1.50E+02}$	$5.28E+03_{4.03E+02}$	$1.98E+04_{1.24E+04}$	$5.04E+03_{3.58E+01}$
F28	$\mathbf{3.00E+03_{1.39E-12}}$	$4.22E+03_{3.15E+01}$	$6.74E+03_{9.09E+02}$	$5.39E+03_{3.66E+02}$	$3.58E+04_{9.45E+03}$	$1.44E+04_{5.87E+02}$
F29	$\mathbf{3.10E+03_{0.00E+00}}$	$2.49E+07_{1.10E+07}$	$1.25E+08_{8.03E+07}$	$3.80E+08_{2.49E+07}$	$6.02E+09_{2.38E+09}$	$9.15E+08_{8.33E+07}$
F30	$1.06E+05_{1.86E+05}$	$\mathbf{3.43E+04_{8.31E+03}}$	$8.76E+05_{6.43E+05}$	$1.65E+07_{1.26E+06}$	$2.43E+08_{1.62E+08}$	$9.88E+06_{2.26E+06}$

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