Online intensification of search around solutions of interest for multi/many-objective optimization

Tapabrata Ray*, Hemant Kumar Singh*, Ahsanul Habib*, Tobias Rodemann†, Markus Olhofer†

* School of Engineering and Information Technology, University of New South Wales (UNSW), Canberra, Australia.
† Honda Research Institute, Carl-Legien-Strasse 30, 63073 Offenbach/Main, Germany
Email: *{t.ray, h.singh, a.habib}@adfa.edu.au, †{tobias.rodemann, markus.olhofer}@honda-ri.de

***Abstract*—In practical multi/many-objective optimization problems, a decision maker is often only interested in a handful of solutions of interest (SOI) instead of the entire Pareto Front (PF). It is therefore of significant research interest to design algorithms that can automatically detect SOIs and search around them instead of attempting to find the entire PF. However, this is challenging for a number of reasons. First and foremost, the interpretation of the underlying measures in terms of quantifying trade-off information for SOIs is not straightforward. Scalability is also an issue for most of such existing measures. Additionally, for many-objective algorithms that rely on decomposition, adaptation of reference directions and appropriate means to scale the objectives to maintain solution density around SOIs is not trivial. Lastly, constraints and decision-space are often overlooked in the existing studies but are important for practical applications. In this work, we present a simple approach to identify SOIs, using normalized net gain over nadir point and angle of influence. We illustrate the utility of the measure for offline and online identification of SOIs using a range of unconstrained and constrained benchmarks and practical design problems spanning up to 5 objectives. We also show further analysis in decision-space for an application problem to aid decision-making in practical scenarios.**

***Index Terms*—Decision-making, Solutions of interest, Many-objective optimization**

I. Introduction and background

In real-world design problems, it is common to encounter situations where multiple conflicting objectives need to be simultaneously optimized. This could involve, for example, minimizing weight, displacement and stress in a structural design [1], maximizing power and fuel efficiency for automobile design [2], minimization of drag and maximization of lift for airfoil design [3], etc.Such problems are referred to as multi-objective optimization problems (MOP). The optimum of such problems consists of not one, but a set of designs known as Pareto-optimal Set (PS). The image of PS in the objective space is called Pareto-optimal Front (PF), and represents the best possible trade-off among the objectives. The MOPs with more than three objectives are further categorized as 'many'-objective optimization problems (MaOP), since they pose additional challenges to conventional non-domination based ranking, visualization and selection of solutions from the PF [4]. Given that multiple (potentially unlimited) Pareto-optimal solutions exist, and not all can be implemented in the a real-world design, identification of one (or a few) solutions of interest is an inherent part of multi-objective design process [5]. Incorporation of such preferences can be done a priori, a posteriori, or interactively [6].

An a priori approach involves imposing a preference structure before the optimization is conducted. However, since the nature of PF is unknown, such preferences are hard to specify in the beginning and may result in undesirable outcomes. A simple example is the scalarization of MOP into a single-objective problem using weighted sum. Contrary to intuition, an equal weighting of the objectives will not result in a solution in the middle of the PF, if the PF is non-convex or if the objectives span different orders of magnitude.

In an a posteriori approach, a good representation of the PF is first sought through an appropriate algorithm (typically an evolutionary multi-objective (EMO) algorithm). Thereafter, the required SOIs are identified through application of expert knowledge, visual or quantitative criteria. This approach is also known as *offline* identification. Though this method is more comprehensive, the computation effort required to obtain the approximation of full PF is substantial, which might make it nonviable for practical problems where each design evaluation is resource intensive. Moreover, for MaOPs, the designs required to cover the PF with sufficient density grows with the number of objectives. It is also widely reported that the performance of Pareto-ranking based EMO algorithms in obtaining a good PF approximation does not scale well with the number of objectives [4].

A potential solution to overcome the above issue is to conduct the search in an interactive manner [7]–[10] so as to come up with a few solutions directly, without spending substantial computational effort in obtaining the remainder of the PF. For interactive optimization, a decision-maker could be presented with certain intermediate solutions and asked to rank them. This ranking can then be incorporated in the subsequent search to bias it towards more preferred solutions.

In case the decision-makers have sufficient know-how about the problem based on prior experience, their preference specifications can drive the search towards the regions of interest. However, this may often be too optimistic a scenario to assume. Without prior knowledge, in presence of large number of alternatives and multiple conflicting criteria, it becomes a formidable task to even define which solutions should be the preferred ones. To aid informed decision-making in such scenarios, an important research direction is to come up with quantifiable measures that can suggest a few preferred solutions from a given set of non-dominated solutions *without* inputs from the decision-makers. Such measures can also be incorporated in the environmental selection of the evolutionary algorithms to design *online* SOI identification [5], [6].

The SOIs have been characterized using various measures in the previous studies. Among some of the earlier works [11], the "knee" solution was suggested as the one that has the maximal convex bulge (longest perpendicular distance from the hyperplane). This concept was further utilized in [12] which used mobile reference points to intensify search in regions of knees for MOPs and their spacing was controlled with a user defined parameter. The measure was also incorporated in other non-domination based stochastic algorithms [13] and a decomposition-based framework in [14]. In [6], the focus towards knee solutions was attained using an angle measure and an expected marginal utility (EMU). In [15], a modified angle based ranking and a trade-off approach was used to identify knees in bi-criteria problems.

While the angle and trade-off approach are specific to bi-objective problems, the challenges in scaling the other measures such as EMU were observed in [5], [16]. Mainly, when the number of objectives is large, most of the solutions obtain a zero value of EMU, making them indistinguishable. A modified version with recursive calculation of EMU was further proposed for complete ordering of the solutions, and demonstrated on a range of multi/many-objective problems in [5], [16].

In [17], a method for posteriori knee selection was proposed by observing the density of the projected solutions on the hyperplane passing through extremities of the PF. In this method, the obtained knees can also be from non-convex region, whereas some of the other methods (e.g. [6], [11]) do not consider such solutions preferable. The reason behind this is that in convex regions, a small improvement in one objective comes at a large sacrifice in at least one other (hence moving away from convex knee is not preferred). In [18], a two-stage approach was adopted, where an initial estimate of the PF was first obtained using a conventional MOEA, followed by a focused search around certain identified knee regions using a weighted sum approach. The approach was demonstrated on bi-objective problems. Another offline selection technique was proposed in [19], which calculated the minimum Manhattan distance (MMD) of the given solutions from the ideal point in the normalized objective space. This was later extended to online knee identification ($Evoknee^r$) in [19] and demonstrated on two and three-objective unconstrained problems. It is important to take note that $Evoknee^r$ employs MMD as a measure and recursively subdivides the objective space. Such a scheme is likely to face challenges with increasing number of objectives since it involves creating divisions along each objective. Furthermore, only corners and knee solutions being used as parents might affect the search adversely in terms of convergence as the parents are likely to be distant from each other.

The online knee solution identification has also been used to improve the search for overall PF in some of the recent studies. For example, the solutions with higher hypervolume contributions were tagged as knee points and ranked higher in [20]. The same approach was extended to particle swarm optimization and generalized differential evolution in [21]. In [22], the knee points were considered as best points within weighted sub-populations. There have also been reports on use of knees to guide dynamic optimization [23].

In order to encourage further research on identification of solutions of interest, a set of new benchmark problems with knee-regions have also been proposed in [24]. The problems are unconstrained and scalable with respect to objectives and the knee characteristics can be controlled. Performance of some existing algorithms from [13], [18] were compared. Approaches for specifically visualizing the test problems with knees have also been proposed in [25].

In this study, we aim to further the studies on this topic, and in particular investigate some of the aspects that have not been considered in the above discussed literature.

- We discuss a simple and scalable approach to identify SOIs that can be easily interpreted by the decision-makers. While the first SOI is the solution with maximum normalized net gain over the nadir vector (similar to MMD from ideal vector), subsequent ones are identified using angle of influence as opposed to objective space partitioning and close knee discard mechanism employed in $EvoKnee^r$.
- We implement the above approach within an evolutionary algorithm in order to intensify the search around the SOIs. That is, instead of only providing the knee solutions, we attempt to provide a range of solutions in the vicinity of the SOIs to consider.
- Unlike current studies that only consider the objective space in terms of decision-making, we also discuss the obtained solutions in terms of their variable space properties to assist in selection.
- The previous studies have considered only unconstrained problems in online knee identification. Since practical problems often have constraints, we investigate the performance of the proposed method on constrained many-objective problems, in addition to the commonly studied unconstrained examples.

The proposed approach is discussed in Section II, followed by results and analysis on mathematical benchmarks and practical problems in Section III. The summary and concluding remarks are then presented in Section IV.

II. Proposed Approach

In this section, we present a scheme to identify SOIs from a set of non-dominated solutions, first offline and then online.

A. *Offline identification of SOIs*

The first step is to normalize the given non-dominated solutions between 0 and 1, so that the method does have an inherent bias towards any particular objective due to the scale of the values. In order to do so, the ideal point Z^I and the nadir point Z^N are identified, which consist of the minimum and maximum values of each objective, respectively. Thus, any objective value f_i gets translated to $\frac{f_i - f_{i,min}}{f_{i,max} - f_{i,min}}$, where $f_{i,min}$ and $f_{i,max}$ are the minimum and maximum values of the i^{th} objective. In a minimization sense, the ideal point translates to the co-ordinates $\{0, 0, \dots 0\}$, while the nadir point translates to the co-ordinates of $\{1, 1, \dots 1\}$ in the normalized objective space.

For any solution, its normalized net gain in the performance of the objectives over nadir point's objective vector is calculated using L_1 norm in the objective space. This can be calculated as $L = \sum 1 - f_i; i = 1, 2 \dots M$, where M is the number of objectives. In order to illustrate the process, we take an existing benchmark, bi-objective problem DEB2DK4 [6] as an example. Let us assume that we are given a set of N non-dominated solutions (198 in this case) on the PF of the problem and the decision-maker is interested in identifying 4 SOIs from the set. The normalized net gain can be calculated as $L = (1 - f_1) + (1 - f_2)$ for each given point and the resulting values are presented using colormap in Fig. 1.

The top solution with the highest normalized net gain is marked as solution A in red. This point (with highest L) in most cases is *likely to be* unique in the objective space [1]. Note that this point will be the same as identified by an existing measure MMD [19] which calculates minimum Manhattan distance from the ideal point. If the number of SOI ($nSOI$) required is exactly one, then this point will be the highest ranked solution in the set and is provided as the output. However, if $nSOI > 1$, then further differentiation is required in order to rank the solutions for identifying other potential knee solutions. We propose doing so using an *angle of influence* Φ, the calculation of which is outlined below.

A reference vector is constructed from the nadir point to every given point under consideration. Let's say R_A is the reference vector corresponding to solution A. The angle of influence Φ for a solution is defined as the smallest angle between its own reference vector and the weight vector of a solution with a *higher value* of L. Since solution A has the highest L value among all available solutions, Φ_A will be the highest as labeled in Fig. 1. For solution B which has the second highest L value, its angle of influence Φ_B is the enclosed angle between the reference vectors R_A and R_B. From the above definitions, it can be seen that $L_A > L_B$ implies that $L_{AB} = (1 - f_1(A) + 1 - f_2(A)) - (1 - f_1(B) + 1 - f_2(B)) > 0$, i.e. $L_{AB} = f_1(B) - f_1(A) + f_2(B) - f_2(A) > 0$. This quantity L_{AB} can be seen as the net gain that the solution A has over solution B. It follows from the interpretability perspective that if the solution A (with highest L) is chosen as an SOI, no other solution in the given set will have a net positive gain in the normalized objective values with respect to A, and its angle of influence will be highest among all the solutions. When only one solution is required as an SOI, and all objectives are equally important to the decision-maker, then A would be an obvious choice by this trade-off criteria (both L and Φ are highest for it). Since the decision-maker is interested in 4 SOIs, there is still a need to identify three additional SOIs. They are selected based on a *descending values of angle of influence*. All 4 selected SOIs are presented in Fig. 2, where the size of the marker indicates the order of SOIs, i.e., largest marker size corresponds to solution A. The pseudo-code of SOI identification process is presented in Algorithm 1. Since the proposed approach uses normalized net gain and angle of influence for SOI identification, for ease of reference, we abbreviate it as NNGA in the subsequent text.

[1] A few exceptions can be envisioned for some peculiar geometries, for example a linear simplex perpendicular to the line joining ideal and nadir points; or discontinuous patches with symmetric farthest points from the nadir. In such cases, any (or all) such points can be considered as SOIs under the current definition.

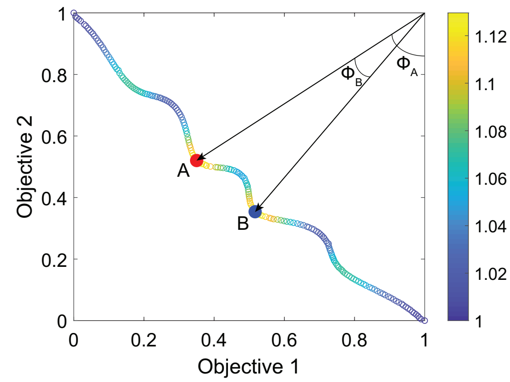


Fig. 1. Illustration of calculating the measures L and Φ. The colorbar represents the values of L calculated for the points in the given non-dominated set.

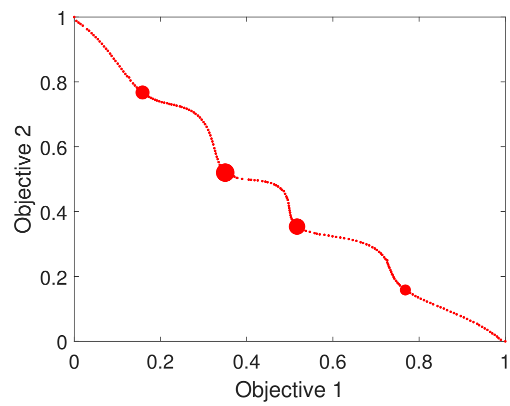


Fig. 2. The 4 SOIs selected using the proposed NNGA approach

B. Online identification of SOIs

During the online search, the SOIs identified using NNGA from the non-dominated (Rank 1) solutions are considered to be the anchor points, towards which rest of the solutions need to be driven in order to intensify the density in their vicinity. Therefore, during the environmental selection process, the remaining solutions (other than the SOIs) of the population are ranked simply based on their proximity to their closest SOIs. The proximity measure is based on

Algorithm 1 Proposed NNGA approach for identifying SOIs

Input: Given set of N non-dominated solutions NDF (objective values of all unique non-dominated solutions (NDX) for an M objective problem), $nSOI$ (number of SOI required)
Output: SOI_F (Final $nSOI$ solutions selected)

Identify ideal point Z^I and nadir point Z^N of NDF.
Normalize NDF using the ideal & nadir points to obtain F_n.
for $i = 1$ to N **do**
    Calculate $L_i = \sum_{j=1}^{M} 1 - f_{i,j}$
    Calculate $R_i = -(1 - f_{i,1}, 1 - f_{i,2} \dots 1 - f_{i,M})/L_i$ {Reference vectors}
end for
for $i = 1$ to N **do**
    for $j = 1$ to N **do**
        Calculate $\theta_{ij} = \cos^{-1} \frac{R_i . R_j}{||R_i|| ||R_i||}$ {Angles b/w reference vectors}
        Calculate $L_{ij} = L_i - L_j$ {Difference between L values}
    end for
    $O =$ Sort $\theta_{ij}, j = 1 \dots N$ in ascending order
    Let S be the sequence of solution ids in the above sorted order
    Identify $\Phi_i = O_{i(j-1)}$, where $j = \arg\min L_{iS_j} < 0$ {Angle of influence}
end for
Sort Φ_i in descending order and select top $nSOI$ solutions as SOI_F

normalized Euclidean distance in the objective space between the solution and its closest SOI. This process yields a complete order among the set of given solutions for any prescribed number of SOIs. For example, complete ordering a set of 198 and 1611 non-dominated solutions of problems DOD2K and DEB3DK (taken from [6]), respectively, is presented for various numbers of SOIs in Fig. 3.

It is clear from Fig. 3 that the approach is able to correctly identify the prescribed number of SOIs from the given set of tradeoff solutions. In addition to identifying them, the approach also generates a total order among the set of non-dominated solutions, where the SOIs themselves and solutions close to them are preferred more over others.

Although in the examples presented above we have generated a total order among the non-dominated set of solutions for various numbers of SOIs, it is possible to extend the approach further to order any given set of solutions (a mix of feasible and infeasible solutions, a set of all feasible solutions or a set of all infeasible solutions). In the event the population contains less than $nSOI$ feasible solutions, the order among the solutions is based on a *feasibility-first scheme*. In the event the number of feasible solutions are more than $nSOI$, the solutions in the first non-dominated front are ordered as discussed above followed by proximity based ordering of all remaining feasible solutions. In the event all the solutions are infeasible, the solutions are ordered based on an increasing order of sum of constraint violations.

Since we now have a means to generate a total order among the solutions, this can be readily integrated in a conventional multi-objective evolutionary algorithm for online search. For demonstrating the process, we use the framework of the widely used non-dominated sorting genetic algorithm II (NSGA-II) [26], and replace the non-dominated sorting+crowding distance based ordering of the feasible solutions with the above discussed NNGA ranking[2]. Use of such an ordering scheme is particularly attractive as it eliminates the need for active reference vector adaptation for search intensification that is commonly used by decomposition based knee search strategies. In order to effectively utilize the information about the objective space already explored, we also make use of the complete archive of solutions at every generation to select members of the population for the next. For consistency and a fair comparison, we use archive based environmental selection in original NSGA-II as well.

III. Numerical Experiments

In this section, the performance of the NNGA for online knee identification is objectively assessed using three unconstrained multi-objective knee benchmarks (DEB2DK4, DEC3DK1, DEB3DK4) [6], a constrained 5-objective water resource management [27] problem and a constrained wind turbine design optimization problem involving 5 objectives and 22 constraints [28]. We compare and contrast the solutions obtained using the traditional ranking (without knee focused search) versus NNGA. Moreover, we show further analysis in variable space for a 4-objective building energy management problem [16] that could be of further relevance in terms of practical decision-making.

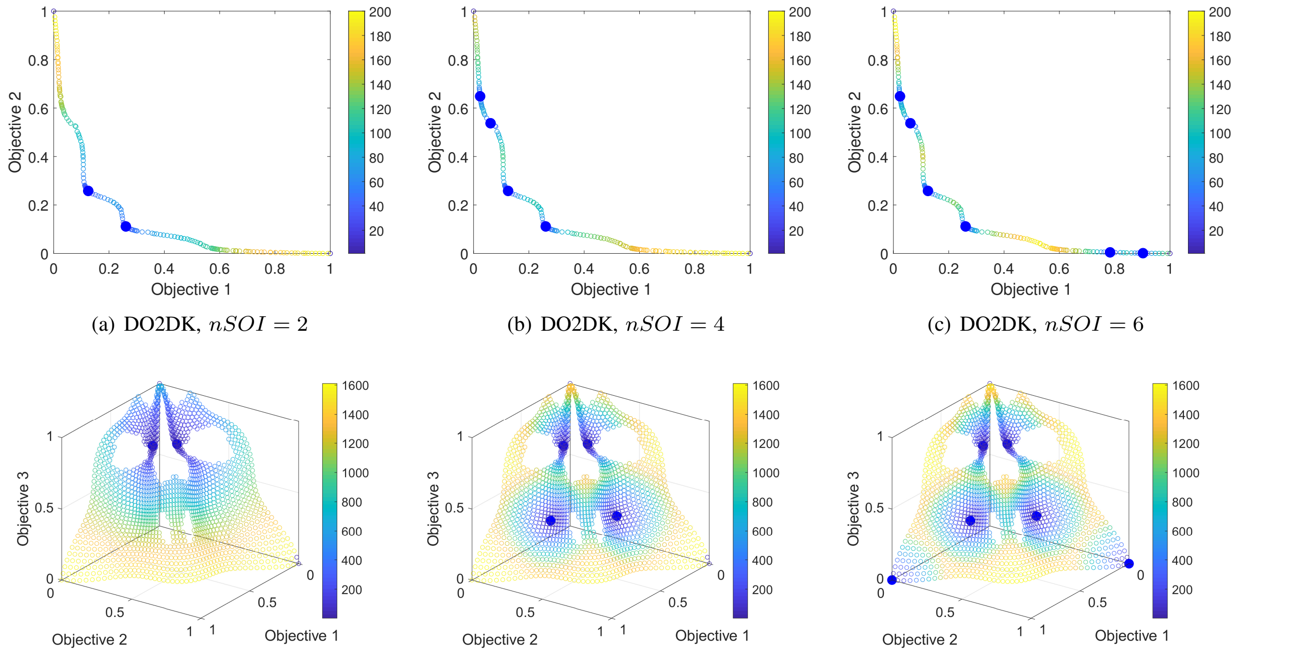
A. Visual comparison for 2/3-objective unconstrained examples

We first solve the 2-objective DEB2DK4 problem (number of variables=30, with the k and s set to 4 and 0 respectively) with an aim to identify 4 SOIs. Thereafter, we solve the tri-objective DEB3DK problem with an aim to identify 1 and 4 SOIs. The k and s parameters are set to [1,0] and [2,0] for DEB3DK which corresponds to $nSOI$=1 and $nSOI = 4$ respectively. All the above problems have been solved using a population of 100 solutions with a total computing budget limited to $10,000$ function evaluations. The probability of simulated binary crossover (SBX) and probability of polynomial mutation (PM) was set to 1 and 0.1 respectively for all the problems studied in this paper. The distribution indices of SBX and PM were set to 20.

In order to visually observe the behavior of the underlying search strategies, we first collect all solutions delivered by the NNGA and NSGA-II from a single run. Thereafter, we generate the non-dominated set of solutions from the combined set of solutions and compute the ideal and nadir coordinates. With this ideal and nadir, the complete archive of solutions obtained by the proposed algorithm and NSGA-II is scaled and the results are presented in Figs. 4(a) through to Fig. 4(f).

In DEB2DK4 problem with 4 knees, one can observe that the proposed algorithm conducts an intensified search around all 4 of them. Similar behavior is also demonstrated by the proposed algorithm for DEB3DK problem with 1 and 4 knees. On the other hand, NSGA-II delivers scattered set of solutions, where density varies across the PF approximation but is not particularly high around SOIs.

[2]Note: For simplicity, we refer to both offline and online versions as NNGA



(a) DO2DK, $nSOI = 2$ (b) DO2DK, $nSOI = 4$ (c) DO2DK, $nSOI = 6$

(d) DEB3DK, $nSOI = 2$ (e) DEB3DK, $nSOI = 4$ (f) DEB3DK, $nSOI = 6$

Fig. 3. Results for DO2DK and DEB3DK problems. The selected SOIs are indicated using the large blue dots, whereas the colormap shows the order of solutions with a lower value indicating a more preferable solution.

B. *Performance on constrained 5-objective examples*

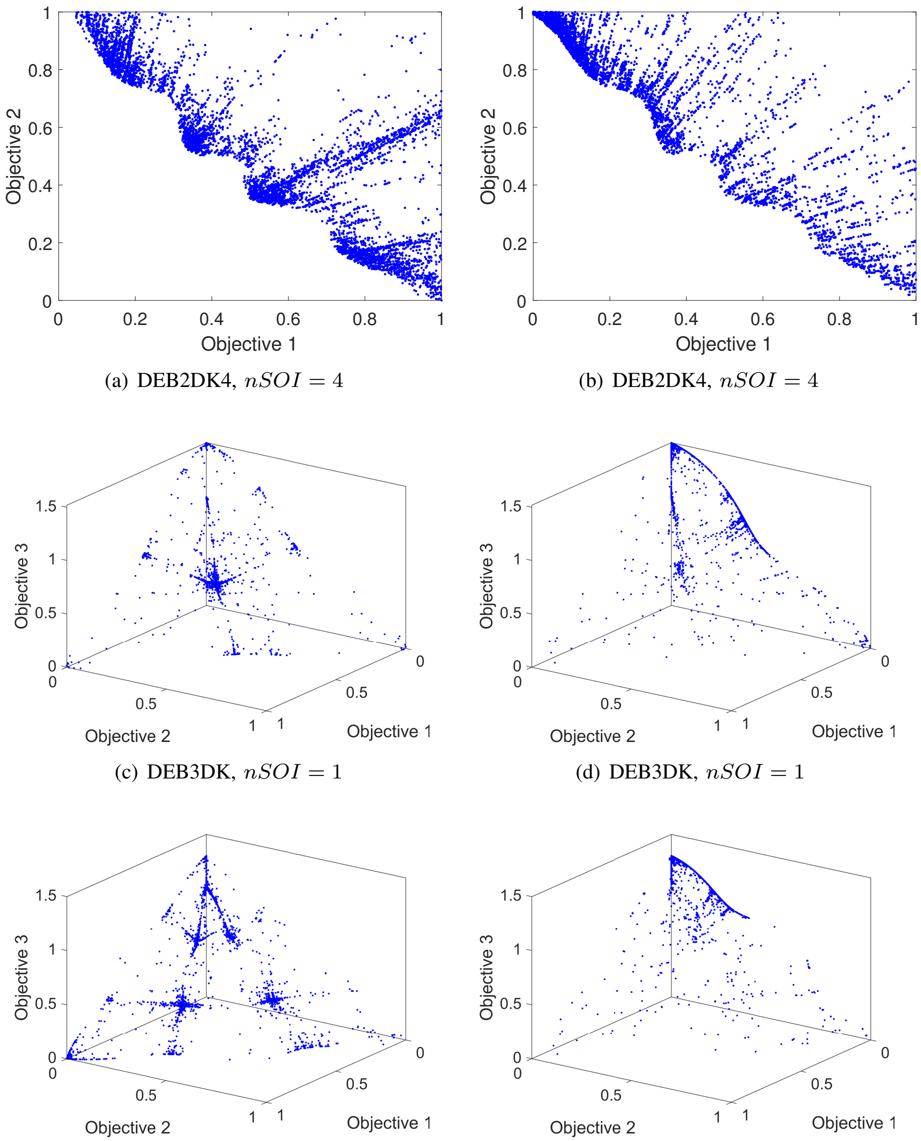
Having visually demonstrated the performance of the NNGA on the above knee benchmarks, we move on to investigate its performance on two constrained optimization problems, namely the water resource management problem [27] and wind turbine optimization problem [28]. For both problems we attempt to solve for $nSOI = 1$ purely because the quality of the solution and behavior of the search strategy can be unambiguously assessed. A number of metrics have been proposed in [24] to objectively assess the performance of knee based search strategies, such as KGD, KIGD and KD derived from generational distance/inverted generational distance counterparts. We adopt a variant of KIGD that measures the density of solutions over a range of distances in both objective and variable space (discussed further in the next subsection). For the water resource problem, a population of 210 solutions were evolved over 100 generations. For $nSOI = 1$, the results of both the algorithms (i.e. the archives) were combined and non-dominated sorting was used to identify the overall set of non-dominated solutions. These solutions were then normalized using the coordinates of the ideal and the nadir to yield a combined set of normalized non-dominated set of solutions. The first SOI, i.e. the solution with the highest L measure in combined set was one which was delivered by the proposed algorithm. Since the proposed algorithm was tasked to search for a single SOI, it is expected that there would be significant number of solutions searched/generated around this SOI. The histogram of L measures of all non-dominated solutions delivered by the proposed algorithm and NSGA-II are presented in Fig. 5(a) and Fig. 5(b). It is clear that a large number of solutions were generated by the proposed algorithm around the SOI as opposed to far fewer solutions generated by NSGA-II.

While the water resource optimization problem is still a relatively simple problem with well described mathematical equations of the objectives and constraints, the next problem involves use of numerical simulations to estimate the objectives and constraints. The problem involves optimization of a wind turbine design, and the detailed formulation can be found in [28]. A population of 330 solutions were evolved with an evaluation budget of $10,000$. The histogram of L measures of all non-dominated solutions delivered by the proposed algorithm and NSGA-II are presented in Fig. 6(a) and Fig. 6(b), respectively. It is clear that a large number of solutions were generated by the proposed algorithm around the SOI (a total of 1128 non-dominated solutions) while NSGA-II had solutions with varying L measure spanning the tradeoff surface (a total of 1755 non-dominated solutions).

C. *Further observations in the variable space*

So far we have focused on the performance of the proposed algorithm and illustrated its ability to deliver solutions in and around SOIs with higher intensity. While most of the works including the one presented above only considered objective space in selecting SOIs, in a practical scenario, a decision-maker may also be interested in observing the decision space of the corresponding solutions. In order to present some relevant solutions to the decision-maker in this regard, two different scenarios are presented.

1) The first is where the decision-maker is interested in seeing multiple solutions which give similar performance



(a) DEB2DK4, $nSOI = 4$ (b) DEB2DK4, $nSOI = 4$

(c) DEB3DK, $nSOI = 1$ (d) DEB3DK, $nSOI = 1$

(e) DEB3DK, $nSOI = 4$ (f) DEB3DK, $nSOI = 4$

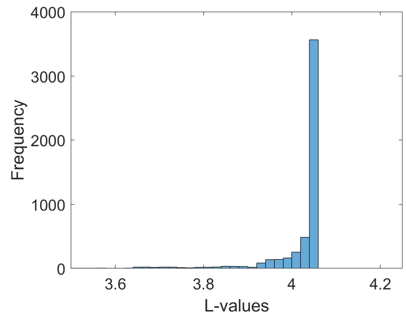
Fig. 4. Results of NNGA (left) and NSGA-II (right) for the test problems.

as the SOIs in the objective space and are close to the SOIs in the decision-space as well. This will allow for variations and options in a particular design. We refer to this output as Type 1 solutions.

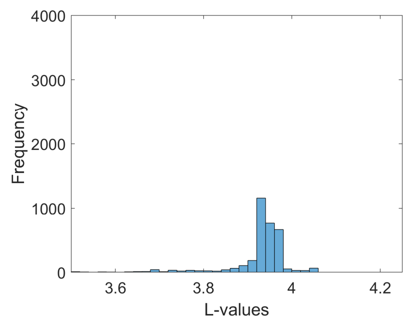
2) The second is where the decision-maker is interested in seeing multiple solutions which give similar performance in the objective space but are far apart in the decision-space. The idea is to examine a diverse set of designs very different from each other that are able to deliver similar performance. We refer to this output as Type 2 solutions.

In order to provide Type 1 and Type 2 solutions, we consider different thresholds on the neighborhood in normalized objective and variable spaces. Consider a given threshold th and a given SOI. We identify the set of all solutions S_o in the objective space which are within th normalized distance from the given SOI in the objective space. Next, we identify the set of solutions S_{x1} that are within th normalized distance from the given SOI in the decision space. Lastly, we identify the set of solutions S_{x2} that are more than th normalized distance from the given SOI in the decision space. Type 1 solutions can then be obtained by the set intersection $S_o \cap S_{x1}$, whereas Type 2 solutions can be obtained by the set intersection $S_o \cap S_{x2}$. This can be repeated for different distance thresholds to expand the neighborhood definition, and a distribution of the number of Type 1 and Type 2 solutions obtained for each case can be plotted.

We illustrate this using the database of solutions from the 4-objective building energy management problem studied earlier in [16]. From the available set of 100 solutions, 2 SOIs are identified using NNGA. Then, the number of Type 1

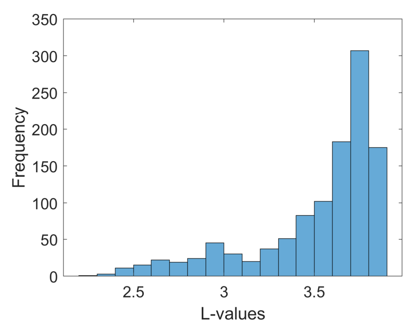


(a) NNGA, $nSOI = 1$

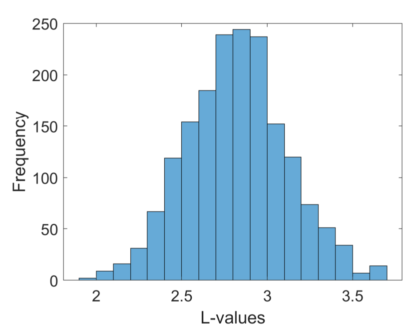


(b) NSGA-II, $nSOI = 1$

Fig. 5. The distribution of the L values of solutions obtained using the NNGA and NSGA-II for the water resource management problem



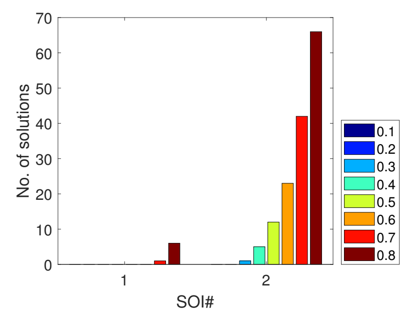
(a) Proposed, $nSOI = 1$



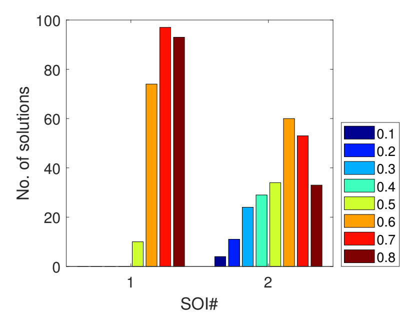
(b) NSGA-II, $nSOI = 1$

Fig. 6. The distribution of the L values of solutions obtained using NNGA and NSGA-II for the wind turbine design problem

and Type 2 solutions for different values of th (normalized neighborhood radius) is plotted, as shown in Fig. 7. For example, the first SOI has no Type 1 solutions for the th value as large as 0.6, which signifies that no similar designs can be found in the neighborhood. On closer examination of objective values, it also happens to be that the solution is isolated from the rest of the solutions in the objective space. The second SOI on the other hand has a good choice of designs around its decision space which also give similar performance in objective space. Alternate options are available for even with the lowest th value of 0.1 and increases as th is increased further. With Type 2 solutions, a high value of $th = 0.5$ is needed to obtain solutions that can achieve similar performance as the first SOI. In contrast, for the second SOI, once again it is possible to get diverse solutions that can obtain close performance, starting from the smallest value of $th = 0.1$. Thus, practically it may be attractive to pick the second SOI, especially in case there are uncertainties in the design process in which case there are many more options around it to chose from; and similar design performance is achievable using similar as well as different other designs.



(a) No. of Type 1 solutions



(b) No. of Type 2 solutions

Fig. 7. The distribution of solutions obtained for two SOIs identified for building energy management problem data

IV. Summary and Conclusions

In this paper we have introduced an approach that can automatically identify and search for and around a handful of SOIs that are easy for a decision-maker to comprehend. At the core of the algorithm lies a selection mechanism that orders the set of solutions such that the SOIs and their neighbors are preferred more over others. From a set of non-dominated solutions, one with the maximum normalized net gain over nadir is selected as the first SOI. Subsequent SOIs are selected based on a descending order of their angle of influence. The feasible solutions in subsequent fronts are selected based on their proximity to the closest SOIs in terms of Euclidean distance in objective space. The behavior of the selection scheme and its ability to order various trade-off set of solutions for different numbers of SOIs was presented visually using standard multi-objective benchmarks with knees (DEB2DK4, DEC3DK1, DEB3DK4).

Since most practical problems involve constraints, we further observed the performance of the proposed algorithm for such classes of problems. To this end, we solved two 5-objective constrained problems. The first one, water resource management problem, involves empirical equations formulated in the literature; whereas the second one is a recently introduced wind turbine design optimization problem with simulation-based evaluations. Both the instances were solved for $nSOI = 1$ to objectively assess the ability of the algorithm to deliver a high density of solutions near the SOI.

Since the approach delivers a relatively high number of solutions around the SOIs, it is possible to mine this information to uncover the characteristics of the SOIs for better informed decision making. This is demonstrated using a building energy management problem. Ideally there is a need to consider both objective and variable space density during the course of search which have practical implications on robustness, uniqueness and of potential family of solutions. Towards this end, incorporation of variable space information for online SOI identification will be explored by the authors in the future extension of this work.

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