Hybrid PSO Algorithm with Adaptive Step Search in Noisy and Noise-free Environments

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***Abstract*—Particle Swarm Optimizer (PSO) is a population-based algorithm applied to many applications due to its competitive performance. As a pioneering variant, Dual-Environmental Particle Swarm Optimizer (DEPSO) solves optimization problems both in noisy and noise-free environments. This paper employs Adaptive step search (ASS) as an improvement of DEPSO by enhancing the information utilization. ASS is an efficient scheme to solve the stochastic point location (SPL) problem. It magnifies or shrinks the step size of a learning mechanism (LM) adaptively according to historical success or failure. This method allows each particle to search with an adaptive step size by enhancing the utilization of historical information. Experimental results performed on CEC2013 benchmark functions indicate that DEPSO-ASS outperforms DEPSO and other the state-of-art PSO variants in both noise-free and noisy environments.**

***Index Terms*—Evolutionary Algorithms, Swarm Intelligence, Particle Swarm Optimizer, Adaptive Step Search, Dual Environment.**

I. Introduction

Particle Swarm Optimization Algorithm (PSO) is first proposed by Kennedy and Eberhart inspired by the foraging of a bird flock or fish school [1]. It considers a swarm with n particles in a D-dimensional search space and the i-th particle at the t-th generation maintains two vectors : a velocity vector and a position vector. The update of particle i includes the update of its position X_i and velocity V_i, and it is given by:

$$v_i^d = v_i^d + c_1 r_1^d (pbest_i^d - x_i^d) + c_2 r_2^d (gbest^d - x_i^d), \quad (1)$$

$$x_i^d = x_i^d + v_i^d \quad (2)$$

where D is the dimension of the search space and $d \in \{1, ..., D\}$. $X_i = (x_i^1, x_i^2, ..., x_i^D)$ denotes the position of the ith particle; $V_i = (v_i^1, v_i^2, ..., v_i^D)$ represents the velocity of the ith particle; $pbest_i = (pbest_i^1, pbest_i^2, ..., pbest_i^D)$ is the personal best position for the ith particle; $gbest = (gbest^1, gbest^2, ..., gbest^D)$ is the best position discovered by the swarm; c_1 and c_2 are accelerate coefficients determining the relative importance of $pbest_i$ and $gbest$; and r_1^d and r_2^d are two random numbers within $[0, 1]$.

In [2], Shi and Eberhart introduced a new parameter called the inertia weight denoted as w into the original PSO in order to balance global and local search abilities. A linearly decreasing inertia weight is given as

$$w(t) = w_{max} - (w_{max} - w_{min}) \times \frac{t}{T} \quad (3)$$

where w_{max} and w_{min} denotes the upper and lower bounds of w and they are usually set to 0.9 and 0.4, respectively. t is the current generation and T represents the max generation.

Since PSO is introduced in [1], it has attracted more and more interest and many variants are proposed. These approaches can be briefly classified into the following categories: tuning the control parameters [2]–[4], designing different neighborhood topologies [5], [6], hybridizing PSO techniques [7]–[9], using multiswarm techniques [10], applying PSO to pratical scenes [11]–[15], solving multi-objective problems [16]–[20].

Most of them use a series of strategies to improve the optimization capability of PSO in noisy environment or noise-free environment. However, noise emerges irregularly and unpredictably in many real applications. It is required that a PSO algorithm can work well both in noisy and noise-free environment automatically. Dual-Environmental Particle Swarm Optimizer (DEPSO) [21] is proposed to solve this problem by constructing a weighted search center based on k elite particles in current iteration to guide the swarm.

In this paper, Hybrid PSO Algorithm combined with Adaptive step search (ASS) in Noisy and Noise-free Environments (DEPSO-ASS) is proposed to further enhance the information utilization. ASS [22] is a pioneer scheme to solve the stochastic point location (SPL) problem, which is introduced by Oommen [23] which uses a learning mechanism (LM) to find a point on the line with stochastic feedback from an environment. The key of ASS is the search step size which can be magnified or shrinked in different situations on the basis of a decision table. A decision table is designed according to the historical information from constant communication with the environment.

Overall, our contributions are as follows:

1) A reasonable and efficient decision table is redesigned to adjust the search step size of particle adaptively.

2) For each particle, DEPSO-ASS utilizes the historical information to adjust the search step size according to the

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decision table, which improves the guiding effect of the weighted center.

The rest of this paper is organized as follows. Section II reviews the related work of DEPSO and ASS. Section III describes DEPSO-ASS in detail. Section IV presents the experimental settings and comparison results on 28 benchmark functions with DEPSO and other PSO variants. Conclusions are drawn in Section V.

II. Related Work

Without loss of generality, we consider the following continuous optimization problem throughout this paper:

$$min_{x\in\mathbb{R}^D} f(x), \tag{4}$$

where x is a vector in the D-dimensional Euclidean space. The optimization problem aims to find an optimal x with minimal evaluation (fitness) value.

A. Dual Environmental Particle Swarm Optimizer

DEPSO abandons the historical information and generates an elite subswarm including top-k particles according to the fitness values in the current iteration. In each iteration, DEPSO consists of the following steps:

1) Step 1 : Generate the positions and velocities of all particles randomly.

2) Step 2 : Calculate the fitness values of all the particles and select the top-k particles to synthesize the elite set E^t according to their fitness values.

3) Step 3 : Construct a new position called search center $\theta(t)$ according to the equations as follows.

$$f_{max}(t) = \max_{i\in E^t} f_i(t), \tag{5}$$

$$f_{min}(t) = \min_{i\in E^t} f_i(t), \tag{6}$$

$$m_i(t) = \mathrm{e}^{\frac{f_{max}(t)-f_i(t)}{f_{max}(t)-f_{min}(t)}}, \tag{7}$$

$$W_i(t) = \frac{m_i(t)}{\sum_{i=1}^{k} m_i(t)}. \tag{8}$$

$$\theta(t) = \sum_{i=1}^{k} W_i(t) \times \boldsymbol{X}_i(t). \tag{9}$$

where $f_i(t)$ is the current fitness value of member i at t-th iteration and $\sum W_i(t) = 1$.

4) Step 4 : Update the positions and velocities of all the particles according to Eq. (9) and Eq. (10).

$$v_i^d(t+1) = wr_1^d v_i^d(t) + \alpha r_2^d(\theta^d(t) - x_i^d(t)), \tag{10}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{11}$$

where c is the accelerate coefficient, r_1^d and r_2^d are two uniformly distributed random numbers in $[0,1]$, w is the inertia weight given in Eq. (3), where w_{max} and w_{min} are set to 0.9 and 0.4 respectively.

5) Step 5 : Repeat Step 2 to Step 4 until termination criteria is met.

B. Adaptive Step Search

Adaptive Step Search (ASS) [22] is an efficient scheme to solve the stochastic point location (SPL) problem [24]. It shows good performance on the SPL problems. The ASS aims to adapt the step size denoted by $1/N_t$ in different situations during the searching according to the last three decisions of direction that learning mechanism (LM) made. The decision table is given in Table I. Digit "0" and "1" represents that the moving positions of left and right. $\lambda(t)$ represents the position in t-th iteration.

TABLE I
Decision table to choose the current step size

Condition	Current Resolution	Current value
000	$N_t = N_{t-1}/2$	$\lambda(t) = \lambda(t-1) - 1/N_t$
001	$N_t = N_{t-1}$	$\lambda(t) = \lambda(t-1) - 1/N_t$
010	$N_t = N_{t-1} * 2$	$\lambda(t) = \lambda(t-1) - 1/N_t$
011	$N_t = N_{t-1}$	$\lambda(t) = \lambda(t-1) - 1/N_t$
100	$N_t = N_{t-1}$	$\lambda(t) = \lambda(t-1) + 1/N_t$
101	$N_t = N_{t-1} * 2$	$\lambda(t) = \lambda(t-1) + 1/N_t$
110	$N_t = N_{t-1}$	$\lambda(t) = \lambda(t-1) + 1/N_t$
111	$N_t = N_{t-1}/2$	$\lambda(t) = \lambda(t-1) + 1/N_t$

The principle behind the decision table is as follows:

1) If the current position is far away from the optimal position, the last feedbacks tend to be the same and a bigger step size is reasonable to track the optimum faster;

2) If the current position is around the location of optimum, the roundabout situation probably happens and the last three decisions are more likely 010 or 101. In this situation, the step size should shrink and carry out a fine search.

3) If the last three decisions are $001, 100, 011, 110$, the step size remain unchanged.

III. The Proposed Algorithm

Dual-Environmental Particle Swarm Optimizer (DEPSO) [21] is proposed to solve optimization problems in noisy and noise-free environment by constructing a weighted search center based on k-elite particles in current iteration to guide the swarm. It abandons the historical information like $pbest$ and $gbest$. In order to utilize more information, a hybrid PSO (DEPSO-ASS) is proposed as an improvement of DEPSO using ASS strategy in order to enhancing information utilization.

As the Eq. (10) shows, the velocity of a particle depends on a distance from the weighted center and the old velocity. The accelerate coefficient α control the distance regularly. On the one hand, a larger α can magnify the search step size and strengthen exploration capability of a particle consequently. On the other hand, a smaller α shrinks the search step size and facilitates exploitation capability. In DEPSO, α is set to a constant and controls the search step size regularly.

In order to adjust the search step size adaptively, a restriction factor δ is introduced in DEPSO-ASS and δ is calculated on the basis of a new decision table given in Table II.

It should be noted that:

1) Letter "T" represents that the move direction of (t-1)-th iteration in dimension i is the same as the t-th iteration. Such as 00 or 11 is denoted by "T".

TABLE II
NEW DECISION TABLE TO CHOOSE THE CURRENT STEP SIZE

Condition	Fitness	Restriction factor	Current α
TT	success	$\delta_t = \delta_{t-1} * 2$	$\alpha_t = \alpha_{t-1} * \delta_t$
TF	success	$\delta_{t-1} < B \quad and \quad \delta_t = \delta_{t-1} * 2$	$\alpha_t = \alpha_{t-1} * \delta_t$
FT	success	$\delta_{t-1} < B \quad and \quad \delta_t = \delta_{t-1} * 2$	$\alpha_t = \alpha_{t-1} * \delta_t$
FF	success	$\delta_{t-1} < B \quad and \quad \delta_t = \delta_{t-1}/2$	$\alpha_t = \alpha_{t-1} * \delta_t$
TT	failure	$\delta_t = \delta_{t-1}/2$	$\alpha_t = \alpha_{t-1} * \delta_t$
TF	failure	$\delta_t = \delta_{t-1}/2$	$\alpha_t = \alpha_{t-1} * \delta_t$
FT	failure	$\delta_t = \delta_{t-1}/2$	$\alpha_t = \alpha_{t-1} * \delta_t$
FF	failure	$\delta_t = \delta_{t-1}/2$	$\alpha_t = \alpha_{t-1} * \delta_t$

2) Letter “F” represents that the move direction of (t-1)-th iteration in dimension i is different from the t-th iteration. Such as 01 or 10 is denoted by “F”.

3) “success” denotes the quality of i-th particle becomes better under the guidance of search center. “failure” indicates the quality of i-th particle becomes worse.

4) δ is a restriction factor to control the factor α and is designed to control the search step size adaptively. δ ranges in $[lb, ub]$. In DEPSO-ASS, $lb = 1/4$ and $ub = 2$.

5) B is a preset constant to restrict the increase of δ and in the following experiments we set $B = \frac{1}{2}$.

The new formula to update the velocity of a particle is as follows:

$$v_i^d(t+1) = wr_1^d v_i^d(t) + \alpha_i^d(t+1)\delta_i^d(t+1)r_2^d(\theta^d(t) - x_i^d(t)), \tag{12}$$

The framework of DEPSO-ASS is shown in Algorithm.

Algorithm 1 The framework of DEPSO-ASS

```
Initialize n particles
while termination criteria are not met do
    Evaluate the fitness of particles
    Construct a search center θ(t) according to W_i(t) as
    Eq. (8) and Eq. (9)
    for each particle do
        Update velocity and position according to Eq. (12)
        and Eq. (11)
    end for
end while
return the optimal position with the minimum fitness
value
```

IV. EXPERIMENTAL RESULTS AND COMPARISONS

In this section, a series of experiments are carried out on the CEC 2013 benchmark [25] suite including 28 benchmark functions to validate the performance of the proposed algorithm DEPSO-ASS in different environments.

A. Benchmark and Experimental Settings

The experiments are conducted on the CEC 2013 benchmark suite including 28 benchmark functions.

This benchmark suite includes unimodal functions, multimodal functions, and composition functions, shown in Table III. In the following experiments, the dimensionality of these functions is D = 50 and the population size is $n = 50$. All the algorithms are run 51 times for each function and the maximal number of evaluations of each run is $5.0 * 10^5$. Other parameters are set the same as the ones in DEPSO:

1) Inertia weight varies within $[0.4, 0.9]$;

2) The size of elite set changes from 20 to 15.

3) α = 3.2.

TABLE III
28 TEST FUNCTIONS USED IN THIS PAPER

	Function Notation	Optimum (f_i^*)	Function Name
Unimodal	f_1	-1400	Sphere Function
	f_2	-1300	Rotated High Conditioned Elliptic Function
	f_3	-1200	Rotated Bent Cigar Function
	f_4	-1100	Rotated Discus Function
	f_5	-1000	Different Powers Function
Basic Multimodal	f_6	-900	Rotated Rosenbrocks Function
	f_7	-800	Rotated Schaffers F7 Function
	f_8	-700	Rotated Ackleys Function
	f_9	-600	Rotated Weierstrass Function
	f_{10}	-500	Rotated Griewanks Function
	f_{11}	-400	Rastrigins Function
	f_{12}	-300	Rotated Rastrigins Function
	f_{13}	-200	Non-Continuous Rotated Rastrigins Function
	f_{14}	-100	Schwefel’s Function
	f_{15}	100	Rotated Schwefel’s Function
	f_{16}	200	Rotated Katsuura Function
	f_{17}	300	Lunacek Bi_Rastrigin Function
	f_{18}	400	Rotated Lunacek Bi_Rastrigin Function
	f_{19}	500	Expanded Griewanks plus Rosenbrocks Function
	f_{20}	600	Expanded Scaffers F6 Function
Composition	f_{21}	700	Composition Function 1
	f_{22}	800	Composition Function 2
	f_{23}	900	Composition Function 3
	f_{24}	1000	Composition Function 4
	f_{25}	1100	Composition Function 5
	f_{26}	1200	Composition Function 6
	f_{27}	1300	Composition Function 7
	f_{28}	1400	Composition Function 8

* All test functions are described in [25]. Please refer to [25] and also the website http://www.ntu.edu.sg/home/EPNSugan/index_files/CEC2013/CEC2013.htm

B. Comparisons in Noisy and Noise-Free Environment

In order to illustrate the effectiveness of DEPSO-ASS, DEPSO-ASS is compared with other PSO variants in noise-free environment including FIPS [5], CLPSO [6], ALCPSO [26], SPSO [27], DEPSO [21]. The parameters for these algorithms are set to the suggested values and the parameter configurations of all PSO variants are given in Table IV. σ is the standard deviation reflecting the degree of noise. When σ=0, the environment is noise-free and the the fitness value of a particle is equal to the real fitness value.

Thus, in summary, DEPSO shows its superior convergence accuracy and speed among its noise-free PSO variant competitors.

Their means and standard deviations are shown in Table V and Wilcoxon signed-rank test is used to validate their performance improvement (with confidence level at least 95%). The better results are highlighted. According to the experimental results, it can be seen that on the most test functions the DEPSO-ASS outperforms the other PSO algorithm including DEPSO significantly.

TABLE IV
NOISE-FREE AND NOISY PSO VARIANTS USED IN COMPARISONS

Algorithm	Population topology	Parameter Setting	Type
FIPS	Local URing	$\chi = 0.72984$, $\sum c_i = 4.1$	Noise-free
CLPSO	Global Star	$w = 0.9 \to 0.4$, $c = 1.49445$	Noise-free
SPSO	Local Ring	$w = 0.72984$, $c_1 = c_2 = 1.49617$, $K = 3$	Noise-free
ALCPSO	Global Star	$w = 0.4$, $c_1 = c_2 = 2.0$, $\Theta_0 = 60$, $T = 2$	Noise-free
IILPSO	Local Ring	$w = 0.6 \to 0.4$, $c_1 = 2.5 \to 0.5$, $c_2 = 0.5 \to 2.5$, $c_1' = c_2' = 1$, $c_3 = 2$, $k = 15$	Noise-free
PSOER	Global Star	$w = 0.9 \to 0.4$, $c_1 = c_2 = 1.49617$, $B_0 = 5$, $B_\Sigma = 50$	Noisy
PSOERN	Global Star	$w = 0.9 \to 0.4$, $c_1 = c_2 = 1.49617$, $B_0 = 5$, $B_\Sigma = 50$, $N = 2$	Noisy
PSOLA	Global Star	$w = 0.72984$, $c_1 = c_2 = 1.49617$, $B_0 = 5$, $B_\Sigma = 50$, $Threshold = 0.7$	Noisy
DEPSO	Global Star	$w = 0.9 \to 0.4$, $c = 1.49445$	Dual
DEPSO-ASS	Global Star	$w = 0.9 \to 0.4$, $c = 1.49445$	Dual

C. Comparison in dual Environment

In dual environment in which σ switch between 0 and 0.24 (noise-free and noisy) every 3000 evaluations. The experimental results are shown in Table VI and the best results are highlighted. It can be seen that DEPSO-ASS shows the best performance among the PSO variants.

D. Comparsion with other algorithms

In this section, we compare DEPSO-ASS with six representative swarm intelligence techniques including DE [28], HS [29], ABC [30], GSA [31], GSO [32] and CS [33]. Parameter configurations of all these algorithms are in accordance with their corresponding literatures. The performance on the solution accuracy of each algorithm listed above is compared with that of DEPSO as shown in Table VII. The experimental result demonstrates the superiority of the proposed method over the representative swarm intelligence algorithms both in solution accuracy and stability in noise-free environment. Since the above six swarm intelligence techniques do not have the noise reduction ability, thus we do not present the performance comparison in a noisy environment.

V. CONCLUSION

In this paper, a hybrid PSO (DEPSO-ASS) is introduced to provide an adaptive search step size with the modified decision table. In the decision table, the step restriction factor will increase or decrease according to the historical success and failure of particles. As an improvement of DEPSO by enhancing the information utilization , DEPSO-ASS performs better in the dual environment. In addition, DEPSO-ASS also shows the competitve performance compared with the existing PSO variants. The intrinsic property of the improvements is to raise the information utilization level. In the future, we will focus on investigating other strategies to improve the performance of PSO by enhancing the information utilization.

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TABLE V

PSO Accuracy in Different Environments with Different σ Values.

Func		$\sigma = 0$						$\sigma = 0.12$				$\sigma = 0.24$			
		FIPS	CLPSO	SPSO	ALCPSO	DEPSO	DEPSO-ASS	PSOER	PSOERN	DEPSO	DEPSO-ASS	PSOER	PSOERN	DEPSO	DEPSO-ASS
f_1	Mean	**-1.40e+03**	**-1.40e+03**	**-1.40e+03**	**-1.40e+03**	**-1.40e+03**	**-1.40e+03**	-1.24E+03	-1.28E+03	-1.32E+03	**-1.35E+03**	-1.16E+03	-1.22E+03	-1.27E+03	**-1.30E+03**
	Std.	6.94E-13	**0.00e+000**	3.22E-14	1.54E-13	2.25E-13	3.29E-13	3.21E+01	3.72E+01	6.40E+00	**4.99E+00**	6.16E+01	4.75E+01	1.31E+01	**6.73E+00**
	Rank	1	1	1	1	1	1	4	3	2	1	4	3	2	1
f_2	Mean	1.96E+06	6.68E+07	1.43E+07	2.23E+07	3.53E+06	**7.83E+05**	9.11E+07	7.94E+07	3.25E+06	**1.98E+06**	1.13E+08	9.18E+07	3.55E+06	**2.30E+06**
	Std.	1.81E+06	8.63E+06	1.24E+07	1.45E+07	3.80E+06	**2.81E+05**	3.74E+07	3.24E+07	1.11E+06	**4.06E+05**	4.03E+07	3.64E+07	1.22E+06	**9.00E+05**
	Rank	2	6	4	5	3	1	4	3	2	1	4	3	2	1
f_3	Mean	2.25E+09	2.40E+09	2.74E+08	9.89E+08	3.15E+07	**2.04E+06**	1.96E+10	1.51E+10	5.35E+07	**1.32E+07**	3.26E+10	2.10E+10	7.15E+07	**1.38E+07**
	Std.	2.64E+09	6.86E+08	2.12E+08	1.25E+09	8.82E+07	**3.95E+06**	1.13E+10	1.27E+10	7.01E+07	**1.36E+07**	1.63E+10	1.12E+10	9.18E+07	**1.45E+07**
	Rank	5	6	3	4	2	1	4	3	2	1	4	3	2	1
f_4	Mean	3.54E+04	2.41E+04	1.37E+04	**2.33E+03**	1.03E+05	8.37E+04	7.77E+04	**5.26E+04**	1.11E+05	1.03E+05	1.08E+05	**7.65E+04**	1.12E+05	1.03E+05
	Std.	5.06E+03	3.68E+03	2.68E+03	**8.61E+02**	1.80E+04	1.57E+04	2.63E+04	1.43E+04	1.66E+04	**1.20E+04**	3.29E+04	2.57E+04	**1.37E+04**	1.91E+04
	Rank	4	3	2	1	6	5	2	1	4	3	3	1	4	2
f_5	Mean	**-1.00E+03**	**-1.00E+03**	**-1.00E+03**	**-1.00E+03**	**-1.00E+03**	**-1.00E+03**	-7.77E+02	-8.35E+02	-9.32E+02	**-9.51E+02**	-7.32E+02	-7.62E+02	-8.77E+02	**-9.12E+02**
	Std.	1.79E-04	1.83E-13	1.18E-13	2.01E-13	**0.00E+00**	2.92E-13	3.79E+01	3.90E+01	**9.05E+00**	9.46E+00	4.27E+01	4.54E+01	1.76E+01	**1.58E+01**
	Rank	1	1	1	1	1	1	4	3	2	1	4	3	2	1
f_6	Mean	-8.24E+02	-8.55E+02	-8.50E+02	-8.34E+02	**-8.57E+02**	-8.56E+02	-7.18E+02	-7.25E+02	-8.05E+02	**-8.20E+02**	-6.77E+02	-6.87E+02	-7.75E+02	**-7.99E+02**
	Std.	2.73E+01	**4.17E-01**	1.38E+01	2.94E+01	1.04E+01	5.47E-02	5.25E+01	5.07E+01	**4.48E+00**	3.87E+00	4.61E+01	4.79E+01	**7.77E+00**	5.13E+00
	Rank	6	3	4	5	1	2	4	3	2	1	4	3	2	1
f_7	Mean	-6.98E+02	-7.14E+02	-7.18E+02	-7.03E+02	**-7.97E+02**	-7.95E+02	-6.56E+02	-6.74E+02	-7.22E+02	**-7.33E+02**	-6.34E+02	-6.46E+02	-6.92E+02	**-7.08E+02**
	Std.	1.12E+01	8.92E+00	1.21E+01	1.92E+01	**4.61E+00**	3.75E+00	2.30E+01	2.26E+01	**9.31E+00**	4.55E+00	2.01E+01	2.33E+01	**1.16E+01**	1.05E+01
	Rank	6	4	3	5	1	2	4	3	2	1	4	3	2	1
f_8	Mean	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**
	Std.	4.18E-02	**3.30E-02**	3.95E-02	3.78E-02	4.95E-02	4.25E-02	4.48E-02	4.21E-02	3.69E-02	**3.84E-02**	3.79E-02	4.11E-02	3.58E-02	**2.43E-02**
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1
f_9	Mean	-5.43E+02	-5.44E+02	-5.45E+02	-5.50E+02	-5.79E+02	**-5.82E+02**	-5.27E+02	**-5.30E+02**	-5.30E+02	**-5.33E+02**	-5.26E+02	-5.26E+02	-5.26E+02	**-5.28E+02**
	Std.	3.37E+00	**2.19E+00**	3.34E+00	6.05E+00	3.28E+00	3.56E+00	3.17E+00	3.10E+00	**2.52E+00**	2.45E+00	2.45E+00	2.50E+00	**2.17E+00**	1.28E+00
	Rank	6	5	4	3	2	1	2	2	2	1	2	2	2	1
f_{10}	Mean	-4.90E+02	-4.96E+02	**-5.00E+02**	**-5.00E+02**	**-5.00E+02**	**-5.00E+02**	-2.89E+02	-3.08E+02	-4.69E+02	**-4.77E+02**	-1.55E+02	-2.63E+02	-4.50E+02	**-4.66E+02**
	Std.	5.08E+00	1.35E+00	6.19E-02	2.67E-01	1.28E-01	**2.25E-02**	4.06E+01	4.15E+01	4.13E+00	**2.49E+00**	2.35E+02	1.15E+02	6.32E+00	**2.73E+00**
	Rank	3	2	1	1	1	1	4	3	2	1	4	3	2	1
f_{11}	Mean	-2.18E+02	-2.69E+02	-2.42E+02	**-3.97E+02**	-3.71E+02	-3.88E+02	-2.40E+02	-2.40E+02	-3.34E+02	**-3.36E+02**	-2.21E+02	-2.22E+02	-2.97E+02	**-3.00E+02**
	Std.	2.56E+01	1.07E+01	2.89E+01	7.38E+00	4.72E+00	**3.44E+00**	2.48E+01	2.86E+01	7.14E+00	**3.94E+00**	2.57E+01	2.48E+01	1.09E+01	**8.61E+00**
	Rank	5	3	4	1	3	2	3	3	2	1	4	3	2	1
f_{12}	Mean	7.58E+01	6.17E+01	-1.23E+02	-3.18E+01	-2.35E+02	**-2.57E+02**	-8.18E+01	-7.55E+01	-2.22E+02	**-2.40E+02**	-2.79E+01	-2.99E+01	-1.83E+02	**-2.09E+02**
	Std.	3.03E+01	2.06E+01	2.95E+01	8.84E+01	1.10E+01	**7.88E+00**	4.36E+01	6.06E+01	1.28E+01	**9.06E+00**	1.05E+02	8.12E+01	1.82E+01	**1.57E+01**
	Rank	6	5	3	4	2	1	3	4	2	1	4	3	2	1
f_{13}	Mean	2.43E+02	1.87E+02	1.16E+02	1.63E+02	-4.80E+01	**-9.73E+01**	2.03E+02	1.83E+02	-3.70E+00	**-8.14E+01**	2.53E+02	2.52E+02	9.11E+01	**5.06E+01**
	Std.	2.89E+01	**2.04E+01**	3.41E+01	8.17E+01	3.03E+01	2.84E+01	6.25E+01	7.29E+01	**3.18E+01**	3.36E+01	**5.67E+01**	7.37E+01	5.76E+01	7.10E+01
	Rank	6	5	3	4	2	1	4	3	2	1	4	3	2	1
f_{14}	Mean	9.96E+03	8.15E+03	4.89E+03	5.22E+02	9.84E+02	**3.69E+02**	6.33E+03	5.41E+03	**3.53E+03**	7.00E+03	1.20E+04	**7.44E+03**	1.15E+04	1.24E+04
	Std.	8.67E+02	3.97E+02	6.46E+02	3.94E+02	**3.20E+02**	2.71E+02	**1.10E+03**	**1.10E+03**	1.17E+03	1.89E+03	1.35E+03	1.40E+03	**8.28E+02**	6.20E+02
	Rank	6	5	4	2	3	1	3	2	1	4	3	1	2	4
f_{15}	Mean	1.36E+04	1.39E+04	8.63E+03	7.60E+03	**4.22E+03**	4.25E+03	1.33E+04	1.15E+04	5.74E+03	**5.48E+03**	1.46E+04	1.22E+04	**1.11E+04**	1.25E+04
	Std.	**3.66E+02**	4.00E+02	1.15E+03	9.34E+02	7.30E+02	6.34E+02	1.19E+03	1.22E+03	7.99E+02	**6.22E+02**	**6.66E+02**	1.19E+03	1.30E+03	1.03E+03
	Rank	5	6	4	3	1	2	4	3	2	1	4	2	1	3
f_{16}	Mean	2.03E+02	2.03E+02	2.03E+02	2.02E+02	**2.00E+02**	2.03E+02	2.04E+02	2.04E+02	**2.03E+02**	**2.03E+02**	2.04E+02	2.04E+02	**2.03E+02**	**2.03E+02**
	Std.	2.95E-01	2.62E-01	3.74E-01	5.49E-01	**1.27E-01**	8.26E-01	3.56E-01	5.78E-01	2.44E-01	**1.75E-01**	3.63E-01	3.88E-01	3.68E-01	**2.14E-01**
	Rank	3	3	3	2	1	3	2	2	1	1	2	2	1	1
f_{17}	Mean	7.16E+02	5.83E+02	5.11E+02	3.87E+02	3.89E+02	**3.64E+02**	6.87E+02	6.85E+02	**6.73E+02**	7.12E+02	1.10E+03	**8.40E+02**	9.35E+02	8.86E+02
	Std.	2.67E+01	1.38E+01	2.79E+01	1.81E+01	7.91E+00	**2.61E+00**	6.22E+01	7.34E+01	4.43E+01	**1.86E+01**	2.04E+02	9.50E+01	4.92E+01	**2.27E+01**
	Rank	6	5	4	2	3	1	3	2	1	4	4	1	3	2
f_{18}	Mean	8.88E+02	8.55E+02	7.52E+02	7.27E+02	5.35E+02	**5.17E+02**	9.42E+02	9.27E+02	8.76E+02	**8.65E+02**	1.26E+03	1.10E+03	1.08E+03	**1.00E+03**
	Std.	2.68E+01	1.57E+01	5.41E+01	7.60E+01	**1.49E+01**	2.90E+01	5.55E+01	6.46E+01	2.84E+01	**9.07E+00**	1.93E+02	1.12E+02	3.84E+01	**1.92E+01**
	Rank	6	5	4	3	1	2	4	3	2	1	4	3	2	1
f_{19}	Mean	5.42E+02	5.23E+02	5.14E+02	5.12E+02	**5.06E+02**	**5.06E+02**	5.58E+02	5.50E+02	5.38E+02	**5.34E+02**	5.89E+02	5.64E+02	5.41E+02	**5.36E+02**
	Std.	7.69E+00	1.32E+00	3.17E+00	4.07E+00	1.11E+00	**5.83E-01**	8.70E+00	7.98E+00	1.45E+00	**8.67E-01**	2.67E+01	1.06E+01	**1.82E+00**	2.13E+00
	Rank	5	4	3	2	1	1	4	3	2	1	4	3	2	1
f_{20}	Mean	6.22E+02	6.22E+02	**6.21E+02**	6.22E+02	**6.21E+02**	**6.20E+02**	**6.25E+02**	**6.25E+02**	**6.25E+02**	**6.25E+02**	**6.25E+02**	**6.25E+02**	**6.25E+02**	**6.25E+02**
	Std.	6.52E-01	**3.32E-01**	7.45E-01	1.30E+00	1.48E+00	1.84E+00	1.20E-02	3.19E-02	2.73E-02	1.37E-01	1.99E-02	**1.95E-02**	4.52E-02	8.95E-02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1
f_{21}	Mean	1.68E+03	1.59E+03	**1.20E+03**	1.53E+03	1.58E+03	1.47E+03	2.31E+03	1.90E+03	1.78E+03	**1.73E+03**	4.74E+03	2.76E+03	2.59E+03	**2.11E+03**
	Std.	**1.53E+02**	3.24E+02	3.74E+02	3.58E+02	1.70E+02	3.05E+02	6.86E+02	5.47E+02	3.15E+02	**1.76E+02**	9.73E+02	8.81E+02	6.39E+02	**4.74E+02**
	Rank	6	5	1	3	4	2	4	3	2	1	4	3	2	1
f_{22}	Mean	1.21E+04	9.49E+03	7.44E+03	3.16E+03	2.00E+03	**1.22E+03**	1.11E+04	8.73E+03	**6.37E+03**	1.02E+04	1.62E+04	**1.14E+04**	1.49E+04	1.51E+04
	Std.	9.76E+02	6.05E+02	8.63E+02	8.60E+02	4.17E+02	**1.41E+02**	2.61E+03	2.28E+03	1.55E+03	**1.05E+03**	9.48E+02	2.42E+03	1.19E+03	**5.96E+02**
	Rank	6	5	4	3	2	1	4	2	1	3	4	1	2	3
f_{23}	Mean	1.58E+04	1.53E+04	1.13E+04	1.09E+04	**5.94E+03**	7.38E+03	1.55E+04	1.39E+04	9.42E+03	**8.18E+03**	1.69E+04	1.58E+04	**1.55E+04**	1.58E+04
	Std.	7.03E+02	**4.58E+02**	1.14E+03	1.19E+03	1.44E+03	1.43E+03	**1.22E+03**	1.68E+03	1.88E+03	1.25E+03	8.60E+02	1.34E+03	1.02E+03	**4.97E+02**
	Rank	6	5	4	3	1	2	4	3	2	1	3	2	1	2
f_{24}	Mean	1.32E+03	1.33E+03	1.33E+03	1.36E+03	1.23E+03	**1.22E+03**	1.43E+03	1.41E+03	1.41E+03	**1.39E+03**	1.46E+03	**1.42E+03**	1.44E+03	**1.42E+03**
	Std.	1.40E+01	**9.17E+00**	1.20E+01	1.61E+01	1.34E+01	1.33E+01	1.41E+01	1.40E+01	**8.56E+00**	1.12E+01	2.02E+01	1.09E+01	1.23E+01	**8.16E+00**
	Rank	3	4	4	5	2	1	3	2	2	1	3	1	2	1
f_{25}	Mean	1.51E+03	1.48E+03	1.49E+03	1.49E+03	1.42E+03	**1.41E+03**	1.61E+03	**1.57E+03**	1.60E+03	1.59E+03	1.69E+03	**1.60E+03**	1.65E+03	1.65E+03
	Std.	1.20E+01	**5.99E+00**	1.03E+01	1.35E+01	2.12E+01	1.00E+01	2.83E+01	1.69E+01	1.58E+01	**1.40E+01**	4.42E+01	2.22E+01	2.43E+01	**1.69E+01**
	Rank	5	3	4	4	2	1	4	1	3	2	3	1	2	2
f_{26}	Mean	1.43E+03	**1.41E+03**	1.46E+03	1.61E+03	1.52E+03	1.52E+03	1.68E+03	1.67E+03	**1.64E+03**	1.66E+03	1.68E+03	1.67E+03	**1.62E+03**	1.62E+03
	Std.	6.56E+01	**1.66E+00**	1.00E+02	7.74E+01	5.88E+01	6.07E+01	**5.26E+01**	6.27E+01	1.02E+02	7.96E+01	**6.70E+01**	7.76E+01	1.13E+02	1.22E+02
	Rank	2	1	3	5	4	4	1	3	4	2	1	2	3	4
f_{27}	Mean	3.01E+03	3.02E+03	3.05E+03	3.05E+03	2.13E+03	**2.08E+03**	3.40E+03	3.28E+03	3.02E+03	**2.89E+03**	3.64E+03	3.43E+03	3.47E+03	**3.39E+03**
	Std.	1.14E+02	**5.72E+01**	9.40E+01	1.13E+02	1.47E+02	1.64E+02	1.06E+02	1.39E+02	**1.00E+02**	1.44E+02	1.05E+02	1.19E+02	**8.86E+01**	9.50E+01
	Rank	3	4	5	5	2	1	4	3	2	1	4	2	3	1
f_{28}	Mean	2.08E+03	**1.80E+03**	1.86E+03	2.47E+03	**1.80E+03**	**1.80E+03**	3.55E+03	2.92E+03	**2.31E+03**	2.35E+03	5.53E+03	3.62E+03	2.83E+03	**2.70E+03**
	Std.	9.74E+02	4.58E-01	4.52E+02	1.36E+03	8.04E+02	**2.03E-13**	1.76E+03	1.55E+03	**8.76E+02**	1.06E+03	2.50E+03	1.79E+03	**1.01E+03**	1.22E+03
	Rank	1	3	4	2	1	1	4	3	1	2	4	3	2	1
Overall	A.R.	4.14	3.71	3.07	2.89	1.96	1.57	3.28	2.53	1.92	1.46	3.35	2.21	2	1.5

TABLE VI
DEPSO ACCURACY IN THE DUAL ENVIRONMENT

Func		FIPS	CLPSO	SPSO	ALCPSO	IILPSO	PSOER	PSOERN	PSOLA	DEPSO	DEPSO-ASS
f_1	Mean	5.38E+04	3.43E+04	3.83E+04	2.18E+04	-1.14E+03	-1.14E+03	-1.21E+03	4.27E+02	**-1.40E+03**	**-1.40E+03**
	Std.	1.03E+04	5.23E+03	9.36E+03	1.32E+04	2.75E+02	4.63E+01	4.88E+01	1.08E+04	**5.90E-05**	6.71E-01
	Rank	10	8	9	7	5	4	3	6	1	2
f_2	Mean	1.66E+08	1.91E+08	1.49E+08	2.31E+08	6.72E+07	1.05E+08	8.46E+07	1.15E+08	1.55E+06	**1.00E+06**
	Std.	6.01E+07	3.72E+07	3.35E+07	9.56E+07	1.59E+07	3.78E+07	3.27E+07	4.27E+07	7.33E+05	**2.69E+05**
	Rank	8	9	7	10	3	5	4	6	2	1
f_3	Mean	5.85E+10	6.41E+10	5.22E+10	7.79E+10	3.06E+10	3.38E+10	1.96E+10	6.24E+10	1.53E+07	**9.81E+06**
	Std.	1.06E+10	6.98E+09	1.18E+10	3.10E+10	8.06E+09	2.05E+10	1.21E+10	2.87E+10	2.94E+07	**1.18E+07**
	Rank	7	9	6	10	4	5	3	8	2	1
f_4	Mean	8.83E+04	8.70E+04	1.05E+05	1.39E+05	**3.62E+04**	1.12E+05	6.46E+04	1.13E+05	1.11E+05	9.74E+04
	Std.	1.22E+04	1.13E+04	1.56E+04	2.97E+04	**8.12E+03**	2.90E+04	1.74E+04	3.92E+04	1.57E+04	1.56E+04
	Rank	4	3	6	10	1	8	2	9	7	5
f_5	Mean	3.70E+03	5.38E+01	-6.97E+02	-5.13E+01	-7.67E+02	-7.30E+02	-7.59E+02	-7.15E+02	**-1.00E+03**	-9.96E+02
	Std.	1.57E+03	5.61E+02	4.48E+02	1.25E+03	7.28E+01	4.43E+01	3.65E+01	5.38E+01	**1.76E-04**	5.16E-01
	Rank	10	9	7	8	3	5	4	6	1	2
f_6	Mean	7.20E+01	-7.04E+02	-6.42E+02	-6.14E+02	-6.17E+02	-6.66E+02	-6.91E+02	-5.86E+02	**-8.54E+02**	-8.52E+02
	Std.	7.37E+02	1.89E+01	4.81E+01	6.18E+01	4.48E+01	4.86E+01	5.19E+01	6.45E+01	**6.99E+00**	7.90E-01
	Rank	10	3	6	8	7	5	4	9	1	2
f_7	Mean	-6.59E+02	-6.42E+02	-6.63E+02	-6.37E+02	-6.88E+02	-6.31E+02	-6.42E+02	-6.22E+02	**-7.85E+02**	-7.83E+02
	Std.	1.12E+01	8.09E+00	1.57E+01	2.51E+01	1.97E+01	2.51E+01	2.15E+01	2.45E+01	**4.38E+00**	2.64E+00
	Rank	5	7	4	8	3	9	6	10	1	2
f_8	Mean	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	-6.79E+02
	Std.	3.68E-02	3.57E-02	3.96E-02	3.26E-02	4.28E-02	4.20E-02	3.58E-02	4.13E-02	**3.05E-02**	4.18E-02
	Rank	4	3	5	2	1	10	9	8	7	6
f_9	Mean	-5.27E+02	-5.28E+02	-5.27E+02	-5.30E+02	-5.41E+02	-5.24E+02	-5.26E+02	-5.24E+02	-5.63E+02	**-5.69E+02**
	Std.	2.40E+00	1.79E+00	**1.59E+00**	2.59E+00	3.99E+00	2.17E+00	2.57E+00	2.68E+00	3.53E+00	2.44E+00
	Rank	6	5	7	4	3	9	8	10	2	1
f_{10}	Mean	4.25E+03	3.03E+03	1.38E+03	1.76E+03	-6.73E+01	-1.83E+02	-2.74E+02	2.56E+02	**-5.00E+02**	-4.96E+02
	Std.	1.09E+03	6.21E+02	9.44E+02	1.01E+03	1.00E+02	1.83E+02	5.00E+01	6.95E+02	**2.69E-01**	6.11E-01
	Rank	10	9	7	8	5	4	3	6	1	2
f_{11}	Mean	4.51E+02	3.52E+02	3.12E+02	2.61E+02	-1.05E+02	-2.08E+02	-2.27E+02	-1.61E+02	-3.62E+02	**-3.65E+02**
	Std.	8.44E+01	5.90E+01	1.13E+02	2.24E+02	4.71E+01	2.64E+01	2.88E+01	5.49E+01	7.06E+00	**4.18E+00**
	Rank	10	9	8	7	6	4	3	5	2	1
f_{12}	Mean	6.03E+02	5.11E+02	5.09E+02	5.30E+02	1.07E+02	-4.06E+00	-5.70E+01	8.88E+01	-2.17E+02	**-2.41E+02**
	Std.	7.24E+01	5.28E+01	8.76E+01	1.28E+02	6.83E+01	9.88E+01	6.60E+01	1.71E+02	1.92E+01	**1.47E+01**
	Rank	10	8	7	9	6	4	3	5	2	1
f_{13}	Mean	6.99E+02	6.37E+02	6.71E+02	6.66E+02	2.89E+02	2.77E+02	2.32E+02	3.44E+02	-4.61E-01	**-6.43E+01**
	Std.	1.01E+02	5.09E+01	8.02E+01	1.67E+02	7.85E+01	8.39E+01	7.58E+01	1.03E+02	4.92E+01	**3.09E+01**
	Rank	10	7	9	8	5	4	3	6	2	1
f_{14}	Mean	1.36E+04	1.34E+04	1.34E+04	1.25E+04	5.24E+03	1.22E+04	7.12E+03	9.11E+03	**2.27E+03**	4.10E+03
	Std.	4.82E+02	**2.91E+02**	5.03E+02	6.05E+02	9.67E+02	1.56E+03	1.36E+03	1.76E+03	5.00E+02	8.45E+02
	Rank	10	9	8	7	3	5	4	6	1	2
f_{15}	Mean	1.43E+04	1.43E+04	1.43E+04	1.43E+04	9.55E+03	1.46E+04	1.27E+04	1.37E+04	6.11E+03	**4.95E+03**
	Std.	**3.48E+02**	**3.48E+02**	3.77E+02	4.63E+02	1.23E+03	5.50E+02	1.19E+03	1.24E+03	8.43E+02	7.45E+02
	Rank	7	9	8	6	3	9	4	5	2	1
f_{16}	Mean	2.03E+02	2.03E+02	2.04E+02	2.03E+02	2.02E+02	2.04E+02	2.04E+02	2.04E+02	**2.01E+02**	2.03E+02
	Std.	2.96E-01	3.22E-01	**2.15E-01**	2.96E-01	5.72E-01	3.83E-01	4.78E-01	3.36E-01	2.75E-01	2.67E-01
	Rank	4	3	7	6	2	9	10	8	1	5
f_{17}	Mean	1.69E+03	1.97E+03	1.79E+03	1.82E+03	6.69E+02	1.13E+03	8.20E+02	1.10E+03	**4.43E+02**	6.03E+02
	Std.	1.84E+02	1.22E+02	1.47E+02	2.95E+02	5.79E+01	1.95E+02	9.21E+01	1.86E+02	2.38E+01	**2.19E+01**
	Rank	7	10	8	9	3	6	4	5	1	2
f_{18}	Mean	1.81E+03	2.12E+03	1.89E+03	1.93E+03	8.76E+02	1.30E+03	1.03E+03	1.34E+03	**6.54E+02**	8.02E+02
	Std.	1.62E+02	1.31E+02	1.97E+02	2.74E+02	5.85E+01	1.83E+02	9.85E+01	2.71E+02	**3.96E+01**	1.46E+01
	Rank	7	10	8	9	3	5	4	6	1	2
f_{19}	Mean	2.65E+04	5.95E+02	6.29E+02	7.82E+02	8.08E+02	5.82E+02	5.69E+02	6.61E+02	**5.08E+02**	5.16E+02
	Std.	2.26E+04	1.16E+01	4.11E+01	1.90E+02	2.00E+02	2.15E+01	1.30E+01	7.48E+01	**1.45E+00**	1.72E+00
	Rank	10	5	6	8	9	4	3	7	1	2
f_{20}	Mean	6.25E+02	6.25E+02	6.25E+02	6.25E+02	6.23E+02	6.25E+02	6.25E+02	6.25E+02	6.24E+02	**6.22E+02**
	Std.	1.08E-01	4.52E-02	5.93E-02	6.28E-02	1.01E+00	2.28E-02	1.70E-02	**8.11E-03**	1.46E+00	1.06E+00
	Rank	4	5	6	7	2	8	9	10	3	1
f_{21}	Mean	5.51E+03	5.89E+03	5.74E+03	5.72E+03	2.66E+03	4.52E+03	2.70E+03	5.20E+03	**1.53E+03**	1.60E+03
	Std.	2.54E+02	2.76E+02	2.76E+02	5.14E+02	6.58E+02	8.53E+02	9.19E+02	5.24E+02	3.42E+02	**2.46E+02**
	Rank	7	10	9	8	3	5	4	6	1	2
f_{22}	Mean	1.68E+04	1.58E+04	1.62E+04	1.55E+04	8.36E+03	1.64E+04	1.15E+04	1.53E+04	**3.35E+03**	5.86E+03
	Std.	4.67E+02	**3.75E+02**	5.83E+02	8.33E+02	1.54E+03	9.71E+02	2.28E+03	2.22E+03	5.75E+02	1.02E+03
	Rank	10	7	8	6	3	9	4	5	1	2
f_{23}	Mean	1.70E+04	1.63E+04	1.67E+04	1.67E+04	1.27E+04	1.72E+04	1.49E+04	1.71E+04	8.95E+03	**8.14E+03**
	Std.	**3.83E+02**	4.05E+02	4.50E+02	5.22E+02	1.57E+03	6.21E+02	1.61E+03	5.83E+02	1.59E+03	1.06E+03
	Rank	8	5	7	6	3	10	4	9	2	1
f_{24}	Mean	1.58E+03	1.43E+03	1.49E+03	1.40E+03	1.37E+03	1.46E+03	1.43E+03	1.47E+03	1.29E+03	**1.26E+03**
	Std.	6.20E+01	9.61E+00	2.78E+01	1.45E+01	1.65E+01	2.17E+01	1.39E+01	2.53E+01	2.20E+01	**4.62E+00**
	Rank	10	6	9	4	3	7	5	8	2	1
f_{25}	Mean	1.76E+03	1.65E+03	1.72E+03	1.49E+03	1.52E+03	1.69E+03	1.61E+03	1.72E+03	1.46E+03	**1.43E+03**
	Std.	3.62E+01	1.49E+01	3.63E+01	**4.66E+00**	2.33E+01	4.67E+01	2.98E+01	4.74E+01	1.10E+01	2.90E+01
	Rank	10	6	8	3	4	7	5	9	2	1
f_{26}	Mean	1.59E+03	**1.51E+03**	1.55E+03	1.65E+03	1.63E+03	1.67E+03	1.65E+03	1.69E+03	1.56E+03	1.57E+03
	Std.	7.22E+01	**2.67E+01**	6.79E+01	7.48E+01	7.47E+01	7.82E+01	9.35E+01	4.26E+01	8.31E+01	5.63E+01
	Rank	5	1	2	8	6	9	7	10	3	4
f_{27}	Mean	3.93E+03	3.61E+03	3.75E+03	3.55E+03	3.25E+03	3.63E+03	3.42E+03	3.68E+03	2.57E+03	**2.39E+03**
	Std.	1.30E+02	**4.66E+01**	7.68E+01	8.59E+01	1.41E+02	9.05E+01	1.40E+02	1.46E+02	1.23E+02	1.43E+02
	Rank	10	6	9	5	3	7	4	8	2	1
f_{28}	Mean	9.54E+03	8.25E+03	8.97E+03	9.15E+03	4.21E+03	6.13E+03	3.72E+03	7.04E+03	2.41E+03	**2.13E+03**
	Std.	8.25E+02	**4.21E+02**	7.25E+02	1.17E+03	1.77E+03	2.15E+03	1.87E+03	2.20E+03	1.26E+03	9.40E+02
	Rank	10	7	8	9	4	5	3	6	2	1
Overall	A.R.	7.07	5.86	6.18	6.18	2.89	5.54	3.64	6.18	2	1.96

TABLE VII
MEAN FUNCTION VALUES AND STANDARD DEVIATIONS ON CEC2013 COMPARED WITH EAS.

Func		DE	HS	ABC	GSA	GSO	CS	DEPSO
f_1	Mean	**-1.40E+03**	-6.25E+02	**-1.40E+03**	**-1.40E+03**	**-1.40E+03**	**-1.40E+03**	**-1.40E+03**
	Std.Dev.	1.56E-03	2.17E+04	2.37E-08	**0.00E+00**	2.58E-11	**0.00E+00**	2.25E-13
	Rank	5	7	6	**1**	4	**1**	3
f_2	Mean	4.51E+07	2.99E+08	5.37E+07	**6.76E+05**	2.87E+06	9.49E+06	3.68E+06
	Std.Dev.	5.71E+14	8.98E+15	1.94E+14	5.41E+10	5.41E+11	2.90E+12	**3.80E+06**
	Rank	5	7	6	**1**	2	4	3
f_3	Mean	2.00E+09	2.09E+10	3.00E+10	2.87E+08	1.74E+09	1.00E+10	**7.60E+07**
	Std.Dev.	1.57E+18	5.22E+19	1.71E+20	7.12E+16	2.09E+18	**0.00E+00**	8.82E+07
	Rank	4	6	7	2	3	5	**1**
f_4	Mean	9.31E+04	1.77E+05	1.93E+05	8.98E+04	**9.54E+03**	5.27E+04	9.99E+04
	Std.Dev.	1.72E+08	6.16E+08	2.87E+08	5.09E+07	6.77E+06	8.76E+07	**1.80E+04**
	Rank	4	6	7	3	**1**	2	5
f_5	Mean	**-1.00E+03**	-8.95E+02	**-1.00E+03**	**-1.00E+03**	**-1.00E+03**	**-1.00E+03**	**-1.00E+03**
	Std.Dev.	1.34E-02	3.45E+02	4.13E-06	2.60E-25	6.59E-08	6.15E-22	**0.00E+00**
	Rank	6	7	5	2	4	3	**1**
f_6	Mean	-8.53E+02	-7.66E+02	-8.50E+02	-8.22E+02	-8.18E+02	**-8.56E+02**	-8.53E+02
	Std.Dev.	**9.32E-01**	1.25E+03	3.30E+01	5.99E+02	9.80E+02	1.91E+00	1.04E+01
	Rank	7	6	3	4	5	**1**	2
f_7	Mean	-7.22E+02	-6.79E+02	-5.59E+02	-7.61E+02	-6.29E+02	-6.70E+02	**-7.90E+02**
	Std.Dev.	3.09E+02	2.55E+02	1.26E+03	7.24E+01	1.52E+03	1.90E+02	**4.61E+00**
	Rank	3	4	7	2	6	5	**1**
f_8	Mean	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**
	Std.Dev.	1.95E-03	1.66E-03	1.37E-03	1.27E-03	2.78E-03	**1.20E-03**	4.95E-02
	Rank	5	4	3	2	6	**1**	7
f_9	Mean	-5.28E+02	-5.23E+02	-5.35E+02	-5.61E+02	-5.33E+02	-5.41E+02	**-5.78E+02**
	Std.Dev.	9.60E+00	4.21E+00	5.19E+00	2.32E+01	1.95E+01	3.29E+00	**3.28E+00**
	Rank	6	7	4	2	5	3	**1**
f_{10}	Mean	-4.90E+02	5.51E+01	-4.82E+02	**-5.00E+02**	-4.99E+02	**-5.00E+02**	**-5.00E+02**
	Std.Dev.	3.78E+01	2.11E+04	6.66E+01	**3.75E-05**	1.89E-02	1.75E-04	1.28E-01
	Rank	5	7	6	**1**	4	2	3
f_{11}	Mean	-2.95E+02	-3.27E+02	**-3.95E+02**	-1.35E+02	-3.85E+02	-2.93E+02	-3.78E+02
	Std.Dev.	1.45E+03	8.25E+01	7.30E+00	8.47E+02	1.21E+01	2.19E+02	**4.72E+00**
	Rank	5	4	**1**	7	2	6	3
f_{12}	Mean	1.36E+02	1.66E+02	6.47E+02	1.58E+02	4.43E+02	6.96E+01	**-2.44E+02**
	Std.Dev.	3.77E+02	4.66E+02	8.46E+03	2.78E+03	1.80E+04	2.95E+03	**1.10E+01**
	Rank	3	5	7	4	6	2	**1**
f_{13}	Mean	2.31E+02	2.55E+02	7.62E+02	4.21E+02	5.89E+02	2.52E+02	**-6.80E+01**
	Std.Dev.	4.95E+02	8.40E+02	1.08E+04	3.82E+03	1.12E+04	2.07E+03	**3.03E+01**
	Rank	2	4	7	5	6	3	**1**
f_{14}	Mean	3.49E+03	3.17E+02	3.09E+02	4.99E+03	**1.64E+02**	5.09E+03	9.34E+02
	Std.Dev.	2.20E+06	7.84E+03	2.66E+04	2.14E+05	2.18E+04	2.34E+05	**3.20E+02**
	Rank	5	3	2	6	**1**	7	4
f_{15}	Mean	1.43E+04	1.53E+04	1.06E+04	7.65E+03	8.97E+03	9.09E+03	**4.68E+03**
	Std.Dev.	8.17E+04	2.39E+05	5.33E+05	3.40E+05	1.59E+06	1.31E+05	**7.30E+02**
	Rank	6	7	5	2	3	4	**1**
f_{16}	Mean	2.03E+02	2.04E+02	2.03E+02	**2.00E+02**	2.02E+02	2.03E+02	**2.00E+02**
	Std.Dev.	1.09E-01	1.64E-01	1.40E-01	**1.80E-06**	3.11E-01	6.00E-02	1.27E-01
	Rank	5	7	6	**1**	3	4	2
f_{17}	Mean	5.66E+02	5.39E+02	**3.64E+02**	3.89E+02	3.74E+02	6.23E+02	3.87E+02
	Std.Dev.	2.67E+03	4.11E+02	**4.16E+00**	9.62E+01	7.34E+01	1.61E+03	7.91E+00
	Rank	6	5	**1**	4	2	7	3
f_{18}	Mean	8.98E+02	9.66E+02	1.36E+03	**4.96E+02**	1.32E+03	8.60E+02	5.29E+02
	Std.Dev.	4.32E+02	9.48E+02	6.65E+03	**1.29E+02**	4.32E+04	2.86E+03	1.49E+01
	Rank	4	5	7	**1**	6	3	2
f_{19}	Mean	5.28E+02	5.51E+02	5.06E+02	**5.05E+02**	5.09E+02	5.25E+02	5.06E+02
	Std.Dev.	3.08E+01	1.30E+02	2.15E+00	**7.52E-01**	4.35E+00	2.09E+01	1.11E+00
	Rank	6	7	2	**1**	4	5	3
f_{20}	Mean	6.23E+02	6.23E+02	6.25E+02	6.25E+02	6.24E+02	6.23E+02	**6.21E+02**
	Std.Dev.	4.38E-02	8.47E-02	**2.87E-03**	1.58E-01	3.11E-01	2.13E-01	1.48E+00
	Rank	2	3	6	7	5	4	**1**
f_{21}	Mean	1.09E+03	1.87E+03	1.09E+03	1.62E+03	1.66E+03	**1.08E+03**	1.62E+03
	Std.Dev.	1.32E+05	2.11E+05	2.49E+04	2.95E+04	9.10E+04	9.31E+04	**1.70E+02**
	Rank	3	7	2	5	6	**1**	4
f_{22}	Mean	4.62E+03	1.52E+03	1.46E+03	1.10E+04	**1.29E+03**	7.24E+03	2.00E+03
	Std.Dev.	2.02E+06	4.43E+04	3.93E+04	2.02E+06	6.16E+04	3.11E+05	**4.17E+02**
	Rank	5	3	2	7	**1**	6	4
f_{23}	Mean	1.54E+04	1.63E+04	1.34E+04	1.13E+04	1.28E+04	1.17E+04	**8.20E+03**
	Std.Dev.	1.44E+05	1.90E+05	5.36E+05	1.50E+05	1.67E+06	3.96E+05	**1.44E+03**
	Rank	6	7	5	2	4	3	**1**
f_{24}	Mean	1.31E+03	1.35E+03	1.38E+03	1.29E+03	1.40E+03	1.37E+03	**1.24E+03**
	Std.Dev.	8.89E+02	4.34E+02	6.45E+01	3.62E+02	3.57E+02	5.35E+01	**1.34E+01**
	Rank	3	4	6	2	7	5	**1**
f_{25}	Mean	1.45E+03	1.51E+03	1.49E+03	1.59E+03	1.57E+03	1.50E+03	**1.43E+03**
	Std.Dev.	1.38E+03	2.57E+02	7.94E+01	1.10E+03	6.00E+02	3.79E+01	**2.12E+01**
	Rank	2	5	3	7	6	4	**1**
f_{26}	Mean	1.46E+03	1.69E+03	1.41E+03	1.57E+03	1.63E+03	**1.40E+03**	1.53E+03
	Std.Dev.	1.14E+04	1.39E+02	3.20E+00	9.36E+03	9.99E+03	**1.68E-01**	5.88E+01
	Rank	3	7	2	5	6	**1**	4
f_{27}	Mean	3.37E+03	3.51E+03	2.83E+03	2.78E+03	3.53E+03	3.23E+03	**2.16E+03**
	Std.Dev.	6.23E+04	6.38E+03	5.21E+05	2.56E+04	1.34E+04	2.54E+04	**1.47E+02**
	Rank	5	6	3	2	7	4	**1**
f_{28}	Mean	**1.80E+03**	2.74E+03	1.82E+03	7.52E+03	7.10E+03	**1.80E+03**	2.03E+03
	Std.Dev.	3.97E-01	1.83E+06	6.38E+01	2.19E+05	1.80E+06	**3.46E-17**	8.04E+02
	Rank	2	5	3	7	6	**1**	4
	Avg.Rank	4.39	5.54	4.43	3.39	4.32	3.46	**2.43**

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