Memetic Multi-agent Optimization with Problem Reformulation by Coordinate Rotation

Sun Mingyang¹, Hou Yaqing*¹, Zhang Qiang¹, Ge Hongwei¹, Yang Xin¹, Feng Liang², Abhishek Gupta³, Li Xianneng⁴

¹School of Computer Science and Technology, Dalian University of Technology(DLUT), China

²College of Computer Science, Chongqing University(CQU), China

³Agency for Science Technology and Research (A*STAR), Singapore

⁴School of Economics and Management, Dalian University of Technology(DLUT), China

E-mail: ¹mingysun1@gmail.com, ¹{houyq, zhangq, hwge, xinyang}@dlut.edu.cn, ²liangf@cqu.edu.cn,

³abhishek gupta@simtech.a-star.edu.sg, ⁴xianneng@dlut.edu.cn

Abstract-Memetic multi-agent system (MeMAS) is recently proposed as an enhanced version that integrates meme concept into multi-agent system (MAS) wherein all meme-inspired agents have an improvement in learning performance via meme evolution independently or social interaction. In the process of solving the black box optimization problem, the potential advantages of MeMAS have not been utilized well, which makes it a fertile area for further exploration. This paper presents a memetic multiagent optimization paradigm through coordinate rotation (MeMAO-R) to combine MeMAS with evolutionary algorithms (EAs) to improve optimization efficiency. Based on MeMAS, the particular interest of MeMAO-R is placed on assisting original complex optimization task with new tasks generated by coordinate rotation. Further, MeMAO-R constructs the social interaction mechanism which facilitates to improve their convergence speed for solving the target optimization problem by utilizing meaningful information transferred across multiple agents with differing views of the target problem. Besides, MeMAO-R employs one or more classical EAs as the fundamental population based evolutionary solvers for multiple agents to optimize multiple tasks in a multi-agent scenario. Lastly, to testify the efficacy of the proposed MeMAO-R, comprehensive empirical studies on basic optimization problems are provided.

Keywords—Memetic multi-agent system, Coordinate rotation, knowledge transfer.

I. INTRODUCTION

Multi-agent evolutionary computation (MAEC) has emerged as a new hybrid scheme that is capable of leveraging the underlying benefits from the integration of multi-agent systems (MAS) and evolutionary computation (EC) [1]. In past few years, it has shown great excellence for solving a wide range of complex problems effectively [2][3]. In particular, memetic multi-agent system (MeMAS), which takes inspiration from Darwin's theory of memes, has been proposed as an enhanced version of MAS where all meme-inspired agents are capable of obtaining the increasing learning performance via social interaction or meme evolution independently [4]. Notably, memes are defined as the basic building block of knowledge in the mind of multiple agents and can be transferred to others for more effective problems solving.

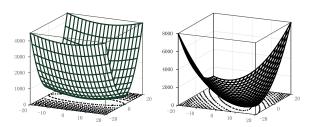


Fig. 1. 3D and contour plots of the function $f(x_1, x_2) = x_1^2 + 10x_2^2 + 100$ with (right) and without (left) a rotation of the coordinate system. It is seen that the function maintains its original structure after rotation

Existing studies on MeMAS have shown many profits on solving complex learning problems including multi-agent planning problems in a minefield navigation domain [5], human-like AI designs in first person shooting games [6], and so on. However, these work has mainly focused on improving the online learning capability of memetic agent by evolving selfcontained memes or learning from each other in a complex setting. Current studies have started to investigate the performance of MeMAS for solving black-box optimization problems. For example, MeMAS has been applied to enhance the search efficiency of EAs to solve high-dimensional optimization problems by reformulating the original problem into different yet related alternative problems [7]. By assigning genetic materials for multiple agents to optimize all generated problems together. The convergence speed for solving the original optimization problem can be improved. However, in this work, the alternative problems are re-formulated via random embedding methods. It assumes that the original optimization problem possesses a "low effective dimensionality", meaning only a small fraction of dimensions are effective, hence restrict it applications.

In this work, we propose a memetic multi-agent optimization paradigm based on the coordinate rotation (MeMAO-R). Specifically, coordinate rotation introduces a linear and orthogonal transformation matrix \mathbf{M} such that $f(\mathbf{M}\vec{x})$ is calculated [8]. The transformation matrix \mathbf{M} visualizes the effect of \mathbf{M} in Fig. 1, showcasing the 2-dimensional version of a

polynomial function with (right) and without (left) a rotation. It is observed that the function maintains its original structure after rotation. Therefore, through coordinate rotation, an alternative problem with the same search structure as the original problem can be generated. As the original function structure is maintained, the optimization process of the new task has great reference value for the original task.

In our proposed method, multiple alternative tasks of the original optimization problem are firstly generated through coordinate rotation. Then, each agent in the multi-agent system optimizes one of these tasks including the original optimization problem. Notably, different agents could choose different EAs (i.e., GA and DE) as their respective evolutionary solvers. As different solvers may have diverse perceptron and search strength on the specific optimization landscapes, they can lead to distinct search performance. Therefore, MeMAO-R provides a way for multiple agents to explore the most appropriate EA solver for solving a given problem. Moreover, multiple agents could conduct social communication to leverage useful memetic knowledge transferred from the better performing partners, so as to enhance the search efficiency of the entire system. For progressively definite comprehension, the core contributions of this paper can be summarized as follows:

- Firstly, we use coordinate rotation to reformulate the
 original optimization problem into multiple new
 optimization tasks. MeMAO-R is then applied to solve
 the newly formulated tasks and the original task in a
 multi-agent scenario. In addition, different agents are
 able to employ different EAs at random to optimize their
 tasks respectively.
- Besides, the optimization process of multiple agents in MEMAO-R is mainly composed of *gene internal evolution* and *meme external evolution*. During the gene internal evolution process, each agent could evolve the genetic materials in their mind based on the evolutionary operators (e.g., crossover and mutation) and the self-learning operator (e.g., local search). In meme external evolution, each agent is able to leverage from the beneficial information transferred across multiple agents to improve the search efficacy.
- Lastly, to investigate the performance of the proposed MeMAO-R framework, comprehensive empirical studies are conducted on multiple continuous optimization functions. The results demonstrated that the efficiency of EA solvers for optimizing multiple benchmark functions can be improved significantly.

The organization of this paper is presented as follows: Section II provides a brief introduction to memetic multi-agent system. Further, details of the proposed memetic multi-agent optimization paradigm through coordinate rotation (MeMAO-R) are elaborated in Section III. Section IV provides a comprehensive empirical study of MeMAO-R based on basic optimization functions for verifying the performance of MeMAO-R. Finally, this paper is concluded with a few brief remarks in Section V.

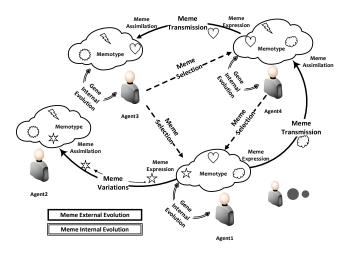


Fig. 2. The generic illustration of Memetic Multi-Agent System.

II. MEMETIC MULTI-AGENT SYSTEM

For the past few decades, the research on memetic science has gained increasing attention and spreads rapidly across multifaceted fields [9][10]. In computational intelligence, memetic computation is inspired by Dawkins' notion of a meme, which is defined as an information unit that reproduces itself as people exchange information and knowledge [11][12]. Memetic multi-agent system is proposed by taking memes as the basic building blocks of knowledge filling the agents' mind universe [4][13]. Also, the social behaviors are modeled by transferring useful information across multiple agents mainly via an imitation process. In Fig. 2, the basic framework of MeMAS is clearly depicted. Notably, meme representation and meme evolution are the two most important aspects of the framework.

Firstly, meme representation provides the specification of a meme in the system. Memes are categorized into memotypes and sociotypes. Internally, memotypes (depicted by the different LEGO-like block objects lying in the agents mind universe of Fig. 2) are described as the agents' information and knowledge captured as memory items or generalized abstractions inside the agents' mind universe. Externally, sociotypes are defined as information obtained from observable behaviors of others. In memetic expression, individual agents express their internal knowledge in the form of external behaviors that are accessible to the others. Meme assimilation, on the other hand, provides a way for agents to capture the information contained in these behaviors and updates memes in their own mind universe accordingly.

Further, meme evolution, namely meme internal evolution and meme external evolution, is the core behavior learning aspects of MeMAS. For each agent, it updates its knowledge through self-learning during the internal evolution of meme knowledge. Meme external evolution is applied to model the social interaction between agents. The pseudo-code outlined in Alg.1 shows the basic structure of MeMAS. As can be seen, each memetic agent firstly conducts meme internal evolution in the environment of interest (see line 5). After an agent carries out the meme selection operation, the external evolution of meme will instruct the agent to obtain social knowledge from

the better performing agent through the meme transmission process (see lines 6-10). In meme evolution, agents will perform a series of learning trials until the stop conditions are achieved.

In existing studies, memory items have been considered as a manifestation of neuronal memes to fill agents' mind universe [14]. In such case, agents interact with the environment mainly through reinforcement learning to gain the incremental knowledge of a particular environment domain [15]. Different from existing works, MeMAO-R assigns different tasks to multiple agents and employs the evolutionary concepts (e.g., crossover and mutation GA) as the basic problem solvers to be used by each agent. Besides, MeMAO-R improves optimization performance of all tasks by disseminating helper knowledge gained by agents when optimizing different yet related tasks in the form of meme external evolution.

Alg. 1 Pseudo Code of MeMAS

1 Initialization: Generate multiple agents.

2 Begin:

3 while stopping conditions are not satisfied do

4 for each current agent A(c) do

5 **Perform** *meme internal evolution*

6 if conduct meme selection to identify

a teacher agent A(t) then

7 Conduct meme expression with A(t) to activate its knowledge.

activate its knowledge.

8 Conduct meme transmission to transfer

knowledge from A(t) to A(c).

9 **Conduct** *meme variation* to promote the diversity of the transferred knowledge.

10 end for

11 end while

12 End

III. THE PROPOSED MEMAO-R

This section introduces the generic framework and its detailed realization of the proposed MeMAO-R.

A. The Structure of MeMAO-R

Recall that our interest is placed on leveraging from multiple tasks generated by coordinate rotation to improve the search efficiency of traditional EAs. Specifically, coordinate rotation means that a rotation matrix **M** is applied such that

$$f(\vec{x}) = f_0(\mathbf{M}\vec{x}) \tag{1}$$

where f_0 is the fitness function of original optimization problem. Rotation matrices are square matrices with real elements and can be characterized as orthogonal matrices with determinant 1; that is, a square matrix \mathbf{M} is a rotation matrix if and only if $\mathbf{M}^T = \mathbf{M}^{-1}$ and $\det \mathbf{M} = 1$. By generating N rotation matrices and applying them to the original problem, we can create N alternative tasks of the original problem.

The general framework of MeMAO-R is clearly depicted as Fig. 1. As can be observed, first of all, N + 1 agents are initialized to solve the original task and the newly generated

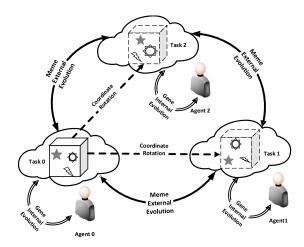


Fig. 3. The generic illustration of MeMAO-R. The target optimization task is reformulated into 2 new tasks via coordinate rotation.

tasks respectively in the multi-agent environment. In MeMAO-R, the optimization behaviors of each agent are mainly carried out by two important evolutionary processes, namely, *gene internal evolution* and *meme external evolution*. Specifically, in gene internal evolution, each individual agent will evolve the genetic materials inside its mind universe independently based on the specific EA solvers (i.e., GA and DE). On the other hand, meme external evolution facilitates to transfer beneficial information or sociotype memes (in terms of chromosomal materials of agents) across multiple agents, thereby accelerating the convergence speed of the entire system. In the following subsection, more details about the realization of the framework are discussed for a clearer understanding.

B. Realization of MeMAO-R

Alg. 2 outlines the pseudo-code of MeMAO-R. To begin, N rotation matrices are generated randomly to build N alternative tasks. After that N+1 agents are initialized to solve all optimization tasks respectively. In our case, different agents could choose different EAs (i.e., GA and DE) as their respective evolutionary solvers. And each agent is attributed with the same size of chromosome individuals in its mind universe during the optimization process.

Meme evolution is the core process of the algorithm (step 5 in Alg. 2), which serves to model the self-gene evolution of each agent and the social interactions among multiple agents. In MeMAO-R, the knowledge and genetic materials of each agent exist in their minds in the form of population-based chromosome individuals. With the proceeding of the evolution process, the gene information in the agent's mind universe tends to become stable and it is difficult to produce better new individuals, especially when it is trapped in the local optimum. Besides, agents have quite different search behaviors, so new individuals in each agent with external genetic information have the probability of obtaining more promising performance. Taking this cue, meme transmission is designed wherein agents share their chromosome information to obtain helpful knowledge from one another during the meme external evolution. As given by the pseudo code of the meme evolution procedure in Alg. 3, meme transmission can be achieved by

activating chromosome individuals from different agents to perform evolutionary operations. To be specific, we first introduce a meme transfer probability p to control the frequency of information transmission between agents during the evolution (see Alg. 3, line 4). A value of p close to 0 means there is small chance for agents to share information with others, while a value close to 1 permits completely random evolutionary manipulation between chromosome individuals in all agents. In detail, a random value $rand \in [0,1]$ was generated at first. Then according to the result of rand against p, we decide whether to select individuals from other agents for meme transmission. Further, each chromosome individual in the current agent A(c) will perform crossover and mutation with the selected individual $(v_r \text{ or } v_c)$ to produce the offspring v_o based on the employed EA solver (see Alg. 3, line 4-9). In particular, to enhance the capability of agents, local search can be applied as an important part of the meme internal evolution for enhancing the search efficiency (see Alg. 3, line 10). Further, all the offspring individuals generated in the evolution process are stored in set O for the selection of agents (see Alg. 2, line 6).

Alg. 2 Main Framework of the Proposed MeMAO-R

Input: N (number of newly generated tasks), p (the probability of meme external transmission), Algs (alternative EAs).

1 Initialization: Generate the initial N+1 agent A and corresponding N rotation metrics.

```
2 Begin:
3 Each agent employs an EA solver from Algs.
4 while stopping conditions are not satisfied do
5 for each current agent A(c) do
6 O ← Meme_Evolution(A, c, p)
7 A(c) ← Environmental_Selection(A(c), O)
8 end for
9 Update_Agents(A)
10 end while
11 End
```

Alg. 3 Meme Evolution in MeMAO-R

```
1 Begin:
 2 Get the current agent A(c).
      for each chromosome individual v_c^i of A(c) do
 3
         if rand(0, 1) < p then
 4
 5
            Get a random individual v_r from another agent.
 6
            v_o \leftarrow \text{Crossover} and mutation(v_c^i, v_r)
 7
         Else
 8
            Get a random individual v_c^j from A(c)
 9
            v_o \leftarrow \text{Crossover} and mutation(v_c^i, v_c^j)
10
         Local Search(v_o)
11
      end for
12 End
```

Note that in MeMAO-R, the alternative tasks are generated by the original task through coordinate rotation, which means the solutions in all tasks can be mapped into mind universe of others by their own corresponding rotation matrix. Taking this cue, each agent can replace the worst chromosome in their mind by transforming the best solution from other agents into a chromosome individual suitable for its own task, as an important part of the meme external evolution with meme selection. This process is easy to implement and requires no additional fitness evaluation (see Alg. 2, line 8).

IV. EMPIRICAL STUDY

In this section, comprehensive empirical studies are conducted to investigate the performance of our proposed MeMAO-R. Particularly, we first give the parameter configurations in the present study, which is followed by the discussions of empirical results obtained.

A. Experimental Configuration

To empirically investigate the performance of the proposed MeMAO-R framework, we consider both classical GA and DE algorithms as our baseline EA solvers. In particular, GA uses simulated binary crossover operator and gaussian mutation operator, and DE employs a mutation strategy of DE/rand/1. Notably, a local search strategy can be applied to GA and DE as well. In our current study, we consider Quasi-Newton Algorithm as the preferred local search algorithm. In the experiment, maximum generation number GN was set to 500 without local search steps, while in the case with local search GN = 100. The comprehensive search operator settings and parameters in MeMAO-R are outlined in Table I, which are kept consistent across all solvers. Besides, 6 single objective classical optimization functions and their variants are considered here to verify the performance of the proposed MeMAO-R.

B. Results and Discussions

Firstly, the performance of GA, DE, MAO-R and MeMAO-R in terms of averaged objective values on the 6 classical single objective optimization problems over 20 independent runs are summarized in Table II. In particular, MeMAO-R is configured to possess 3 agents in the following experiments. In addition, given the ease with which some functions are optimized, MeMAO-R did not use local search during this phase. The corresponding optimization curves are depicted in Fig. 4. Through the statistical hypothesis test of 95% confidence interval, the experimental results prove that MeMAO-R has better optimization performance than GA, DE and MAO-R on the 6 benchmark functions.

TABLE I. SUMMARY OF PARAMETER SETTINGS

Populati	100	
Number of Inde	20	
Maximum Generation	With Local Search	100
Number GN	Without Local Search	500
Meme Transmi	0.3	
DE parameters	F	0.5
	CR	0.9
GA parameters	Simulated Binary Crossover mu	10
	Gaussian Mutation σ	0.02

TABLE II.

THE AVERAGED OBJECTIVE VALUES OBTAINED BY GA, DE, MAO-R AND MEMAO-R FOR SOLVING OPTIMIZATION FUNCTIONS AFTER COMPLETING 500 GENERATIONS ACROSS 20 INDEPENDENT RUNS. THE BEST VALUES ARE HIGHLIGHTED IN **BOLD**.

Functio	ons	Shifted Sphere	Rotated Weierstrass	Rosenbrock	Griewank	Rastrigin	Shifted Rotated Rastrigin	Ackley	Shifted Rotated Ackley
GA	Mean	3.59E+02	2.94E+01	1.51E+05	1.07E+00	3.28E+02	4.98E+02	1.94E+01	1.99E+01
	Std.	1.42E+02	2.51E+00	6.83E+04	1.76E-02	3.68E+01	5.08E+01	1.30E-01	1.82E-01
DE	Mean	5.38E+03	3.75E+01	3.54E+07	5.75E+00	2.96E+02	4.11E+02	1.51E+01	1.72E+01
	Std.	3.13E+03	1.00E+00	4.21E+07	1.21E+00	6.23E+01	9.33E+01	1.01E+00	1.33E+00
MAO-R	Mean	5.19E+02	2.92E+01	2.74E+04	1.26E+00	2.24E+02	4.51E+02	1.99E+01	2.02E+01
	Std.	8.45E+01	2.33E+00	2.07E+04	2.81E-02	8.13E+01	7.28E+01	1.47E-01	1.91E-01
MeMAO-R	Mean	5.44E+00	2.65E+01	3.25E+03	6.52E-01	1.72E+02	3.29E+02	5.14E+00	1.49E+01
	Std.	1.50E+00	1.84E+00	2.65E+03	1.07E-01	3.58E+01	2.88E+01	4.73E-01	5.53E+00

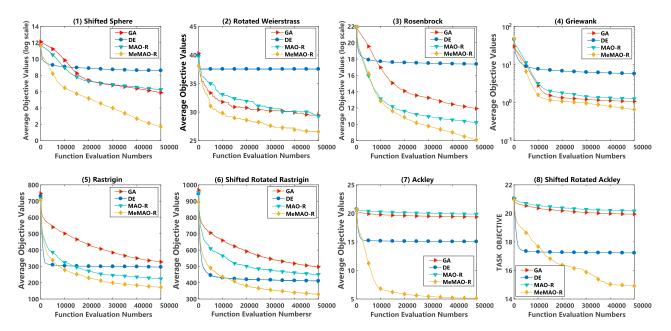


Fig. 4. Averaged convergence trends obtained by GA, DE, MAO-R and MeMAO-R on respective optimization functions.

As can be seen in Table II, when compared to GA and DE, MeMAO-R achieved competitive or superior performance in terms of solution quality on all optimization tasks. For example, on shifted Sphere and Ackley functions, the average best fitness values obtained by MeMAO-R are 5.44 and 5.14, significantly better than GA (i.e., 358 and 194) and DE(i.e., 5830 and 15.1). In addition, to assess the efficiency of MeMAO-R, Fig. 3 depicts the convergence graphs of GA and MeMAO-R on all testing problems, where Y-axis denotes the averaged fitness values and X-axis represents the number of generations. It can be observed that MeMAO-R employing GA tend to have better performance than GA and DE in the early stage of the optimization process. In particular, on function Ackley in Fig. 4(7), etc., MeMAO-R takes 10,000 FEs to arrive at solutions of higher quality(i.e. around 7), which is much better than the average best fitness

values obtained by DE and GA. These results clearly highlight the significance of the proposed MeMAO-R in accelerating convergence speed.

In addition, we can see that MeMAO-R has achieved comparatively better performance when compared to MAO-R wherein the probability of meme transmission is set to 0 on solving optimization problems after completing 50,000 function evaluations. It can be clearly seen from the convergence curves that on all the 8 optimization tasks, MeMAO-R has significantly better fitness performance than MAO-R during the whole optimization process. This demonstrates the effectiveness of the meme external evolution mechanism we proposed, hence leading to higher optimization rate.

However, it can be found from table II that due to the limited ability of GA and DE employed by MeMAO-R, the optimization results of several functions are unsatisfactory (e.g. Shifted Rotated Ackley). Therefore, we employ local search to improve the optimization ability of agents in the following experiments, and the statistical results are shown in table III and table IV. It is worth noting that during this phase of the experiment, agents in MeMAO-R choose the same evolutionary solver. In the tables, superior performance is obtained by MeMAO-R when compared to GA and DE with local search in terms of solution quality. For example, on Ackley function, with the help of a local search, GA reported the best fitness values of 8.6306. On the other hand, in multi-agent senario, MeMAO-R obtained significantly improved solutions with the fitness values of 1.48E-05, which is close to the global optimum 0. Subsequently, Fig. 5 and Fig. 6 provide the convergence graphs of DE, GA and MeMAO-R with local search on the representative optimization functions. As can be observed, in Fig. 5, in the optimization process, the optimization speed of MeMAO-R significantly increased compared with GA. For example, on Shifted Rotated Rastrigin function, MeMAO-R takes less than 50 generations to obtain solutions of higher quality than those obtained by GA where the latter exhausted over 90 generations to arrive at the same solution quality.

To conclude, these results clearly highlight the significance of the proposed MeMAO-R in improving the search efficacy of classical GA and DE. From the analysis of the experimental results, the efficacy of the designed meme external evolution mechanism of MeMAO-R in utilizing benefitial information transferred across multiple agents with differing views of the target problem has also been proven.

TABLE III. THE AVERAGED OBJECTIVE VALUES OBTAINED BY GA AND MEMAO-R WITH LOCAL SEARCH

Functions		Rastrigin	Shifted Rotated Rastrigin	Ackley	Shifted Rotated Ackley	
GA	Mean	2.23E+01	1.14E+02	8.63E+00	1.16E+01	
	Std.	7.79E+00	1.63E+01	2.27E+00	2.24E+00	
MeMA O-R	Mean	9.26E+00	3.00E+01	1.48E-05	3.92E+00	
	Std.	3.51E+00	9.41E+00	4.69E-05	8.55E-01	

TABLE IV. THE AVERAGED OBJECTIVE VALUES OBTAINED BY DE AND MEMAO-R WITH LOCAL SEARCH

Functions		Rastrigin Shifted Rotated Rastrigin		Ackley	Shifted Rotated Ackley
DE	Mean	2.38E+02	2.39E+02	1.37E+01	1.28E+01
	Std.	5.52E+01	4.26E+01	4.95E-01	1.27E+00
MeMA O-R	Mean	9.80E+01	1.09E+02	5.60E-01	7.82E+00
	Std.	2.29E+01	1.48E+01	1.24E+00	2.83E+00

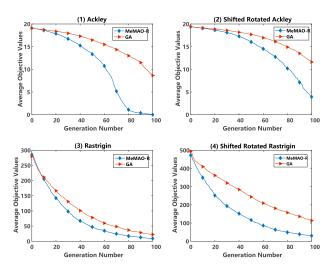


Fig. 5. Averaged convergence trends obtained by MeMAO-R and GA with local search on respective optimization functions.

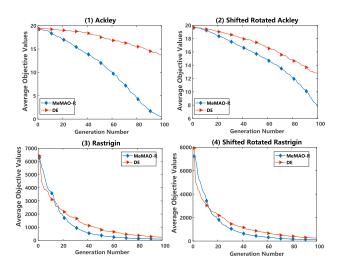


Fig. 6. Averaged convergence trends obtained by MeMAO-R and DE with local search on respective optimization functions.

V. CONCLUSION

Memetic multi-agent system integrates meme-inspired computational principles into the traditional multi-agent system to instruct the effective social interaction across multiple agents. In this paper, a memetic multi-agent optimization paradigm through coordinate rotation (MeMAO-R) is proposed, which employs MeMAS as essential backbone to enhance the searching efficiency of EAs for solving optimization problems. Specifically, multiple alternative formulations of the original optimization problem are firstly generated through coordinate rotation. Then, different EA solvers (i.e., GA and DE) are employed by multiple agents to optimize the original and alternative generated problems respectively. To make this clearer, we elaborate the detailed realization of MeMAO-R, including the process of knowledge transmission among multiple agents in the meme external evolution and the details of agent self-learning in the meme internal evolution.

Computational experiments are carried out on a variety of benchmark optimization problems. The experimental results demonstrate the efficiency and effectiveness of MeMAO-R for accelerating the optimization process.

Immediate future work may consider to enhance either the generality or applicability of the present study. In the present work, the effectiveness of MeMAO-R is only verified on the optimization of benchmark problems. However, there are a variety of complex optimization problem domains of practical interest, both in continuous as well as discrete optimization, where similar methods can be of significant use. In addition, it is by assigning agents with different tasks that MeMAO-R combines MeMAS with EA solvers. In the near future, we plan to explore more underlying benefits of multi-agent systems to enhance the performance of EAs in problem solving.

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