Trend Mining 2.0: Automating the Discovery of Variable Trends in the Objective Space

Henrik Smedberg, Sunith Bandaru, Amos H.C. Ng
Department of Production and Automation Engineering
School of Engineering Science, University of Skövde
Skövde, Sweden
firstname.lastname@his.se

Kalyanmoy Deb
Department of Electrical and Computer Engineering
Michigan State University
East Lansing, USA
kdeb@egr.msu.edu

***Abstract*—Practical multi-criterion decision making not only involves the articulation of preferences in the objective space, but also a consideration of how the variables impact these preferences. Trend mining is a recently proposed visualization technique that offers the decision maker a quick overview of the variables' effect on the structure of the objective space and easily discover interesting variable trends. The original trend mining approach relies on a set of predefined reference directions along which an *interestingness score* is measured for each variable. In this paper, we relax this requirement by automating the approach to find optimal reference directions that maximize the interestingness for each variable. Additional extensions include the use of an Achievement Scalarizing Function (ASF) for ranking solutions along a given reference direction, and an updated interestingness score formulation for more appropriately handling discrete variables. We demonstrate the working of the extended approach on DTLZ2 and WFG2 benchmarks for up to five objectives and on a biobjective engineering design problem. The results show that the ability of the proposed approach to detect variable trends in high dimensional objective spaces is heavily dependent on the quality of the solutions used.**

***Index Terms*—multi-objective optimization, variable trends, objective space, decision making.**

I. Introduction

The generation of a set of Pareto-optimal solutions does not mark the end of solving a practical multi-objective optimization problem (MOOP). The analysis of the trade-off front is an important step before a final solution can implemented in practice, a process referred to as multi-criterion decision-making (MCDM).

A. Multi-Objective Optimization and MCDM

Multi-objective optimization problems involve the simultaneous optimization of multiple conflicting objectives and can be formulated as:

$$\begin{aligned} \text{Minimize} \quad & \mathbf{F}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})]^T \\ \text{Subject to} \quad & g_j(\mathbf{x}) \geq 0 \;\; \forall\, j = 1, 2, \ldots, J \\ & h_k(\mathbf{x}) = 0 \;\; \forall\, k = 1, 2, \ldots, K \\ & \mathbf{x}^{(L)} \leq \mathbf{x} \leq \mathbf{x}^{(U)} \end{aligned}$$

where $\mathbf{F}(\mathbf{x})$ is a set of M (≥ 2) objective functions, $g_j(\mathbf{x})$ represents J inequality constraints, and $h_k(\mathbf{x})$ represents K equality constraints, $\mathbf{x}$ is a vector of n decision variables representing a solution ($\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$) to be optimized within the bounds of $[\mathbf{x}^{(L)}, \mathbf{x}^{(U)}]$. $\mathbf{F}(\mathbf{x})$ is a mapping from the decision space to the objective space of the problem.

Multiple conflicting objectives lead to multiple optimal solutions which lie on the so called Pareto-optimal front. In practical MOOPs, often a decision maker (DM) with in-depth problem knowledge is involved who can provide preference information. The field of MCDM deals with the development of methods that use such preference information to narrow down the Pareto-optimal set to one or a few solutions for further consideration or implementation. In MCDM there are four broad classes of methods [1]: (i) *no preference methods*, where the DM's preferences are not considered, (ii) *a priori methods*, where the DM's preferences are known beforehand and are used to focus the search towards desirable Pareto-optimal solutions, (iii) *a posteriori methods*, where the DM's preferences are used after the generation of a representative set of Pareto-optimal solutions to select desirable solutions, and (iv) *interactive methods*, where the DM's preferences are incorporated into the search process to converge towards desirable region(s) of the Pareto-optimal front.

This paper concerns a posteriori analysis of the solutions generated by multi-objective evolutionary algorithms (MOEAs), algorithms that find a representation of the Pareto-optimal front by applying principles of natural evolution to drive the search.

B. Decision Space and Objective Space

The presence of both a decision space and an objective space invokes several interesting questions about the relationship between the variables and the objective functions of the MOOP, such as "how do the preferences affect the variable values?". Many current MCDM methods disregard the decision space altogether and only seek knowledge about the objective space. However, in practical decision making, a better understanding of the relationship between the decision space and the objective space can benefit the decision making process. In other words, the better informed a DM is about the impact of variables on decisions and vice versa, the higher are the quality of decisions, and the confidence associated with them. With this motivation, in this paper we propose an extension to a recently proposed approach called trend mining.

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C. Trend Mining

Trend mining was proposed as a way to identify interesting variable trends, along predefined reference directions in the objective space [2]. An interestingness score is defined and calculated for each variable-reference direction pair, which is visualized as an interactive heatmap. Selecting a cell (i.e. a variable-reference direction pair) in the heatmap, shows the corresponding trend line as well as a scatter plot of the objective space color-mapped to the selected decision variable for cases with $M < 4$ objectives. Thus, trend mining offers a quick intuition to the DM about how the variables affect the objective space. The procedure is described in more detail in Section III.

II. Literature Review and Related Works

A posteriori decision making often employs common multidimensional visualization methods [3], [4] such as scatter plots, parallel coordinate plots, bar plots, radial coordinate plots, etc. to visualize the objective space. Graphical visualization methods that have been specifically developed for MOOPs also exist and a survey of these can be found in [5]. However, none of these methods take the decision space into account. The application of knowledge discovery techniques in the decision space to better understand the objective space is a rather new approach to decision making. A survey of data mining techniques that can be used for the purpose of knowledge discovery in multi-objective optimization can be found in [5]. In [6], the same authors propose a method for finding rules that describe the decision space, for selected solutions in the objective space.

Clustering is one form of knowledge discovery which has been applied on MOOP solutions, [7] and [8] applied clustering in the analysis of the decision space of a turbine blade cooling passage optimization problem. In [9], biclustering was applied on a network design problem. In [10] clusters that were separated in both the decision- and the objective space were discovered. In [11] clustering in both the decision- and objective space was performed on cantilever topology optimization problems.

Manifold learning techniques can be used to learn the structure of, and reduce the dimensionality of data. [12] uses a method based on self organizing maps (SOMs) to visually reveal trade-offs in the non-dominated solutions of real world aerodynamic optimization problems. Recently, [13] used a similar method for knowledge-extraction from a ship hull-form design problem.

III. Original Trend Mining Procedure

The original trend mining was proposed as a method to inform the DM about interesting variable trends in the objective space of a MOOP and giving the DM a quick intuition about structure of the objective space irrespective of the number of solutions (N), the number of objectives (M) and the number of variables (n). The original trend mining involves five steps summarized in the following subsections.

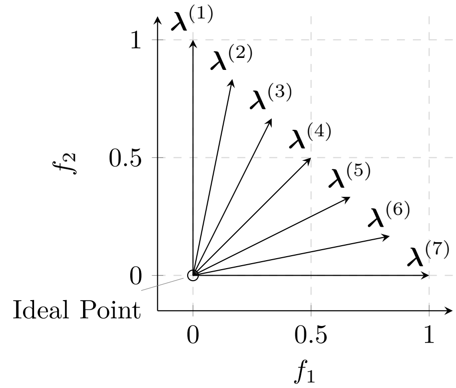

Fig. 1: $P = 7$ reference directions originating from the ideal point of a two-dimensional objective space.

A. Creation of Reference Points and Reference Vectors

Trend mining accounts for the fact that different regions of the objective space may be affected differently by each variable. Therefore the first step is to define various regions of interest in the objective space. To achieve this automatically without the involvement of a real DM, reference directions originating from the *ideal point* of the normalized objective space are predefined to represent several regions of interest on the Pareto-optimal front. Trend mining uses the technique described in [14] to generate P uniformly spaced points on a standard $(M-1)$-simplex. The P reference directions $(\{\boldsymbol{\lambda}^{(1)}, \boldsymbol{\lambda}^{(2)}, \ldots, \boldsymbol{\lambda}^{(P)}\})$ connect the ideal point to each of the points on the simplex. This step is illustrated in Fig. 1 for $M = 2$ and $P = 7$.

B. Projection of Solutions Onto Reference Vectors

In order to assess the interestingness of a reference vector, the N solutions generated by a MOEA are projected orthographically onto the reference vector after having been normalized to lie between $[0, 1]$. The projected solutions are then ordered by their distance from the ideal point along the reference direction.

C. Generation of Variable Trend Lines

A variable's trend is defined as the variance in the consecutive solutions ordered by the projection along the reference vector. Plotting the ordered solutions on a line plot generates the so called *trend line*. It is through analysis of the trend line that an interestingness score can be determined. [2] notes that the trend line can be treated as a time series but that a much simpler approach was used to determine the interestingness score.

D. Calculation of Interestingness Scores

Trend mining generates in total, $n * P$ trend lines for all variables (n) and all reference vectors (P). For MOOPs with many variables, manually analyzing each trend line would be cumbersome, so a metric to define a trend line's *interestingness* was determined.

The interestingness score is defined as a monotonic change in the trend line, and is calculated by counting the number of consecutive increases (upticks) or decreases (downticks) in the variable values over different levels of smoothing. [2] notes that any appropriate smoothing method may be used, however, the simple moving average was selected. The range of window sizes was chosen such that the number of non-overlapping windows increases in powers of two. Pseudo code for the interestingness score calculation can be found in Algorithm 1.

Algorithm 1 Calculation of Interestingness Score, S_{ij}

Input: Variable trend lines $x_i^{I_{j,k}}$ for $k = 1, 2, \ldots N$
Output: S_{ij}

$S_{ij} \leftarrow 0$
for $s \leftarrow 1$ to $\lfloor \log_2 N \rfloor$ **do**
  $ws \leftarrow \lfloor N/2^s \rfloor$ ▷ window size
  $y_k \; \forall \; k \leftarrow$ SimpleMovingAverage($x_i^{I_{j,k}} \; \forall \; k, ws$)
  $UpTicks \leftarrow 0, DownTicks \leftarrow 0$
  for $k \leftarrow 1$ to $N-1$ **do**
    if $y_{k+1} > y_k$ **then**
      $UpTicks \leftarrow UpTicks + 1$
    if $y_{k+1} < y_k$ **then**
      $DownTicks \leftarrow DownTicks + 1$
  $S_{ij} \leftarrow S_{ij} + |UpTicks - DownTicks| * 100/(N-1)$
$S_{ij} \leftarrow S_{ij}/\lfloor \log_2 N \rfloor$

The interestingness score S_{ij} for a given variable i and a given reference vector j is calculated for a trend line $x_i^{I_{j,k}}$ for all k projected solutions. The score is determined as the mean difference between the upticks and downticks over all $\lfloor \log_2 N \rfloor$ window sizes and is expressed as a percentage.

E. Heatmap Visualization of Interestingness Scores

The interestingness score of each variable along all reference vectors can be visualized through an interactive heatmap of $n * P$ cells, where each cell represents the score of a variable-reference vector pair. The use of colors allows for an easy holistic overview of the interestingness scores. By clicking in a cell on the heatmap, the DM can select a variable-reference vector pair and the trend line plot is shown. If the MOOP consist of $M < 4$ objectives, it is also possible to show a scatter plot of the solutions in the objective space, color-mapped to the chosen variable.

IV. Extensions to the Trend Mining Procedure

In this paper we extend trend mining to find the most *interesting* general reference direction in the objective space. Instead of using a predefined set of P reference vectors, we use single-objective optimization to find the most *interesting* reference direction for each variable.

A. Finding an Optimized Reference Direction

The original trend mining procedure allows a quick overview of the interestingness of the variables, but is not a rigorous metric for the *true* interestingness of the variables. To address this, in this paper, we define a reference direction as a reference point and a reference vector. The reference point is constrained to the normalized objective space, mapped to $[0, 1]$, and the reference vector is a unit vector. To find the most *interesting* reference direction for each variable, we use single-objective optimization by the particle swarm optimization algorithm [15], however, any continuous optimization algorithm can be used.

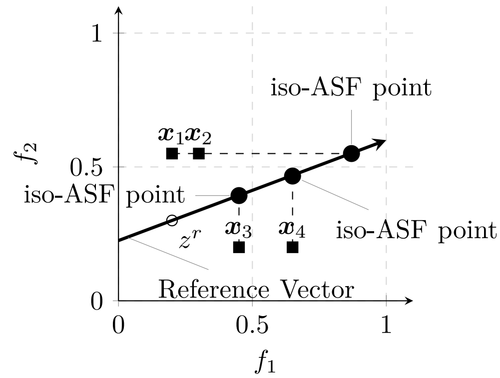

Fig. 2: Illustration of the ASF projection and ordering, x_3 would be ordered before x_4, and x_4 before x_1 and x_2, however x_1 and x_2 would have the same ordering.

B. Projection of Solutions onto the Reference Direction

The solutions are projected onto the reference direction to determine their order. In the original trend mining method the solutions are orthographically projected onto the reference vector, however in this paper we use an achievement scalarizing function (ASF) in order to mimic the preferences of a DM. We use the weighted Chebyshev function, however other ASF forms may also be applied. The weighted Chebyshev function can be formulated as:

$$\text{Minimize} \quad ASF(x \mid z^r, w) = \max_{1 \le i \le M} \frac{f_i(x) - z_i^r}{w_i}$$

for any solution x, reference point z^r and weight vector w. w determines the relative importance between the objectives and in our case is the reference vector. Based on the ASF, each solution x has a corresponding iso-ASF point x^l on the reference vector, which is calculated as: $x^l = z^r + \delta w$ where $\delta = \max_{1 \le i \le M}[x_i - z_i^r/w_i]$, and illustrated in Fig. 2. Using ASF projections offers several advantages over projecting the solutions orthographically, this approach more suited for a greater number of objectives by avoiding the so called *curse of dimensionality* which refers to distance relationships in higher dimensions.

C. Calculation of Interestingness Scores

In this paper, we still define the interestingness score by the monotonic change in the variables along the optimized reference direction. However, where the original trend mining

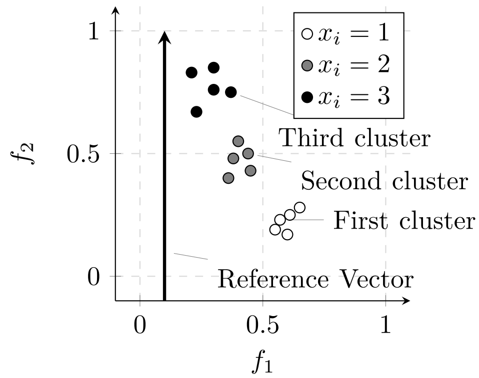

Fig. 3: Illustration of solutions in the objective space color-mapped to a discrete variable x_i in the decision space.

only considered consecutive increase (upticks) and decrease (downticks) of variable values, we now also consider the case where consecutive values remain unchanged. These zero ticks are appropriate to consider when applying trend mining on MOOPs with discrete variables where many consecutive non-changing values on the trend line would signify a cluster. Consider Fig. 3, where the discrete variable under consideration determines three separate clusters. This is obviously an interesting characteristic of the objective space that would not be as apparent by the original interestingness calculation.

Pseudo code for the new calculation can be found in Algorithm 2. Each trend line x_i^I for a variable i produces an interestingness score S_i. S_i is now determined by the median interestingness score of the different window sizes. Algorithm 2 is expressed to handle continuous and discrete variables, while omitting lines 8 and 10, it is also able to handle categorical variables. The concept of monotonic change has no meaning for categorical variables since the values they take on cannot be compared, however, identifying separated clusters is our interpretation.

Algorithm 2 Calculation of Interestingness Score, S_i

Input: Variable trend lines $x_i^{I_k}$ for $k = 1, 2, \ldots N$
Output: S_i
1: $S_i \leftarrow 0,\ I \leftarrow \emptyset$
2: **for** $s \leftarrow 1$ to $\lfloor \log_2 N \rfloor$ **do**
3: $\quad ws \leftarrow \lfloor N/2^s \rfloor$ ▷ window size
4: $\quad y_k\ \forall\ k \leftarrow$ SimpleMovingAverage($x_i^{I_k}\ \forall\ k, ws$)
5: $\quad UpTicks \leftarrow 0, DownTicks \leftarrow 0, ZeroTicks \leftarrow 0$
6: $\quad$ **for** $k \leftarrow 1$ to $N-1$ **do**
7: $\quad\quad$ **if** $y_{k+1} > y_k$ **then**
8: $\quad\quad\quad UpTicks \leftarrow UpTicks + 1$
9: $\quad\quad$ **if** $y_{k+1} < y_k$ **then**
10: $\quad\quad\quad DownTicks \leftarrow DownTicks + 1$
11: $\quad\quad$ **if** $y_{k+1} = y_k$ **then**
12: $\quad\quad\quad ZeroTicks \leftarrow ZeroTicks + 1$
13: $\quad I_s \leftarrow (ZeroTicks + |UpTicks - DownTicks|) * 100/(N-1)$
14: $S_i \leftarrow$ Median(I)

D. *Boxplot Visualization of Interestingness Score*

To ensure robustness in the optimized reference direction generated for each variable, several replications of the trend mining procedure should be performed. Replication can be shown graphically via an interactive boxplot to give the DM a clear holistic overview of the results. The DM selects a variable to investigate by clicking on a column in the plot, which displays the corresponding variable's optimized trend line. For MOOPs with $M \leq 4$ objectives a scatter plot of the objective space, color-mapped to the selected variable can also displayed, along with the optimized reference direction.

V. Results and Discussions

We demonstrate the performance of the extended trend mining procedure on three MOOPs, (i) DTLZ2, a test problem from a suite of seven scalable test problems [16]. (ii) WFG2, a test problem from a suite of nine scalable test problems [17]. And (iii) Clutch-break design problem (CLUTCH), a mechanical design problem with an analytical problem formulation in two objectives.

Both the DTLZ and the WFG test problems are scalable in the number of objectives and the number of search variables, where all variables are continuous. Both problem suites employ the concept of having k number of *positional* variables, and l number of *distance* variables to determine the outcome in the objective space. We apply trend mining on cases for two, three, four and five objectives and reflect on the results for these two problems.

The PSO algorithm used in the optimization of reference directions take on the recommended parameter settings for all cases, and was replicated ten times for each variable.

The solutions for all cases with two objectives were generated by NSGA-II [18] and the solutions for all remaining cases were generated by NSGA-III [19].

A. *DTLZ2*

DTLZ2 has $k = M - 1$ positional variables, and $l = 10$ directional variables. For the two-objective case, the boxplot in Fig. 4a, shows that x_1 is the most interesting variable, with a median score of 50.0. In Fig. 5a a monotonic decrease in x_1 is shown along the optimized reference direction which can be found in Fig. 6a.

Fig. 4b shows the boxplot for the three-objective case. The two positional variables x_1 and x_2 are identified as the most interesting, with a median scores of 43.7 and 24.8 respectively. Fig. 5b shows the trend line for x_1, and compared to the trend line in Fig. 5a, the three-objective case also shows a monotonic decrease along the optimized reference direction. Fig. 6b shows the optimized reference direction for x_1 in the objective space.

Fig. 4c shows the boxplot for the four-objective case, which identifies x_1, x_2 and x_3 as the most interesting with median scores of 42.5, 25.3 and 22.5 respectively. The boxplot shows

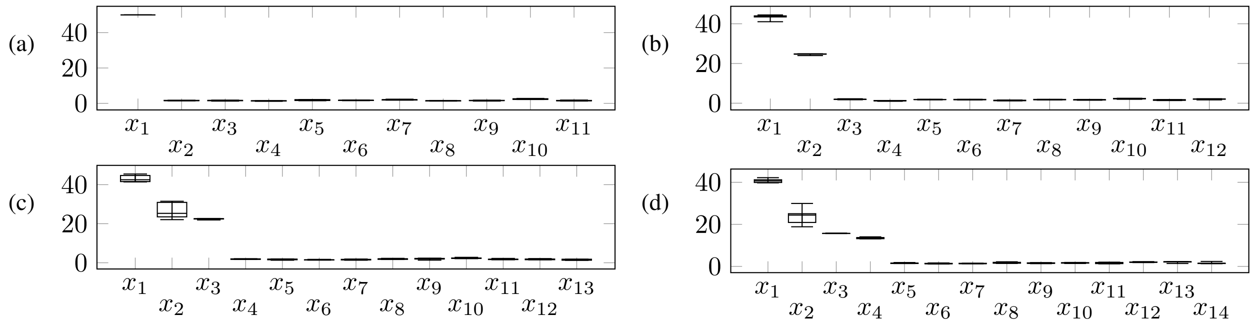

Fig. 4: Boxplots of the interestingness scores for the variables of DTLZ2 with, (a) two objectives, (b) three objectives, (c) four objectives, (d) five objectives.

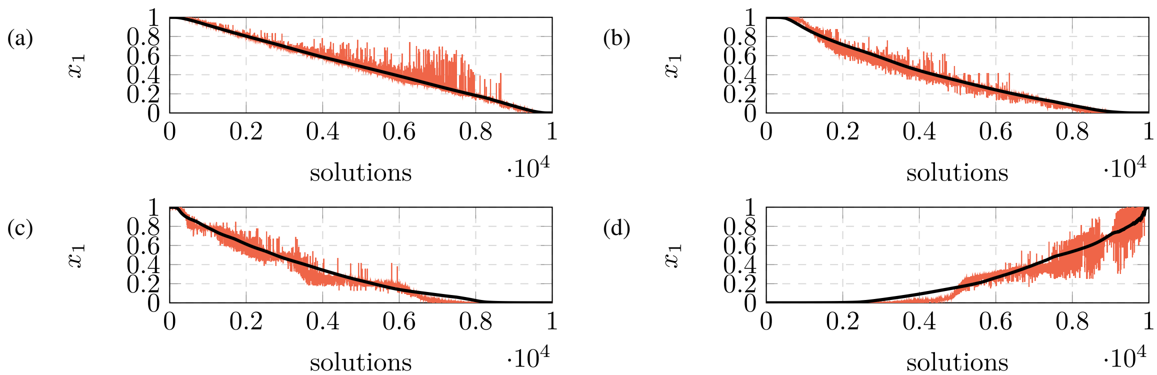

Fig. 5: Trend lines for x_1 for DTLZ2 with, (a) two objectives, (b) three objectives, (c) four objectives, (d) five objectives, along with smoothed version (black line).

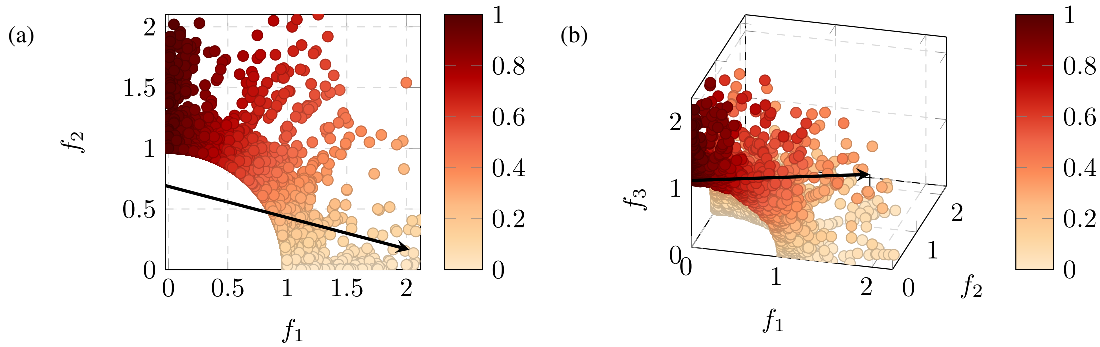

Fig. 6: Objective spaces of DTLZ2 with, (a) two objectives, (b) three objectives, with the optimized reference vector. Solutions are color-mapped to x_1 values.

a relatively large spread for x_2, which suggests that that the complexity of the problem has an affect on the outcome of the trend mining procedure. Fig. 5c shows the trend line for x_1, which still shows a monotonic decrease, however the curve is not as smooth as in previous cases, which may be a result of the optimizer failure to find a diverse set of solutions.

Fig. 4d shows the boxplot for the five-objective case, which identifies x_1-x_4 as the most interesting, with median scores of 40.6, 24.4, 15.7 and 13.7 respectively. As in the four-objective case, a larger spread can be observed for x_2. Fig. 5d shows the trend line for x_1, which for this case shows a monotonic increase, however, testing on different datasets generated by the same optimizer revealed a sensitivity in trend mining's performance related to the optimizer's ability to converge on the Pareto-optimal front. Certain datasets resulted in decreasing trends as well.

It is a known property of DTLZ2 that x_1 determines a solutions position along the Pareto-front along the M^{th} objective, trend mining was able to identify x_1 as the most interesting variable in all cases, suggesting proficiency in finding trends in the variables.

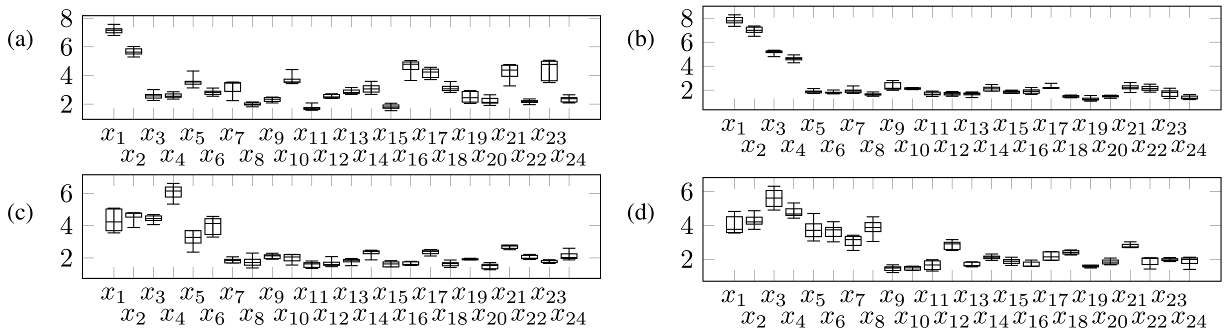

Fig. 7: Boxplots of the interestingness scores for the variables of WFG2 with, (a) two objectives, (b) three objectives, (c) four objectives, (d) five objectives.

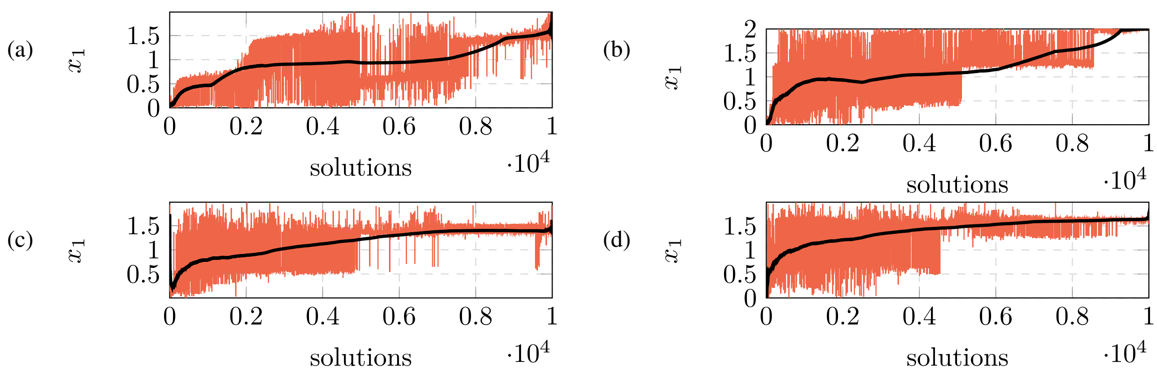

Fig. 8: Trend lines for x_1 for WFG2 with, (a) two objectives, (b) three objectives, (c) four objectives, (d) five objectives, along with smoothed version (black line).

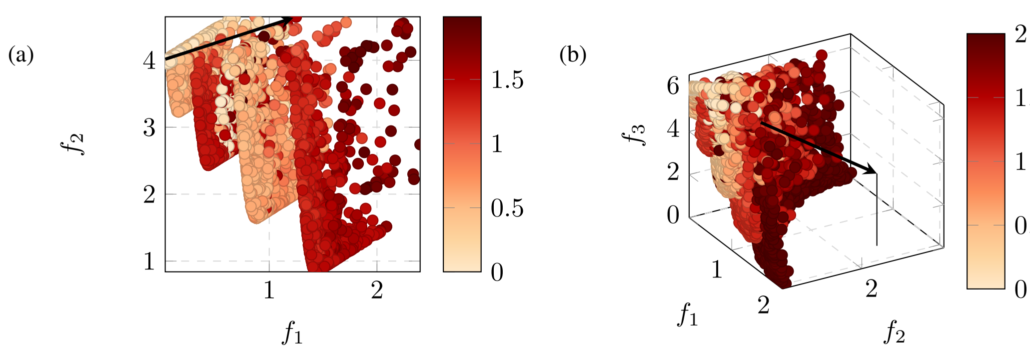

Fig. 9: Objective spaces of WFG2 with, (a) two objectives, (b) three objectives, with the optimized reference vector. Solutions are color-mapped to x_1 values.

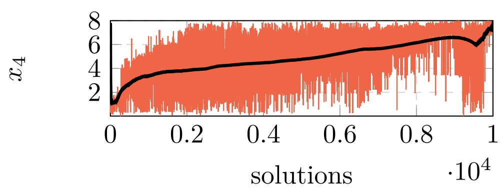

Fig. 10: Trend line for x_4 for WFG2 with four objectives along with smoothed version (black line).

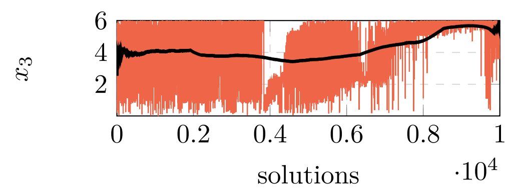

Fig. 11: Trend line for x_3 for WFG2 with five objectives along with smoothed version (black line).

B. WFG2

The number of positional variables was chosen as $k = 2(M-1)$, and the total number of variables was 24 for each

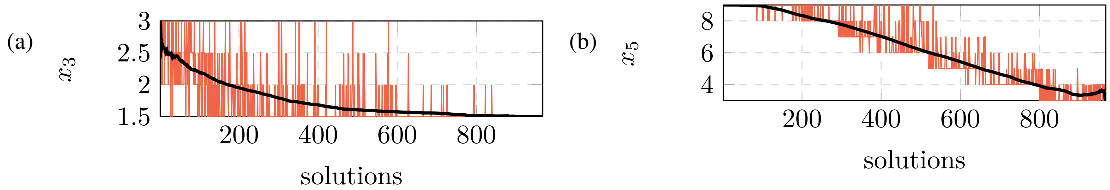

Fig. 12: Trend line for CLUTCH for, (a) x_3, (b) x_5 along with smoothed version (black line).

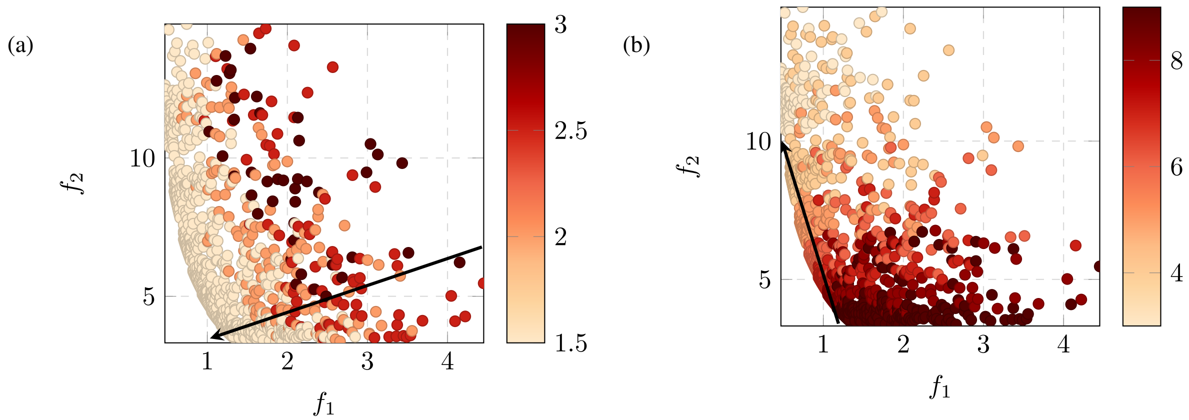

Fig. 13: Objective space of CLUTCH with the optimized reference vector, and color-mapped to, (a) x_3, (b) x_5.

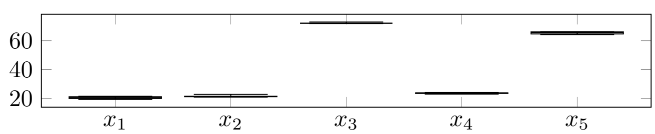

Fig. 14: Boxplot of the interestingness scores for the variables of CLUTCH.

case. The boxplot for the two-objective case, shown in Fig. 7a, illustrates how trend mining was able to identify the positional variables x_1 and x_2 as the most interesting, with median scores of 7.2 and 5.6 respectively. WFG2 is more complex than DTLZ2, and show more spread in the results. The difference in score between the positional variables and the distance related variables is also much lesser. The trend line for x_1 in Fig. 8a does not show a monotonic trend, instead, there appears to be different segments present. The objective space color-mapped to x_1 shown in Fig. 9a, confirms that x_1 is responsible for different segments in the objective space.

Fig. 7b shows the boxplot for the three-objective case, which identifies the positional variables x_1-x_4 as the more interesting, with median scores of 7.8, 7.0, 5.2 and 4.6 respectively. The trend line for x_1 is shown in Fig. 8b, which again does not show a clear monotonic trend. However, past 5000 solutions, the presence of segments can again be observed. Fig. 9b shows that there are segments along the Pareto-optimal front, but not further from it.

The boxplot in Fig. 7c shows that trend mining was able to identify the positional variables x_1-x_6 as the more interesting with median scores of 4.2, 4.8, 4.4, 6.1, 3.3 and 4.1 respectively. In this case, x_4 was the most interesting. Fig. 8c shows that the segmented behavior present in x_1 in previous cases is not clearly be present in this case. Fig. 10 shows that the trend for x_4 is not smooth. WFG2 is known to be difficult, which possibly explains the trends as a result of poor convergence from the optimizer.

Fig. 7d shows the boxplot for the five-objective case, which illustrates that trend mining is able to identify the positional variables x_1-x_8 as the most interesting, with median scores of 3.8, 4.2, 5.6, 3.7, 3.7, 3.2 and 3.9 respectively. x_3 was found to be the most interesting. The trend line for x_1 is shown in Fig. 8d, which displays a similar trend to the four-objective case. Fig. 11 shows the trend line for x_3. The trends are not very clear, which suggests that there are no clear monotonic trends present.

Trend mining is able to identify the positional variables as the most important ones, however the chosen configuration does not seem to generate monotonic trends.

C. CLUTCH

The problem involves the minimization of two objectives, f_1 system mass, and f_2 stopping time, with five discrete decision variables, x_1: inner disk radius, x_2: outer disk radius, x_3: disk thickness, x_4: actuating force, and $x5$: number of disks. The complete problem description can be found in [20]. Fig. 14 shows the boxplot for CLUTCH, which illustrates that x_3 and x_5 are the most interesting, with median scores of 72.0 and 65.6 respectively. The trend lines for x_3 and x_5 can be found in Figs. 12a and 12b. Fig. 13a shows the objective space color-mapped to x_3, and also clearly illustrates how a lower value of x_3 is leads to a more optimal solution. Fig. 13b shows the objective space color-mapped to x_5, which illustrates how x_5 is responsible for a solution's position along f_2.

By considering all solutions, trend mining is able to provide the structure of the MOOP from a representation of the entire search space. This would not be possible by considering only the Pareto-optimal solutions, as illustrated in Fig. 13a where if only considering the Pareto-optimal solutions, no trend would be found since all Pareto-optimal solutions have the same value for x_3.

Trend mining offers a holistic overview of the interestingness of the variables. The relative difference of the interestingness scores is more important than the actual value since different MOOPs cannot be compared. We regard only monotonic changes, even though other trends may be of equal interest to the DM, as for WFG2, where no monotonic trend was found.

VI. Conclusions

In this paper, we extended the trend mining procedure to address several shortcomings of the original method and to further present trend mining as a tool in decision making. The updated method finds optimized reference directions instead of considering only a predetermined set of vectors all originating from the ideal point. The updated method also projects the solutions onto the reference vector using an ASF to regard the relationships between the objectives in the projection. The method was updated to better regard discrete variables that cause clusters in the objective space since many real-world MOOPs involve both continuous, discrete and categorical values.

Trend mining is presented as a procedure to produce knowledge about a MOOP, by identifying which direction the solutions will move along in the objective space, as the value of the variable changes in the decision space.

Given that trend mining is able to autonomously find optimized reference directions for all variables, we believe that the method can be used *online* by a MOEA, during the optimization, to effect the convergence behavior. Following reference directions may for example help populate spares regions of the Pareto front. Directing more computational effort in investigating more interesting variables, may also lead to faster convergence.

As future work, it would also be interesting to further extend trend mining to regard different kinds of trends, such as cyclic trends, and to perform user studies in order to assess the practical performance of trend mining as a tool in decision making.

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