Cartesian Genetic Programming Hyper-Heuristic with Parameter Configuration for Production Lot-Sizing

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***Abstract*—** The design of well-performing heuristic-based algorithms is a tedious and time-consuming task. For this purpose, several approaches to automate the algorithm design have been investigated over the last years. Among them, Genetic Programming-based Hyper-Heuristics (GPHH) are able to design algorithms of varying lengths and permutations of algorithm's operators. Although the parametrization of generated algorithms is crucial for further adaptation of its behavior to the underlying problems, it has been neglected or only considered to a limited extent in many applications of GPHHs. This work proposes a way to integrate GPHH with parameter configuration. In particular, a Cartesian Genetic Programming Hyper-Heuristic (CGPHH) is developed. The proposed method is applied to the multi-level capacitated production lot-sizing problem, a complex and relevant problem in production planning. Conducted experiments evaluate both its efficacy and the quality of the generated algorithms. Finally, results are compared to the CGPHH without parameter configuration and a human-designed algorithm, indicating the ability of CGPHH-PC to generate algorithms that have the best overall-performance for different cutoff times.

Keywords—Hyper-heuristics, Genetic Programming, Algorithm Generation, Parameter Configuration, Lot-sizing Problem.

I. Introduction

Heuristic-based methods, such as meta-heuristics, have been successfully applied to solve a multitude of complex problems. To achieve good results for a particular domain, its operators and parameters need to be adapted, and the performance assessed on the specific problem instances. This design process must be repeated if results are not satisfactory or in case the problem configuration has changed. It is a very laborious and time expensive task, particularly considering the vast amount of methods and different design options. Thus, several approaches for automating the design of heuristic-based algorithms have been developed over the last years.

Methods for automating the configuration and generation of algorithms can be classified into: parameter configuration approaches (e.g., iRace [1]) and hyper-heuristics (HH) approaches, the latter are high-level methodologies for selecting or generating heuristics (be it simple heuristics or complete algorithms) [2]. In particular, HH for algorithm generation typically use Genetic Programming (GP) and favors a great spectrum of different combinations of components and thus different algorithm designs.

Several Genetic Programming Hyper-Heuristics (GPHH) have been applied to generate algorithms, such as tree-based GP [3], Grammatical Evolution (GE) [4], and Cartesian Genetic Programming (CGP) [5]. Although they produce competitive or even better algorithms than human-design ones, the parameter configuration is often neglected or considered to a limited extent (except the GE [4]). Since heuristic-based methods are sensitive to their settings, both the search for the most suitable algorithm design and the parameters of generated algorithms are crucial tasks to be performed by an automated algorithm configuration.

Thus, this work proposes a way of integrating parameter configuration into a GPHH, in particular, a CGPHH. Different from other GP approaches, CGP does not suffer from bloat, has an efficient decoding and fast convergence [6] and, has already been used to evolve algorithms for different problems. In a recent work [5], it was used to generate algorithms for the production lot-sizing problem (LSP), a crucial decision in production planning, that seeks to determine optimal production quantities that satisfy the demand, for each period, at minimal costs.

The investigation of automated configuration of heuristic-based algorithms for LSP is particularly important, as changes in the planning requirements or production configuration can affect the model, and consequently require a re-configuration of the methods used to solve them. With the introduction of reconfigurable and flexible production [7], the environment becomes even more susceptible to changes.

The multi-level capacitated lot-sizing problem (MLCLSP) is a particular problem within the general LSP models. Due to its multi-level structure and capacity restrictions, it portrays one of the most difficult problems in lot-sizing. Thus, this paper applies the proposed method to generate parametrized algorithms to solve the MLCLSP. The specific objectives are:

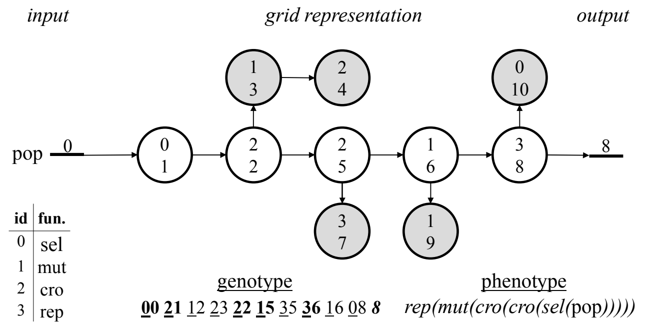
1) To integrate the parametrization of generated algorithms with the CGPHH.
2) To use the approach to evolve algorithms and their parametrization for the MLCLSP.
3) To analyze the contribution of parameter configuration on the evolution of algorithms.
4) To analyze the performance of the algorithms derived by the CGPHH with parameter configuration and compare them with the performance achieved without the parameter configuration.

Section 2 presents an overview of GPHHs, CGP, and introduces the approach for integrating parameter configuration with the CGPHH. Section 3 introduces the MLCLSP and gives an overview of related heuristic-based approaches. In section 4, the configuration of experiments is explained and results are analyzed. Finally, in section 5, conclusions are drawn and future research directions are pointed out.

II. CGP-based Hyper-Heuristic for Algorithm Configuration

Hyper-Heuristics are high-level methodologies that ultimately aim at increasing the generalization ability of heuristic-based methods. They are employed either to automatically select heuristics or algorithms from a set of existing ones or generate heuristics or algorithms from their components. Within generative HHs, Genetic Programming (GP) is the key ingredient and has been widely used because it can combine the components in a multitude of different forms, enabling the discovery of novel heuristic-based methods that can outperform hand-crafted methods for a given problem. GP-based Hyper-Heuristics (GPHHs) favor an ample design space by generating algorithms of varying lengths, sequences, and allowing the repetition of components [2]. GPHHs are generally characterized by the GP method employed, the structure of generated heuristic methods (e.g., simple heuristics, Genetic Algorithms), the set of components (i.e., the operators, variables, and constants that define the function and terminal sets), and how the performance of generated heuristic methods is measured (i.e., the fitness function) [8].

Among the GP methods, Cartesian Genetic Programming (CGP) does not require any parsimony pressure to control bloat, has fast convergence, and efficient decoding [6]. In contrast to the generally used tree representation, CGP uses direct acyclic feed-forward graphs to represent programs. The genotype represents the inputs and outputs for each node within a two-dimensional grid of function nodes, which can be connected to the preceding function or input nodes. Fig. 1 illustrates a CGP representation of an incumbent algorithm. In this example, all functions have arity one (i.e., has one argument/input), and the program has one output only. In order to use CGP, it is necessary to set: (i) the number of columns *c* and rows *r* of the grid; (ii) the maximum number of levels-back *l* any two connected nodes can be apart from each other; (iii) the function set, their respective functions arities and ids in the lookup table; (iv) the terminal set; (v) the definition of the fitness function for the generated programs (items iii-v are also required by others GP methods). Additionally, the genotypes are usually evolved by a (1+4) Evolutionary Strategy (ES) [9].



id	fun.
0	sel
1	mut
2	cro
3	rep

00 **21** 12 23 **22** **15** 35 **36** 16 08 ***8***

*rep(mut(cro(cro(sel(*pop*)))))*

Fig. 1. Example of CGP representation. Grey nodes are inactive; underlined numbers in the gene denotes the *function id* in the lookup table and correspond to the numbers at the top of each node; the *node id* is the number at bottom of each node; genes of active nodes are bold in the genotype. The last gene corresponds to the node directly connected to the output.

Even though the number of columns and rows defines fixed grid dimensions, the CGP can generate programs with variable number and combinations of operators (i.e., function nodes). The actual program (i.e., the phenotype) is decoded by traversing all (active) nodes that connect the program output(s) to the input node(s). In case *l* is equal to *c* (i.e., a node can be connected to any preceding function and input nodes), programs can vary from zero to the maximum number of nodes. Nodes that are not connected to the program output(s) are called introns (i.e., inactive nodes). The redundancy of nodes and the presence of introns in the genotype are of high importance for the performance of the evolutionary process [10].

A. CGP-based Hyper-Heuristic

CGP has been already used as a HH method in previous works to solve the Boolean Satisfiability Problem (SAT) [11], the Traveling Salesman Problem (TSP) [12, 13], and the Multi-level Capacitated Lot-sizing Problem (MLCLSP) [5]. Although CGP has been adapted to incorporate iterative flows [13], most of the existing approaches employ the standard CGP method, however using different algorithm templates and function and terminal sets.

In this work, the CGPHH employs the standard CGP to evolve algorithms and includes the selection and replacement operators in the function set. It allows multiple occurrences of selection operators within the generated algorithm, i.e., multiple sampling of individuals at different points of the evolutionary process. Additionally, it is possible to generate algorithms that correspond to an interleaving of two distinct evolutionary processes, i.e., contains one or more sequences of operators encompassed by selection and replacement operators. Additionally, the environment variable passed to the execution of each operator, is composed of the current population, the pool of parents (i.e., individuals that have been selected for breeding), and the pool of offspring (i.e., individuals that have been changed through mutation or crossover).

Although all operators receive the environment as argument for their execution, each one operates on a different list of individuals (TABLE I.). A crossover takes the individuals from the pool of parents and generates the offspring in the pool of offspring. A mutation is applied to all individuals in the pool of parents and the pool of offspring. Mutated parents are included in the pool of offspring, while the offspring are updated with their mutated counterparts. Local search and mathematical heuristics try to improve the best individual from the current environment. Replacement updates the current population with the individuals from the pool of offspring, according to the replacement type, and removes all individuals from the pool of offspring. The fitness function is only computed when required, e.g.,s to identify the best individual.

The generation of algorithms follows the template depicted in Fig. 2. The current population (populated by an initialization

TABLE I. OPERATORS ARGUMENTS AND OUTPUTS [5].

Input	Operator Type	Output[a]
population	Selection	(+) *pool_parents*
pool_parents	Crossover	(+) *pool_offspring*
pool_parents *pool_offspring*	Mutation	(+, *) *pool_offspring*
best_individual	Math. Heuristic / Local Search	(*) *best_individual*
population *pool_offspring*	Replacement	(/) *population* (-) *pool_offspring*

[a] Individual(s) are: (+) added; (-) removed; (*) modified; (/) replaced.

method) and the two pool of individuals (empty created) compose the environment variable *ENV*, which is passed as argument for the execution of each operator. In case a generated algorithm contains evolutionary operators that are not encompassed by selection and replacement operators, they are repaired by inserting a random selection or replacement, at the beginning or the end of the algorithm, respectively. Afterward, each operator of the generated algorithm is executed until the maximum execution time is exceeded.

This CGPHH is able to generate competitive algorithms for the MLCLSP that presented a much faster convergence compared to the human-designed algorithm, from which the majority of components were extracted [5]. However, like the other CGPHHs, the parameters of generated algorithms were set to their static default-values, thus limiting the configuration search-space to the sequencing of operators only. Since the static parameter-values might not be suitable for all types of generated algorithms nor all kinds of problems, the search for good parametrizations is crucial for improving the performance and adapting the algorithms to the problem instances under consideration.

B. CGPHH with Parameter Configuration (CGPHH-PC)

The proposed approach comprises of two tasks: (i) finding a good combination of algorithm components and (ii) searching for a proper parametrization of the generated algorithms. While the CGP evolves the algorithm`s composition, the parameter-configuration process assigns parameter values to the generated algorithms and optimizes the parameters of the best algorithms

```
1:  pop = InitializePopulation()  //Current population
2:  $pop_p$ = {}  //Pool of parents
3:  $pop_o$ = {}  //Pool of offspring
4:  $ENV_t$ = {pop, $pop_p$, $pop_o$}
5:  if listOperators not valid do
6:      listOperators = Repair(listOperators)
7:  end if
8:  while execTime < maxExecTime do
9:      for-each operator in listOperators do
10:         $ENV_{t+1}$ = operator.Execute($ENV_t$)
11:     end for-each
12:     $pop_p$ = {}; $pop_o$ = {}
13: end while
```

Fig. 2. Template of generated algorithms (highlithed lines are evolved by the CGP) [5, Fig. 3].

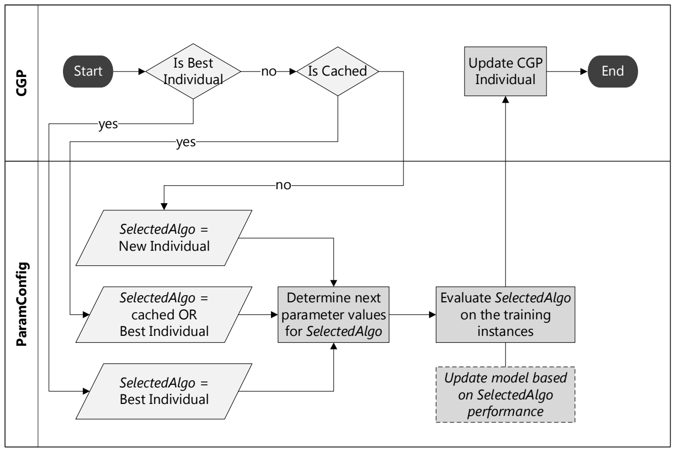

CGP
ParamConfig
Start
Is Best Individual
no
Is Cached
Update CGP Individual
End
yes
yes
no
SelectedAlgo = New Individual
SelectedAlgo = cached OR Best Individual
SelectedAlgo = Best Individual
Determine next parameter values for SelectedAlgo
Evaluate SelectedAlgo on the training instances
Update model based on SelectedAlgo performance

Fig. 3. Flow chart of the CGPHH integrated with parameter configuration.

found so far. In addition, idle computational-resource(s) that have been allocated to the CGP are shared with the parameter-configuration process and used for evaluating further parametrizations. Computational resources become idle when a neutral drift occurs, i.e., the phenotype is not affected by changes in the genotype, resulting in an already evaluated algorithm. The presence of introns causes neutral drifts and it has been proved beneficial for the diversity of the generated programs [10]. In order to identify individuals with equal phenotypes and reuse computational resources, a cache mechanism is employed.

Fig. 3 depicts how the parameter configuration is integrated with the CGP. At each CGP generation, all the produced algorithms plus the best individual – in case of a $(1+\lambda)$ ES, the parent – are passed to the parameter configuration processes, which consists of (i) defining the next parameter values to use for a given algorithm, (ii) evaluating the parametrized algorithm on the training instances, and (iii) updating the model used for generating the next parameter values based on the performance of the parametrized algorithm (necessary when using a model for the parameter configuration). The algorithm considered for parameter configuration depends on the current individual being evaluated. New parameterizations are sampled for the algorithms represented by the best or newly generated individuals. In case an individual has already been evaluated (i.e., it is in the cache), a new parameter setting is sampled for either the cached or the best individual. This strategy aims at giving a configuration budget to both cached and best individuals, hopefully improving either performance and driving them to achieve the best overall fitness.

The proposed approach enables the use of a variety of methods for searching better parameter settings for a given algorithm, such as sampling techniques (e.g., Latin hypercube, orthogonal, random), optimization methods (e.g., local search methods), learning and statistical models (e.g., reinforcement learning). Since the algorithms change over time, the employed method should take this aspect into account (either by adapting its model or by constructing a new one). This design flexibility allows adjusting the parameter configuration approach for the needs and complexity of the problem under consideration.

In order to assure that neutral drifts are not impacted by the integration of the parameter configuration approach, the CGP individual and its update procedure have been adapted. Besides the genotype and fitness, individuals also store the parameter setting used. The best individual as a whole is replaced if an offspring has equal or better fitness. Otherwise, if an offspring has the same phenotype of the best individual, only the genotype is replaced.

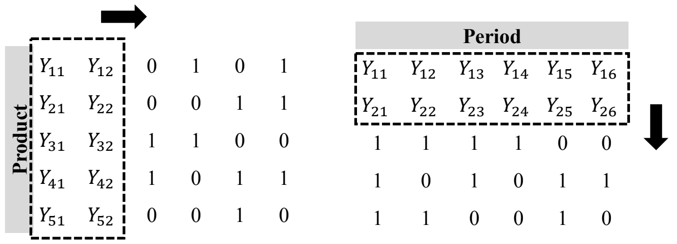
III. Multi-Level Capacitated Lot-Sizing Problem

The MLCLSP is a generalization of lot-sizing problems and one of the most difficult models of central production planning. It takes several characteristics of the production environment into account, thus reflecting the real-world closer and producing more feasible plans. The production is modeled as a sequence of production levels that transform raw materials into one or more intermediate products, ultimately converting them into the final products. Additionally, MLCLSP model considers dynamic demand for final products, multiple final products, multiple machines, capacity restrictions regarding the workers/machines, costs and time to change the production setup between different products, and inventory holding costs. Due to fluctuations in demand and limited capacities, final products can be produced and stocked to satisfy future demand. Besides that, some variations of the MLCLSP consider backorders, overtime costs, and setup carryover. The goal is the determination of the optimal amount of products to produce at each period to satisfy the demand and minimize costs. The standard MLCLSP is NP-Complete [14].

Even though exact methods have also been applied to solve MLCLSPs, most methods employed are heuristic-based, so that feasible, near-optimal solutions can be found even for large problem instances (e.g., genetic algorithms, tabu search, relaxing or fixing variables of the mathematical programming model, greedy heuristics) [15]. Among the solution approaches, the hybridization of meta-heuristics and mathematical heuristics has led to very good results. These hybrid methods, a.k.a. matheuristics, take advantage of the good exploration capabilities of meta-heuristics and the powerful exploitation of mathematical heuristics to search for optimal solutions.

The fix-and-optimize (FaO) mathematical heuristic has been often employed by matheuristics applied to MLCLSP [16–18]. FaO attempts to find optimal solutions by decomposing the problem into sub-problems and iteratively solving them. The sub-problems are defined by fixing some setup variables to constant values (i.e., a binary matrix – *products* x *periods* – if one, a product is produced in that period, incurring setup costs; and zero, otherwise). Fig. 4 illustrates the FaO heuristic. A window determines which values are set as variables and constant. Then, the sub-problem is optimized and variables fixed with the optimal values found. Subsequently, the window is rolled (over the products or periods) and the process repeats until a stop criterion is satisfied. Several variants of the FaO heuristics have been proposed and decompose the problem based on the products, periods, production processes, resources, or a combination of them [16–19].

The multi-population genetic algorithm with fix-and-optimize (MGAFO) [16] is a prominent matheuristic method for MLCLSP. It has produced very good solutions for the considered problem instances and contains several components,



Product
Y_{11} Y_{12} 0 1 0 1
Y_{21} Y_{22} 0 0 1 1
Y_{31} Y_{32} 1 1 0 0
Y_{41} Y_{42} 1 0 1 1
Y_{51} Y_{52} 0 0 1 0
Period
Y_{11} Y_{12} Y_{13} Y_{14} Y_{15} Y_{16}
Y_{21} Y_{22} Y_{23} Y_{24} Y_{25} Y_{26}
1 1 1 1 0 0
1 0 1 0 1 1
1 1 0 0 1 0

Fig. 4. Fix-and-optimize heuristic with rolling windows over products and periods.

besides the FaO, that fits well in the CGPHH framework. Since most components used in this work were extracted from MGAFO, this approach is further explained and latter compared against the generated algorithms. The MGAFO uses three GA populations, hierarchically organized in a quaternary-tree structure of depth three: the root is the best individual of the population and the leaves present individuals with worse fitness than the respective parent nodes. It uses the setup variables as solution representation and initializes them with a hot-start procedure. The fitness is computed by solving the linear programming model with CPLEX, given the setup variables.

Parents are selected based on their tree-hierarchy: only the root and its immediate children are eligible for mating. An offspring only enters the population if its fitness is better than that of its parents. The evolutionary process is performed by four crossover and three mutation variants, which are randomly selected at each iteration. The crossover variants are (i) uniform; (ii) sector: the setup matrix is randomly divided into four partitions and interchanged between the parents at random; (iii) one point period; and, (iv) one point product. Mutation variants comprise (i) random one flip; (ii) random two flips for the same period; (iii) random two flips for the same product; and (iv) random two flips.

After populations have converged, the best individual of each population migrates to the next population in a round-ring fashion, the FaO used with products and periods attempts to improve the best individual among all populations. Next, the improved individual is inserted into the populations. If the FaO failed at improving the best individual, its window size is increased (i.e., a larger set of variables is considered). After this process, the evolutionary process starts again and this cycle continues until the execution time is reached.

IV. Experimental Results

In order to enable comparison, the experimental setting is the same as defined for the CGPHH with static parameter values (CGPHH-S) [5]. The CGP is configured with: one row, 200 columns, levels back = 200, 1000 generations, a (1+4) ES with 4% mutation rate to evolve algorithms, which are in turn executed five times for each training instance and their fitness computed as the mean fitness over the different instances and runs. The same mathematical models for the fitness function and FaO heuristics are used, as well as the same version of CPLEX, running with two threads limit. Thus, at maximum, a total of 5000 different combinations of algorithms and parameters are tested at each CGP execution, due to the parameter-configuration procedure. The time budget of the algorithms is 180s. The CGPHH-PC is also implemented in Java 8 and uses

TABLE II. LIST OF PARAMETERS TO BE CONFIGURED.

Mathematical Heuristics			
FAO		*FAO non-deterministic*	
prod. min. window	(2, 4; 1) [2]	min. products	(2, 4; 1) [2]
prod. max. window	(4, 7; 1) [7]	max. products	(4, 7; 1) [7]
prod. inc. window	(1, 3; 1) [1]	inc. products	(1, 3; 1) [1]
perio. min. window	(2, 5; 1) [2]	min. periods	(2, 5; 1) [2]
perio. max. window	(5, 10; 1) [7]	max. periods	(5, 10; 1) [7]
perio. inc. window	(1, 3; 1) [1]	inc. periods	(1, 3; 1) [1]
rolling span	(1, 3; 1) [1]	max. no. trials	(2, 8; 1) [4]
FAO Convergence			
improvement rate	(0.005, 0.05; 0.005) [0.05]		
n. evaluations	(1000, 5000; 1000) [1000]		
Algorithm		**Mutation**	
n. of indiv.	(20, 80; 30) [50]	prob. mutation	(0.005, 0.1; 0.01) [0.015]
init. meth.	{MGAFO, random+MGAFO} [MGAFO]	mutation rate	(0.75, 1; 0.05) [0.95]
Selection		**Crossover**	
tourn. size	(5, 20; 5) [10]	unif. crossover	(0.5, 0.9; 0.05) [0.6]
n. of indiv.	(2, 4; 1) [2]	crossover rate	(0.75, 1; 0.05) [0.95]

the OpenMPI Java binding. Each tuple (*algorithm*, *training instance*) is distributed to an MPI process, which further distributes the different runs of the algorithm to threads. Thus, ten cores per MPI process are allocated to account for the five runs with two CPLEX threads each. Experiments are performed in the PALMAII cluster at the University of Münster and use the same training (10) and test (19) instances as the CGPHH-S. Instances are extracted from class B+ of Stadtler dataset [20].

Besides the operators included in the function set of CGPHH-S, some experiments consider additional FaO heuristics based on the non-deterministic setting of setup variables: at each iteration, this heuristic selects a random number of columns (i.e., periods) or rows (i.e., products), fix all but the selected periods and products as constants, and tries to optimize the remaining set of setup variables. This process repeats until an improved solution is found or the maximum number of trials is reached. If no enhanced solution is found, the number of trials increases. Three non-deterministic FaO approaches have been conceived: product-wise, period-wise, and product- and period-wise.

The parameters of each operator used in the function set have been extracted and modeled, as shown in TABLE II. The permissible values of numeric and integer parameters are modeled as (*lowerBound, upperBound; interval*) [*defaultValue*], where values between the bounds (inclusive) are discretized in equal intervals. Several parameters are used by the mathematical heuristics, e.g., to configure different behaviors of rolling windows product- and period-wise. By increasing the configuration space of those heuristics, better heuristic designs might be discovered.

A random sampling without repetition is used to set the next parameter setting for the generated algorithms. Since this is the most naïve method to use, it can represent a theoretical lower-bound on the performance evaluation of the CGPHH-PC. The use of any other more advanced method for parameter configuration will likely perform the search in the parameter space more effectively. New generated algorithms are randomly assigned with the default parameter-values or the parameter setting of the best individual so far. Further configurations for the best or cached algorithms are assigned so that the combination (*algorithm*, *parameters*) is unique.

TABLE III. SUMMARY OF CGPHH-PC RESULTS.

	N. Algos.	**N. Params.**	**Fitness**	**Gap(%)**	**Freq. Opers.**[a]
ALLLS	499	4501	141942.5	-1.58	0,0,0,1,0,0
ALLNR	1538	3462	141188.8	-2.11	0,0,0,3,0,0
CROSLS	1752	3248	141465.3	-1.92	1,1,0,3,2,1
CROSNR	2243	2757	141182.5	-2.11	2,4,0,7,0,1
LS	1195	3805	141002.9	-2.24	0,0,0,6,3,0
LSNR	877	4123	140961.9	-2.26	0,0,0,11,0,0
MUTLS	1521	3479	141341.5	-2.00	1,0,1,4,2,1
MUTNR	1582	3418	140929.9	-2.29	1,0,1,5,0,1

a. Sequence regards to: Selection, Crossover, Mutation, FaO, non-deterministic FaO, Replacement.

The CGPHH-PC is executed with function sets that consider all mathematical heuristics (-LS) and only the FaO with rolling windows (-NR). The subsets of functions used in the experiments are: all functions (ALLLS and ALLNR), only mutation and mathematical heuristics to improve solutions (MUTLS and MUTNR), only crossover and mathematical heuristics (CROSLS and CROSNR), and only mathematical heuristics (LS and LSNR). Experiments aim at (i) assessing the contribution of the parameter configuration in the evolution of the algorithms, (ii) compare the performance of the algorithms generated by the CGPHH-PC and CGPHH-S, and (iii) assess the performance of algorithms for different cutoff times.

A. Results

TABLE III. gives an overview of the CGPHH-PC execution for different function sets (fitness, gap, and frequency of operators refer to the best algorithm found). The column *gap* refers to the fitness deviation of the generated algorithms to the results originally reported for MGAFO: $gap = \left(\frac{fitnAlgo - fitnMGAFO}{fitnMGAFO}\right)$. The best algorithms evolved in each experiment uses a different parameter configuration than the default parameterization. In addition, many parameter configurations for the best algorithm have been tested along the CGPHH-PC execution (Fig. 5) and demonstrated to have a great impact on the algorithm's performance (both time- and fitness-wise). As depicted by the frequency of operators, the length (i.e., number of operators) of the best algorithms vary from one to fourteen with always one or more FaO heuristics used. The

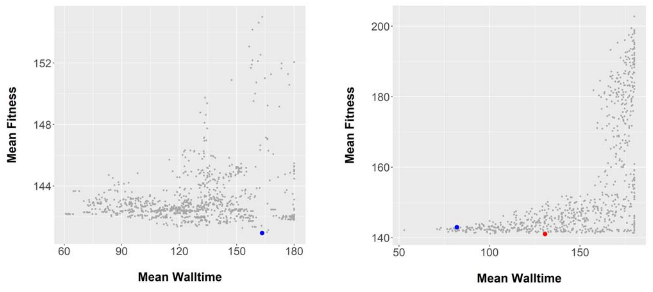


Fig. 5. Different parametrizations tested during the execution of CGPHH-PC for the best algorithms of MUTNR (left) and LS (right). Blue – initial parametrization; red – best parametrization found after configuration (if any). Fitness in order of 1,000.

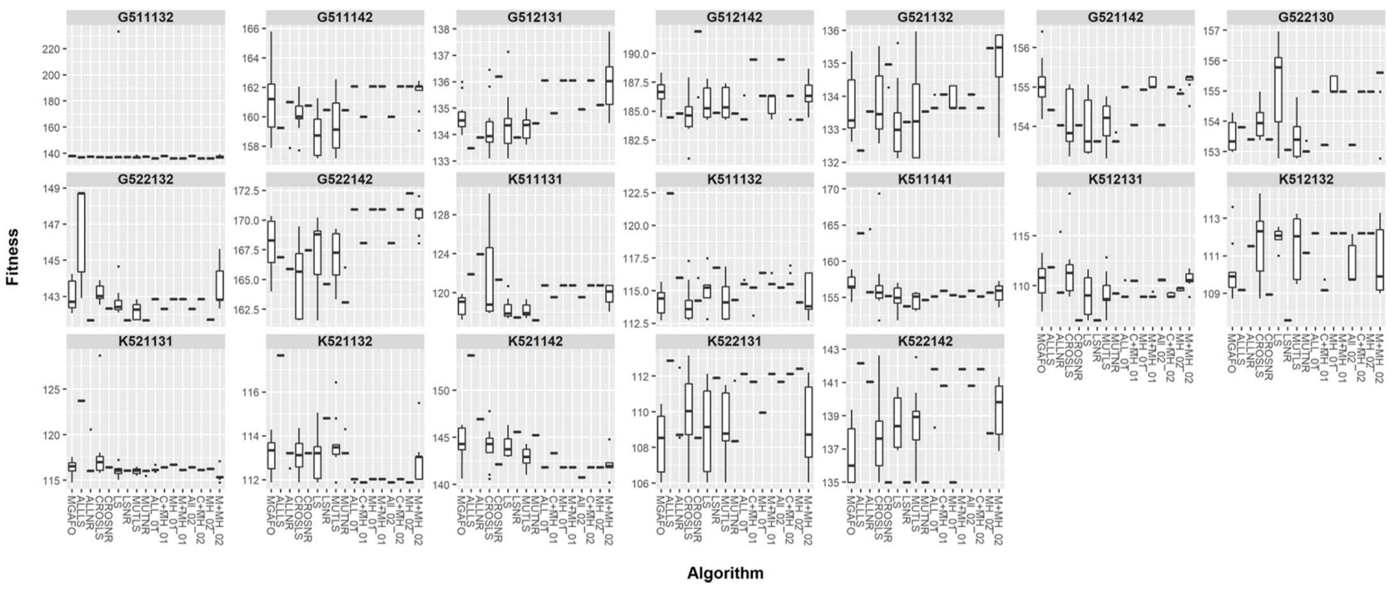


Fig. 6. Fitness variation of MGAFO and best algorithms generated by CGPHH-PC and CGPHH-S for each problem instance.

algorithms resulting from the CGPHH-PC have better fitness for the training instances compared to the outcome of the CGPHH-S, when parameters were set to default values.

The number of distinct algorithms (i.e., distinct sequence of operators) generated per experiment varies considerably. This is likely attributed to the particular graph-structures and neutral drifts. The generation of fewer distinct algorithms, on the other side, implies giving a higher budget for parameter configuration, which in turn can significantly improve the algorithm's performance. For instance, LSNR examines only 877 different algorithms, the second-lowest amount, and was able to find a good parametrized algorithm that produced one of the best fitness for the training instances.

TABLE IV. shows the gap and respective fitness standard-deviation of the MGAFO (re-executed on the same computational environment) and the best algorithms generated by the CGPHH-PC for the instances of the test set. MUTNR, LSNR, MUTLS, and CROSNR present a higher overall improvement and less fitness deviation compared to MGAFO, while ALLLS failed to improve the overall result, but achieved the best improvement for three instances. Only MUTNR and MGAFO consistently improve solutions for all cases. The Friedman's aligned rank test for multiple comparisons rejects the hypothesis that all algorithms are equal (p-value=0.0072), but the Nemenyi posthoc test (a non-parametric equivalent of the Tukey test) does not show a significant difference between the algorithms at 5% significance level (alpha = 5%).

TABLE IV. RESULTS FOR THE BEST ALGORITHMS. TIME BUDGET OF 180 S.

	MGAFO	*ALLLS*	*ALLNR*	*CROSLS*	*CROSNR*	*LS*	*LSNR*	*MUTLS*	*MUTNR*
G511132	-0.01 (637)	-0.65 (0)	-0.37 (0)	-0.47 (575)	**-0.70 (0)**	6.37 (30513)	-0.54 (0)	-0.58 (667)	-0.37 (0)
G511142	-0.11 (2265)	-1.25 (0)	-0.36 (988)	-0.70 (1201)	-0.31 (0)	**-1.49 (1567)**	-0.50 (0)	-1.17 (1958)	-0.51 (0)
G512131 [a]	-0.52 (666)	**-1.43 (0)**	-1.13 (0)	-0.86 (1099)	0.57 (0)	-0.72 (1146)	-1.13 (0)	-0.85 (456)	-0.74 (0)
G512142 [a]	-0.60 (1190)	**-1.73 (0)**	-1.55 (0)	-1.81 (2132)	1.94 (1801)	-1.08 (1377)	-1.52 (0)	-1.09 (1338)	-1.55 (0)
G521132 [a]	-0.67 (962)	**-1.69 (0)**	-0.81 (0)	-0.67 (1030)	0.15 (292)	-1.04 (1131)	-1.05 (0)	-0.92 (1342)	-0.81 (0)
G521142	-0.01 (641)	-0.06 (0)	-0.32 (0)	-0.24 (747)	-0.32 (0)	-0.36 (720)	**-0.58 (0)**	-0.27 (544)	-0.55 (99)
G522130	-0.28 (530)	-0.08 (0)	-0.34 (0)	0.04 (519)	-0.34 (0)	0.80 (1571)	-0.57 (0)	-0.32 (664)	**-0.58 (112)**
G522132 [a]	-0.52 (880)	2.24 (2810)	**-1.45 (0)**	-0.40 (465)	-1.00 (0)	-0.73 (764)	**-1.45 (0)**	-1.08 (471)	**-1.45 (0)**
G522142 [a]	-1.26 (2273)	-1.89 (0)	-2.46 (0)	-2.97 (3156)	-1.55 (0)	-1.71 (2838)	-3.22 (0)	-1.87 (2144)	**-3.95 (936)**
K511131	-0.48 (992)	2.16 (0)	3.88 (0)	1.86 (4902)	1.68 (0)	-0.84 (1065)	-1.53 (0)	-0.96 (702)	**-1.80 (0)**
K511132	-1.35 (1012)	5.83 (0)	0.24 (0)	**-1.51 (1507)**	-1.12 (555)	-0.71 (1366)	0.91 (0)	-1.28 (1454)	-1.24 (0)
K511141 [a]	-3.68 (1377)	0.65 (0)	-3.26 (3676)	-3.64 (4713)	-4.66 (0)	-4.84 (1711)	**-5.54 (0)**	-5.03 (1137)	-5.01 (0)
K512131 [a]	-0.80 (1874)	0.30 (0)	-1.42 (1932)	0.10 (2991)	**-4.45 (0)**	-2.17 (2031)	**-4.45 (0)**	-2.08 (1970)	-2.05 (0)
K512132	-1.09 (1462)	-1.99 (0)	0.11 (0)	0.20 (1908)	-2.21 (0)	0.57 (423)	**-3.45 (0)**	0.07 (1630)	-0.21 (0)
K521131	-1.37 (780)	4.81 (0)	-0.94 (1925)	-0.06 (3854)	-1.37 (0)	-1.72 (617)	-1.69 (0)	-1.73 (323)	**-1.76 (172)**
K521132	-1.87 (819)	2.11 (0)	-1.82 (220)	-1.88 (803)	-1.76 (0)	**-1.90 (1052)**	-0.38 (0)	-1.36 (1206)	-1.67 (346)
K521142 [a]	-2.08 (1741)	4.20 (0)	-0.40 (0)	-2.36 (2103)	-3.64 (0)	-2.31 (1231)	-1.31 (0)	**-3.08 (1069)**	-1.56 (0)
K522131 [a]	**-1.51 (1735)**	2.60 (0)	-0.86 (1199)	0.09 (2144)	-1.34 (0)	-0.83 (2439)	1.71 (0)	-0.73 (1880)	-1.19 (1071)
K522142 [a]	-1.68 (1713)	2.34 (0)	1.53 (0)	-0.85 (2230)	**-2.83 (0)**	-0.24 (1542)	**-2.83 (0)**	-0.15 (2004)	-2.82 (0)
Overall:	**-1.05**	**0.87**	**-0.62**	**-0.85**	**-1.23**	**-0.79**	**-1.53**	**-1.29**	**-1.57**

[a] Instances used by the CGPHH-PC for evolving the algorithms.

TABLE V. RESULTS OF BEST ALGORITHMS FOR DIFFERENT CUTOFF TIMES.

Time	***MGAFO***	***MUTNR***	***LSNR***	***MUTLS***	***LS***	***CROSNR***	***C+MH_01***	***ALL_02***	***ALLNR***	*p-value*
180s	-1.05 (6.10)	**-1.57 (3.39)**	-1.53 (3.53)	-1.29 (4.21)	-0.79 (4.89)	-1.23 (5.18)	-0.82 (5.92)	-0.86 (5.97)	-0.62 (5.79)	*0.02872*
120s	0.04 (6.74)	**-1.14 (2.87)**	-1.16 (3.95)	-0.96 (4.16)	-0.30 (5.16)	-0.82 (5.18)	-0.58 (5.37)	-0.56 (5.53)	1.02 (6.05)	*0.01219*
60s	6.92 (7.89)	1.34 (3.97)	**0.19 (3.21)**	5.03 (5.21)	20.58 (7.26)	6.36 (4.21)	0.08 (3.34)	-0.04 (3.82)	2.97 (6.08)	*8.47E-10*
30s	13.2 (6.52)	2.80 (4.10)	10.16 (3.97)	45.94 (6.42)	60.42 (8.42)	21.86 (4.84)	**0.64 (2.37)**	0.62 (2.63)	19.18 (5.71)	*3.45E-13*
20s	19.46 (5.58)	4.64 (3.66)	22.43 (4.05)	57.06 (6.84)	67.39 (7.53)	63.91 (7.16)	2.11 (2.31)	**1.71 (1.95)**	35.63 (5.92)	*< 2.2E-16*
10s	**33.16 (2.68)**	32.75 (3.68)	43.10 (4.18)	65.32 (4.63)	71.45 (5.79)	82.94 (7.87)	66.16 (4.71)	58.60 (5.76)	57.34 (5.68)	*1.74E-08*

Fig. 6 depicts the fitness variation of the MGAFO and the best algorithms generated by the CGPHH-PC and CGPHH-S. The performance of algorithms varies significantly across different instances. The algorithms produced by CROSSLS, LS, and MUTLS include the non-deterministic FaO and present high deviation of fitness values. Instance-wise, at least one of the generated algorithms presented less variance and better mean fitness than the MGAFO, except for the instances K511132 and K522131. Algorithms generated by CGPHH-PC usually perform similar or better than those produced by CGPHH-S, particularly for instances with general production structure (starting with G). However, for K521132 and K521142 they are worse. Friedman's aligned rank test applied to all algorithms and instances rejects the hypothesis of performance similarity (p-value=0.014). Nonetheless, the Nemenyi test does not show a significant difference between them.

A detailed analysis of the algorithms' performance for different cutoff times is carried out for the eight best-ranked algorithms. TABLE V. presents the gap and rank of the different algorithms alongside with the p-value of the Friedman's aligned rank test, which rejects the hypothesis of similar performance for all cutoff times. The gap for each cutoff time reveals that the generated algorithms need a distinct amount of time to find good results, due to their different algorithm composition and parameters. For example, MUTLS requires initially much more time than C+MH_01 and ALL_02 to find good solutions, but it achieves better results than the other two from 120s onwards (Fig. 7). MUTNR is the only algorithm that presents a similar or better gap than MGAFO for all cutoff times. C+MH_01 and ALL_02 show fast improvement and excellent solutions for short execution times.

The Nemenyi posthoc test is applied to each cutoff time to identify pairwise differences. Fig. 8 shows the relations between algorithms. Although, the analysis for the 180 seconds run does not identify significant distinction, the rank difference between MGAFO and MUTNR (2.71) is very close to the critical difference (CD) computed by the method (2.79). Considering the performance over different time limits, ALLNR is the worst-performing algorithm (in the group of best algorithms only at 180s), followed by MGAFO (within the best algorithms at 10s and 180s). For short execution times, particularly for 20s and 30s, C+MH_01 and ALL_02 achieve very good results and a greater rank distance to other best-performing algorithms. Additionally, only MUTNR and LSNR make into the group of best-performing algorithms for all cutoff times, thus presenting the best anytime behavior among all algorithms.

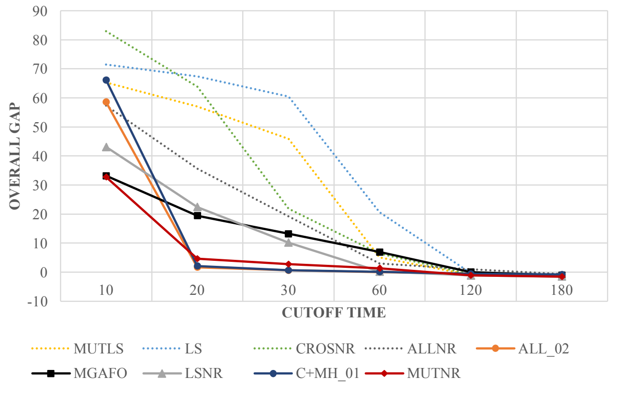


Fig. 7. Parallel plot of overall gap obtained for the different cutoff times.

V. CONCLUSIONS AND OUTLOOK

In this work, the CGPHH-PC is proposed: a CGPHH for algorithm generation with parameter configuration. It is based on the CGPHH with static parameters for generated algorithms (CGPHH-S) developed previously [5]. The approach to integrate parameter configuration into CGPHH framework is flexible and enables several methods for searching a better parameter setting for a given algorithm to be plugged-in, thus allowing adaptations of its design to the needs and complexity of the problem under consideration. The CGPHH-PC was used to design parameterized algorithms for MLCLSP, a complex problem with practical relevance in production planning, and employed a very naïve approach to configuring parameters, based on random sampling without repetition. Experiments were performed to analyze the contribution of the parameter

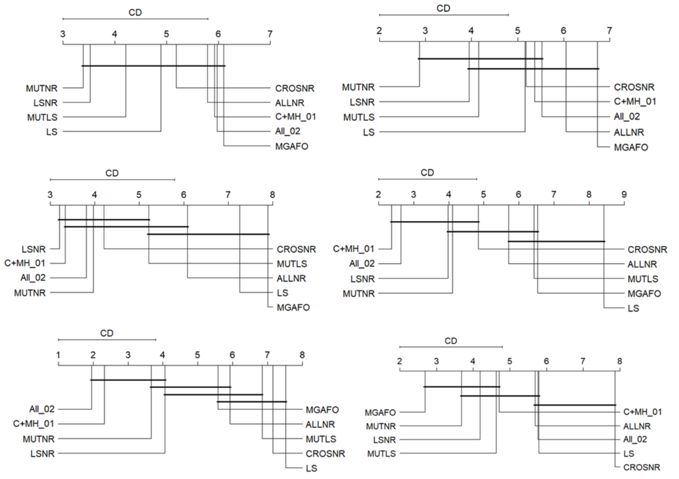


Fig. 8. Groups of algorithms based on the critical distance computed by the Nemenyi test. CD (critical difference); bar connects groups of simmilar algorithms (i.e., within the critical difference). 180s (top left), 120s (top right), 60s (midlle left), 30s (middle right), 20s (bottom left), 10s (bottom right).

configuration and to contrast the performance of algorithms generated by CGPHH-PC with those produced by CGPHH-S.

The analysis of results indicates that the parameter configuration was able to find very good parametrized algorithms compared to the attribution of static default-values, even though the method employed for parameterization is quite rudimentary. Thus, it indicates that even better results can be achieved if more robust methods of parameterization are employed. The large number of different parameter configurations performed during the CGPHH-PC execution shows that it considerably succeeded in reusing computational resources, which would have been wasted due to neutral drifts. In addition, the non-deterministic FaO heuristics produced good results when executed together with deterministic FaO operators. However, algorithms that use these new operators have a higher standard deviation, given their non-deterministic nature.

All the best algorithms generated by CGPHH-PC were executed for a wide range of different parameter values. Although not statistically different from the algorithms generated by CGPHH-S, they have achieved a considerable overall-fitness improvement. Additionally, the best algorithms of CGPHH-PC achieved the biggest improvement in comparison to the human-designed MGAFO for eighteen out of nineteen instances used in the test set. CGPHH-PC was also able to generate two algorithms (MUTNR and LSNR) with significant best overall-performance for any budget imputed on their execution time. Thus, the proposed approach is able to evolve algorithms that have a considerably higher fitness improvement and significant best overall-performance for different cutoff times.

Results also revealed that generated algorithms perform considerably different across the problem instances. Thus, an exciting research direction is an investigation of how features of MLCLSP impact the performance of algorithms. At first, an analysis of relevant features of MLCLSP should be carried out. Then, the acquired knowledge can be used to cluster the instances according to the features that have greater impact on performance and use the CGPHH-PC to generate parameterized algorithms automatically for the different groups of instances. The set of generated algorithms and problem features could then be used by an automatic, instance-based, algorithm-selection method. Furthermore, future research should focus on the comparison of different parameter configuration methods and evaluate their impact on the quality of generated algorithms and the overhead caused on the CGPHH-PC for different problem classes.

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