Different Approaches to Multi-Objective Sparse Array Problem with Social Network Optimization

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***Abstract*—Multi-objective problems with two or more conflicting objectives are very common in every engineering fields, also for antenna optimization. Evolutionary Optimization Algorithms are important tools due to their effectiveness, flexibility and applicability especially for multi-objective problems because they can provide directly the non-dominated set. Among Evolutionary Algorithms, Social Network Optimization (SNO) shows very good optimization performance.**

In this paper three different approaches for solving a multi-objective problem are tested with SNO: the first one is the weighted sum method, the second is the epsilon-constrained method and the third one is the simultaneous search with a multi-objective implementation of SNO. The analysed application is the design of a sparse-array antenna.

I. Introduction

Evolutionary Optimization algorithms (EAs) are important tools due to their flexibility and applicability: in fact, they can work properly on several type of common benchmark functions that can hardly managed by means of traditional techniques, like multi-modal problems [1], constrained problems [2] and discontinuous problems.

In many engineering problems the performance of the systems can be expressed in terms of more than one parameters: when these parameters are conflicting, the problem is called multi-objective [3]. The aim of a multi-objective problem is not just the identification of a single solution, but the identification of the Pareto front. A solution belong to the Pareto front if there is not any other feasible solution that has better values for all the benchmarks [4].

In the field of antenna optimization, Evolutionary Optimization Algorithms have been widely applied due to their capability to face multimodal problems [5].

Among EAs, Genetic Algorithm (GA) is the most popular one: it can be easily formulated either for real-coded (like in the design of time-modulated linear arrays), binary-coded (applied for also thinned antennas and wire antenna design) and mixed integers problems (like for linear array design, thinned subarrays, and circularly polarized patch antenna) [6]. Another important EAs is Particle Swarm Optimization (PSO): this algorithm is native for real-coded problems [7] but it has been also implemented and successfully adopted for binary-coded problems [8].

In addition to these algorithms, recently other EAs has been applied to antenna optimization. One of the most interesting is Differential Evolution that shows a very good convergence rate even in very large-scale electromagnetic problems [9]. Another adopted algorithm is the Evolutionary Strategies that, due to its high exploration capability, is able to overcome the problem of local minima [10].

In this field, in [11] a new Evolutionary Optimization algorithm has been introduced. This algorithm, named Social Network Optimization (SNO), takes its inspiration from the interaction of social network members and it has been successfully applied to several single objective optimization problems, from the design of tubular permanent magnet linear generators [12] to electromagnetic problems [13].

A very recent trend of the application of EAs in electromagnetic is the design of all the optimization environment: this can take into account the use of surrogate models, of several optimization strategies and it can give very good performance in terms of convergence rate and accuracy of the final solution. The System-by-Design approach is one of the most applied in antenna optimization [14], [15]. For what concerns multi-objective optimization problems in antenna design, they are often faced with different EAs, like PSO [16] or GA [17]

In this paper the optimization of a planar sparse array is faced using Social Network Optimization. The problem has been formulated with two objectives and it has been faced with three different approaches: the first one is the solution of several scalarized problems, the second one is the *epsilon constrained method* in which many constrained single objective problem are solved and, finally, the last approach is the contemporaneous search of the entire non-dominated set.

The paper is organized as follows: Section II contains a description of SNO and its modification for multi-objective problems. This implementation has been also preliminary tested on three standard multi-objective benchmarks in Section III.

In Section IV the two approaches to multi-objective problems used in this paper are described and in Section SNO has been tested on In Section V the optimization antenna problem is described, the objectives defined and the results of the optimization by means of SNO are shown and compared. Finally, in Section VI some conclusions are drawn.

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II. Social Network Optimization

Social Network Optimization (SNO) is a population-based Evolutionary Optimization Algorithm that takes its inspiration from the idea sharing mechanism of online social networks.

The basic data structure of SNO is the social network, the virtual space in which the interactions take place and in which people exchange ideas and opinions. It contains the two basic elements: the users, that is the population of the algorithm, and the posts, that are the structures that drive the interaction between users. The population size of the algorithm is the number of users in the Social Networks.

At each iteration, the users express their *opinions* by means of posts. Each post contained the *status*, that is the real post content, the name of the user that have posted it, the time in which it is posted and a visibility value.

The process of passing from *opinions* to a post *status* is called *linguistic transposition*. In the basic implementation of SNO it is a gaussian mutation of the opinion vector.

The status correspond to the candidate solution of the optimization problem, while the visibility value is created by means of a proper mapping of the cost function associated to the specific candidate solution, as shown in Figure 1.

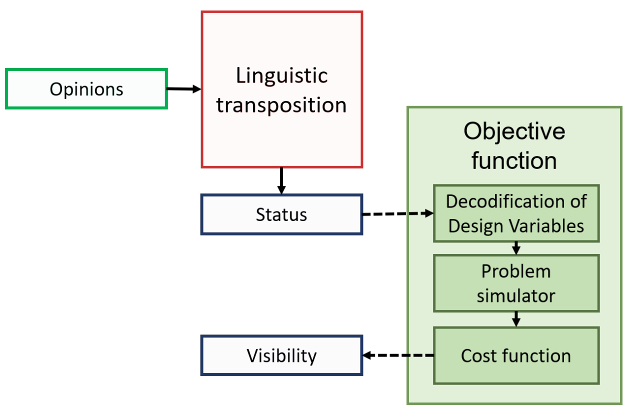


Fig. 1. Schematization of the basic interaction between SNO and the optimization problem.

The visibility values of the entire population are used to update the reputations of all the users of the social network. This operation transfers global information among the entire population.

The reputations are used for creating one of the two interaction structures of SNO: the trust network.

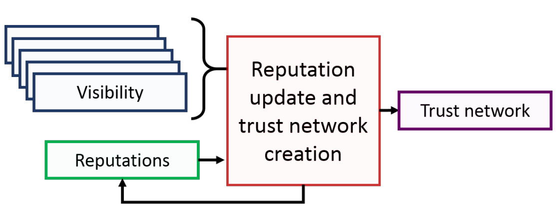


Fig. 2. Schematization of the process of reputation update and creation of trust network.

The second interaction network is the friend network. These two networks are very different one from the other: friend network leads to more consistent interactions, its variability is lower and its modifications are related to out-of-the-social elements. On the other hand, trust network creates weaker connections, it varies fast and it it modified according to the visibility values.

From the interaction networks each user extracts some ideas that composed the attraction point that is used for composing the new opinions. The operator implemented emulates the assumption of a *complex contagion*, that guarantees a better tradeoff between exploration of the domain and the exploitation of the acquired knowledge:

$$\mathbf{o}_u(t+1) = \mathbf{o}_u(t) + \alpha[\mathbf{o}_u(t) - \mathbf{o}_u(t-1)] + \\ + \beta[\mathbf{a}_u(t) - \mathbf{o}_u(t)] \quad (1)$$

$\mathbf{o}_u$ is the opinion of the u-th member of the social network and it is function of the iteration, and $\mathbf{a}_u$ is its desired option. α and β are two user-defined parameters.

This equation has been modified for the binary implementation using logical operators:

$$\mathbf{o}_u(t+1) = \mathbf{o}_u(t) \oplus w \odot [\mathbf{o}_u(t) \otimes \mathbf{o}_u(t-1)] \oplus \\ \oplus c \odot [\mathbf{a}_u(t) \otimes \mathbf{o}_u(t)] \quad (2)$$

where $\odot$ represents the and, $\oplus$ the *or* and $\otimes$ is the operator xor. w, c are random vectors of 1s and 0s where the probability of 1 is given by the user-defined parameters α and β.

All these operators makes SNO a quite complex algorithm, but they give to it the possibility to work very well in different kind of problems.

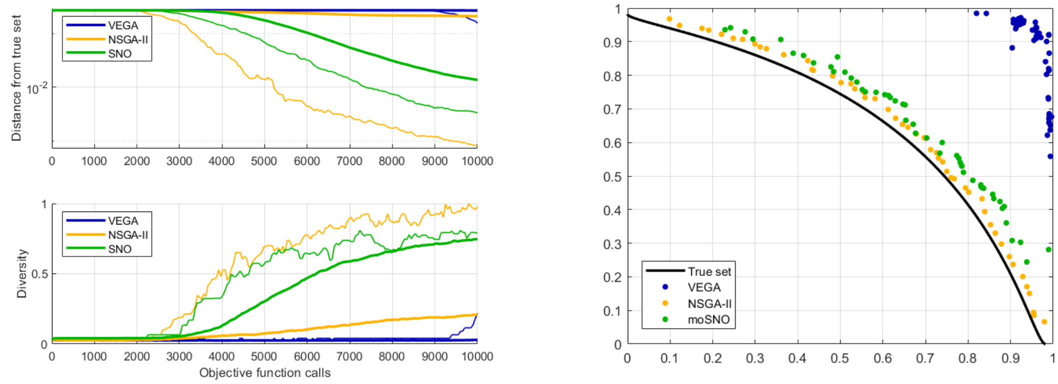
For example, it has been tested on the design of a thinned array antenna [8], on the design of beam scanning reflectarray [18], and on the design of a tubular permanent magnet linear generator [12].

The structure of SNO can be easily extended to multi-objective functions: in fact, just two modifications can be done for obtaining a very effective algorithm. The first one is the selection of the influencers in peer and trust groups, that is performed with a non-linear selection based on a scalarization of the cost values. The second one is the selection criterion for the Social Network: in this case, the solutions have been selected according to the dominance rank and with a greedy algorithm for maximising the crowding distance [19].

In the following Section, the performance of the multi-objective implementation of SNO is tested on some standard benchmarks.

III. SNO test on multi-objective problems

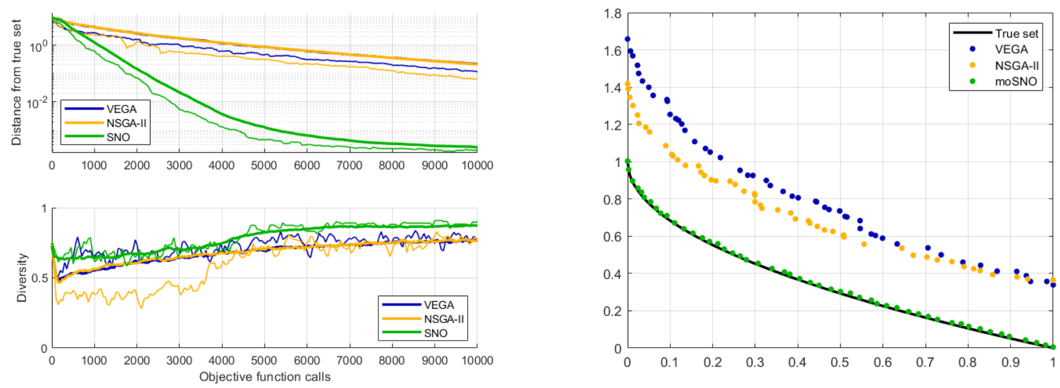
For analysing the optimization capabilities of SNO in multi-objective problems, three standard benchmarks have been here used [20], [21]: the Fonseca-Fleming, the ZDT1, and the ZDT3 functions. Using these benchmarks, SNO has been compared with two state-of-the-art multi-objective optimization algorithms: the VEGA [22] and the NSGA-II [23].



(a) Convergence curves

(b) Non-dominated sets

Fig. 3. Results of the optimization of Fonseca-Flaming function: convergence curves (a) and best non-dominated set (b).



(a) Convergence curves

(b) Non-dominated sets

Fig. 4. Results of the optimization of Zitzler-Deb-Thiele's function 1: convergence curves (a) and best non-dominated set (b).

The solutions obtained have been compared according to two quantitative criteria: the first one is the distance from the true set, while the second is the diversity; both these two metrics have been implemented as described in [24].

Figure 3 shows the results on the Fonseca-Flaming function. In the convergence curves, the thick line is the average value of 50 independent trials and the thin one the best trial. In this function the NSGA-II has the best optimal solution, but the average convergence of SNO is much better, showing the reliability of this algorithm.

In ZDT1 function (Figure 4), the quality of the solution of SNO is really clear, for both the distance from the true non-dominated set and for the distribution of the solutions on it.

Same considerations can be done on the ZDT3 function (Figure 5) in which SNO shows its capabilities to find also discontinuous sets.

These results have proven the performance of SNO on some standard benchmarks, in the next section the results of the optimization of the sparse array will be shown.

IV. APPROACHES TO MULTI-OBJECTIVE OPTIMIZATION

Multi objective problems are defined as the simultaneous solution of a set of problems [25]. The aim of this kind of problems is the identification of the Pareto front.

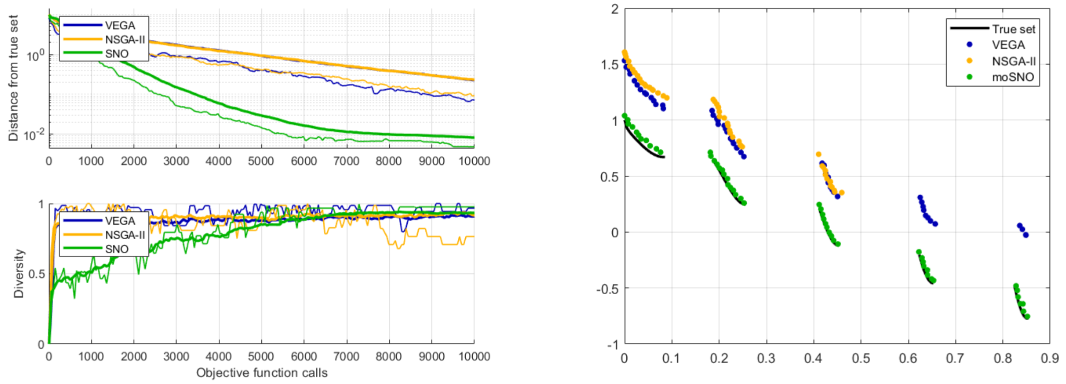
Many classical methods are used to solve this kind of problem. The most commons are the weighted sum method and the epsilon-constraint methods. In both, the Pareto set is reconstructed repeating several times a single objective optimization problem: the solution of each of these problems is one of the points of the non-dominated set.

The weighted sum method solves several single objective problems in which the original performance parameters $f_i(\mathbf{x})$ are combined in a single cost value:

$$c(\mathbf{x}) = \sum_{i=1}^{N_p} \lambda_i f_i(\mathbf{x}) \tag{3}$$

where λ_i are several scalarization parameters and N_p is the number of performance parameters.

On the other hand, the epsilon constrained problems solves a single objective problem in which the cost is one of the performance parameters and the other are used as constraints.



(a) Convergence curves

(b) Non-dominated sets

Fig. 5. Results of the optimization of Zitzler-Deb-Thiele's function 3: convergence curves (a) and best non-dominated set (b).

There are several disadvantages to these approaches, but the most critical one is that they require the solution of a very large number of scalar problems for finding the entire non-dominated set, especially in problems with more than two objectives.

EAs provide the possibility to have a simultaneous search in which the output of the optimizer is directly the non-dominated set.

All these three approaches are used in the next Section for solving the sparse array problem.

V. Sparse array optimization

In this section, the two methods described above will be applied to the optimization of the array factor of a sparse array.

A. Problem and objectives definition

In this section, the test case used to compare the two approaches to multi-objective optimization by means of SNO will be briefly described.

This problem is the optimization of the array factor of a sparse array compose by a grid on 12×12 equal elements in fixed position that can be turned on or off. Thus, it is represented in the optimization frame as a set of boolean variables that represent the presence or the absence of the radiating element. The optimizer output is a candidate solution with 144 elements that can be 0 or 1. The array factor has been calculated accordingly to [26].

Given this data and knowing the position of the elements turned on, it is possible to evaluate the array factor by means of array theory as:

$$AF = \sum_k \sum_l e^{2j\pi d u_k (k-1)} \cdot e^{2j\pi d v_l (l-1)} \tag{4}$$

where v and u indicates the position of the element in the array. in the following manner:

$$u_i = \sin\theta_i \cdot \cos\phi_i \tag{5}$$

$$v_i = \sin\theta_i \cdot \sin\phi_i \tag{6}$$

The two objective functions that should minimized are:

$$\begin{cases} f_1(\mathbf{x}) = SLL \\ f_2(\mathbf{x}) = N \end{cases} \tag{7}$$

where SLL is the maximum values of the side lobe levels and N is the number of elements in the array.

Due to the fact that the SLL objective shows an hard convergence even solved alone, the first objective has been implemented in the optimization process using a slightly different approach: in fact, the minimization of the SLL has been associated with the minimization of the radiation pattern exceeding a mask with SLL equal to -20dB.

While dealing with multi-objective optimization problems, it is important to verify that the objectives are really conflicting and not linearly correlated otherwise the problem can be rephrased in a single objective problem. To assess this point, 10,000 random points have been selected and evaluated.

To analyse the relation between the two objective, the Pearson correlation coefficient has been evaluated: the linear correlation coefficient is -0.17 and the p-value is $8.52 \cdot 10^{-37}$. The p-value is almost zero: it means that the hypothesis of a linear correlation between the variables is false, so the problem cannot be simplified as a single-objective problems.

In the following, the single objective problem composed by only the function f_1 has been analysed. Then the three methods for finding the non-dominated set have been tested and compared.

B. Solution of a single-objective problem

As a first analysis of the problem, the single objective formulation with cost function f_1 has been solved using SNO. 5,000 objective function calls has been set as the termination criterion and 20 independent trials have been performed.

Figure 6 shows the convergence curves of SNO: from these curves, it is possible to see that the algorithm has a very stable behaviour and its results are reliable.

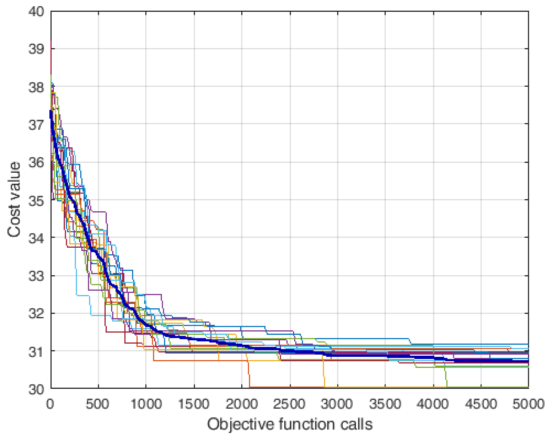


Fig. 6. Pareto front sound by the weighted sum method: each dot correspond to a solution found by SNO and the bigger dots are the non-dominated set.

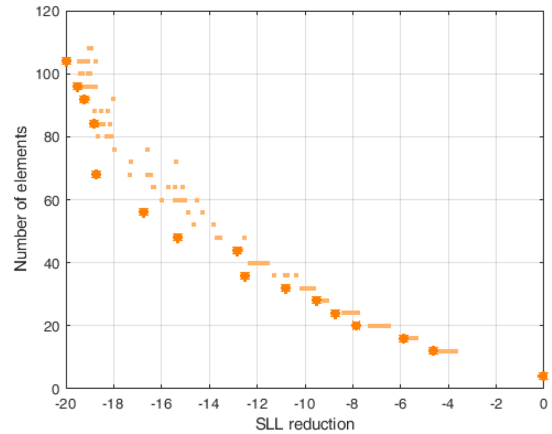


Fig. 7. Pareto front sound by the weighted sum method: each dot correspond to a solution found by SNO and the bigger dots are the non-dominated set.

This is an important aspect for computationally intensive problems because it means that a lower number of independent trials are required for assessing the final solution. Moreover, it shows the capabilities of SNO in solving the sparse array design.

C. Weighted sum method

The first method used for solving the multi-objective problem is the weighted sum method. As described before, in this approach the two objectives have been combined in a single cost value by means of a scalarization coefficient.

In this problem, 36 scalarization coefficients have been used for solving the entire non-dominated set: they have been extracted uniformly from the interval [0.5, 1].

Each problem has been solved with SNO: the termination criterion has been set to 5,000 objective function calls and 20 independent trials have been performed for each scalar coefficient.

Figure 7 shows results of this optimization process: each dot correspond to a solution found by SNO and the bigger dots are the non-dominated set.

From this Figure, it is possible to see that with the scalarization method is not possible to find a solution with less than 12 elements having a SLL reduction: this is due to the fact that the antenna becomes too small.

It is possible to achieve -10dB of SLL reduction with 32 elements, for obtaining -15dB it is required to have 48 elements while it is not possible to achieve -20dB with less than 104 elements.

D. Epsilon constrained method

The second method implemented is the epsilon constrained method: in this approach, SNO has been used for solving a set of scalar problems in f_1 with a constrain on the maximum number of radiating elements (that are the function f_2).

Due to the symmetries of the problem, 36 constrained single-objective problems have been considered. Also in this case, the termination criterion of SNO has been set to 5,000 objective function calls and 20 independent trials have been performed.

Figure 8 shows results of this optimization process: each dot correspond to a solution found by SNO and the bigger dots are the non-dominated set.

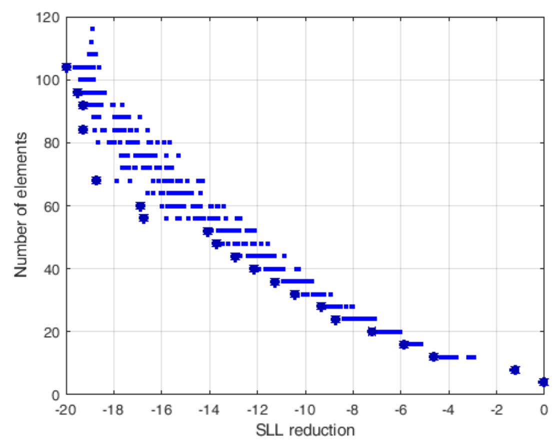


Fig. 8. Pareto front sound by the epsilon constrained method: each dot correspond to a solution found by SNO and the bigger dots are the non-dominated set.

This Pareto front is quite similar to the one obtained with the weighted sum method, but in this case the points are more equally distributed among the front.

In particular, it is possible to notice that it is able to find a solution on the pareto set with 8 elements. As seen in the previous non-dominated set, -10dB can be obtained with 32 elements, while in this case -15dB can be obtained with no less than 56 elements. The solution with -20dB of SLL is the same seen before.

E. Simultaneous search

Finally, the simultaneous search of the pareto front with multi-objective SNO has been approached. 20 independent trials have been done 150,000 fitness function calls. The population of SNO has been set to 200 individuals, resulting in 750 iterations of optimization.

The time required by this search is comparable to the previous methods, and the final non-dominated set is shown in Figure 9

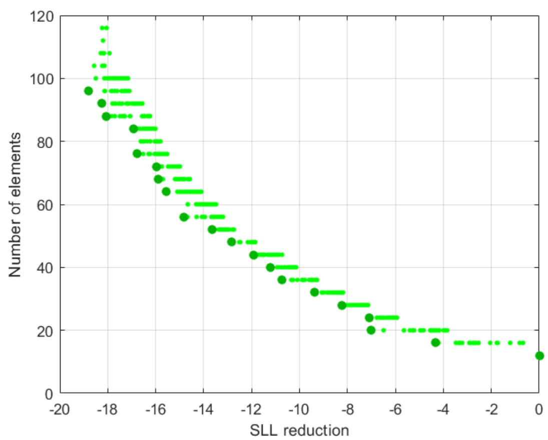


Fig. 9. Results of the simultaneous solution of the multi-objective problem.

The diversity of the non-dominated set is very good, even if the reduction of the SLL is not as good as in the previous optimization. This is due to the fact that the pressure toward the SLL minimization is reduced with respect to standard SNO. This behaviour can be shown better in the following, in which the comparison between the three approaches is shown.

F. Comparison between the approaches and solution analysis

In this Section, the three methods are compared and some of the solutions found are compared.

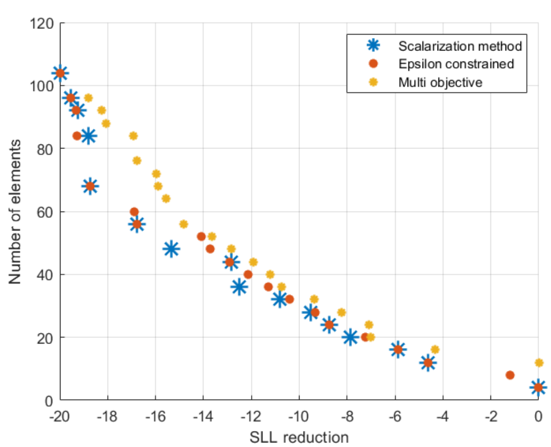


Fig. 10. Comparison between the two Pareto fronts.

Figure 10 shows the three non-dominated sets in the same plot. In this way, the comparison is very easy.

The distribution of the solutions is very good for all the three methods: the simultaneous search has more points in the front because it has no constraint on the number of solutions.

The weighted sum method is generally better than the others in reducing the SLL, even if it often coincide with the epsilon constrained for high number of elements in the antenna. As said also before, the simultaneous search has lower performance.

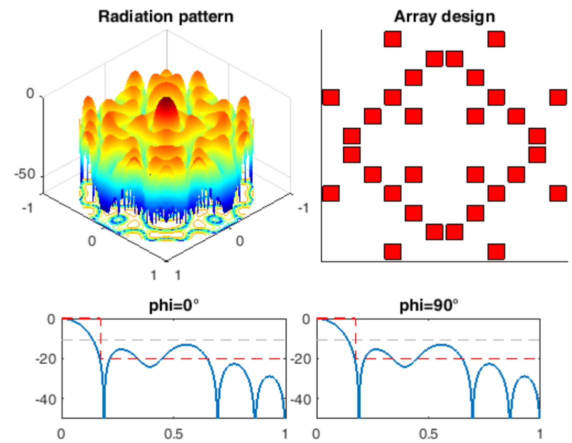


Fig. 11. Analysis of the solution found in the pareto front with -10dB SLL.

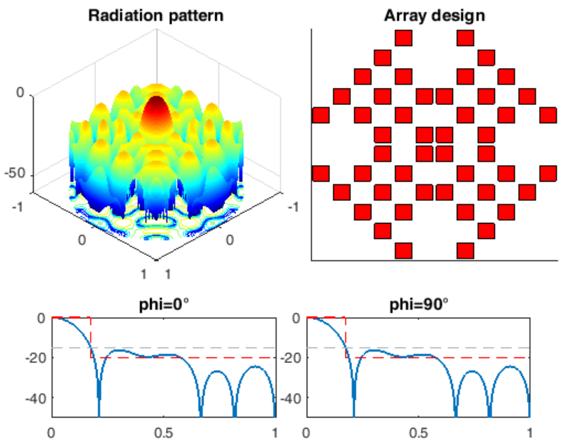
Three different solutions have been analysed from the pareto front: the first one, shown in Figure 11 shows the solution with SLL equal to -10dB. In the figure, it is possible to see the 3D radiation pattern, the disposition of the elements of the array and the two principal planes: here, the red dashed line is the mask used for improving the convergence, while the grey line is the SLL.

Figure 12 shows other two solutions: the one with -15dB obtained with the scalarization method and the solution with SLL -20dB.

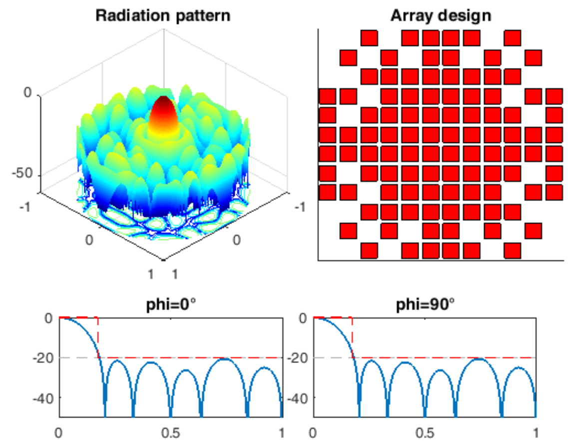
VI. Conclusions

In this paper, three approaches to multi-objective formulation of the problem of sparse array design are tested using Social Network Optimization. The first two methods (the weighted sum and the epsilon constrained) are based on the solution of scalar single-objective problems, while the third is based on the simultaneous search of the entire pareto front.

The test performed on standard benchmarks shows that the multi-objective formulation of SNO is capable to find effectively pareto fronts, but in the application to the thinned array problem the solution is not very effective: this means that the operators of this algorithms should be further improved for multi-objective problems in which one of the two objectives is much simpler than the other.



(a) Solution with -15dB reduction.



(b) Solution with -20dB reduction.

Fig. 12. Analysis of two solutions found in the pareto front: the first one (a) achieve a SLL of -15dB, while the second one of -20dB.

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