

PSO-based Optimal Formation of Multiple Biomimetic Underwater Vehicles

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Abstract—This paper aims to investigate optimal formation solutions of multiple biomimetic underwater vehicles (BUVs). The BUV is propelled by undulatory fins on both sides, and can perform various locomotion patterns, especially turning in situ and diving vertically. Firstly, the optimal formation problem is formulated, followed by theoretical analysis of a special case of optimal line formation. Then, a solution is proposed from the perspective of evolutionary computation. In particular, the coordinates and the slope of the desired line formation, together with the pairings between initial positions and target positions, are obtained based on particle swarm optimization. Furthermore, we demonstrate the validity of this method by comparing the simulation results with the results of theoretical analysis. Finally, simulations results of multiple BUVs verify the feasibility of the proposed optimal formation methods.

Index Terms—Biomimetic underwater vehicle, undulatory propulsion, line formation, particle swarm optimization, evolutionary computation.

I. INTRODUCTION

Biomimetic underwater vehicles (BUVs) have the advantages of high efficiency, stability, high maneuverability, adaptability to complex turbulence, and good concealment. They have broad application prospects in the civil and military fields [1]. In recent years, with the development and integration of biological sciences, mechanical sciences, control sciences, robotics, researchers and engineers developed a variety of

BUVs [2]–[6]. Although it has good motion performance, a single BUV has limited capabilities in terms of information acquisition and processing. When facing some large and complex marine tasks that need to be efficiently performed in parallel, multiple BUVs are often required to cooperate in a formation.

There are a few studies that have focused on coordinated planning and control of multiple BUVs. Ryuh et al. developed a multi-agent system composed of multiple autonomous robotic fish, which were used to monitor and cover a large scale sea coast cooperatively [7]. Yu et al. designed a coordination control system of multiple biomimetic robotic fish. Experiments on 2vs2 water polo game are carried out to verify the feasibility of the proposed control scheme [8]. However, the formation control of BUVs is rarely studied or implemented, probably due to high complexity of the underwater environment and technical bottlenecks of underwater communication and positioning. Formation of multiple BUVs will be a challenging research direction.

Over the past few years, formation problem has been a hot research area and receives a lot of attention from worldwide researchers. Hubbard et al. considered an individual based, discrete and stochastic model for the formation of fish [9]. Dorfler et al. conducted global stability analysis by a differential geometric approach considering invariant manifolds and their local stability properties [10]. Li et al. proposed a model-based control law to achieve multilayer formation [11]. Ge et al. designed a cluster formation method of multi-agent systems under aperiodic sampling and communication delays

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[12]. But this method is relatively complicated and not easy to be implemented.

On the other hand, formation problem can be seen as a typical optimization problem which may include one or more objectives, which is difficult to obtain precise analytic solutions. This paper conducts preliminary research on the optimal formation of BUVs from the perspective of evolutionary computation. We build the mathematical model of the optimal formation and analyze a special case of optimal line formation problem using the partial derivative method. But in this way, it is hard to obtain the optimal solution when the formation includes more than three BUVs. Then, we propose an optimal formation solution based on particle swarm optimization (PSO), obtaining target positions of the optimal formation, and solving the pairing problem between initial positions and target positions. The comparative study demonstrates the effectiveness of the proposed method.

In the remainder of this paper, the modeling of the formation problem of BUVs are described in Section II. Some theoretical results of a special case are also presented in this section. Section III details the proposed solutions of the optimal formation problem. Simulations results are provided in Section IV. Finally, the conclusion and future work are summarized in Section V.

II. FORMATION PROBLEM

In this section, we briefly introduce the biomimetic underwater vehicle first. Then the formation problem of the BUV is modeled, and the theoretical analysis of a simple case is given.

A. Introduction to the BUV

Inspired by Stingrays, Fig.1 illustrates the biomimetic underwater vehicle designed in our previous work [14]–[16]. The BUV can perform multiple locomotion modes by control the propagating waves on bilateral fins, including forward/backward swimming, diving/floating, turning maneuver and even rotating in situ. Compared with traditional underwater vehicles propelled by axial propellers, the BUV is less likely to cause entanglement of aquatic weeds, and could move flexibly in a narrow underwater space with environmental friendliness. It has broad application prospects in underwater search and rescue, biological observation, and underwater operations.

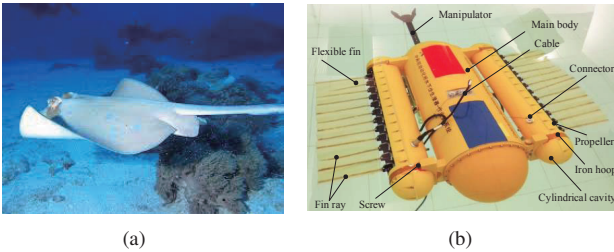


Fig. 1. (a) Stingray. (b) The biomimetic underwater vehicle.

B. Modeling of the Formation Problem

As shown in Fig. 2, consider a scenario in which multiple BUVs need to form a formation towards the target (here is a ship). This paper considers the line formation problem on two-dimensional plane. Assume that there are n BUVs, whose initial positions are $IP = \{P_1, P_2 \cdots, P_n\}$. Their target positions are $TP = \{P'_1, P'_2 \cdots, P'_n\}$, which are desired to form a line formation towards the goal G and the distance between each two adjacent BUVs is d . The problem is that how they could form the desired formation in the least time (or the shortest distance).

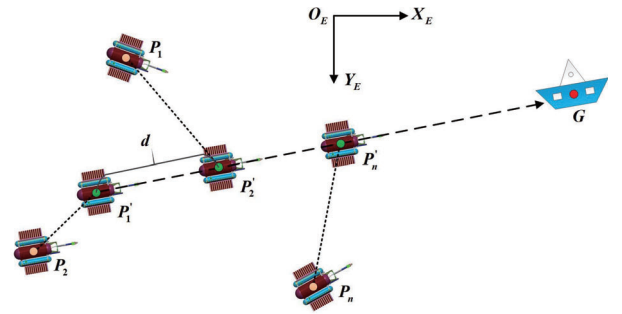


Fig. 2. Formation problem diagram.

Considering that the BUVs can turn in situ, when calculating the moving distance between the initial positions and the target positions, we assumed the underwater vehicle as a mass point. Thus it's easy to build the model of above formation problem:

$$\min TotalDis = \alpha d_{mean}(IP, TP) + (1 - \alpha) d_{max}(IP, TP) \quad (1)$$

$$s.t. \begin{cases} P'_i, G \in l & i = 1, 2 \cdots n \\ d(P'_i, P'_{i-1}) = d & i = 2, 3 \cdots n \end{cases} \quad (2)$$

where $d(P'_i, P'_{i-1})$ denotes the distance between point P'_i and point P'_{i-1} . The mean distance $d_{mean}(IP, TP)$ can be defined as

$$d_{mean}(IP, TP) = \frac{d(P_1, P'_1) + d(P_2, P'_2) + \cdots + d(P_n, P'_n)}{n} \quad (3)$$

The maximum distance $d_{max}(IP, TP)$ is defined as

$$d_{max}(IP, TP) = \max\{d(P_1, P'_1), d(P_2, P'_2) \cdots, d(P_n, P'_n)\} \quad (4)$$

The line l can be expressed as $y = kx + b$. Coefficient α denotes the crucial coefficient between the mean distance and the maximum distance BUVs will travel, which is used to avoid much too long distance that some BUVs will move.

C. Theoretical Analysis of A Special Case

In this subsection, some theoretical analyses are given based on the above model. For simplicity, we take $n = 3$, $\alpha = 1$ for an example. Assuming that the first target point of line formation is x'_1 , and the slope of the line is k . Then other final

points are functions of x'_1 and k . Mathematically speaking, those points can be expressed by:

$$P_i' = \begin{cases} x'_{i+1} = x'_i + \frac{id}{\sqrt{1+k^2}} & i = 1, 2 \cdots n-1 \\ y'_i = k(x'_i - x_0) + y_0 & i = 1, 2 \cdots n \end{cases} \quad (5)$$

where $n = 3$. The line of final formation is denoted as $y(x) = k(x - x_0) + y_0$ for that goal point $G(x_0, y_0)$ is on the line.

Since there are only two parameters x'_1, k to be optimized, we can solve the problem using the partial derivative method:

$$TotalDis = \frac{1}{3} \sum_{i=1}^3 \sqrt{(x_i - x'_i)^2 + (y_i - y'_i)^2} \quad (6)$$

$$\begin{cases} \frac{\partial TotalDis}{\partial k} = -\frac{1}{3} \sum_{i=1}^3 \frac{(x_i - x'_i) \frac{\partial x'_i}{\partial k} + (y_i - y'_i) \frac{\partial y'_i}{\partial k}}{\sqrt{(x_i - x'_i)^2 + (y_i - y'_i)^2}} = 0 \\ \frac{\partial TotalDis}{\partial x'_1} = -\frac{1}{3} \sum_{i=1}^3 \frac{(x_i - x'_i) \frac{\partial x'_i}{\partial x'_1} + (y_i - y'_i) \frac{\partial y'_i}{\partial x'_1}}{\sqrt{(x_i - x'_i)^2 + (y_i - y'_i)^2}} = 0 \end{cases} \quad (7)$$

Then we can obtain the analytic solutions of x'_1 and k . Set $d = 10$, $G = (50, 50)$, $P_1 = P_2 = P_3 = (0, 0)$, we can get $x'_1 = -7.07$, $k = 1$ and $F_{obj} = 6.67$. The target positions of these three BUVs are $P'_1 = (-7.07, -7.07)$, $P'_2 = (0, 0)$, $P'_3 = (7.07, 7.07)$. Obviously, the results are correct.

From the analysis shown above, we can find that even if there are only three BUVs, the computation process of the partial derivative method is complicated. When the number of the BUVs increases, it will be harder or even impossible to obtain analytic solutions. Furthermore, this method cannot solve matching problems between initial and final points since the objective function is discrete and we cannot obtain the partial derivative of discrete functions if maximum distance is taken into consideration. Therefore, we should seek other method to deal with this problem and PSO algorithm meets the requirement.

III. SOLUTIONS

A. Particle Swarm Optimization

The development of particle swarm optimization dates from the basic particle swarm optimization which was raised by Eberhart and Kennedy [13]. Basic particle swarm optimization can be expressed as

$$V_{ij}(t+1) = V_{ij}(t) + c_1 r_1 (P_{ij}(t) - X(t)) + c_2 r_2 (G_{ij}(t) - X(t)) \quad (8)$$

$$X_{ij}(t+1) = X_{i,j}(t) + V_{ij}(t+1)$$

where $V_{ij}(t)$ and $X_{ij}(t)$ are the j th velocity and position of i th particle at t th iteration, respectively. c_1 and c_2 are so-called acceleration coefficients. r_1 and r_2 are the random variable in the range of $[0, 1]$. $P_{ij}(t)$ and $G_{ij}(t)$ denote the j th previous best position of i th particle.

In order to improve the convergence performance of basic particle swarm optimization, Shi et al. added inertia weight to the basic PSO [17]. This modified PSO algorithm with inertia weight can be denoted as follows

$$V_{ij}(t+1) = \omega V_{ij}(t) + c_1 r_1 (P_{ij}(t) - X(t)) + c_2 r_2 (G_{ij}(t) - X(t)) \quad (9)$$

TABLE I
MATCHING EXAMPLE, N=5

	1	2	3	4	5
Value	0.8147	0.9058	0.1270	0.9134	0.6324
Sorted order	3	5	1	2	4

$$X_{ij}(t+1) = X_{i,j}(t) + V_{ij}(t+1)$$

Later, Clerc found that by introducing a constriction factor, the PSO could be prevented from falling into a local optimal solution [18], which can be denoted as

$$V_{ij}(t+1) = \chi [V_{ij}(t) + c_1 r_1 (P_{ij}(t) - X(t)) + c_2 r_2 (G_{ij}(t) - X(t))] \quad (10)$$

$$X_{ij}(t+1) = X_{i,j}(t) + V_{ij}(t+1) \quad (11)$$

where constriction factor χ is a function of parameters c_1 and c_2 and can be expressed as

$$\chi = \frac{2}{|2 - l - \sqrt{l^2 - 4 * l}|}, \quad l = c_1 + c_2, l > 4 \quad (12)$$

Because particle swarm optimization, which doesn't need to adjust too many parameters and doesn't require gradient information, is easy and succinct to be implemented, it has become a practical and classical tool in dealing with optimization problems. And it has been widely used in many fields such as neural network training [19], [20], control [21], and electric power systems [22]. In this paper, we will use this modified PSO with constriction factor to solve the problem of optimal line formation.

B. PSO-based optimal formation Algorithm flow

According to aforementioned modeling and analysis, the x -coordinate of the first target point and the slope of the optimal line formation are two important parameters that PSO algorithm need to optimize. In addition, the pairing problem between initial points and target points also need to be handled by PSO, which is one of main shortcomings of theoretical analysis in section II. The main flow of the PSO-based optimal formation solution can be summarized as follows.

Step 1: Initialize the problem of line formation, such as the initial points of n BUVs and the coordinate of the goal point. As for PSO algorithm, it is important to set appropriately the parameters including the number of all particles, the initial velocities and positions of particles, acceleration coefficients, the maximum loop time, the dimension of solution space, etc.

Step 2: Calculate the objective function (fitness) according to (1), before which we should match the initial points and target points by the sort-based method.

Take five vehicles for an example. As shown in the Table I, firstly, we generate five random numbers and sort them in ascending order. The pairing of initial index of these random numbers with the corresponding index of sorted numbers are matching results. It is obvious that there should be n parameters to sort if the number of BUVs is n .

If the fitness of new position is better than that of the previous best position $Pbest_i(t)$, the $Pbest_i(t)$ is replaced by the new position.

$$Pbest_i(t+1) = \begin{cases} X_i(t) & F_{obj}(X_i(t)) < F_{obj}(Pbest_i(t)) \\ Pbest_i(t) & F_{obj}(X_i(t)) \geq F_{obj}(Pbest_i(t)) \end{cases} \quad (13)$$

where $X_i(t)$ denotes the new position of the i th particle.

Step 3: Calculate the best fitness in $Pbest_{1,\dots,n}(t)$, and the best position is modified by

$$Gbest(t+1) = \arg \min \{F_{obj}(Pbest_1(t)), \dots, F_{obj}(Pbest_n(t))\} \quad (14)$$

Step 4: Update the velocities and the position of each particle according to (10), (11) and (12).

Step 5: Determine whether the positions and the velocities of particles are beyond the minimum and maximum range. If a particle go beyond the range of $[X_{min}, X_{max}]$ or $[V_{min}, V_{max}]$, the particle should be reset based on

$$X_{ij}(t) = \begin{cases} X_{max} & X_{ij}(t) > X_{max} \\ X_{min} & X_{min} < X_{min} \end{cases} \quad (15)$$

$$V_{ij}(t) = \begin{cases} V_{max} & V_{ij}(t) > V_{max} \\ V_{min} & V_{min}(t) < V_{min} \end{cases} \quad (16)$$

Step 6: If iterations run up to setting value, the PSO algorithm exits or else go to **Step 2**.

IV. NUMERICAL SIMULATIONS RESULTS

A. Formation of Three BUVs

In order to demonstrate the validity of the proposed method, the simulation results of the special case introduced in section II are firstly compared with the results of theoretical analysis.

The number and dimension of particles are set to 200 and 5, respectively, together with the iterations 150. Both c_1 and c_2 are set to 2.05 in the whole evolutionary process. To keep the uniformity of the objective function, we still choose (6) as objective function. Moreover, the ranges of x -coordinate of first target point and k are set to $[-10, 10]$ and $[0, 2]$. The random numbers o_1, o_2, o_3 used to deal with the pairing problem are set to $[0, 1]$. The maximum and minimum range of velocity of each dimension can be expressed by a matrix:

$$A = \begin{bmatrix} -5 & 5 \\ -1 & 1 \\ -0.5 & 0.5 \\ -0.5 & 0.5 \\ -0.5 & 0.5 \end{bmatrix} \quad (17)$$

In order to illustrate the evolutionary process, the evolution of optimized parameters and objective fitness are depicted in Fig. 3, Fig. 4, and Fig.5, respectively. It is seen from the figures that the values of optimized parameters have large changes from 1th generation to 25th generation, while in the latter stage, the values are relatively stable. In the end, the x -coordinate of the first target point and the slope of the line formation converge to -7.07 and 1 . Due to the particularity of this case (all three initial points are located in the same position), the matching between initial points and

target points are arbitrary. And the global optimized objective fitness converges to 6.67 . The final formation is plotted in Fig.6.

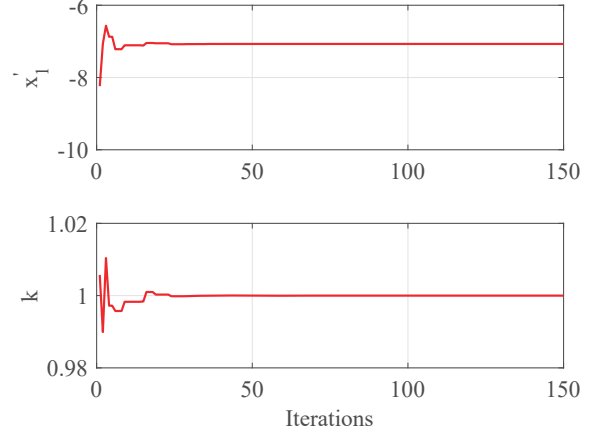


Fig. 3. Evolution of the x'_1, k .

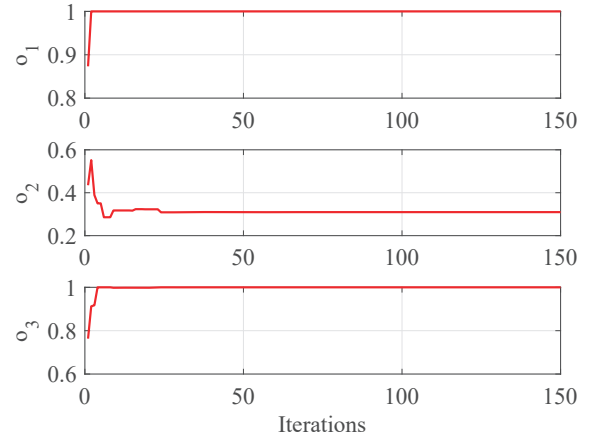


Fig. 4. Evolution of the parameters used for sorting.

In Table II, we make a comparison between the simulation and the theoretical analysis shown in section II. Both methods obtains the same results, which demonstrates the validity of the proposed algorithm.

TABLE II
COMPARISON OF THEORETICAL ANALYSIS AND SIMULATION RESULTS.

	Theoretical analysis	Proposed method
First final position	$(-7.07, -7.07)$	$(-7.07, -7.07)$
Second final position	$(0, 0)$	$(0, 0)$
Third final position	$(7.07, 7.07)$	$(7.07, 7.07)$
The slope k	1	1
Objective fitness	6.67	6.67

B. Formation of Many BUVs

In the previous simulation, we discuss the formation of only three BUVs, whose initial positions are identical. Furthermore,

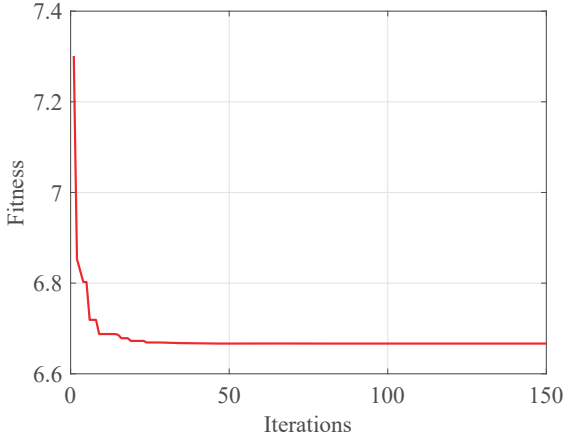


Fig. 5. Evolution of the objective fitness.

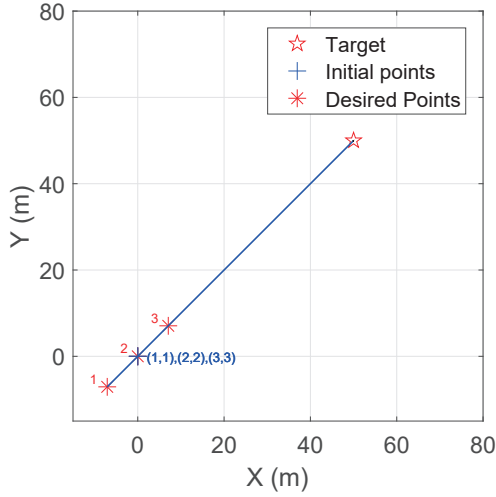


Fig. 6. Final formation of three BUVs

we only consider the sum distance all BUVs have to move, and don't limit the maximum distance each BUV moves, which is obviously unreasonable.

In this part, we consider the formation of ten BUVs (i.e. $n = 10$). The initial positions of all BUVs are generated randomly. The same parameter setting is used except that particle dimension is 12 ($n + 2$), the range of $x_1 = [150, 250]$, $G = (1000, 1000)$, and $\alpha = 0.5$.

The x'_1 , k are two crucial parameters which decide the final shape of formation. In Fig. 7, they converge to $x'_1 = 178.6835$ and $k = 0.9268$ in the end, respectively. Therefore, the equation of the final line formation is

$$y = 0.9258(x - 1000) + 1000 = 0.9258x + 74.2 \quad (18)$$

The evolution of first three parameters used for pairings is illustrated in Fig. 8. Considering length reasons, the evolutionary process of other parameters are omitted. The pairing results are shown in Table III.

TABLE III
PAIRING RESULTS BETWEEN INITIAL POINTS AND TARGET POINTS

No.	$Init(x, y)$	$Final(x, y)$	Pairing
1	(231.4724, 215.7613)	(186.0118, 244.2249)	2
2	(240.5792, 297.0593)	(229.9813, 285.0499)	8
3	(162.6987, 295.7167)	(200.6683, 257.8332)	4
4	(241.3376, 248.5376)	(215.3248, 271.4416)	6
5	(213.2359, 280.0280)	(222.6530, 278.2457)	7
6	(159.7540, 214.1886)	(178.6835, 237.4207)	1
7	(177.8498, 242.1761)	(193.3400, 251.0291)	3
8	(204.6882, 291.5736)	(207.9965, 264.6374)	5
9	(245.7507, 279.2207)	(237.3096, 291.8541)	9
10	(246.4889, 295.9492)	(244.6378, 298.6582)	10

In addition, the value of objective fitness decreases nonlinearly, and converges to 39.8659 eventually as shown in Fig. 9. The final formation is depicted in Fig. 10.

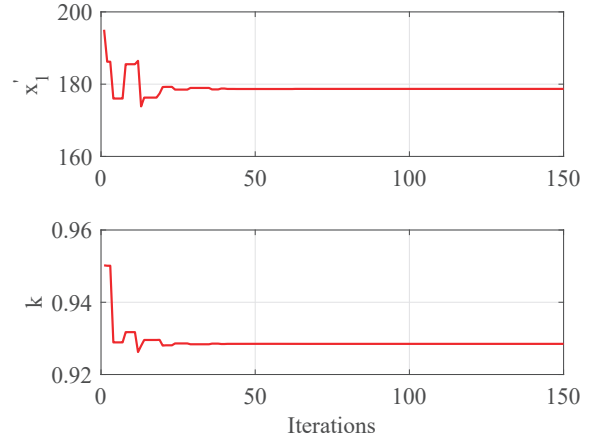


Fig. 7. Evolution of the x'_1 , k .

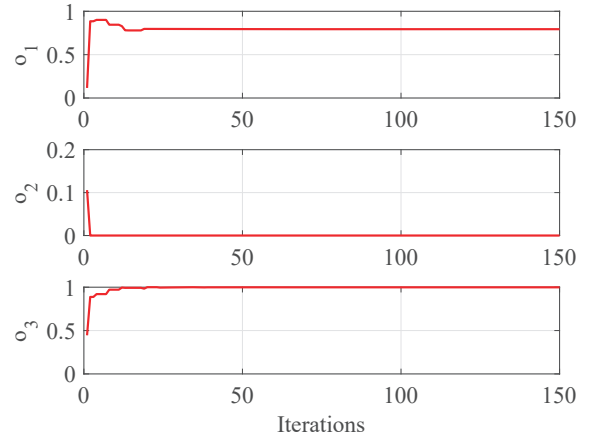


Fig. 8. Evolution of the parameters used for sorting.

V. CONCLUSION AND FUTURE WORK

The optimal line formation problem of multiple BUVs have been solved from the perspective of evolutionary computation.

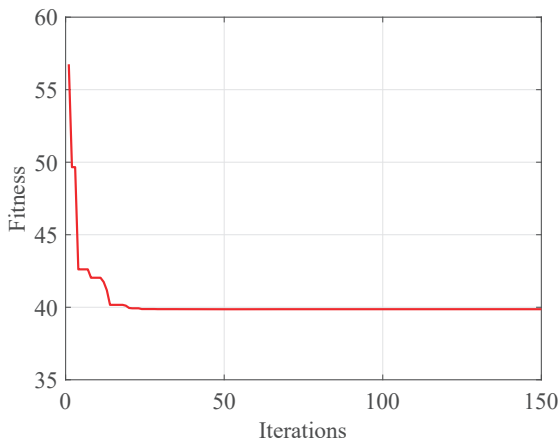


Fig. 9. Evolution of the objective fitness.

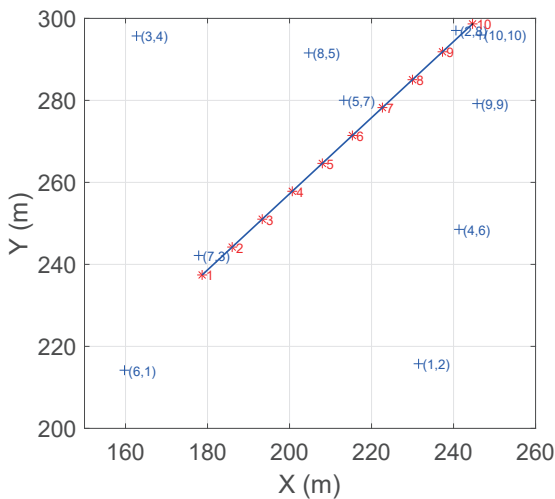


Fig. 10. Final formation of ten BUVs

The mathematical model of optimal formation problem is firstly built and a special case is theoretically analyzed by partial differential equations. In order to solve the shortcomings of the method of partial differential equations, we propose a PSO-based optimal formation method. The validity of the proposed method is demonstrated by comparing the results of theoretical analysis with the simulations results.

The ongoing research will consider formation tracking, formation target round-up and other practical issues, and seek to perform experimental research and verification of multiple BUVs. In addition, we only study the formation problem in a two-dimensional plane in this paper. The formation of BUVs in three-dimensional space is one of challenging problems.

REFERENCES

[1] A. Raj and A. Thakur, "Fish-inspired robots: Design, sensing, actuation, and autonomy-A review of research," *Bioinspiration and Biomimetics*, vol. 11, no. 3, pp. 031001, 2016.

[2] M. M. Rahman, S. Sugimori, H. Miki, R. Yamamoto, Y. Sanada, and Y. Toda, "Braking performance of a biomimetic squid-like underwater robot," *Journal of Bionic Engineering*, vol. 10, no. 3, pp. 265-273, 2013.

[3] Z. Wu, J. Yu, J. Yuan, and M. Tan, "Towards a gliding robotic dolphin: design, modeling, and experiments," *IEEE/ASME Transactions on Mechatronics*, vol. 24, no. 1, pp. 260-270, 2019.

[4] J. Zhu, C. White, D. K. Wainwright, V. Di Santo, G. V. Lauder, and H. Bart-Smith, "Tuna robotics: A high-frequency experimental platform exploring the performance space of swimming fishes," *Science Robotics*, vol. 4, no. 34, pp. eaax4615, 2019.

[5] G. Roza, M. Minas, N. Spyridakis, and M. Sfakiotakis, "Development of a bio-inspired underwater robot prototype with undulatory fin propulsion," in *Proceedings of the 9th International Conference on New Horizons in Industry, Business and Education*, 2015.

[6] R. Wang, S. Wang, Y. Wang, M. Tan, and J. Yu, "A paradigm for path following control of a ribbon-fin propelled biomimetic underwater vehicle," *IEEE Transactions on Systems, Man and Cybernetics: Systems*, vol. 49, no. 3, pp. 482-493, 2019.

[7] Y. Ryuh, G. Yang, J. Liu, and H. Hu, "A school of robotic fish for mariculture monitoring in the sea coast," *Journal of Bionic Engineering*, vol. 12, pp. 37-46, 2015.

[8] J. Yu, C. Wang, and G. Xie, "Coordination of multiple robotic fish with applications to underwater robot competition," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 2, pp. 1280-1288, 2016.

[9] S. Hubbard and B. Babak, "A model of the formation of fish schools and migrations of fish," *Ecological Modelling*, vol. 174, pp. 35-37, 2004.

[10] F. Dorfler and B. Francis, "Geometric analysis of the formation problem for autonomous robots," *IEEE Transactions on Automatic Control*, vol. 55, no. 10, pp. 2379-2384, 2010.

[11] D. Li D, S. Ge, W. He, G. Ma, and L. Xie, "Multilayer formation control of multi-agent systems," *Automatica*, vol. 109, no. 108558, 2019.

[12] X. Ge, Q. Han, and X. Zhang, "Achieving cluster formation of multi-agent systems under aperiodic sampling and communication delays," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 4, pp. 3417-3426, 2018.

[13] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proceedings of IEEE International Conference on Neural Networks*, pp. 1942-1948, 1995.

[14] R. Wang, S. Wang, and Y. Wang, "A hybrid heading control scheme for a biomimetic underwater vehicle," in *Proceedings of the 26th International Ocean and Polar Engineering Conference*, pp. 619-625, 2016.

[15] R. Wang, S. Wang, Y. Wang, C. Tang, and M. Tan, "Three-dimensional helical path following of an underwater biomimetic vehicle-manipulator system," *IEEE Journal of Oceanic Engineering*, vol. 43, no. 2, pp. 391-401, 2018.

[16] R. Wang, S. Wang, Y. Wang, M. Cai, and M. Tan, "Vision-based autonomous hovering for the biomimetic underwater robot - RobCutt-II," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 11, pp. 8578-8588, 2019.

[17] Y. Shi and R. Eberhart, "A modified particle swarm optimization," in *Proceedings of IEEE International Conference on Evolutionary Computation*, pp. 69-73, 1998.

[18] M. Clerc, "The swarm and the queen: Towards a deterministic and adaptive particle swarm optimization," in *Proceedings of the Congress of Evolutionary Computation*, pp. 1951-1957, 1999.

[19] S. Sengupta, S. Basak, R. Peters II, "Particle swarm optimization: A survey of historical and recent developments with hybridization perspectives," *Machine Learning and Knowledge Extraction*, vol. 1, no. 1, pp. 157-191, 2019.

[20] S. Chatterjee, S. Sarkar, S. Hore, N. Dey, A. Ashour, and V. Balas, "Particle swarm optimization trained neural network for structural failure prediction of multistoried RC buildings," *Neural Computing and Applications*, vol. 28, no. 8, pp. 2005-2016, 2017.

[21] G. El-Saady, E. A. Ebrahim, H. I. Abdul-Ghaffar, Y. S. Mohamed, A. El-Sayed, "Particle-swarm optimization control of active-power filter for harmonic mitigation of hybrid electric-unbalanced traction-system," in *Proceedings of the 2018 IEEE Congress on Evolutionary Computation*, 2018.

[22] M. AlRashidi and M. Hawary, "A survey of particle swarm optimization applications in electric power systems," *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 4, pp. 913-918, 2009.