N−ary norm operators and TOPSIS

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***Abstract*—New Technique for Order Preference by Similarity to Ideal Solution, (TOPSIS), variant is presented, where $n-$ary norm operators are used in creating ideal solutions. We show that we are gaining different ranking results with this new proposal. Also we address reasons for why this variant is giving different ranking orders compared to original TOPSIS and propose a way to try to select suitable $n-$ary norm operations and how to try to select a suitable parameter in case of parametrized norm operators. New method is examined with the patent selection problem.**

***Index Terms*—$N-$ary norms, TOPSIS, ranking order, preference modelling, patent selection problem**

I. Introduction

One of the most well known multicriteria decision making methods is TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [1]. There the aim is to choose alternative that simultaneously has the closest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS).

TOPSIS has been studied by many researchers. To mention few, Fuzzy TOPSIS was introduced by Chen [2] for triangular fuzzy number and later extented to trapezoidal fuzzy numbers [3]. Similarity based fuzzy TOPSIS was introduced by Luukka [4] and later examined with different fuzzy similarities for R&D evaluation problem by Collan and Luukka [24]. Also aggregation over the criterians with more general aggregation operators has been examined by Luukka and Collan [6] by using Bonferroni mean. Method has been applied to many real world problems by many researchers. To mention few Doukas et al. [7] applied TOPSIS to assess the sustainability of renewable energy options. Identification of influential nodes in complex networks using TOPSIS were assessed by Du et al. [8]. Feia et al. [9] applied modified TOPSIS based on D numbers to human resources selection problem. Goh et al. examined load shedding scheme for large pulp mill electrical systems using combination of TOPSIS and AHP [10]. Green supplier selection problem was under investigation by Kannan et al. [11] with fuzzy TOPSIS. Nouri et al. [12] had a technology selection problem where they applied fuzzy ANP and fuzzy TOPSIS. Database system cutting with machining features and TOPSIS was under investigation by Peng et al [13]. Rashid et al. [14] used generalized interval-valued fuzzy numbers with TOPSIS for robot selection. Personel selection for knowledge-intensive enterprise was examined with fuzzy TOPSIS by Sang et al. [15]. Wang [16] used fuzzy TOPSIS to evaluate financial performance for Taiwan container shipping companies. Consumer credit decision making was under investigation by Zhu et al. [17] with C-TOPSIS. For more information about TOPSIS variants and their applications see [18] for a survey.

Eventhough TOPSIS is much studied method subject of having preference over PIS or NIS in considering both ideal solutions has not been much considered so far. Several researchers have commented on the fact that basic TOPSIS does not take into account preference over PIS or NIS (see i.e. [19]), but only few provide meaningful solution to the subject [20]. Most of these address this problem from relative closeness (or closeness coefficient) point of view as in [20]. Here we concentrate on addressing this problem already at the point where we create the ideal solutions by generalizing standard intersection/union operators to a stricter/weaker form by using $n-$ary norm operators.

II. $N-$ary norm operators

In this section we first fastly go through intersection (T-norm) and union (T-conorm) operators and then go into their n−ary extensions. Following Klir & Yuan [23], we can define the T-norm as follows:

Definition 1: An aggregation operator $T:[0,1]^2 \rightarrow [0,1]$ is called a T-norm if it is commutative, associative, monotonic, and satisfies the boundary conditions. That is, for all $x,y,z \in [0,1]$ we have that

- $T(x,y)=T(y,x)$(commutativity)
- $T(x,T(y,z))=T(T(x,y),z)$ (associativity)
- $T(x,y) \leq T(x,z)$ whenever $y \leq z$ (monotonicity)
- $T(x,1)=x$ (boundary condition)

These are minimum requirements for a norm operator to be a T-norm. Besides this often one can introduce further axioms to have even stricter norm operators. For example subidempotency and continuity [21].

Definition 2: A T-norm is said to be an *Archimedean t-norm* if it is also continuous and $T(x,x)<x, \forall x \in (0,1)$.

Due to the associativity of T-norms, it is possible to extend the operation to the $n-$ary case, $n \geq 2$. E.g. for $n=3$ the T-norm can be computed from $T(x_1,x_2,x_3)=T(T(x_1,x_2),x_3)$. For example with algebraic product ($T(x_1,x_2)=x_1x_2$) we would get $T(x_1,x_2,x_3)=x_1x_2x_3$. Klement et al. [22] gave following definition for $n-$ary case.

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Definition 3: Let T be a T-norm and $(x_1, x_2, ..., x_n) \in [0,1]^n$ be any n-ary tuple, we define $T(x_1, x_2, ..., x_n)$ as;

$$T(x_1, x_2, ..., x_n) = T(T(x_1, x_2, ..., x_{n-1}), x_n) \quad (1)$$

In following you can see some examples from $n-$ary T-norms derived using (1)

The standard n-ary $T-$norm, T_M can be obtained for all $(x_1, x_2, ..., x_n) \in [0,1]^n$ by:

$$T_M(x_1, x_2, x_3, ..., x_n) = \min(x_1, x_2, x_3, ..., x_n) \quad (2)$$

Probabilistic n-ary $T-$ norm:

$$T_P(x_1, x_2, x_3, ..., x_n) = \prod_{k=1}^{n} x_k \quad (3)$$

Łukasiewicz n-ary T-norm:

$$T_{Ł}(x_1, x_2, x_3, ..., x_n) = \max[0, (1 - \sum_{k=1}^{n}(1 - x_k))] \quad (4)$$

Drastic product n-ary T-norm:

$$T_D(x_1, x_2, x_3, ..., x_n) = \begin{cases} x_i, & if\ x_{i+1} = 1 \\ x_{i+1}, & if\ x_i = 1 \\ 0, & otherwise. \end{cases} \quad (5)$$

Similarly, for a norm operator to be a T-conorm following axioms needs to be satisfied.

Definition 4: An aggregation operator $T_{co} : [0,1]^2 \to [0,1]$ is called a T-conorm if it is commutative, associative, monotonic, and satisfies the boundary conditions. That is, for all $x, y, z \in [0,1]$ we have that

- $T_{co}(x, y) = T_{co}(y, x)$(commutativity)
- $T_{co}(x, T_{co}(y, z)) = T_{co}(T_{co}(x, y), z)$ (associativity)
- $T_{co}(x, y) \leq T_{co}(x, z)$ whenever $y \leq z$ (monotonicity)
- $T_{co}(x, 0) = x$ (boundary condition)

Notice that difference between T-norm and T-conorm is in the boundary condition. Again these axioms are minimum requirements for norm operator to be a T-conorm and further stricter requirements can be imposed. For example superidempotency and continuity.

Definition 5: A T-conorm is said to be an *Archimedean T-conorm* if it is also continuous and $T(x, x) > x, \forall x \in [0,1]$.

Triangular conorms can be also extended to $n-$ary conorms [21], [22] due to their associativity. Klement et al. in [22] defined $n-$ary T-conorms as follows.

Definition 6: Let T_{co} be a T-conorm and $(x_1, x_2, ..., x_n) \in [0,1]^n$ be any n-ary tuple, then $T_{co}(x_1, x_2, ..., x_n)$ is given by,

$$T_{co}(x_1, x_2, ..., x_n) = T_{co}(T_{co}(x_1, x_2, ..., x_{n-1}), x_n) \quad (6)$$

Some examples from $n-$ary T-conorms derived using (6) are as follows

Let $(x_1, x_2, ..., x_n) \in [0,1]^n$ be an n-ary vector, then the standard union can be calculated,

$$T_{coM}(x_1, x_2, ..., x_n) = \max(x_1, x_2, ..., x_n) \quad (7)$$

The $n-$ary probabilistic t-conorm:

$$T_{coP}(x_1, x_2, x_3, ..., x_n) = 1 - \prod_{k=1}^{n}(1 - x_k) \quad (8)$$

The n-ary Łukasiewicz t-conorm:

$$T_{coŁ}(x_1, x_2, ..., x_n) = \min[1, \sum_{i=1}^{n} x_i] \quad (9)$$

The n-ary drastic sum t-conorm:

$$T_{coD}(x_1, x_2, x_3, ..., x_n) = \begin{cases} x_i, & if\ x_{i+1} = 0 \\ x_{i+1}, & if\ x_i = 0 \\ 1, & otherwise. \end{cases} \quad (10)$$

III. Basic TOPSIS and relative importance of two ideal solutions

To apply TOPSIS we require a specification of the decision matrix for a set of alternatives over a set of criteria. Given a set of alternatives $A = \{a_i | i = 1, 2, \cdots, m\}$, a set of criteria $C = \{c_j | j = 1, 2, \cdots, n\}$ and a set of weigths $W = \{w_j | j = 1, 2, \cdots, n\}$, $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$, where w_j denotes the weight of the criteria c_j, let $X = \{x_{ij | i=1,2,\cdots,m, j=1,2,\cdots,n}\}$ denote the decision matrix where x_{ij} is the performance measure of the alternative a_i with respect to the criteria c_j. Given the decision matrix, the TOPSIS involves six steps.

1. Normalize the decision matrix. The normalized value z_{ij} is calculated as

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \quad (11)$$

2. Compute the weighted normalized decision matrix. The weighted normalized value v_{ij} is calculated as

$$v_{ij} = w_j z_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \quad (12)$$

3. Determine the positive ideal solution (PIS) and the negative ideal solution (NIS).

$$PIS = \{v_1^+, v_2^+, \cdots, v_n^+\}$$
$$= \{max_{\forall i} v_{ij} | j \in J_1, min_{\forall i} v_{ij} | j \in J_2\} \quad (13)$$

$$NIS = \{v_1^-, v_2^-, \cdots, v_n^-\}$$
$$= \{min_{\forall i} v_{ij} | j \in J_1, max_{\forall i} v_{ij} | j \in J_2\} \quad (14)$$

Here, J_1 is the set of benefit criteria, and J_2 is the set of cost criteria.

4. Calculate the separation measures using the n-dimensional Euclidean distance. The separation measures D_i^+ and D_i^- of an alternative a_i from the PIS and NIS are

$$D_i^+ = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^+)^2}, i = 1, 2, \cdots, m \quad (15)$$

$$D_i^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2}, i = 1, 2, \cdots, m \quad (16)$$

5. Calculate relative closeness (RC) of the alternative a_i.

$$RC_i = \frac{D_i^-}{D_i^+ + D_i^-} \quad (17)$$

$0 \leq RC_i \leq 1$, $i = 1, 2, \cdots, m$

6. Arrange the ranking indexes in a descending order to obtain the best alternative.

As mention the ranking index of TOPSIS seems reasonable, but as such it is also criticised by several authors from the fact that the relative importance of the two separations is not considered in step 5. One of the researchers pointing this out where Opricovic and Tzeng [19] where they pointed out that the original TOPSIS simply sums D_i^+ and D_i^- without using any parameter that could represent the relative importance of these two separations. They analyzed the index of the relative closeness to the ideal solution, and pointed out that when a_1 is assumed to be the alternative with $D_1^- = D_1^+$, then all alternatives a_2 with $D_2^- > D_2^+ > D_1^+$ are ranked preceding to a_1, even though a_1 is closer than a_2 to the PIS [19]. This as such is true, eventhough not that surprising because a_1 is also closer than a_2 to the NIS. From this seminal work question arised that can the ranking order of alternatives a_1 and a_2 be decided by using the relative information of separation measures defined in step 4? This was further examined by Kuo [20] by examining differences $D_2^+ - D_1^+$ and $D_2^- - D_1^-$ and ranking indexes. In short Kuo addressed this by examining four cases shown in Table I.

TABLE I
FOUR CASES FOR DIFFERENCES BETWEEN DISTANCES TO POSITIVE AND NEGATIVE IDEAL SOLUTIONS

	Case 1	Case 2	Case 3	Case 4
$D_2^+ - D_1^+$	> 0	> 0	< 0	< 0
$D_2^- - D_1^-$	> 0	< 0	> 0	< 0
Decidable?	No	Yes	Yes	No
Result	?	$RC_2 < RC_1$	$RC_1 < RC_2$	?

In case that distance to PIS is considered more important than distance to NIS or vice versa in cases 1 and 4 decision can be either one ($RC_2 \prec RC_1$ or $RC_1 \prec RC_2$) depending on actual value of a_2. From this became clear that it is reasonable in such cases to try to introduce weights that reflect the relative importance of the two separation measures. In Kuo 2017 [20], solution to the problem was proposed by introducing relative weights to the D_i^+ and D_i^-.

$$RCnew_i = w^+ \left(\frac{D_i^-}{\sum_{i=1}^{m} D_i^-} \right) - w^- \left(\frac{D_i^+}{\sum_{i=1}^{m} D_i^+} \right) \quad (18)$$

With (18) by introducing proper weights cases 1 and 4 can be solved so that preference follows importantness of the particular ideal solution. For example consider following decision matrix shown in Table II.

TABLE II
DECISION MATRIX 1

	C_1	C_2	C_3	C_4	C_5
A_1	46	19	11	18	10
A_2	29	23	9	20	10
A_3	33	24	10	18	9
A_4	43	26	8	10	8

If we calculate a_1 (where $D_1^- = D_1^+$) and do comparison with a_1 and $A_1 - A_4$ we find out that A_1 and A_3 are decidable cases where as A_2 and A_4 are non decidable. This example of course can be solved using $RCnew_i$ with proper weights.

IV. TOPSIS WITH $n-$ARY NORM OPERATORS

A. Motivation

In this paper we examine can we address this problem already at earlier point in TOPSIS than in weighting relative closeness proposed by Kuo [20]. As an alternative way of addressing this problem we propose the use of $n - ary$ norm operators in creation of positive and negative ideal solutions. Basic idea is very simple: we simply replace min and max operators in Step 3 with their more general counterparts: $n - ary$ $T-$norms and $T-$conorms. In order to apply $n - ary$ norm operators in TOPSIS we also need to modify normalization done in step 1. $N - ary$ norm operators are doing mapping $[0, 1]^n \rightarrow [0, 1]$ so normalization is now done to the unit interval.

Justification for replacing minimum and maximum in positive and negative ideal solution is quite intuitive. The standard fuzzy intersection (minimum) is the weakest fuzzy intersection and hence producing largest set from among those produced by all possible fuzzy intersections (T-norms) [23]. Similarly standard fuzzy union (maximum) is the strongest fuzzy union and hence it produces the smallest set among the sets produced by all possible fuzzy unions (T-conorms). Now by changing the norm operator we can also address relative importantess type of a problem in TOPSIS already at the point where we are creating Positive and Negative ideal solutions.

B. Step by step algorithm

Let T and T_{co} denote $n-$ary $T-$norm and $T-$conorm. Given a set of alternatives $A = \{a_i | i = 1, 2, \cdots, m\}$, a set of criteria $C = \{c_j | j = 1, 2, \cdots, n\}$ and a set of weigths $W = \{w_j | j = 1, 2, \cdots, n\}$, $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$, where w_j denotes the weight of the criteria c_j, let $X = \{x_{ij|i=1,2,\cdots,m,j=1,2,\cdots,n}\}$ denote the decision matrix where x_{ij} is the performance measure of the alternative a_i with respect to the criteria c_j. Given the decision matrix, the $n-$ary norm based TOPSIS involves following steps.

1. Normalize the decision matrix into unit interval.

$$z_{ij} = \frac{x_{ij} + min_i(x_{ij})}{max_i(x_{ij}) - min_i(x_{ij})}, \quad (19)$$

$$i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$$

2. Let z_{ij} denote normalized decision matrix. Compute the weighted normalized decision matrix. The weighted normalized value v_{ij} is calculated as

$$v_{ij} = w_j z_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \quad (20)$$

3. Determine the positive ideal solution (PIS) and the negative ideal solution (NIS) using chosen $n-$ary T-norm and T-conorm.

$$PIS = \{v_1^+, v_2^+, \cdots, v_n^+\}$$
$$= \{T_{\forall i}(v_{ij}) | j \in J_1, Tco_{\forall i}(v_{ij}) | j \in J_2\} \quad (21)$$

$$NIS = \{v_1^-, v_2^-, \cdots, v_n^-\}$$
$$= \{Tco_{\forall i}(v_{ij}) | j \in J_1, T_{\forall i}(v_{ij}) | j \in J_2\} \quad (22)$$

Here, J_1 is the set of benefit criteria, and J_2 is the set of cost criteria.

4. Calculate the separation measures using the n-dimensional Euclidean distance. The separation measures D_i^+ and D_i^- of an alternative a_i from the PIS and NIS are

$$D_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2}, i = 1, 2, \cdots, m \quad (23)$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, i = 1, 2, \cdots, m \quad (24)$$

5. Calculate relative closeness (RC) of the alternative a_i.

$$RC_i = \frac{D_i^-}{D_i^+ + D_i^-} \quad (25)$$

$0 \leq RC_i \leq 1$, $i = 1, 2, \cdots, m$

6. Arrange the ranking indexes in a descending order to obtain the best alternative.

Now returning back to example shown in Table II if we examining it with $D_2^+ - D_1^+$ and $D_2^- - D_1^-$ and ranking indexes and select Łukasiewicz n-ary T-norm and standard n-ary T-conorm we are able to get the result that we are in decidable region for all four alternatives where as originally this was not the case with A_2 and A_4. Ranking order gained by using proposed norm operators is $A_1 \prec A_2 \prec A_3 \prec A_4$ where as with original TOPSIS ranking order is $A_1 \prec A_3 \prec A_2 \prec A_4$.

C. Numerical example: Patent selection problem

The problem presented here deals with ranking of patents and the selection of the best ranking patents to be included in a patent portfolio, this problem has been previously presented in more details by Collan et al. [24]. The initial (financial) evaluation of the patents has been done by a group of managers and has resulted in consensual fuzzy pay-off distributions for each patent. These possibilistic mean, standard deviation and skewness are reported in Table IV

TABLE III

RANKING ORDERS OF TWO ALTERNATIVES WHEN SAMPLE IS COMPARED TO a_1 (WHERE $D_1^+ = D_1^-$), 1=DECIDABLE, 0=NON DECIDABLE. IN PROPOSED METHOD ŁUKASIEWICZ N-ARY T-NORM AND STANDARD N-ARY T-CONORM WAS USED

	Orig TOPSIS	$n-$ary TOPSIS
A_1	1	1
A_2	0	1
A_3	1	1
A_4	0	1

TABLE IV

POSSIBILISTIC MOMENTS FROM THE CONSENSUS PAY-OFF DISTRIBUTIONI FOR EACH PATENT [24]

Patent	Mean	Standard	Skewness
1	0.3175	0.0081	0.0005
2	0.3593	0.0111	0.0011
3	0.3203	0.0044	-0.0001
4	0.3038	0.0081	-0.0004
5	0.2665	0.0010	0.0000
6	0.4546	0.0174	-0.0018
7	0.4447	0.0143	0.0014
8	0.3504	0.0092	0.0013
9	0.3301	0.0083	0.0004
10	0.3297	0.0022	0.0002
11	0.3187	0.0067	0.0012
12	0.3638	0.0037	0.0001
13	0.2900	0.0038	0.0002
14	0.3352	0.0049	0.0001
15	0.3536	0.0069	0.0008
16	0.4187	0.0127	0.0011
17	0.5409	0.0169	0.0003
18	0.3934	0.0126	0.0012
19	0.4125	0.0111	-0.0001
20	0.3042	0.0050	0.0005

The possibilistic moments of these pay-off distributions (that are fuzzy numbers) are used as the criteria for TOPSIS ranking of the patents. When we examine are there undecidable cases present in the patents we can find three cases, patents number 5, 10 and 12. For this problem it was decided to use Yager's T-norm and T-conorm [23] $T_{Yager}(x_1, x_2) = 1 - min(1, ((1-x_1)^p + (1-x_2)^p)^{\frac{1}{p}})$, $T_{coYager}(x_1, x_2) = min(1, (x_1^p + x_2^p)^{\frac{1}{p}})$ and their $n-$ary form using recursive formulas (1), (6). Reasons for selecting Yager's norms are that it is well known that it holds for Yager's T-norm that $T_D \leq T_{Yager} \leq T_{min}$ meaning that by changing the parameter p value we can get T-norm to approach Drastic minimum which is most strict T-norm there can be and also toward minimum which is most weakest T-norm. Similarly for Yager's T-conorm it holds that $T_{coM} \leq T_{coYager} \leq T_{coD}$ meaning that by proper parameter p selection we can approach strictest T-conorm maximum and weakest T-conorm drastic sum. In order to select suitable parameter value p for the problem we simply tested suitable values from range $0 < p < 100$ using for loops. From the results we computed the number of undecidable cases present for different p values and selected the case with fewest possible amount of undecidable cases for the problem. It turned out that fewest possible amount of undecidable cases for doing the computations this way was one. This time patent number six being undecidable. In the

Table V one can see the results from the experiments with regular TOPSIS and $n-$ary based TOPSIS where the amount of undecidable cases are minimized. These results where found when parameter p was set to be $p = 2$ for $n-$ary T-norm and $p = 10$ for $n-$ary T-conorm.

TABLE V
RELATIVE CLOSENESS (RC) VALUES FOR TOPSIS AND $n-$ARY TOPSIS AND INFORMATION ABOUT DECIDABLE VERSUS NONDECIDABLE CASES.

Patent	RC (TOPSIS)	Decidable	RC (Proposed)	Decidable
1	0.6451	1	0.8072	1
2	0.8024	1	0.9760	1
3	0.4710	1	0.6379	1
4	0.4283	1	0.6522	1
5	0.4625	0	0.5655	1
6	0.3168	1	0.5779	0
7	0.9081	1	0.8850	1
8	0.7915	1	0.9140	1
9	0.6275	1	0.7990	1
10	0.5208	0	0.6325	1
11	0.7386	1	0.8297	1
12	0.5160	0	0.6617	1
13	0.5314	1	0.6553	1
14	0.5243	1	0.6804	1
15	0.6940	1	0.8236	1
16	0.8426	1	0.9394	1
17	0.7017	1	0.7747	1
18	0.8477	1	0.9466	1
19	0.5447	1	0.7559	1
20	0.6077	1	0.7254	1

To further examine the results from patent portfolio forming problem in Table VI one can see ranking orders of the patents from both TOPSIS and $n-$ary TOPSIS. As can be seen ranking results have somewhat changed. For example if we look at five top candidates TOPSIS gives ordering $7 \prec 18 \prec 16 \prec 2 \prec 8$ where as $n-$ary TOPSIS gives $2 \prec 18 \prec 16 \prec 8 \prec 7$. Eventhough we have same five patents selected order of them is quite different especially between patents no 7 and 2. In case of just selecting three patents we would get a different selection result from these two.

TABLE VI
RANKING ORDERS FOR TOPSIS AND $n-$ARY TOPSIS

Ranking order	TOPSIS	$n-$ary TOPSIS
1.	7	2
2.	18	18
3.	16	16
4.	2	8
5.	8	7
6.	11	11
7.	17	15
8.	15	1
9.	1	9
10.	9	17
11.	20	19
12.	19	20
13.	13	14
14.	14	12
15.	10	13
16.	12	4
17.	3	3
18.	5	10
19.	4	6
20.	6	5

Notable also is that when we are examining undecidable cases none of them seem to be in top 10 selected patents, but rather close to be last ones. This however is not anyway generalizable result.

V. CONCLUSIONS

We have presented new version of TOPSIS where $n-$ary norm operators are used in creation of positive and negative ideal solutions. We have shown that by doing this we are able to create different ranking order. Also by doing so we can address the problem of having different preferences towards positive and negative ideal solutions. This can be used as alternative method instead of setting importance weights for D_i^+ and D_i^- in relative closeness coefficient. By choosing a stricter/weaker norm operator we can change the preference of an ideal solution. Besides changing the preference by imposing stricter/weaker norm operator another way for possible suitable norm operator selection or parameter selection in $n-$ary norm operators would be to used examination of decidable cases proposed by Kuo [20] and minimization of undecidable alternatives.

For the future work, method introduced in this paper can be extented so that it covers different $n-$ary norms for different criterias. In doing so preference order can be altered even more. In the case that we would want to find a regions where there would be least amount of undecidable cases this will create an optimization problem where suitable norms/parameter to norms can be optimized e.g. by minimizing the set of undecidable cases.

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