# Modeling of synchronous weapon target assignment problem for howitzer based defense line

O. Tolga Altinoz
*Department of Electrical and Electronics Engineering*
*Ankara University*
Ankara, Turkey
taltinoz@ankara.edu.tr

***Abstract*—Weapon-target assignment (WTA) is a combinatorial optimization problem in the NP-hard category. The problem is to assign targets to weapons in order to ensure that the targets are eliminated with weapons in the most effective way. This problem, which can be discussed on different scenarios, is called static or dynamic according to the definition of the problem. In this study, the target value and hit probability values will be assigned according to the position of the target and the weapon. The WTA problem will be analyzed in different cases and solved using optimization algorithms. Howitzer will be chosen as weapon and four different types of targets will be offered to get solutions through different scenarios. Three different WTA problems will be examined. The first two scenarios will be solved with single-objective optimization algorithms, and the last case will be modeled as a multi-objective optimization problem.**

***Index Terms*—weapon-target assignment, genetic algorithm, particle swarm optimization, differential evolution, artificial bee colony, bees algorithm, firefly algorithm, simulated annealing, invasive weed optimization, NSGA-II, MOEAD, MOPSO**

## I. Introduction

The Weapon-Target Assignment (WTA) which is in the class of NP-complete problems [7], can be simply defined as systematically assigning the weapons corresponding to the targets in the most effective way possible. These two words (”systematically” and ”effective way”) determine the type of assignment problem. It has been examined in two main categories in the literature. These are: Static WTA and Dynamic WTA problems [5]. In the static WTA problem, a one-time assignment is made from targets to weapons, and the aim is to give the maximum possible hit probability. In general, in the static WTA problem, hit probability, target value values and number of targets and weapons are the most important parameters. Although it is called static WTA, the problem can also be considered as a multi-stage problem. This means that different assignments are made at each stage. These assignments are repeated for a different number of targets, depending on whether the target is eliminated or not. At each stage the weapons are assigned to the targets. But if the weapon can destroy the target at this stage, the number of targets decreases in the next stage. The evaluation of such a multi-stage scenario as static or dynamic will depend on other criteria. These criteria may be time-dependent criteria such as the number of ammunition in the weapons and / or the firing time of the weapons. It is certain that the problem should be called Dynamic WTA if there are connections between the stages that change dynamically, and this dynamic change will affect the dynamics of the system. If these stages are run as semi-independent events and there are values that may vary in the previous stage, such as the number of targets / weapons affecting the stages, such a problem should be considered as Static WTA. When the current studies are examined, it is seen that the calculation areas shown as stages in the multi-stage dynamic WTA problem are evaluated as time windows. In this way, ammunition and time, which are the sources used to neutralize the target, have become an important part of the WTA problem.

In the light of studies in the literature related to WTA, it is recommended to examine the WTA problem in three categories. These categories are synchronous WTA, asynchronous WTA and dynamic WTA problems. Synchronous WTA and asynchronous WTA problems can be evaluated as static WTA. According to a probabilistic value determined in the synchronous WTA problem, the hit probability values are examined, and a comment is made about eliminating the target and the target number is updated in the next stage. Thus, more than one static WTA problem is solved for different number of targets. In the asynchronous WTA problem, not only the target but also the ammo information is shared between the stages. A weapon may be shooting at a target at different consecutive stages and increasing the hit probability. In synchronous and asynchronous WTA problems, the environment is almost static. Other variables such as the number of targets and ammunition, if any, vary. In the dynamic WTA problem, the environment is dynamic. Not only stages, but also weapons and targets can move over time. There may be more than one type of target and weapon (this applies to synchronous and asynchronous WTA). These targets can move through time and eliminate weapons. At the same time, the positions of the weapons may change. All movements are time dependent and the stage returns to time. The more dynamic the environment, the more realistic a scenario can be achieved. At the same time the problem turns to Game Theory as the dynamism increases.

Regardless of whether static and dynamic synchronous or asynchronous in the WTA problem, hit probability and target value are important parameters. Hit probability and target value values are variables that depend on multiple criteria, although they are shown at certain fixed values for targets and

978-1-7281-6929-3/20/$31.00 ©2020 IEEE

weapons in the scenario. The hit probability depends on the type of weapon, the type of target, the type of firing of the weapon, the distance between the weapon and the target and, if any, other characteristics of the weapon and the environment. In addition, positioning the target relative to the weapon for the same target also affects the hit probability value. The surface shapes of the region will affect the hit probability value. Similarly, Target Value depends on the type of target and the location of the target. The target may be moving and there may be important areas close to the target. If the target can move to a place where it can cause harm or damage, the target value should increase.

In this study, it is accepted that the hit probability (and target value) value changes linearly with the distance between the weapon and the target. That is, as the distance between the weapon and target increases, the hit probability will decrease. This distance information and the effect of distance on hit probability (target value) will be added to the WTA problem. The target value depends not only on the distance between gun and target, but also on the target type. This information has been added to the WTA problem.

The WTA problem has been studied as an engineering and military problem since the 70s. Until 70s studies are summarized the WTA problems like naval and aerial missile defense [6]. Today, heuristic methods are used for WTA problem solution and the performance of these methods is discussed. The survey of WTA research was also provided, and It is indicated that the static WTA models are mainly studied, and the dynamic WTA models are not fully/detailed studied [1]. The ballistic missile defense mission is about the counterattack against the incoming missile and selecting the highest hit probability value makes it a WTA problem. But the hit probability value is a value that changes over time due to missile. This value is assumed to be known in many WTA studies, and in that study, the estimation of the hit probability value has been studied by using machine learning model. An interceptor with a hit probability higher than a threshold is lunched. The problem has been run for the scenario where there are five interceptor and up to 100 targets. A 30-minute limit is set for the calculation time [14]. A land-based air defense system is a security system that tries to eliminate incoming air targets with a minimum survival probability. This system can be modeled as WTA system. When modeled as a WTA problem, properties of the environment such as engagement duration, setup duration, target type were added to the WTA problem and solved dynamically using constraint based nonlinear goal programming algorithm. In the scenario applied, three defensive systems were assumed to have a total of 24 targets. In the same time, three types of missiles were assumed to be in addition to multiform shapes. This problem based on a realistic model can be described as a dynamic WTA problem [15].

WTA problem is evaluated from a different perspective, the expected damage of own-force asset is minimized by a definition of a novel GA with greedy eugenics [2]. The results showed the performance of the proposed algorithm when compared with other GA variants. Also, it is possible to compare results from [3]. Also, the parameters of GA investigated and improved [4] where, new crossover operator with greedy reformation improved GA for weapon target assignment (WTA) is proposed and tested on W=10 and T=10 problem case. The implementation results are compared with GA algorithm and results showed that proposed GA converges and gives the best fitness value [4]. In [3], for 10 target (T) and 10 weapon (W) ant colony optimization (ACO)-based algorithm and proposed immunity-based ACO converge to the best fitness value. The proposed algorithm compared with Simulated Annealing (SA), and Genetic Algorithm (GA) for the set of problems $[W, T] = [50, 50]; [80, 80]; [100, 80]; [120, 80]$. The results reported in the paper gives the best to worst ranking of algorithms for all cases is SA, GA, immunity based ACO. From all results it can be observed that the performance of immunity based ACO gives better performance except for (W=50, T=50) problem case [3], [2].

Li et al [9] in their study focused on multi-objective optimization of the multi-stage WTA problem (asynchronous WTA). The aim of the study is to provide the least amount of ammunition while harming the targets. For this purpose, two multi-objective optimization algorithms NSGA-II and MOEAD algorithms were applied to the problem. However, these algorithms have been improved with adaptive mechanism for crossover rate. In this study, a chromosome has been defined as the number of weapon length as encoding method and each genetic value holds the information about the target of the weapon in the corresponding index. Also, a random repair mechanism has been proposed to prevent unfeasible solution caused by the crossover operator. W = 50 and T = 50 were evaluated as 8 stages [9]. The static WTA (synchronous WTA) problem is modeled as a multi objective optimization problem and it is aimed to use the minimum number of ammunition while giving maximum damage to the target [12]. NSGA-III algorithm has been proposed to avoid unnecessary and repetitive solutions by defining the domination matrix; this method is called D-NSGA-II-A. In the study, 4 weapons used 12 ammunition in total and directed to 10 targets. When the results are examined, it is seen that the proposed algorithm gives the best performance. In addition, it has been shown that NSGA-II algorithm produces a more effective solution for this two-objective problem than MOEAD algorithm [12].

In [16] branch and bound algorithm is applied to the WTA problem. W = 80 and T = 160 were evaluated as test problem. A hybrid multi-objective discrete particle swarm optimization algorithm is proposed to solve the dynamic WTA (air combat) problem [10]. This problem can be categorized as asynchronous WTA. The reason for this is that static WTA is run for each step and the target number changes in the next step. In this research, the encoding scheme proposed in the [9] declaration was used together with the Boolean type decision matrix. Constraint expressions are evaluated as penalty function. W = 20 and T = 10 were evaluated as 10 stages [10]. In [8] dynamic WTA (DWTA) was investigated in the research. The improved Particle Swarm optimization

TABLE I
SOLUTIONS FOR THE CASE-1 WEAPON TARGET ASSIGNMENT PROBLEM

| | W=50, T=50 | | W=100, T=100 | | W=200, T=200 | |
|---|---|---|---|---|---|---|
| **Algorithm** | **Mean** | **Std. Dev.** | **Mean** | **Std. Dev.** | **Mean** | **Std. Dev.** |
| Constraint Genetic Algortihm (CGA) | 44.0757 | 0.0369 | 87.3309 | 0.0243 | 174.2738 | 0.0359 |
| Genetic Algorithm (GA) | 44.0736 | 0.0184 | 87.3310 | 0.0705 | **174.2647** | 0.0976 |
| Particle Swarm Optimization (PSO) | 44.0793 | 0.0162 | 87.3349 | 0.0245 | 174.2808 | 0.0288 |
| Differential Evolution (DE) | 44.0842 | 0.0176 | **87.3298** | 0.0287 | 174.2680 | 0.0381 |
| Artifical Bee Colony (ABC) | 44.0808 | 0.0191 | 87.3328 | 0.0256 | 174.2818 | 0.0299 |
| Bees Algorithm (BA) | 44.0750 | 0.0225 | 87.3381 | 0.0293 | 174.2743 | 0.0279 |
| Firefly Algorithm (FA) | **44.0695** | 0.0183 | 87.3331 | 0.0278 | 174.2691 | 0.0328 |
| Simulated Aneealing (SA) | 44.0787 | 0.0161 | 87.3409 | 0.0268 | 174.2797 | 0.0399 |
| Invasive Weed Optimization (IWO) | 44.0731 | 0.0227 | 87.3391 | 0.0291 | 174.2671 | 0.0306 |

algorithm with deterministic initialization and target exchange schemes for increasing solution process and quality. Different heuristics optimization algorithms are implemented for 36 case studies of DWTA where maximum 50 weapons and 200 targets for 5 stages are selected. The results showed the performance of the proposed PSO algorithm [8]. Also in [13], Dynamic WTA problem is solved by using decomposition co-evolution algorithm for cooperative aerial warfare. There are two fighter teams here, which aim to do the most damage and take the least damage. The problem is called antagonistic game WTA. The dynamic WTA problem is defined as a game model between two classes of combat units (red and blue) in [11]. The clonal selection algorithm (CSA) was used for problem solving and the results were interpreted through a relatively small scenario. It was shown that the solutions could not converge to the solution compared GA to CSA algorithm [11].

In this study, WTA problem will be modeled in three different ways. In the first model, the WTA target value will be taken constant and the hit probability will be determined according to the distance between the weapon and the target, and the WTA problem will be solved. In the second case, target value will be determined for four different types of targets. Similarly, the distance between the weapon and the target will be considered. These two situations will be solved using single-objective optimization algorithms. In the last case, it will be included in the problem with the ammunition. In addition to the ammunition, the same target will be considered in case of shooting with a target. This can be called a continuous shooting weapon mode. In this case, the hit probability is assumed to reach 0.9 after 10 shots. As a result, the least amount of ammo will be added as an extra objective. Thus, WTA will be transformed into a multi-objective optimization problem.

This study consists of four chapters following the Introduction. In the second part, WTA problem will be explained, in the third part, optimization algorithms will be summarized and 3 encoding scheme will be given, the results will be presented in part 4 and the result of the research will be presented in the last part.

## II. WEAPON-TARGET ASSIGNMENT (WTA) PROBLEM

The WTA problem is the problem of directing / assigning weapons to target targets for different scenarios. The main objective is to eliminate the target or to cause the most damage. The hit probability concept is used for the damage. In addition, the value of each target can be different for this purpose, the target value variable can be used. The equation used in this study and WTA problems is given below.

$$J = \sum_{i=1}^{T} v_i \prod_{j=1}^{W} (1 - p_{ij})^{x_{ij}} \tag{1}$$

$$\sum_{i=1}^{T} x_{ij} \leq W \tag{2}$$

where $W$ is the number of weapons, $T$ is the number of targets, $x$ is the decision Boolean variable, $v$ is the value of target and $p$ is the hit probability. The constraint expression given in Equation 2 indicates the assignment of each weapon to at least one target. In other words, no weapon will remain idle.

In the studies performed in literature, it was accepted that the hit probability and target value values are known and constant(fixed). The performance of different optimization algorithms was examined according to these values. Although these investigations can be single-objective or multi-objective, it is accepted that the hit probability and target value values are known. The two values, hit probability and target value, are based on two main references. These references are the distance between the weapon and the target and the type of target. Fig 1. shows variation of hit probability and the target value according to the target type and distance between weapon and target.

As shown in Fig 1, four types of targets were used in this study. Hit Probability values selected according to these targets and distance are shown. In the same plot, the target value base value is also given. This base value is multiplied by the target type to obtain the target value. In this study, Land Target: 1, Large Target: 2, Small Target: 3, and Armored Target: 4 values are assigned. However, these values can be divided into different categories and can take different values. Fig 2 gives the positions of targets and weapons.

Two cases were examined in this study. In the first case, a uniform target and a fixed target value for each target were determined. In the other case, 4 types of targets were included

TABLE II
SOLUTIONS FOR THE CASE-2 WEAPON TARGET ASSIGNMENT PROBLEM

| | W=50, T=50 | | W=100, T=100 | | W=200, T=200 | |
|---|---|---|---|---|---|---|
| **Algorithm** | **Mean** | **Std. Dev.** | **Mean** | **Std. Dev.** | **Mean** | **Std. Dev.** |
| Constraint Genetic Algortihm (CGA) | **113.4861** | 0.30344 | 240.5148 | 0.54487 | 457.3442 | 1.1932 |
| Genetic Algorithm (GA) | 113.5557 | 0.28737 | 240.6019 | 0.40388 | 457.2868 | 0.68476 |
| Particle Swarm Optimization (PSO) | 113.5437 | 0.32087 | 240.6845 | 0.39621 | 457.4227 | 0.67554 |
| Differential Evolution (DE) | 113.5864 | 0.32544 | **240.3815** | 0.40768 | 457.4882 | 0.59135 |
| Artifical Bee Colony (ABC) | 113.5258 | 0.32685 | 240.5949 | 0.37365 | 457.2818 | 0.71339 |
| Bees Algorithm (BA) | 113.5815 | 0.25386 | 240.585 | 0.50027 | 457.2846 | 0.588 |
| Firefly Algorithm (FA) | 113.6753 | 0.27389 | 240.4855 | 0.37062 | **457.2309** | 0.68924 |
| Simulated Anealing (SA) | 113.5811 | 0.27389 | 240.5612 | 0.37062 | 457.4561 | 0.68924 |
| Invasive Weed Optimization (IWO) | 113.5259 | 0.27212 | 240.4018 | 0.37882 | 457.3156 | 0.64261 |

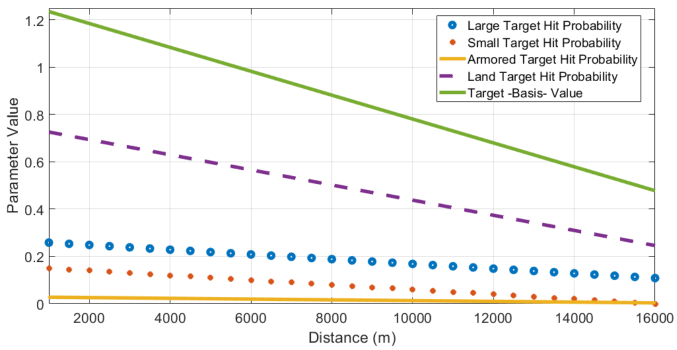


Fig. 1. Variation of hit probability and target value according to target type and distance between weapon and target.

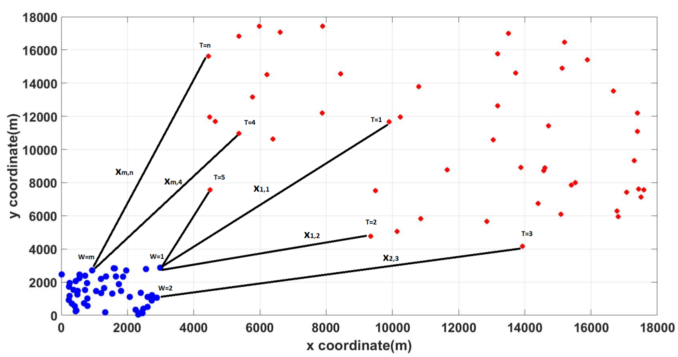


Fig. 2. Position of Targets (T) and weapons (W) on a map.

in the problem by calculating the hit probability and target value according to the target type and distance. The equation given below is the linear equation to be used for both hit probability and target value.

$$p^i = R_1^i - R_2^i * d \tag{3}$$

where $i$ is the number of target types, $d$ is the Euclidean distance between target and weapon (km), $p$ is the hit probability. The values for $(R_1, R_2)$ are selected as $(0.2684, 0.01)$, $(0.1599, 0.01)$, $(0.0291, 0.0016)$, $(0.758, 0.032)$ for Land, Large, Small, and Armored Target, respectively. If you want to use the same equation for the target value $(1.286, 0.0505)$ is selected. In this case, the objective function is written as follows:

$$J = \sum_{i=1}^{T} v_i \prod_{j=1}^{W} \left(1 - \left(R_1^i - R_2^i * d_{ij}\right)\right)^{x_{ij}} \tag{4}$$

In the WTA problem, weapons are assigned to targets. One of the issues to be considered while doing this is the characteristics of the weapons. Howitzer can shoot sequentially when directed at a single target, and these shots increase the hit probability value. In this study, it is accepted that the hit probability value of 0.9 will be reached after 10 shots, regardless of the distance between the target and the weapon. With the addition of ammo to the WTA problem, keeping the ammo number at the minimum value becomes one of the intended values. Therefore, as a second objective, the number of ammunition will be kept to a minimum. The relationship between gunshot number and hit probability is linear.

## III. OPTIMIZATION ALGORITHMS

In this study, eight different single objective optimization algorithms with three different multi-objective optimization algorithms were applied to the WTA problem and the results were interpreted comparatively. GA has been implemented in two different ways: GA (Penalty) and Constraint GA. All the remaining algorithms are added to the goal value by multiplying the constraint with a relatively large value using the penalty method. In multi-objective optimization three layer encoding scheme proposed in this study is applied Constraint GA was used with the Augmented Lagrangian method proposed by Deb [25].

Genetic Algorithm (GA) is the evolutionary algorithm first proposed by Holland in 1975 [24]. GA chromosomes make up a population. This population is used to create new individuals with the crossover operator and mutation operator. These new individuals and the best individuals among their ancestors are chosen to survive for the next generation. Augmented Lagrangian Genetic Algorithm (ALGA) (Constraint Genetic Algorithm) method is an approach proposed by Deb when it is nonlinear constraint [25]. Together with the nonlinear constraint Lagrangian and the objective function, they create the new objective function as sub-problem. Differential Evolution (DE) is an evolutionary algorithm proposed by Storn

TABLE III
HYPERVOLUME METRIC FOR MULTI-OBJECTIVE WEAPON-TARGET ASSIGNMENT PROBLEM

| Problem | Decision Variable | NSGAII | MOEAD | MOPSO |
|---|---|---|---|---|
| W=50,T=50 | 99 | **8.9363e-1 (5.67e-3)** + | 8.6415e-1 (7.20e-3) − | 8.7938e-1 (1.13e-2) |
| W=100,T=100 | 199 | **7.4271e-1 (1.08e-2)** + | 6.8719e-1 (7.79e-3) − | 7.1017e-1 (1.27e-2) |
| W=200,T=200 | 399 | **4.4957e-1 (1.26e-2)** + | 3.8071e-1 (1.80e-2) − | 4.1051e-1 (2.27e-2) |

and Price in 1997 [22]. Algorithm, as in other evolutionary algorithms, is based on obtaining new individuals from the existing population and transferring the best of all individuals to the next generation. Unlike Genetic Algorithm, individuals applied to the mutation operator first are subjected to selection.

Simulated Annealing (SA) algorithm was proposed in 1983 and has been used in engineering problems since then [18]. Simulated annealing is an optimization algorithm based on a phenomenon called solids annealing to optimize a complex system. Annealing means heating one layer and then cooling it slowly. After this process, atoms become the minimum energy state globally. The algorithm starts with a relatively high temperature value. The initially set temperature is cooled slowly as the algorithm progresses. At this stage, a neighbor solution is chosen by making a small change in its current solution. as the last step Whether this neighboring solution will be chosen is decided according to the value of objective. Invasive Weed Optimization (IWO) algorithm is a nature-inspired optimization algorithm proposed in 2006 [17]. The IWO algorithm is based on planting seeds in one region and surviving the best of these propagated seeds. The algorithm begins by randomly distributing seeds (candidate for solution) to the search space. In these seeds, they grow according to their goal values, survive, and their seeds benefit other regions. Seed production depends on its own target value and the smallest and largest target values of all weeds. These nine optimization algorithms are applied to WTA problems.

Particle Swarm Optimization (PSO) is an optimization algorithm in which the herd behavior of animals that move in flocks like birds is modeled [23]. The location and speed information of each member of the herd is defined. Each member acts by considering the best goal value of the herd and the best goal value in their memories. This motion is the solution to the optimization problem, and the place to act is the search space. Bees Algorithm (BE) algorithm is another algorithm that takes the behavior of the bees proposed in 2006 as a model [20]. In this algorithm, it has emerged by modeling the foraging behavior of bees. The algorithm starts with sending scout bees to different areas. Bees with the best objective value are selected. Later, more bees are searched in these areas and their neighborhood. The remaining bees continue to search for different regions randomly. Artificial Bee Colony (ABC) algorithm is an algorithm proposed in 2007, which is based on another bee behavior [21]. There are three types of bees in nature-inspired ABC algorithm: employed bees, onlooker bees and scout bees. Employed bees keep the information of the food recipient in their memories and look for food around it. They also share the information of these food sources with onlooker bees. Onlooker bees choose the source of food from the incoming information. There will be more chances to choose the food source that gives better objective value than the one with smaller objective value. Along with this, scout bees are searching for food sources on a random basis. Firefly Algorithm (FA) algorithm is an algorithm proposed in 2007 and developed on firefly behavior [19]. Fireflies try to influence each other with their lights in order to mate. Light intensities depend on both the firefly and the distance between the fireflies. The firefly moves towards the more intensely insect. if there is no such insect, it acts randomly. The location of the firefly corresponds to the solution. The solution is obtained as the firefly moves.

As multi-objective optimization algorithms, MOEAD [27], NSGA-II [26], and MOPSO [28] algorithms were chosen. NSGA-II and MOPSO algorithms are algorithms developed to find non-dominated solutions. A new population is generated using GA-based operators in the NSGA-II algorithm. The best individuals from the population are used in the next generation. In MOPSO, this population is produced by the location and speed update algorithms used in PSO. MOEAD is a decomposition-based algorithm. In the algorithm, the problem is divided into more than one problem. In this respect, it is like scalarization methods. These problems are then solved using the evolutionary algorithm and solutions on the objective space are used in the next iteration according to their neighborhood and distance.

### A. *Encoding Scheme*

In this study, three-layer encoding scheme is used. The first layer is the basic matrix representation. The second layer is the vector expression of the matrix. It is also called 'permutation coding'. Finally, a method that converts permutation coding into binary (sometimes called real coding) coding has been proposed. The name for variables $x^{L1}$, $x^{L2}$, and $x^{L3}$ are defined for Layer 1, Layer 2, and Layer 3 respectively.

**Layer 1:** Regardless of the optimization algorithm used, defining WTA problem candidates in the algorithm is called encoding scheme. Solution candidates are matrices. The rows of the matrix correspond to the weapons and the columns to the targets. The elements of this matrix can take the value 0 or 1. Also, there is only one 1 per line in the matrix. This is the simplest assignment to be defined.

$$x_{i,j}^{L1} = \begin{cases} 1 & \text{if weapon } i \text{ assigned to target } j \\ 0 & \text{if else} \end{cases} \tag{5}$$

$$x^{L1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{6}$$

**Layer 2:** The preferred method in the studies in the literature is the use of vectors instead of matrices [9], [10], [4], [3], [2]. The size of the vector is the same as the number of weapons. The values of the elements that make up the vector give the information that the weapon corresponding to the index of that element is assigned to the target corresponding to the value of the element. The number of numbers that make up this vector should be the same as the target number and should not repeat each other if the number of weapons and targets is equal. Otherwise, this number should be less than the target number and it should not be repeated. These criteria may be defined as constraint.

$$x^{L2} = (\ 2, 4, 1, 3) \tag{7}$$

**Layer 3:** In this layer, a method will be proposed to make the weapon-target information vectored. Thus, more operator types can be selected instead of the limited number of crossover operators developed for permutation encoding. In addition, a solution will be proposed for the permutation + binary encoding requirement encountered for many applications. Thus, both the vector dimension and the search space dimension are reduced. It can also be used in multi-objective problems. However, it is thought to be applicable to other engineering problems.

The permutation of the elements $(1, 2, 3, 4)$ that make up a cluster is similar to the tree structure made up of units that are connected to each other. If permutation is considered as a vector $x^{L3} = x_1^{L3}, x_2^{L3}, x_3^{L3}$, one of the as many elements in the set can be selected for the first element of the vector. In this case, there are options equal to the number of cluster elements, and the search space length for this variable will be 4 for this example. One of the remaining elements can be selected after an element selected from the set. In this case, there are 3 options. After this step, only 2 options remain, and one of these 2 options is not selected. If it is not selected, it gives the last element. In this case, the search space range for the elements that make up the vector $x_1^{L3} \in [1, 4], x_2^{L3} \in [1, 3], x_3^{L3} \in [1, 2]$. Thus, the length of the vector is reduced, and the search space is narrowed. Integer value can be selected for vector elements and these values can be the same.

In Fig 3, for example $x^{L3} = 1, 3, 2$ gives $x^{L2} = 1, 4, 3, 2$, and that gives $x^{L1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$. It can be written more generally. Assume that there are N weapons and M targets. Suppose you have N weapons and targets. In this case, $x^{L1}$ will be $NxN$ in length. For Layer 2, the length of vector will be $N$, and for layer 3 it will be $N-1$ number of elements. Thus, $x_1^{L3} \in [1, N], x_2^{L3} \in [1, N-1], ...x_{N-1}^{L3} \in [1, 2]$.

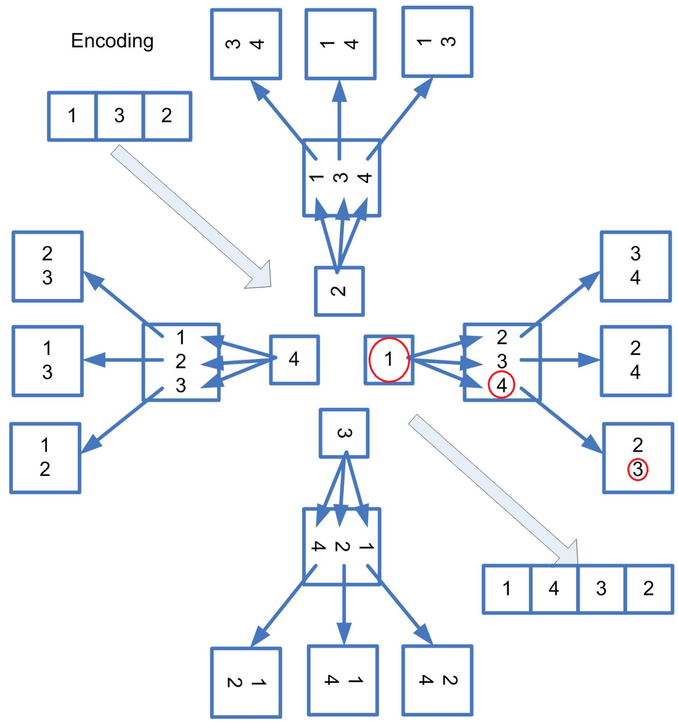


Fig. 3. Position of Targets (T) and weapons (W) on a map.

The most important advantage of this method is when it is included in the problem in the arsenal of weapons. In other words, it is used in mixed programming solution that requires permutation encoding and binary encoding. The number of ammunition to be sent to each target can be given with N elements to be added to the vector created in the third layer. (The code for layer 3 can be downloaded from https://dosyam.ankara.edu.tr/jdwxn)

## IV. Analysis and Results

In this study, three different applications will be made for the WTA problem. As explained in the introduction, the WTA problem is divided into three different categories. These are synchronous (static) WTA, asynchronous WTA and dynamic WTA. Synchronous WTA problem (can be called as static WTA) is the solution of WTA problem that is divided into certain stages for each stage. It is therefore sequentially the same as the static WTA solution. Therefore, in this study, static WTA is called synchronous WTA and it is the subject of this study.

WTA problem will be examined for three different situations. All three situations are improved versions of each other. In the first case, weapons and target positions were determined for the traditional static WTA problem. Hit probability was calculated according to the distance between them. The solution to this problem is given in the Table I. As can be seen from the table, although all algorithms find close results, it cannot be said that a single algorithm performs best for different weapons and target numbers. FA, DE and GA produce the best values for W = T = 50, W = T = 100, and W = T = 200,

TABLE IV
SPREAD METRIC FOR MULTI-OBJECTIVE WEAPON-TARGET ASSIGNMENT PROBLEM

| Problem | Decision Variable | NSGAII | MOEAD | MOPSO |
|---|---|---|---|---|
| W=50,T=50 | 99 | 9.8323e-1 (3.90e-3) $\approx$ | 9.9773e-1 (2.85e-3) $-$ | **9.8155e-1 (4.83e-3)** |
| W=100,T=100 | 199 | 9.9879e-1 (5.54e-4) $\approx$ | 9.9996e-1 (1.67e-4) $-$ | **9.9856e-1 (3.31e-4)** |
| W=200,T=200 | 399 | 9.9873e-1 (5.61e-4) $-$ | 9.9988e-1 (1.62e-4) $-$ | **9.9786e-1 (5.35e-4)** |

respectively. However, when standard deviations are examined, it can be concluded that there is no difference between the results.

In the second case, the target value is added to the problem. This operation also had to be included in the problem of the target type. For this purpose, four different types of problems have been identified. These are land target, small target large target and armored target. Land target is the biggest target. This target has the lowest target value due to its inability to respond to the attacks and its large size. Of course, it should not be forgotten that the target value is related to distance. Small target and large target may correspond to arch units with similar structures. Land target has a larger target value, considering that it can attack against large target and cause more damage due to its size. Armored target is having the largest target value. This target corresponds to armored units such as tanks. In fact, not only the distance between the gun and the target is important for such targets, but also the angle of the target's stance. However, in this study, it is assumed that such goals have an equal value regardless of their posture. WTA problem given in Case 1 as Case 2 was obtained by adding the calculation of target value data. The results given in the table are solutions to this problem. GA, DE and FA algorithms are like the solutions in the table. GA, DE and FA produce the best values for W = T = 50, W = T = 100, and W = T = 200, respectively.

As for Case 3, multi-objective optimization problem was investigated. Unlike the problems studied for Case 1 and Case 2, ammo information has also been added to the WTA problem. Accordingly, if the gun fires multiple shots at the same target, this will increase the hit probability value. Of course, many shots will increase the use of ammo. For this reason, it is desirable to use the least amount of ammo both when destroying the target. In this case, a two-objective problem turns into. The differences mentioned for Case 1 and Case 2 are also found in Case 3.

Data for hyper-volume (Hy) [30] and spread metrics ($\Delta$) [29] obtained as a result of multi-objective optimization algorithms for Case 3 are given in Table 3 and Table 4. A spread metric from these two metrics is a metric that shows the distribution of the approximate Pareto data-set into the objective space.

$$\Delta = \frac{d_{e1} + d_{e2} + \sum |d_i - d'|}{d_{e1} + d_{e2} + (N-1)\, d'} \tag{8}$$

where $\Delta$ is the Spread metric value (smaller is better), $d_{e1}$ and $d_{e2}$ are Euclidean distance between extreme (boundary) solutions, $d_i$ the Euclidean distance between consecutive element and $d'$ is the mean of the $d_i$. Another metric, hyper-volume, is the area of the region where they create a reference point how close the solution set is to its origin. In this case, the larger this area, the closer to the origin and well-distributed solutions.

$$Hy = \bigcup F_i \tag{9}$$

where $Hy$ is the hyper-volume metric value (larger is better), and hypercube $F_i$ is constructed with reference point. Using this metric, the performance of multi-objective optimization algorithms was evaluated (Table III and Table IV for Hyper-volume and Spread, respectively).

When the results are examined, it is seen that the best value of the Hyper-volume metric is obtained with NSGA-II. In addition, it is seen that the performance of other algorithms, which are among the statistical features, are below NSGA-II. Since there are two objectives in this study, the MOEAD performance is expected to remain below NSGA-II. Also, when the Spread metric is examined, it is seen that the MOPSO algorithm shows the best distribution. When the statistical test of the spread metric is examined, it is seen that the results obtained for MOPSO are equal to NSGA-II. In the light of the results in both tables, it is seen that the best solution is obtained with NSGA-II.

In the figure, Pareto fronts of WTA problem with three different features are given for NSGA-II. As can be seen from the figures, the Pareto front of the method has a linear structure and the aim is reduced almost linearly with the ammunition; given as two. In other words, as the number of ammo increases, more damage is done. This is to be expected, as the number of ammo increases, the hit probability increases.

## V. CONCLUSION AND FUTURE WORKS

In this study, the WTA problem is categorized and classified in the light of the studies in the literature. Thus, the WTA problem, which is classified only as static and dynamically, is classified as synchronous and dynamically, and the synchronous (static) WTA problem is solved in this study. For this purpose, three different applications have been made. The previous application has been improved in each application. Accordingly, the hit probability value was obtained according to the distance between the gun and the target for the hit probability value. Then, the target value was added to this study. Here, four different types of targets are accepted, and the problem is solved according to these types of targets. Finally, the knowledge of the arsenal was added to the problem. Here the ammunition information corresponds to the type of shot

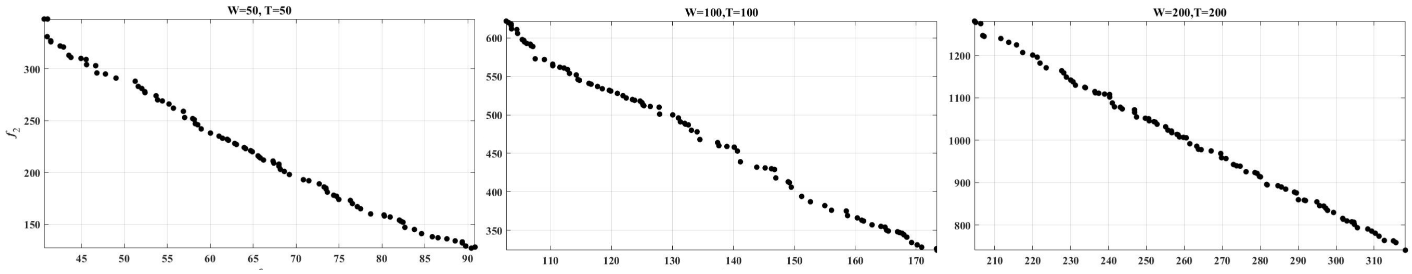


Fig. 4. Graphical Pareto Front Representation of WTA problem obtained from NSGA-II.

for the other. Here, it is considered that sequential shots fired by the same target increase the hit probability value. Thus, the problem has become a multi-objective optimization problem. In studies conducted, it has been observed that the best performance is obtained with GA, DE and FA, although solutions are produced by different algorithms for a single objective problem. However, the difference in value between each other is the same given the standard deviation. In the multi-objective problem, NSGA-II algorithm has obtained better results. In addition, a three-layer method has been proposed for encoding scheme and permutation encoding binary encoding has been converted with this method. In this way, it can be applied / added to different operators and mixed integer-like problems.

In the studies conducted, it was revealed that a more detailed environment and ammunition loading should be added to the optimization problem in the times required for shot and angle change. As the next study (case 5), it is planned to model the problem by considering the moving targets and weapon dynamics on the environment.

## References

[1] C. Huaiping, L. Jingxu, C. Yingwu, and W. Hao, “Survey of the research on dynamic weapon-target assignment problem,” Journal of Systems Engineering and Electronics, vol. 17, no. 3, pp. 559–565, 2006.

[2] Z.-J. Lee , S.-F. Su, and C.-Y. Lee, “Efficiently solving general weapon-target assignment problem by genetic algorithms with greedy eugenics,” IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, vol. 33, no. 1, pp. 113–121, 2003.

[3] Z.-J. Lee, C.-Y. Lee, and S.-F. Su, “An immunity-based ant colony optimization algorithm for solving weapon–target assignment problem,” Applied Soft Computing, vol. 2, no. 1, pp. 39–47, 2002.

[4] Z.-J. Lee , S.-F. Su, and C.-Y. Lee, “A genetic algorithm with domain knowledge for weapon-target assignment problems,” Journal of the Chinese Institute of Engineers, vol. 25, no. 3, pp. 287–295, 2002.

[5] P.A. Hosein, and M. Athans, “Some analytical results for the dynamic weapon-target allocation problem,”

[6] S. Matlin, “A review of the literature on the missile-allocation problem,” Operations Research, vol. 18, no. 2, pp. 334–373,1970.

[7] S.P. Lloyd, and H.S. Witsenhausen, “Weapons allocation is NP-complete,” IEEE Summer Simulation Conference, pp. 1054-1058, 1986.

[8] C.-M. Lai, and T.-H. Wu, “Simplified swarm optimization with initialization scheme for dynamic weapon–target assignment problem,” Applied Soft Computing, vol. 82, no. 1, pp. 105542, 2019.

[9] J. Li, J. Chen, B. Xin, and L. Dou, “Solving multi-objective multi-stage weapon target assignment problem via adaptive NSGA-II and adaptive MOEA/D A comparison study,” IEEE Congress on Evolutionary Computation (CEC) pp. 3132–3139, 2015.

[10] G.Peng, Y. Fang, S. Chen, W. Peng, and D. Yang, “A Hybrid Multi-objective Discrete Particle Swarm Optimization Algorithm for Cooperative Air Combat DWTA,” BIC-TA 2016. Communications in Computer and Information Science, vol 682. pp.ç 114–119, Springer, Singapore, 2016.

[11] Y. Wang, W. Zhang, and Y. Li, “An efficient clonal selection algorithm to solve dynamicweapon-target assignment game model in UAV cooperative aerial combat,” 35th Chinese Control Conference (CCC), pp. 9578–9581, 2016.

[12] C. Gao, Y. Kou, Y. Li, Z. Li, and A. Xu, “Multi-Objective Weapon Target Assignment Based on D-NSGA-III-A,” IEEE Access, vol. 7, pp. 50240–50254, 2019.

[13] Q. Pan, D. Zhou, Y. Tang, and X. Li, “A Novel Antagonistic Weapon-Target Assignment Model Considering Uncertainty and its Solution Using Decomposition Co-Evolution Algorithm,” IEEE Access, vol. 7, pp. 37498–37517, 2019.

[14] J. Jang, H.G. Yoon, J.C. Kim, and C.O. Kim, “Adaptive Weapon-to-Target Assignment Model Based on the Real-Time Prediction of Hit Probability,” IEEE Access, vol 7, pp. 72210–72220, 2019.

[15] M.F. Hocaoglu, “Weapon target assignment optimization for land based multi-air defense systems A goal programming approach,” Computers and Industrial Engineering, vol. 128, pp. 681–689, 2019.

[16] A.G. Kline, D.K. Ahner, and B.J. Lunday, “Real-time heuristic algorithms for the static weapon target assignment problem,” Journal of Heuristics, vol. 25, pp. 377-–397, 2019.

[17] A.R. Mehrabian, and C. Lucas, “A novel numerical optimization algorithm inspired from weed colonization,” Ecological Informatics, vol. 1, pp. 355–366, 2006.

[18] S. Kirkpatrick, C.D. Gelett, and M.P. Vewcchi, “Optimization by simulated annealing,” Science, vol. 220, pp. 621–630, 1983.

[19] X.S. Yang, “Nature-Inspired Metaheuristic Algorithms,” Luniver Press, UK, 2008.

[20] D.T. Pham, A. Ghanbarzadeh, E. Koc, S. Otri, S. Rahim, and M. Zaidi, “The Bees Algorithm,” Manufacturing Engineering Centre, Cardiff University, 2005.

[21] D. Karaboga, and B. Basturk, “A powerful and efficient algorithm for numerical function optimization artificial bee colony (ABC) algorithm,” Journal of Global Optimization, vol. 39, pp. 459–471, 2007.

[22] R. Storn, and K. Price, “Differential Evolution – A Simple and Efficient Heuristic for global Optimization over Continuous Spaces,” Journal of Global Optimization, vol. 11, pp. 341–359, 1997.

[23] J. Kennedy, and R.C. Eberhart, “Particle swarm optimization,” Proc. IEEE int’l conf. on neural networks Vol. IV, pp. 1942–1948, 1995.

[24] J.H. Holland, “Adaptation in Natural and Artificial Systems,” The University of Michigan Press, Ann Arbor, MI, 1975.

[25] K. Deb, “An efficient constraint handling method for genetic algorithms,” Computer Methods in Applied Mechanics and Engineering, vol. 186, pp. 311—338, 2000.

[26] K.Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm NSGA-II,” IEEE Transactions on Evolutionary Computation, vol. 6, no. 2, pp. 182-197, 2002.

[27] Q. Zhang, and H. Li, “MOEA/D A Multiobjective Evolutionary Algorithm Based on Decomposition,” IEEE Transactions on Evolutionary Computation, vol. 11, no. 6, pp. 712-731, 2007.

[28] C.C. Coello, and M.S. Lechuga, “MOPSO: A proposal for multiple objective particle swarm optimization,” IEEE Congress on Evolutionary Computation, pp. 1051–1056, 2002.

[29] K.Deb, “Multi-objective optimization using evolutionary algorithms,” John Wiley and Sons, 2001.

[30] E. Zitzler, and L. Thiele, “Multiobjective evolutionary algorithms a comparative case study and the strength pareto approach,”IEEE Transactions on Evolutionary Computation, vol. 3, no. 4, pp. 257–271, 1999.