A Fast Solution to Two-Impulse Lunar Transfer Trajectory based on Machine Learning Method

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***Abstract*—In this paper, the machine learning method is applied to predict the two-impulse lunar transfer velocity and orbital transfer window in a fast and efficient way. Firstly, the two-impulsive lunar transfer problem is described, and a trajectory design method is proposed to optimize the transfer based on Sequential Quadratic Programming algorithm. Then the geocentric relationship of the parking Low Earth Orbit (LEO) and the Moon is analyzed and the Earth-Centered Moon-to-Earth Plane Coordinate is introduced to transfer the inertial orbital elements into the pseudo ones. In this coordinate frame, domain knowledge suggests that the pseudo orbital elements can also serve as good learning features for the machine learning model. After generating the dataset, two machine learning models are established to estimate the orbital transfer window and orbital transfer velocity impulse respectively. Numerical simulations demonstrate that the machine learning models are efficient to estimate hundreds of transfer trajectories within 0.1 second, which shows much better performance than traditional orbit optimization method. The performance of two different machine learning algorithms is also assessed, where the neural network outperforms the gradient boosting model with the velocity impulse error less than 1m/s. It is also verified that the feature selection of pseudo orbital elements is appropriate which has smaller learning errors than that using the inertial orbital elements.**

Keywords—two-impulse lunar transfer, transfer window, machine learning, regression model, feature selection

I. INTRODUCTION

In 1969 Neil Armstrong took human's first step on the Moon and a number of lunar exploration missions were executed ever since [1]. Lunar transfer orbit (LTO) design is an important factor in the lunar exploration project. In the last decades, the general characteristics of Earth-to-Moon and Moon-to-Earth trajectories were analyzed for the Apollo project [2].

One of the main technical issues in lunar exploration is the trajectory design method. Totally the translunar trajectories could be divided into 4 categories: the Hohmann transfer, low thrust transfer, libration point transfer and the Weak Stability Boundary transfer [3]. The Hohmann transfer is the easiest way in lunar engineering practice and the two-impulse transfer from a low Earth orbit (LEO) to a low Lunar orbit (LLO) has been researched most extensively. Topputo [4]proposed a direct transcription and multiple shooting strategy to solve the optimal two-impulse transfer problem within the planar restricted four-body model and characterized families of the solutions. Liang et al [5]studied the envelop of the direct cislunar transfers and reveal the relationship between Jacobi energy and perilune. Lv et al [6] developed a differential correction approach to optimize the two-impulse translunar trajectory. Yim et al [7] systematically analyzed the launch windows of high accurate translunar trajectories. Yi and Xu [8] investigated the minimum impulse cost in the CR3BP and the conical-patched model respectively. Gao et al [9]designed the two-impulse transfer between LEO and lunar orbital station based on the patched conic approach.

Recently machine learning (ML) method presents great capability of learning the underlying patterns from the training samples if the learning features are well identified. Machine learning originate from computer science but it has attracted attention of researchers in the astrodynamics with some successful applications. Peng and Bai[10] used the Support Vector Machine (SVM) to improve the orbit prediction accuracy. The simulation results showed that the learning model could reduce prediction errors relative to physics-based models. Mereta and Izzo [11] used ML as a regressor to

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estimate the final spacecraft mass in the optimal low-trust transfer problem between two near earth objects. Seven regressors were discussed, all of which gave superior performance to the Lambert estimate. For the asteroid exploration, Shang and Liu [12] employed the Gaussian process regression to evaluate the accessibility of asteroid exploration for the unmanned missions. Zhu et al [13]extended the work to the manned mission using a learning-based method. Li et al [14]applied SVM to classify transit orbits in CR3BP and found that it has a more efficient computing speed than the Fourier series approximation method.

Neural network is a subset of machine learning and with the universal approximation capability for any piecewise continuous function to any degree of accuracy [15], artificial neural networks are exploited and used as the estimator in the areas of astrodynamics. Sanchez and Izzo [16][17] found that DNN could represent the optimal state-feedback control with both high accuracy and good generalization ability. Peng and Bai [18]used neural network to improve the orbit prediction accuracy of a resident space object and validated generalization of the trained model to future epochs. Furfaro et al [19] adopted Convolutional Neural Networks (CNN) and Recurrent Neural Net-works (RNN) to perform autonomous Moon landing, which predicts the fuel-optimal control actions according to the raw images taken by the camera. From the researches above, machine learning or deep neural network shows good regression ability.

For the two-impulsive lunar transfer problem, the trajectory can be optimized one by one but it is time consuming. It is difficult to balance computation time and model precision, and this kind of balance is required for large-scale analysis. Therefore, it is possible to apply the machine learning method to solve the problem in a fast and efficient way. The reminder of this paper is organized as follows. Section II describes the lunar transfer problem and the corresponding optimization approach is proposed to optimize the transfer trajectory. Section III presents the procedure of the dataset generation and the selection of the learning features. Section IV introduces the design of the two machine learning models as well as the as well as the evaluation criterion for the learning performance. Section V shows the detailed simulation results of the machine learning methods and Section VI gives the conclusions.

II. Optimization of Two-Impulse Lunar Transfer Orbit

A. Problem Description

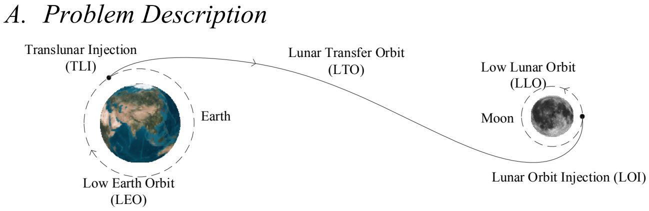


Fig. 1. Description of two-impulse lunar transfer problem

In the lunar transfer process, the spacecraft usually executes two impulsive maneuvers from LEO to LLO. At Translunar Injection (TLI), the first impulse is assumed parallel to the parking velocity of LEO, which places the spacecraft into the LTO. At Lunar Orbit Injection (LOI), the second impulse inserts the spacecraft into LLO. It is noted that the transfer trajectory is not limited to the symmetric free-return orbit which is required by the manned spacecraft. The high-fidelity dynamics model is used for orbit propagation.

B. Design variables and Objective function

For the two-impulse transfer process, the parking LEO and LLO are supposed to be near circular. Therefore, the orbital elements can be described in 3 parameters $E=(h,\Omega,i)$, where h is the orbit altitude, Ω the right ascension of ascending node (RAAN), and i the inclination. The orbital design variables of LTO are selected by

$$x=(v_A,u_A,t_{\mathrm{TLI}}) \tag{1}$$

where v_A is the TLI velocity impulse, u_A the argument of latitude and t_{TLI} the TLI time. The LTO can be determined by the 3 parameters given.

According to the constraint of LLO inclination i_{LLO} and altitude h_{LLO}, the parameters are optimized by minimizing the following objective function

$$J=K_1\left|r_{\mathrm{LTO}}-h_{\mathrm{LLO}}-R_M\right|+K_2\left|i_{\mathrm{LTO}}-i_{\mathrm{LLO}}\right| \tag{2}$$

where r_{LTO} is the selenocentric radius of LTO, i_{LTO} the inclination of LTO and R_M the Moon radius. The Sequential Quadratic Programming (SQP) is employed to solve the optimization problem but the convergence performance depends on the initial guess of the design variables. To provide the appropriate initials for the algorithm, the Earth-Centered Moon-to-Earth Plane Coordinate (ECMEPC) is imported based on the geocentric analysis in the cislunar space.

C. Trajectory Optimization Algorithm

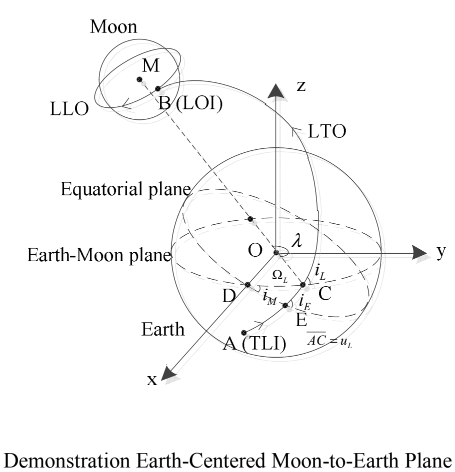


Fig. 2. Demonstration Earth-Centered Moon-to-Earth Plane Coordinate

The ECMEPC is shown in Fig. 2. The Earth center is at point *O*, TLI at point *A*, and LOI at point *B*. The LTO plane ACE intersects the EMP at point *C* and intersects the Earth-Moon plane at point *E*. With frame *o-xyz*, the x axis points from *O* to *D*, the z axis points in the direction with the Moon angular momentum and the y axis makes up a Cartesian coordination system with the other two axis. Orbital elements (Ω_E, i_E) in the J2000 ECI coordinate is firstly transformed into (Ω_L, i_L) in ECMEPC, where Ω_L and i_L are named as the pseudo RAAN and the pseudo inclination respectively.

At LOI time, the intersection angle between $\overline{OC}$ and the Moon position $\overline{OM}$ is $\angle MOC = \lambda_{\text{prl}}$, called the pseudo perilune longitude. With only the Earth gravity taken into consideration, the LTO plane ACE and the LEO plane should be nearly coplanar. Therefore, when the spacecraft arrives at the Moon, the LOI position can be regarded as the intersection between LEO plane and Moon-to- Earth plane and the corresponding pseudo longitude in the ECEMP coordinate can be estimated as

$$\lambda_{\text{prl}} = 180^\circ \tag{3}$$

Then the Moon's longitude $\Omega_{\text{prl}}(t_{\text{prl}})$ at LOI time is obtained as

$$\Omega_{\text{prl}}(t_{\text{prl}}) = \Omega_L + 180^\circ \tag{4}$$

where Ω_L is obtained by (6). The LOI time t_{prl} could be expressed by the inverse function

$$t_{\text{prl}} = \Omega_{\text{prl}}^{-1}(t_{\text{prl}}) \tag{5}$$

where t_{prl} can be obtained by the interpolation method. With transfer time T_{AB} known, the TLI time t_{TLI} is

$$t_{\text{TLI}} = t_{\text{prl}} - T_{AB} \tag{6}$$

Similarly, $\overline{AC} = u_L$ is defined as the pseudo argument of latitude relative in ECMEPC. With only the Earth gravity considered, the corresponding pseudo argument of latitude should be

$$u_L = 0^\circ \tag{7}$$

The argument of latitude u is obtained by

$$u = u_L - \overline{CE} \tag{8}$$

Equations (6) and (8) provide the initial scope of t_{TLI} and u for the orbit design parameters, which are called the pseudo longitude rule and the pseudo argument of latitude rule. For the estimation of the TLI impulse, the initial guess is provided by the two-body model dynamics.

III. Dataset Generation and Learning Features Selection

A. Dataset generation

TABLE I. ORBITAL ELEMENTS GENERATION

Orbital Elements	Range
h_E	200~2000km
Ω_E	0~360°
i_E	0~90°
h_{LLO}	200~800km
i_{LLO}	0~90°
T_{AB}	3~5day

To construct translunar orbit database, the orbital elements $E=(h_E, \Omega_E, i_E)$ of parking LEO is given with the random inclination $0 \le i_E \le 90^\circ$ and the random RAAN $0 \le \Omega_E \le 360^\circ$. The height h_{E} is discretized between 200km and 2000km. For the orbital elements of parking LLO, the orbital altitude h_{LLO} is discretized between 200km and 800km and the inclination i_{LLO} is randomly set between 0° and 90°. The transfer time T_{AB} is limited to 5 days. Given a parking LEO and a target LLO, the lunar transfer orbit is searched using the optimization method in Section II. Then the transfer orbit dataset $\sum_1 = \{(E,x)_j, j = 1:n\}$ is generated, where n represents the length of the set. In this paper, 11000 pieces of LTOs are generated, where 80% and 20% are employed as the training and testing set respectively. The former set is used to train the machine learning model and the latter set is used to test the performance of the trained model.

Then if only the constraint of the LLO altitude h_{LLO} is considered in the objective function (2), one parking LEO $(h,i,\Omega)_{\text{LEO}}$ should have a set of LTOs that satisfies the LLO constraint. It is called the launch window set and is defined as

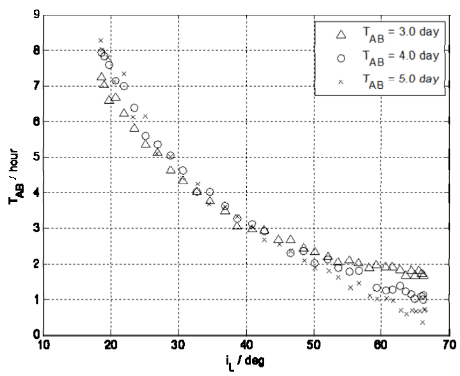
$$y = (T_{\text{prl}}, u_{\min}, u_{\max}) \tag{9}$$

where T_{prl} is the duration of the launch window, $u_{\min}$ the minimum argument of latitude at TLI and $u_{\max}$ the maximum argument of latitude at TLI . For the spacecraft placed at TLI, if the argument of latitude $u_L \notin [u_{\min}, u_{\max}]$, it means that there is no LTO to transfer from the Earth to the Moon. Given a parking LEO and a target LLO height, the transfer window set $\sum_2 = \{(E,y)_j, j = 1:m\}$, where the length of the set m=2150 .

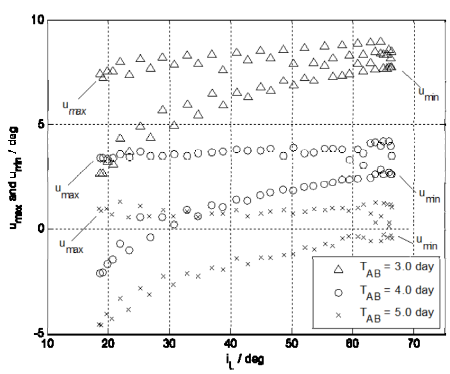
B. Learning Feature Selection

Before applying the machine learning method, the orbital characteristics is analyzed to select appropriate learning features for the two prediction models. Because inappropriate learning features will cause difficulty in the training process of the machine learning model and also lead to the inaccuracy of the trained model. In Ref. [20] it is found that the pseudo inclination i_L could be selected as an important parameter that describes the variation of the T_{prl}, $u_{\min}$ and $u_{\max}$. Especially for the launch window duration T_{prl}, it could be estimated as a quadratic function of the inclination i_L as shown in Fig. 3(a). For the argument of latitude, $u_{\min}$ and $u_{\max}$ increases with i_L but the interval of them $u_{\max} - u_{\min}$

decreases with i_L as shown in Fig. 3(b). Furthermore, the pseudo RAAN Ω_L contributes to the proposed pseudo longitude rule in (11), which provides the initial value of the TLI time. So the pseudo orbital elements of (i_L, Ω_L) are selected as the learning feature for the machine learning models. For the two-impulse lunar transfer problem in this paper, orbital elements are the key learning features for the learning model. In the following part introduces. In the next part, the selection and combination of the learning features will be introduced for the two machine learning models.



(a) Variation of T_{prl}



(b) Variation of $u_{\min}, u_{\max}$

Fig. 3. Relationship between the launch window and the pseudo inclination

IV. Machine Learning Model Design

In the following part, two machine learning models are designed to predict the two-impulse lunar transfer problem. The first model is built to predict the transfer window y and the second model is built to predict the optimization solution x.

A. Orbital Transfer Window Regression

TABLE II. POSSIBLE ATTRIBUTES TO GENERATE THE SURROGATE MODEL

Attributes	Description
h_{LLO}	Aiming orbital height of LLO
h_{E}	Parking orbital height of LEO
Ω_L	Pseudo RAAN of LEO
i_L	Pseudo inclination of LEO
Ω_E	Inertial RAAN of LEO
i_E	Inertial inclination of LEO
u_L	Pseudo argument of latitude at TLI
t_{TLI}	Departure time
v_A	Velocity impulse at TLI
D_M	Distance from Moon to Earth

The first machine learning model is used to predict the orbital transfer window y appropriately. A family of possible learning features are listed in Table II. In the analysis above, the orbital states $(h_{\mathrm{E}}, \Omega_L, i_L, h_{\mathrm{LLO}})$ of parking LEO and LLO determines the orbital launch window. It should be noted that Ω_L and i_L are the orbital elements described in ECMEPC. Also, the transfer time T_{AB} should be included into the learning inputs. The departure time t_{TLI}, however, is not necessary because it is constrained by the spatial relationships according to the pseudo perilune longitude rule in (9). The parameter D_M, the distance from Moon to Earth, should be used which varies from 363300km to 405500km. Note that all parameters are normalized from 0 to 1 in the learning process.

The following features are selected for the first regressor to predict the orbital transfer window by

$$f_1 : R^6 \to y \tag{10}$$

where $R^6 = (h_{\mathrm{E}}, \Omega_L, i_L, h_{\mathrm{LLO}}, T_{AB}, d_M)$ and $y = (u_{\min}, u_{\max}, T_{\mathrm{prl}})$ are inputs and outputs of the surrogate model f_1 respectively.

B. Orbit Transfer Impulse Regression

For the TLI velocity increment, it is related to argument of latitude for every LTO. Especially with departure LEO and aiming LLO known, one LTO is determined by u, v_A and t_{TLI}. Therefore, with $u \in [u_{\min}, u_{\max}]$ added into the learning inputs and then regression model f_2 is built to estimate TLI velocity impulse and TLI time by

$$f_2 : R^7 \to x \tag{11}$$

where $R^7 = (h_{\mathrm{E}}, \Omega_L, i_L, h_{\mathrm{LLO}}, T_{AB}, d_M, u)$ and $x = (v_A \lambda_{\mathrm{prl}})$ are inputs and outputs of f_2.

C. Machine Learning Algorithm

There are a lot of regression algorithms in the machine learning family. Totally it can be divided into two categories. The first one is called the classical machine learning models such as Gradient Boosting (GB), Support Vector Machine, Random Forest and et al. The second one is the neural networks that is inspired by biological neural networks and is widely used in deep learning nowadays . Both the algorithms can construct the "black-box" relationship between the input and output of the training dataset during a learning process.

Therefore, two representative machine learning algorithms of the two categories is employed in this paper to train the dataset, including Gradient Boosting model in the python Machine Learning scikit-learn library and deep neural networks in TensorFlow. The GB has an advantage of overcoming over-fitting and handling skewed variables with good computational robustness. In the scikit-learn library, Gradient Boosting is set with a maximum depth of 6 and estimators of 500. In the tensorflow library, the back-propagation (BP) neural network applied and it is set with 2 hidden layers for model 1 and 3 hidden layers for model 2. The tansig function is selected as the activation function and learning rate is set by 0.001. The output layer uses the linear activation function. The maximum iteration epoch is set as 600.

The Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) are employed as the evaluation criterion and is defined as

$$\varepsilon_{\mathrm{MAE}} = \frac{1}{N}\sum_{j=1}^{N}\left|x_{pred}^{j} - x_{opti}^{j}\right| \tag{12}$$

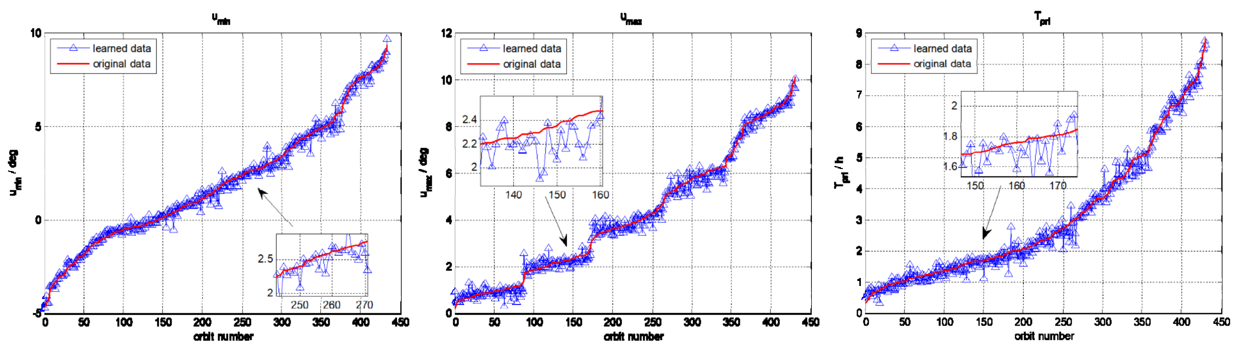
$$\varepsilon_{\mathrm{RMSE}} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(x_{pred}^{j} - x_{opti}^{j}\right)^{2}} \tag{13}$$

where N is the number of the learning samples, x_{pred}^{j} and x_{opti}^{j} are the predicted data and the optimized data of the j transfer.

V. Introduction (*Heading 1*)

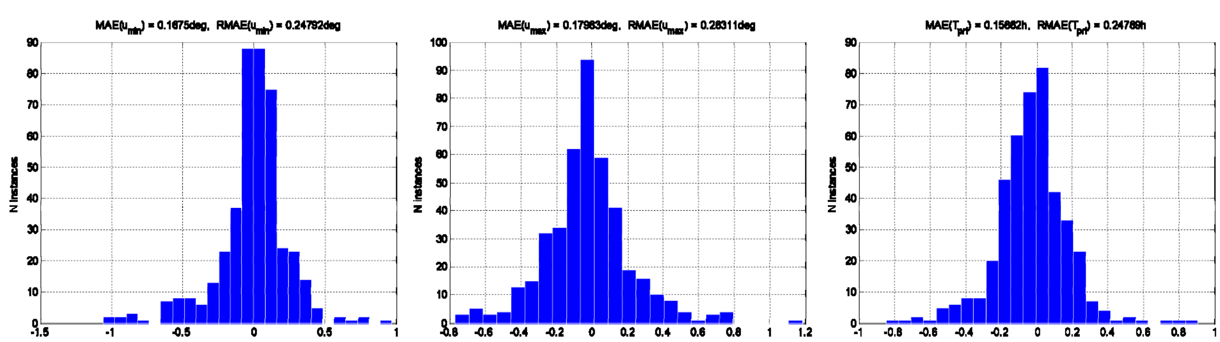
A. Machine Learning Result of Transfer window

The learning results and errors for the transfer window using neural network are shown in Figs. 4 and 5. The red points denote the original data in the test sets and the blue points denote the regression results based on the learning model. In Fig. 4, it is illustrated that the learned data fit the original data well for $u_{\min}$ and $u_{\max}$. The MAE and RMSE for $u_{\min}$ are 0.1675° and 0.2479° respectively. The errors are smaller than MAE and RMSE of 0.1796° and 0.2631° for $u_{\max}$ in Fig. 4(b). For the two variables, it can also be found that the majority of the histograms has $\Delta u_{\min} \in [-0.5^{\circ}, 0.5^{\circ}]$ and $\Delta u_{\max} \in [-0.6^{\circ}, 0.6^{\circ}]$. It is believed that the regression model could be used to estimate the interval of departure position on LEO with acceptable errors. As for the MAE of T_{prl} , it is about 0.1566h . The majority of regression differences concentrate in the interval of $[-0.5\mathrm{h}, 0.5\mathrm{h}]$ for ΔT_{prl} . It is noted that the demonstrated histograms of regression error in Fig. 5 are all similar to a normal distribution.



(a) Umin (b) Umax (c) Tprl

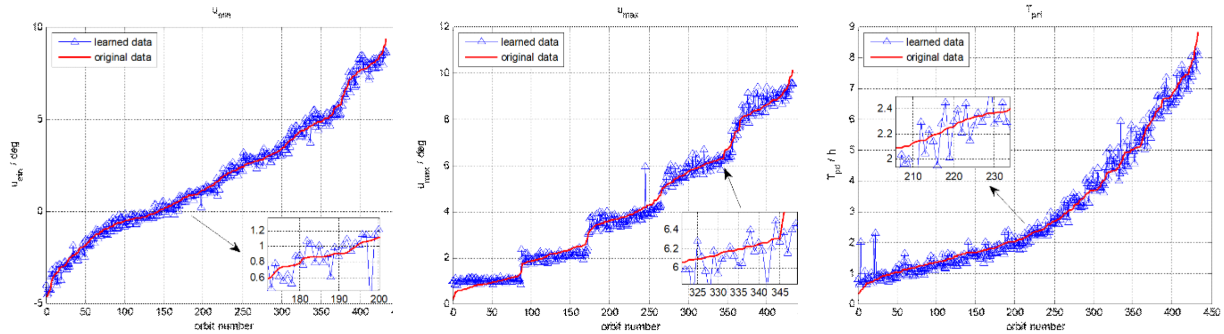
Fig. 4. Machine learning results of the transfer window using neural network



(a) Umin (b) Umax (c) Tprl

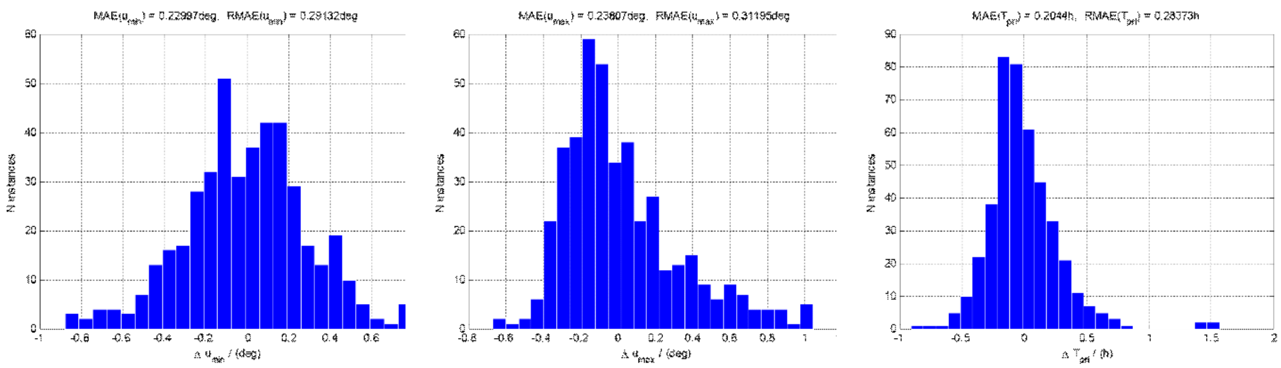
Fig. 5. MAEs of the transfer window using neural network

The regression performance based on the GB model is demonstrated in Figs. 6 and 7. Compared with neural network, the GB model has larger MAEs of 0.2300° and 0.2381° for $u_{\min}$ and $u_{\max}$. There is a small increase in the regression error of T_{prl} with MAE of 0.2044h . The histograms of the regression errors in Fig. 7 are all similar to a normal distribution, too. In Fig. 6, the GB shows more inflection points between the original data and the learned data than the neural network. The regression performance of two algorithms is summarized in Table III. In terms of the learning error, it can be concluded that the neural network learning model outperforms the GB learning model for all the 3 learning parameters.



(a) Umin (b) Umax (c) Tprl

Fig. 6. Machine learning results of the transfer window using gradient boosting



(a) Umin (b) Umax (c) Tprl

Fig. 7. MAEs of the transfer window using gradient boosting

Table III and Fig. 8 also give training time and regression time for the two algorithms. The simulation computation is performed on a desktop computer with Intel Core (TM) i7-4790 CPU at 3.6GHz and 8 GB of memory. For both two regression-based methods, it takes less than 24 seconds to train the database and less than 0.1 second to estimate the trajectories planning solution. However, the use of the numerical optimization algorithm based on dynamical models could cost hundreds of hours (about 20 seconds per LTO) to finish the same task. It can be believed that the regression-based learning method has a good advantage of computation efficiency and simple implementation. Note that database generation and learning feature selection is the foundation of applying machine learning method.

TABLE III. ERROR PERFORMANCE OF TWO ALGORITHMS

Learning variables	Neural Network		Gradient Boosting	
	MAE	RMSE	MAE	RMSE
$u_{\min}$ (deg)	0.1675	0.2479	0.2300	0.2913
$u_{\max}$ (deg)	0.1796	0.2631	0.2381	0.3120
T_{prl} (hour)	0.1566	0.2479	0.2044	0.2837
Training time(s)	23.276349		5.315139	
Regression time(s)	0.020368		0.031003	

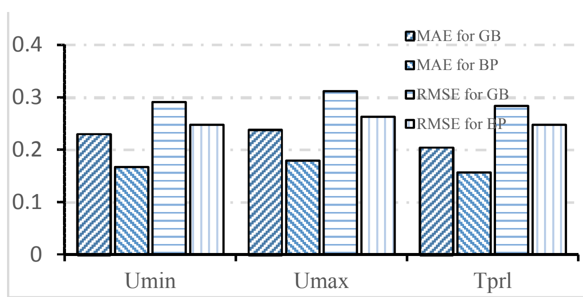


Fig. 8. Performance comparison of GB and BP regressors

B. *Verification of the learning features selection*

To verify the effectiveness of the selected learning features, orbital elements of inclination i_E and RAAN Ω_E in J2000 ECI are used as the learning features. Only the GB regressor is used this time and the results are listed as in Table IV.

It can be found that MAEs of $u_{\min}$ and $u_{\max}$ are 0.9706° and 0.9795° respectively which is approximately 5 times the corresponding errors using the pseudo orbital elements. In

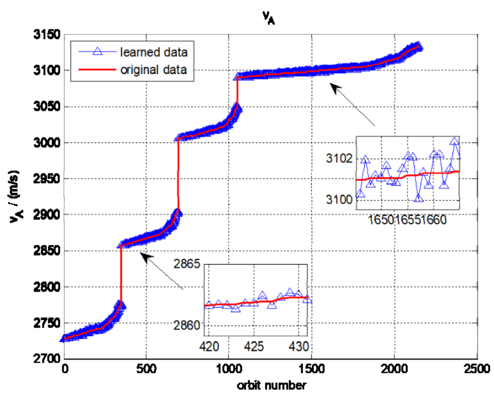
ECI frame, u_{min} and u_{max} has a larger range from -40° to 50° than the range from -5° to 10° in ECEMP frame.

Therefore, it is more difficult for the regressor to reveal the underlying nonlinear relationship between the output and input parameters. Also, MAE of T_{prl} is 0.2865h , which increases by 82.84% than using the pseudo elements. Moreover, it takes more time for the regressor to train the database. It can be concluded that the learning feature selection of pseudo orbital elements in ECMEPC has a better performance than the inertial orbital elements in the J2000 inertial coordinate.

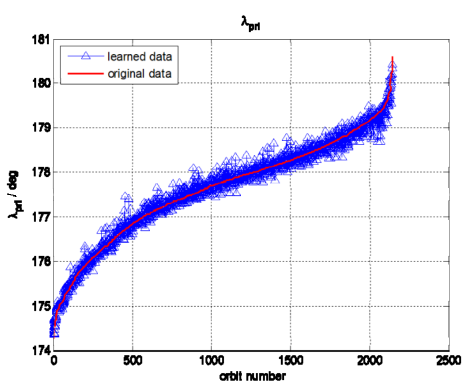
TABLE IV. ERROR PERFORMANCE USING ORBITAL ELEMENTS

Learning variables	Learning Errors	
	MAE	RMSE
u_{min} (deg)	0.9706	1.3334
u_{max} (deg)	0.9795	1.3512
T_{AB} (hour)	0.2865	0.2984
Training time(s)	33.276349	
Regression time(s)	0.120368	

C. Transfer Velocity Impulse Learning Result

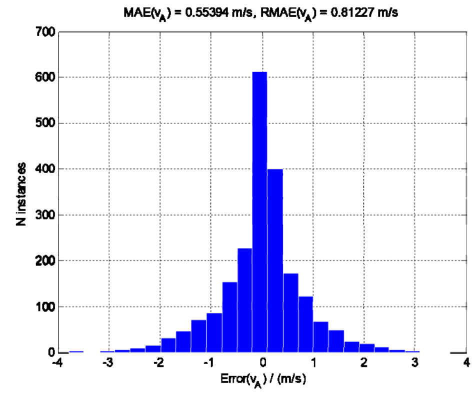


(a) Velocity impulse v_A

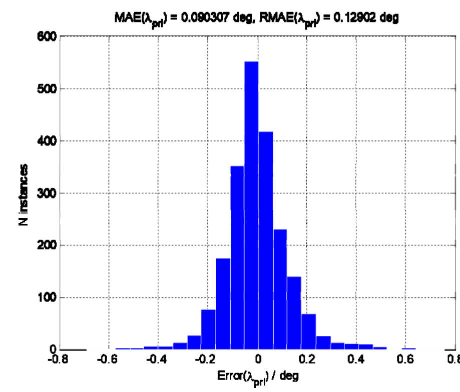


(b) Perilune longitude λ_{prl}

Fig. 9. Regression results of the transfer velocity impulse using neural network



(a) Velocity impulse v_A



(b) Perilune longitude λ_{prl}

Fig. 10. MAE and RMSE results of the transfer velocity impulse using neural network

Table V and Figs. 9 to 10 demonstrate comparison of the regression performance between the two regressors in v_A and λ_{prl} . It is found that MAEs of v_A for the BP and GP regressors are 0.5539m/s and 1.9167m/s respectively and both of them have the Absolute Relative Error (ARE) of less than 0.1% . Especially for histograms of the BP neural network, the majority has the error of $\Delta v_A \in [-2\mathrm{m/s}, 2\mathrm{m/s}]$ and $\Delta\lambda_{prl} \in [-0.2^\circ, 0.2^\circ]$. Note that the TLI velocity is mainly determined by the LEO height and the distance of the Moon to Earth. Therefore, the surrogate model is thought to be effective enough to estimate the TLI velocity impulse and perilune pseudo longitude. Again, the BP method demonstrates better performance than the GB network in the regression error.

TABLE V. PERFORMANCE ERROR OF TWO REGRESSORS FOR v_A AND λ_{prl}

Learning variables	Neural Network		Gradient Boosting	
	MAE	RMSE	MAE	RMSE
v_A (m/s)	0.5539	0.8123	1.9167	2.4216
λ_{prl} (deg)	0.0903	0.1290	0.3008	0.4398
Training time(s)	101.439122		6.9600000	
Regression time(s)	0.1564246		0.0349998	

VI. Conclusions

A fast solution to the two-impulse lunar transfer problem is studied based on machine learning method. The database characteristics analysis and the simulation results verify the correct selection of pseudo inclination and RAAN as the learning features. The learning error performance of two regression algorithms is discussed where BP regressor outperforms GB regressor. Compared with the traditional optimization algorithm, machine learning surrogate model shows great capacity of regression with acceptable errors, where the MAE of transfer window duration and velocity impulse is 0.1556h and 0.5539m/s. The work in this paper can be applied into quickly estimating the translunar trajectories. It should be noted that the obtained surrogate models are limited to the two-impulse transfer problem. Future work will extend the regression models to more complex ones such as the estimate for the three-impulse lunar transfer orbit as well as the symmetric free-return trajectory.

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