A Simulation-Based Approach to Multi-Item Multi-Echelon Spare Parts Inventory System Optimization with Variable Warehouse Roles

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***Abstract*—An optimized supply chain is essential for the success of large-scale industries. In order to meet the high availability level requirements, an efficient inventory system is crucial to reduce downtime when a failure event occurs. In a multi-echelon spare parts inventory system, each warehouse within the system may operate as a hub or a spoke. A hub is a warehouse that fulfills the demands of other warehouses, while a spoke is a warehouse that fulfills the demands of final customers. When the inventory system deals with multiple items, a fixed role (hub or spoke) is commonly assigned to each warehouse. However, this limitation may lead to sub-optimal solutions. If the optimization model allows each warehouse to have a different role for different items, a new degree of freedom is included and more efficient solutions can be found. In this paper, we propose a simulation-based optimization model to define the configuration of a multi-echelon spare parts inventory system of multiple items. The goal is to minimize total inventory costs, subject to a fill rate constraint. We relax the assumption that warehouses have a fixed role for all the items. Two algorithms are used to evaluate the model: the Teaching-Learning Based Optimization (TLBO) and the Simulated Annealing (SA) algorithms. A case study based on a spare parts inventory system of an aircraft manufacturer is used to compare the performance of the proposed model with the performance obtained considered fixed warehouse roles. The results showed that the proposed model provided a reduction of** 6.8% **in total cost, without violating the fill rate constraint.**

***Index Terms*—Optimization, Multi-Echelon Inventory System, Simulation-Based Optimization, Simulated Annealing, Metaheuristics, Teaching-Learning Based Optimization.**

I. Introduction

At the beginning of the 20th century, solving an integrated logistic problem was mainly considered in the military sector [1]. In contrast, companies used to have fragmented logistics activities in different departments, which resulted in sub-optimal solutions. This field of study has evolved over the years and researchers have developed several works with practical applications. An example of an inventory system optimization model is presented in [2].

An efficient supply chain strategy is crucial for the success of big companies. However, supply chain optimization is a challenging task. Inventory systems represent an important segment of supply chain management. An inventory system comprises a set of warehouses and their logistical relationships. The definition of these relationships has a major effect on system performance. An inventory optimization problem consists of defining the role and the optimum stock level of each warehouse within the system.

A multi-echelon inventory system is composed of central warehouses (hubs) that receive parts from suppliers and distribute them to regional warehouses (spokes). Regional warehouses send the parts to final customers [3]. In general, hubs can also send parts directly to customers. Fig. 1 shows an example of a multi-echelon inventory system.

A performance indicator commonly used to assess the performance of an inventory system is the Fill Rate (FR), which is the percentage of demand that is met at the time that the order is placed [4]. Commonly, there is an agreement between the logistic service provider and the customers that establishes the minimum acceptable performance, denoted by Service Level Agreement (SLA) [5].

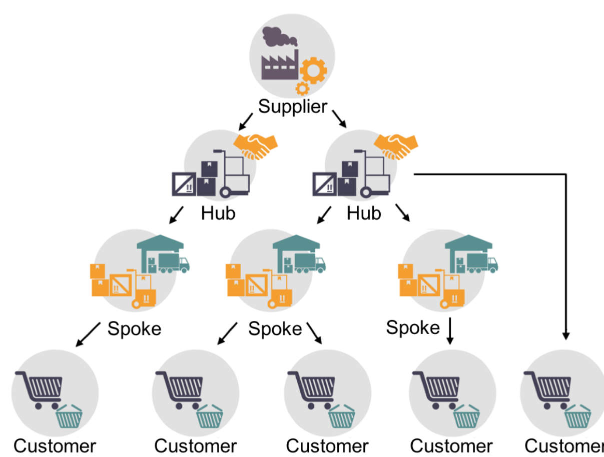

Fig. 1. An example of a multi-echelon inventory system

A seminal work for the inventory optimization research field was presented in [4]. The author proposed the Multi-Echelon Technique for Recoverable Item Control (METRIC), which

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defines the stock level in each base for repairable items. The model aims at reducing the sum of backorders across all the bases, creating an availability-cost curve. The METRIC model served as a base for many other models such as the MOD-METRIC model [6] and the VARI-METRIC model [7].

A different approach consists of optimizing the inventory system using a simulation-based model. In [8], the authors modeled a supply chain to minimize the inventory cost with a response time constraint. Later, the authors in [9] developed a simulation-based model applied to a multi-objective optimization considering inventory cost, inventory level, and frequency of inventory shortage as objective functions. They used the reorder point and the order quantities as parameters for the optimization algorithm. Another simulation-based inventory model combined with an optimization tool is presented in [10].

In this paper, we propose a simulation-based optimization model to define the configuration of a multi-echelon spare parts inventory system for multiple consumable items. The objective function is to minimize the inventory cost, subject to a fill rate constraint. The proposed model relaxes the fixed role assumption and allows each warehouse within the system to play a different role for each part. We evaluate this model using two metaheuristic algorithms: Teaching-Learning Based Optimization (TLBO) and Simulated Annealing (SA).

The remaining sections of this paper are organized as follows. Section II presents a brief overview of TLBO and Simulated Annealing. Section III describes the optimization problem under consideration. Section IV introduces the proposed solution. Section V presents the results obtained in a case study used to illustrate the application of the proposed model. Concluding remarks are given in Section VI.

II. Theoretical Background

A. Simulated Annealing

The Simulated Annealing (SA) algorithm was introduced in [11]. It is inspired by the statistical mechanics of annealing in solids, which is the metallurgical process of heating a material to a high-temperature and then cooling it slowly to increase the size of internal crystals and improve its mechanical and structural properties [12].

The idea of mechanical annealing is to have a controlled cooling to transform the material from a disorganized state into a material with ordered and defect-free crystals. The analogy made for the Simulated Annealing algorithm is to use a controlled "cooling" to transform a bad and disorganized candidate solution into an optimized and proper solution [12]. The cooling process in the SA algorithm is done by reducing the degree of randomness during the optimization process.

The first step of the algorithm is to generate randomly a valid candidate solution. From this solution, new candidate solutions are generated in the neighborhood to possibly replace the current solution. As explained in [12], the algorithm always accepts better solutions. However, with a high-temperature, it is more likely that the algorithm also accepts a solution that is worse than the current one. In this way, the algorithm can explore the problem in different basins of attraction. After some iterations at the same temperature, the temperature value is reduced and a new cycle begins. This is considered the inner loop in the optimization process [13]. At each cycle, the temperature is reduced and the probability of accepting a worse solution decreases. The acceptance criterion is implemented with a Boltzmann distribution.

A worse solution is accepted if a generated random number (between 0 and 1) is less than the Boltzmann factor, $e^{-\Delta E/T}$, where ΔE is the difference in fitness between the current and the new candidate solutions and T is the current temperature [12]. When the temperature is less than a predefined temperature or a maximum number of iterations is completed, the algorithm stops and the last accepted solution is the final output of the optimization process [13]. Fig. 2 shows a flow chart of the SA algorithm.

The key of the SA algorithm is that it escapes from local optima by allowing "hill-climbing" moves during the optimization procedure. But it also refines the candidate solutions by reducing the probability of "hill-climbing" while it reduces the temperature parameter [14]. In [15], the authors presented a study showing the efficiency of SA in the solution of several optimization problems.

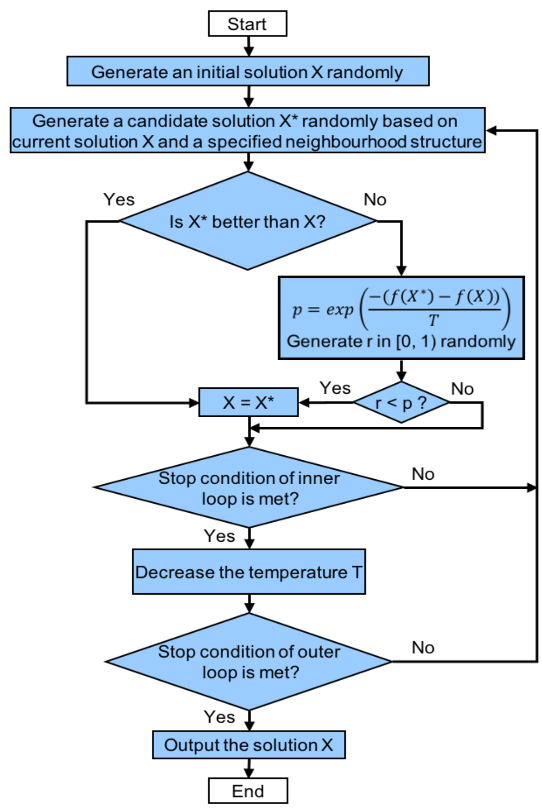

Fig. 2. Simulated annealing flow chart (adapted from [13])

B. Teaching-Learning Based Optimization

The Teaching-Learning Based Optimization (TLBO) algorithm is a population-based metaheuristic algorithm inspired by the teaching-learning process observed in a classroom [16].

This algorithm simulates the influence of a teacher on the output of a group of students in a class. The algorithm has two main phases: the Teacher Phase and the Student Phase [17]. During the Teacher Phase, students learn from the teacher, while in the Student Phase students learn through interactions among themselves. As presented in [18], TLBO shows good potential for combinatorial optimization problems.

Consider a group of N students. Each student X has an associated solution that corresponds to a candidate solution for the optimization problem. The quality of each solution is quantified by a fitness value $f(X)$ that is computed by evaluating the solution X using the objective function.

The student with the best solution in each iteration is called the Teacher. Fig. 3 shows the flowchart for implementing the TLBO algorithm [16]. The Teacher Phase and the Student Phase are described in the next sections.

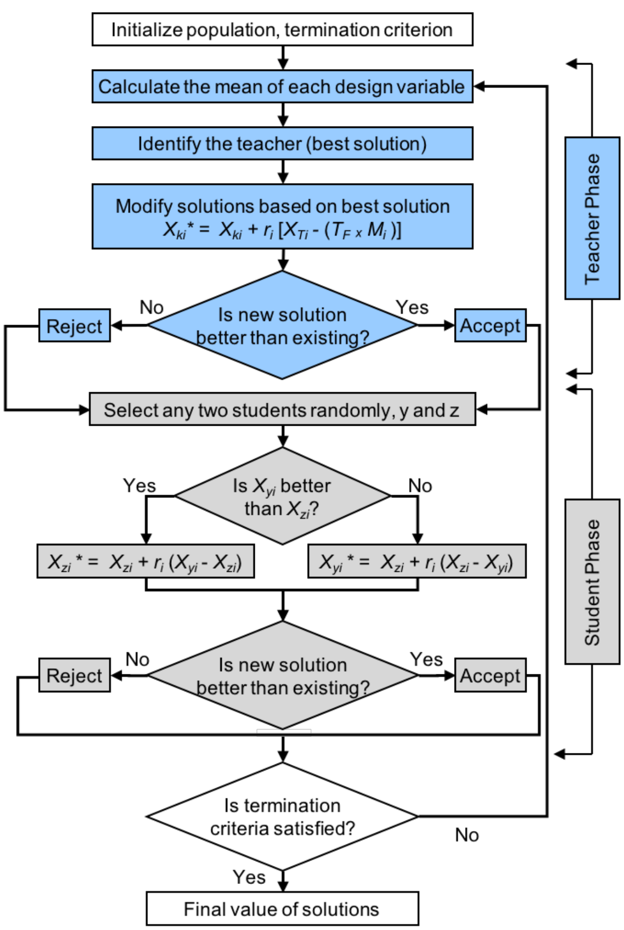

Fig. 3. TLBO flowchart (adapted from [16]

*1) **Teacher Phase**:* During the Teacher Phase, the algorithm simulates the learning of the students from the teacher (best solution). During this phase, the teacher makes an effort to increase the mean result of the class. Let M_i be the mean solution of all the students and T_i be the teacher in the i-th iteration. The teacher T_i will try to move M_i to its level. Knowledge is obtained based on the quality of the teacher and the quality of the students. The difference D_i between the solution of the teacher, denoted by X_{Ti}, and the mean solution of the students, M_i, is expressed according to (1).

$$D_i = r_i(X_{Ti} - T_F \cdot M_i) \tag{1}$$

where r_i is a random number chosen from a standard uniform distribution, and T_F is the teaching factor for iteration i, which is randomly set to either 1 or 2 according to (2).

$$T_F = round(1 + rand(0, 1)) \tag{2}$$

Based on the difference D_i, the current solution associated with each student n in iteration i, denoted by X_{ni}, with $n \in \{1, 2, \dots, N\}$, is updated during the teacher phase according to (3).

$$X^\star_{ni} = X_{ni} + D_i \tag{3}$$

where $X^\star_{ni}$ is the updated value of X_{ni}.

If $f(X^\star_{ni})$ is better than $f(X_{ni})$, then $X^\star_{ni}$ is accepted and replaces X_{ni} for the next iteration. Otherwise, $X^\star_{ni}$ is discarded.

*2) **Student Phase**:* During the Student Phase, TLBO simulates the learning of the students through interactions among themselves. During this phase, students gain knowledge by discussing with other students who have more knowledge [17].

Consider a pair of students y and z. Let X_{yi} and X_{zi} be the solutions of students y and z in iteration i, respectively. If $f(X_{yi})$ is better than $f(X_{zi})$, the solution of student z is updated according to (4). If $f(X^\star_{zi})$ is better than $f(X_{zi})$, $X^\star_{zi}$ is accepted and replaces X_{zi} for the next iteration. Otherwise, $X^\star_{zi}$ is discarded. Similarly, if $f(X_{zi})$ is better than $f(X_{yi})$, the solution of student y is updated according to (5). If $f(X^\star_{yi})$ is better than $f(X_{yi})$, $X^\star_{yi}$ is accepted and replaces X_{yi} for the next iteration. Otherwise, $X^\star_{yi}$ is discarded.

$$X^\star_{zi} = X_{zi} + r_i(X_{yi} - X_{zi}) \tag{4}$$
$$X^\star_{yi} = X_{yi} + r_i(X_{zi} - X_{yi}) \tag{5}$$

At the end of each iteration, the stop criteria are checked. Different stop criteria may be adopted. Some commonly used stop criteria are the maximum number of iterations, the maximum number of successive iterations without any improvement, the maximum simulation time, and the maximum number of objective function evaluations.

III. Problem Definition

The optimization problem addressed in this paper consists of finding a configuration for a multi-item multi-echelon spare parts inventory system that minimizes total annual inventory cost, subject to a fill rate constraint. There is one supplier that provides parts for the system, which is composed of W warehouses. The inventory system serves a set of customers located in R different regions. The total annual inventory cost for each item, denoted by C_T, is computed according to (6).

$$C_T = C_S + C_H + C_P + C_N + C_M \tag{6}$$

where C_S is the shipping cost, C_H is the holding cost, C_P is picking cost, C_N is the income tax cost, and C_M is the importation cost. These cost components are computed according to the expressions in (7) to (11).

$$\begin{aligned} C_S &= \sum_{w=1}^{W}(D_w \cdot s_{sw}) + \sum_{i=1}^{W}\sum_{j=1}^{W}(D_{ij} \cdot s_{ij}) + \\ &\quad + \sum_{w=1}^{W}\sum_{r=1}^{R}(D_{wr} \cdot s_{wr}) \quad (7) \\ C_H &= \sum_{w=1}^{W}(SL_w \cdot P \cdot h_w) \quad (8) \\ C_P &= \sum_{w=1}^{W}(O_w \cdot P \cdot \rho_w) \quad (9) \\ C_N &= \sum_{w=1}^{W}(D_w \cdot P \cdot n_w) \quad (10) \\ C_M &= \sum_{w=1}^{W}(D_w \cdot m_{sw}) + \sum_{i=1}^{W}\sum_{j=1}^{W}(D_{ij} \cdot m_{ij}) + \\ &\quad + \sum_{w=1}^{W}\sum_{r=1}^{R}(D_{wr} \cdot m_{wr}) \quad (11) \end{aligned}$$

where P is the part unit price, D_w is the annual demand generated by warehouse w to the supplier, D_{ij} is the annual demand generated by warehouse i to warehouse j, D_{wr} is the annual demand generated by clients in region r to warehouse w, s_{ij} is the unit shipping cost from warehouse i to warehouse j, s_{sw} is the unit shipping cost from the supplier to warehouse w, s_{wr} is the unit shipping cost from warehouse w to region r, SL_w is the average stock level in warehouse w, O_w is the annual number of orders placed in warehouse w, h_w is the annual holding tax for warehouse w, ρ_w is the picking tax for warehouse w, n_w is the income tax for warehouse w, m_{sw} is the importation tax from the supplier to warehouse w, m_{ij} is the importation tax from warehouse i to warehouse j, and m_{wr} is the importation tax from warehouse w to region r.

Finding a solution for the problem consists of defining the role (hub or spoke) of each warehouse in the system, the hub responsible for meeting the demand generated by each spoke, and the warehouse responsible for meeting the demand generated by each region. Also, a solution must define the stock level for each warehouse.

The role of each warehouse w is represented by a binary variable $isHub_w$ that assumes value 1 if the warehouse acts as a hub and zero otherwise. The hub responsible for each spoke s is represented by an integer variable H_s, and the warehouse responsible for each region r is represented by an integer variable R_r. The number of spare parts to be allocated at each warehouse is represented by an integer variable ROP, with $ROP \in \mathbb{N}$. In an inventory policy, the reorder point (ROP) is the stock level that, whenever reached, triggers the need for placing an order for new parts.

We assume that the Economic Order Quantity (EOQ) is one. Each spoke is served by only one hub. However, a hub may be responsible for multiple spokes and regions. Each customer region is served by only one warehouse.

As mentioned earlier, the goal is to find a solution that minimizes total annual inventory cost C_T subject to a fill rate (FR) constraint. Fill rate is the percentage of demand met at the time the customer places an order, as defined in (12)

$$FR = \frac{MD}{TD} \tag{12}$$

where MD is the demand met at the time the order is placed, and TD is the total demand.

The proposed model considers multiple items, and the warehouses may have different roles (hub or spoke) for each item. The motivation for proposing this approach is that relaxing the fixed warehouse role assumption provides the opportunity for improving the overall economic efficiency of the entire inventory system.

IV. Proposed Approach

Due to the high complexity level of the inventory problem under consideration, in this paper we use a discrete-event simulation model to evaluate the quality of candidate solutions. The proposed model is based on daily events. During each day in the simulation environment, the costs associated with each cost component presented in (6) are computed. Total demand and the demand met at the time that the customer placed an order are also computed, so the fill rate can be calculated at the end of the simulation.

A metaheuristic approach is used for the optimization process. As mentioned earlier, two metaheuristic algorithms are considered in this paper: Teaching-Learning Based Optimization (TLBO), which is a population-based metaheuristic, and Simulated Annealing (SA), which is a single solution algorithm.

The fitness function associated with each candidate solution is obtained through a discrete-event simulation. Some adaptations are implemented in the metaheuristics to deal with the inventory optimization problem. These adaptations are discussed in the next sections.

A. Modifications in Simulated Annealing

In order to use SA with the developed model, some adaptations are implemented. The first modification is related to the inclusion of the fill rate (FR) constraint. After computing the fitness value for a candidate solution, the FR is checked. If the FR constraint is violated, the candidate solution is discarded regardless of the total inventory cost. Also, a valid solution must be used to initialize the algorithm.

The process of randomly generating new candidate solutions in the neighborhood is also modified. Since there are interdependent variables in the model, the random change should follow some rules. The list of rules considered in this paper is as follows.

- If a variable $isHub_w$ associated with a warehouse w changes from 1 to 0 (i.e. the warehouse role changes from hub to spoke), the spokes that were served by that warehouse must be reassigned to another hub or transformed into a hub themselves.
- If a variable $isHub_w$ associated with a warehouse w changes from 0 to 1 (i.e. the warehouse role changes from a spoke to a hub), a check must be performed to assure that the new solution has at least one hub.
- If the current solution has only one hub, then variable H_s that defines which hub is responsible for spoke s cannot be changed.
- The maximum allowed variation in the stock levels between two consecutive steps of the algorithm varies according to the algorithm temperature. High temperatures allow higher variations in stock levels, while low temperatures allow small changes.
- Besides the maximum number of iterations per temperature, the model also considers the maximum number of attempts without success. If the optimization process fails to improve the solution a certain number of attempts, then the temperature decreases and a new cycle begins.

B. Modifications in Teaching-Learning Based Optimization

The first modification in TLBO is implemented to adapt the algorithm to deal with discrete variables. The use of TLBO with discrete variables has already been proposed [19], [20], [21]. We use continuous variables to define the solution associated with each student, and use a conversion step that consists of rounding the values to map the real numbers to integer values before running the discrete event simulation.

Another modification is performed to deal with the binary variable $isHub_w$. We adopted the same approach used in [22], which consists of replacing (3) with (13), and replacing (4) and (5) with (14).

$$X_b = \begin{cases} 0 & \text{, if } X_i + r_i \cdot (X_{Ti} - T_F \cdot M_i) < 0.5 \\ 1 & \text{, if } X_i + r_i \cdot (X_{Ti} - T_F \cdot M_i) \geq 0.5 \end{cases} \tag{13}$$

$$X_b = \begin{cases} 0 & \text{, if } X_{zi} + r_i \cdot (X_{yi} - X_{zj}) < 0.5 \\ 1 & \text{, if } X_{zi} + r_i \cdot (X_{yi} - X_{zj}) \geq 0.5 \end{cases} \tag{14}$$

where X_b is the converted binary variable.

During the teacher phase, if the warehouse is a spoke in the student solution and a hub in the teacher solution, then the teacher solution does not have a responsible hub. In this case, if the student has a valid hub responsible for it, this variable is not modified. Otherwise, a random valid hub is chosen for the warehouse. The same procedure is adopted during the student phase for each pair of students.

The last consideration is about the inclusion of the fill rate constraint. In this paper, we use a tournament selection operator similar to the one used in [23]. The tournament selection operator considers that a feasible solution is always better than an unfeasible solution. When two feasible solutions are compared, the one with the best fitness value (lower total inventory cost in the inventory problem under consideration) is preferred. When two unfeasible solutions are compared, the one with the smaller constraint violation is preferred.

V. Case Study

This section presents a case study to illustrate the application of the proposed model in a multi-item multi-echelon spare parts inventory optimization problem. The case study is based on an aircraft manufacturer global inventory system for multiple consumable items. Ten items were used in the simulations. The inventory system is composed of five warehouses and eight customer regions around the globe. Warehouses W_1 to W_5 are located at regions R_1 to R_5, respectively. There is one supplier for each item. A minimum acceptable fill rate of 90% is considered.

Table I shows the simulation parameters used for each item. Table II shows the picking and the holding rates for each warehouse, as a percentage of the item unit price. Table III shows the importation tax for each region and the income tax for each region containing a warehouse. Table IV shows the shipping cost rate per weight between regions for each item.

TABLE I
Item parameters

Item	Supplier Location	Price	Weight	Item	Supplier Location	Price	Weight
P_1	R_4	700	4	P_6	R_3	500	4
P_2	R_3	450	3	P_7	R_4	100	3
P_3	R_4	350	5	P_8	R_3	900	5
P_4	R_2	100	3	P_9	R_3	950	5
P_5	R_3	150	3	P_{10}	R_4	100	2

TABLE II
Picking and holding costs for each warehouse

Warehouse	Picking Cost	Holding Cost	Warehouse	Picking Cost	Holding Cost
W_1	3%	4%	W_4	5%	7%
W_2	3%	4%	W_5	4%	6%
W_3	4%	6%			

TABLE III
Income and importation and taxes for each region

Region	Income Tax	Importation Tax	Region	Income Tax	Importation Tax
R_1	35%	14%	R_5	17%	0%
R_2	20%	5%	R_6	-	15%
R_3	25%	5%	R_7	-	5%
R_4	25%	10%	R_8	-	4%

Based on experimental results, for the Simulated Annealing algorithm we used a maximum of 20 modification attempts in each temperature, a maximum number of 10 unsuccessfully modification attempts, a temperature reduction factor of

TABLE IV
SHIPPING COSTS BETWEEN REGIONS

Origin	Destination							
	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
R_1	2.5	5.0	2.5	4.0	5.0	4.0	4.0	2.5
R_2	5.0	2.5	4.0	4.0	4.0	4.0	4.0	5.0
R_3	2.5	4.0	2.5	2.5	5.0	4.0	4.0	2.5
R_4	4.0	4.0	2.5	2.5	5.0	2.5	2.5	4.0
R_5	5.0	4.0	5.0	5.0	2.5	5.0	5.0	5.0
R_6	4.0	4.0	4.0	2.5	5.0	2.5	2.5	4.0
R_7	4.0	4.0	4.0	2.5	5.0	2.5	2.5	4.0
R_8	2.5	5.0	2.5	4.0	5.0	4.0	4.0	2.5

0.985, an initial temperature of 4500 degrees, and a minimum temperature of 10 degrees. For the TLBO algorithm, we used a population of 50 individuals, and a maximum number of 75 generations.

A. Simulation Results

This section presents and discusses the results observed in the numerical experiments. Firstly, we compare the performance of SA and TLBO considering a single item multi-echelon inventory system. Then, we consider a system with multiple items and compare the results obtained with the proposed model (that allows warehouses to play different roles for each item) with a traditional model that adopts the assumption of fixed roles for each warehouse.

*1) **Performance Comparison in a Single Item Inventory System**:* In this first experiment, we compare the performance of SA and TLBO considering a single item inventory system. Fig. 4 shows a comparison between the total inventory costs computed with each algorithm for each item. Table V shows a comparison between the fill rates obtained with each algorithm for each item.

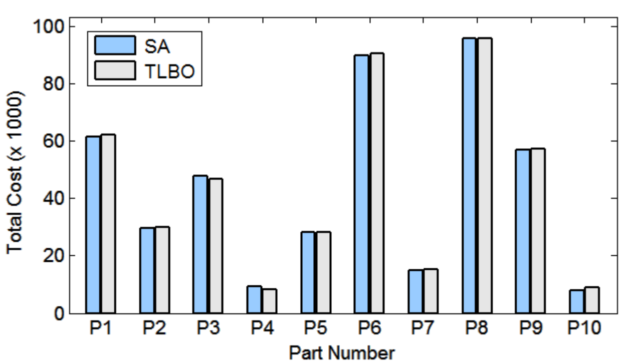

Fig. 4. Total inventory cost computed with each algorithm for each item in a single item system

Based on the results, we can conclude that SA and TLBO presented very similar performances. TLBO found better solutions for four items (P_3, P_4, P_5, and P_8) while SA found better solutions for the remaining items. As expected, both SA and TLBO found solutions that were close to the fill rate constraint of 90%.

TABLE V
FILL RATE COMPARISON IN A SINGLE ITEM INVENTORY SYSTEM

PN	SA	TLBO	PN	SA	TLBO
P_1	90.38%	91.21%	P_6	91.99%	92.70%
P_2	90.06%	90.61%	P_7	91.45%	90.79%
P_3	90.12%	90.12%	P_8	90.85%	90.54%
P_4	90.38%	90.38%	P_9	91.53%	90.40%
P_5	90.53%	90.53%	P_{10}	90.48%	90.48%

For illustration purposes, Figs. 5 and 6 show the inventory system configurations obtained for item P_2 with SA and TLBO, respectively. The number in the circle in each warehouse indicates the warehouse stock level.

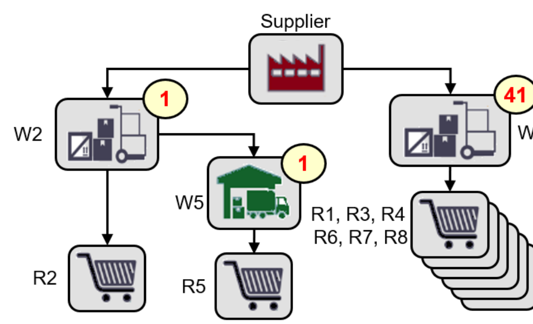

Fig. 5. Inventory system configuration for item P_2 obtained with SA

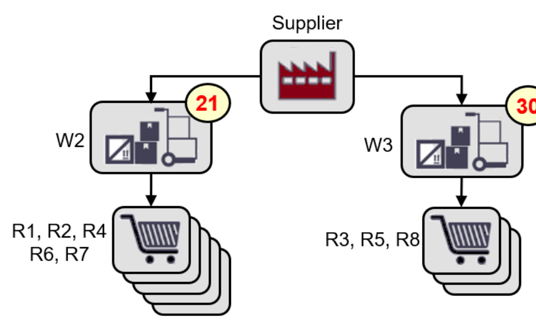

Fig. 6. Inventory system configuration for item P_2 obtained with TLBO

In the configuration obtained with SA, warehouses W_2 and W_3 act as hubs, receiving parts from the supplier, which is located in region R_3. Warehouse W_5 is the only spoke in this configuration, served by W_2. Regions R_2 and R_5 are served by warehouses W_2 and W_5, respectively. All other regions are served by warehouse W_3. Warehouses W_1 and W_4 are not used in this solution. The solution obtained with TLBO does not have any spokes. Warehouses W_2 and W_3 act as hubs, similarly to the solution obtained with SA. Regions R_3, R_5, and R_8 are served by warehouse W_3. The remaining regions are served by warehouse W_2. Warehouses W_1, W_4, and W_5 are not used.

*2) **Impact of Considering Variable Warehouse Roles in a Multi-Item System:*** Traditional inventory models for multiple items assume a fixed role for each warehouse within the system. As mentioned earlier, our goal is to investigate the impact of relaxing the fixed warehouse role assumption. This section presents a comparison between a fixed role model and the proposed variable role model in terms of total inventory cost.

In the first experiment, we observed that both TLBO and SA presented good performances, with very similar responses. In this second experiment, we present the results obtained with the Simulated Annealing algorithm only. Fig. 7 shows the inventory system configuration considering a multiple item system with all the ten different items.

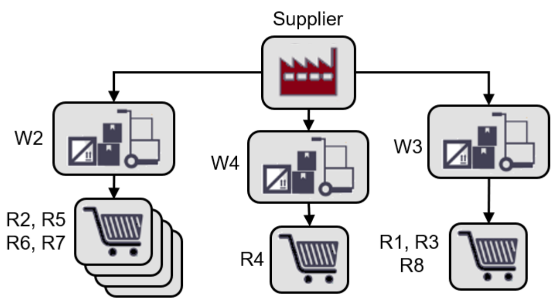

Fig. 7. Inventory system configuration for multiple items considering a fixed role

The configuration obtained for multiple items with fixed warehouse roles has three hubs: warehouses W_2, W_3, and W_4. Regions R_1, R_3, and R_8 are served by warehouse W_3. Warehouse W_4 serves region R_4 only. The remaining regions are served by warehouse W_2. The total inventory cost and the expected fill rate for this configuration are $454,555$ and 90.28%, respectively.

In the solution obtained using the proposed model, i.e. when the fixed warehouse role is relaxed, the total inventory cost and the expected fill rate are $441,377$ and 90.78%, respectively. When compared with the solution that considers a fixed warehouse role, this solution provides a slightly better fill rate and also provides a reduction of 2.90% in total inventory cost. Tables VI and VII describe the system configuration for the variable role solution.

TABLE VI
WAREHOUSE ROLES FOR MULTIPLE ITEMS CONSIDERING VARIABLE WAREHOUSE ROLES

Part	W_1	W_2	W_3	W_4	W_5
P_1	-	Hub	-	Hub	Hub
P_2	-	Hub	Hub	-	Spoke
P_3	-	Hub	-	Hub	Hub
P_4	-	Hub	Hub	Hub	Spoke
P_5	-	-	Hub	Hub	Spoke
P_6	Spoke	Hub	Hub	-	Hub
P_7	-	Hub	Hub	Hub	-
P_8	Hub	Hub	Hub	-	Hub
P_9	-	Hub	Hub	Hub	Hub
P_{10}	-	Spoke	Hub	Hub	-

TABLE VII
WAREHOUSE RESPONSIBLE FOR REGIONS FOR MULTIPLE ITEMS CONSIDERING VARIABLE WAREHOUSE ROLES

Part	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
P_1	W_4	W_2	W_2	W_4	W_5	W_2	W_4	W_5
P_2	W_3	W_2	W_3	W_3	W_5	W_3	W_3	W_3
P_3	W_4	W_2	W_2	W_4	W_5	W_2	W_4	W_5
P_4	W_3	W_4	W_4	W_2	W_2	W_2	W_5	W_3
P_5	W_3	W_5	W_3	W_3	W_4	W_3	W_3	W_3
P_6	W_1	W_2	W_3	W_5	W_2	W_2	W_1	W_5
P_7	W_3	W_2	W_4	W_4	W_3	W_4	W_3	W_4
P_8	W_3	W_5	W_3	W_3	W_2	W_3	W_1	W_3
P_9	W_5	W_5	W_3	W_3	W_2	W_4	W_4	W_3
P_{10}	W_4	W_3	W_4	W_4	W_2	W_4	W_3	W_4

The cost savings obtained by using variable warehouse roles are highly dependent on items data such as price, demand profile, and taxes. However, the use of variable warehouse roles always provides better solutions in comparison with the use of fixed roles. We conducted an additional simulation using a different demand scenario and with some changes in the shipping, holding, importation and income costs, as presented in Tables VIII, IX and X. As a result of the second scenario, it was observed a cost reduction of 6.79% in total inventory cost. In all the cases, the fill rate obtained met the fill rate constraint. Note that inventory systems commonly involve high amounts of money, and a cost reduction of 3% up to 7%, as observed in the simulations, may lead to significant cost savings.

TABLE VIII
PICKING AND HOLDING COSTS - 2ND SCENARIO

Warehouse	Picking Cost	Holding Cost	Warehouse	Picking Cost	Holding Cost
W_1	3%	4%	W_4	5%	20%
W_2	3%	30%	W_5	4%	6%
W_3	4%	3%			

TABLE IX
INCOME AND IMPORTATION AND TAXES - 2ND SCENARIO

Region	Income Tax	Importation Tax	Region	Income Tax	Importation Tax
R_1	35%	14%	R_5	17%	10%
R_2	15%	5%	R_6	-	15%
R_3	25%	1%	R_7	-	5%
R_4	25%	10%	R_8	-	4%

VI. CONCLUSIONS

In this paper, we present a simulation-based model to minimize the total inventory cost in a multi-item multi-echelon spare parts inventory system, subject to a fill rate constraint. In the proposed model, we relax the fixed warehouse role assumption and allow each warehouse within the system to have a different role for each item. Two metaheuristic algorithms were used in the optimization process: Simulated Annealing (SA) and Teaching-Learning Based Optimization (TLBO). Some adaptations were implemented in each metaheuristic

TABLE X
SHIPPING COSTS BETWEEN REGIONS - 2ND SCENARIO

Origin	Destination R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
R_1	2.5	5.0	2.5	4.0	5.0	4.0	4.0	2.5
R_2	5.0	2.5	2.0	5.0	5.0	4.0	4.0	5.0
R_3	2.5	2.0	2.5	2.0	7.0	4.0	4.0	2.5
R_4	4.0	5.0	2.0	2.5	5.0	2.5	2.5	4.0
R_5	5.0	5.0	7.0	5.0	2.5	5.0	5.0	5.0
R_6	4.0	4.0	4.0	2.5	5.0	2.5	2.5	4.0
R_7	4.0	4.0	4.0	2.5	5.0	2.5	2.5	4.0
R_8	2.5	5.0	2.5	4.0	5.0	4.0	4.0	2.5

algorithm to deal with the interactions among the decision variables of the problem.

A case study comprising ten different items was used to evaluate the impact of relaxing the fixed warehouse role assumption. The case study is based on an aircraft manufacturer global inventory system. We compared the performance of SA and TLBO in the solution of a single item multi-echelon spare part inventory system. The results showed that both SA and TLBO presented good results with similar performance. We used SA to compare the solution obtained with a traditional approach that considers a fixed role for each warehouse in the system with the solution obtained with the proposed model.

The results show that allowing the warehouses to have variable roles for different items increased the performance of the inventory system. In the case study described in section V, we observed a reduction of 2.90% in total inventory cost. In the second scenario, the cost savings reached 6.79%. All the solutions provided in the experiments met the fill rate requirement.

By relaxing the fixed role assumption, the proposed model provides cost savings without violating the fill rate constraint. However, the cost savings depend on many variables such as the number of different items, the demand profile for each item, the relative taxes among regions, etc. Future research may extend the scope of this paper by evaluating different scenarios. Another opportunity for future research is related to the use of other metaheuristics to solve the problem, including hybrid algorithms, as discussed in [24], and the automated design of metaheuristics algorithms, as discussed in [25].

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