Admissibility Analysis and Robust Stabilization via State Feedback for Uncertain T–S Fuzzy Descriptor Systems

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***Abstract*—This paper considers the admissibility and robust stabilization via state feedback for continuous-time T–S fuzzy descriptor systems (TSFDS) with a class of uncertainties. First, the admissibility of the nominal system without uncertainties is investigated. An equivalent augmented system is presented to deal with different and singular derivative matrices. Then, admissible conditions for the open-loop and close-loop systems are both derived based on a non-quadratic fuzzy Lyapunov function. Second, the admissibility and robust stabilization of TSFDS with uncertainties in all matrices are investigated. The uncertainty in each derivative matrix is equivalently expressed by a constant matrix left multiplied by an invertible uncertain matrix so that a similar augmented system can be constructed. Then admissible conditions are derived. This paper generalizes existing related results since we consider a wider class of TSFDS with different derivative matrices and different membership functions in each subsystem. All conditions are expressed as strict linear matrix inequalities (LMIs). Finally, a simulation example is provided to show effectiveness of the proposed results.**

***Index Terms*—Admissibility, Uncertain Takagi–Sugeno (T–S) fuzzy descriptor systems, Robust control, LMIs**

I. Introduction

Takagi-Sugeno (T-S) fuzzy systems have experienced an impressive growth in recent years [1]. This approach is described by a set of fuzzy rules and connects local linear input-output relations with nonlinear membership functions (MFs), thus it enables to describe smooth nonlinear systems [2]. T–S fuzzy descriptor systems (TSFDS) are defined by extending the T-S fuzzy systems to the descriptor form [3]. Since TSFDS takes advantages of fuzzy theory and descriptor systems, it is suitable for modeling of complex systems [4]–[6].

The admissibility of descriptor system is important for system analysis and controller design. Current research on admissibility of TSFDS can be divided into two cases, one is with a same derivative matrix in [7]–[9] and the other is with different derivative matrices in [3], [10], [11]. In [7], new types of fuzzy controllers and fuzzy Lyapunov functions were proposed to guarantee the stability of TSFDS, but they are quite conservative since each fuzzy subsystem is required to be stable. In [8], relaxed admissible conditions of TSFDS via a fuzzy Lyapunov function were presented. In [9], admissibility analysis and control synthesis for TSFDS were investigated based on a non-quadratic fuzzy Lyapunov function. Furthermore, the information of MFs was fully considered to relax these conditions. However, these works assumed that derivative matrices of each subsystem were same, which is quite limited. As for different derivative matrices, researchers in [3] presented an augmented system to deal with different derivative matrices and this system was widely used in subsequent works. However, as investigated in [12], this augmented system is not equivalent to the original system when derivative matrices are singular. In [10], the stability of unforced fuzzy descriptor systems was considered. Then both parallel distributed compensation (PDC) controller and fuzzy proportional and derivative state feedback (PDSF) controller were proposed to stabilize the closed-loop system. The admissibility and dissipativity of TSFDS were investigated in [11], which presented a new augmented system to deal with different derivative matrices. This system is equivalent to the original system even if derivative matrices are singular. Although researchers in [10], [11] considered descriptor systems with different derivative matrices, each derivative matrix and state matrix shared the same MFs in each subsystem. It is obvious that TSFDS can represent a wider class of systems when MFs are different. As far as we know, very little attention has been paid to the admissibility analysis of this system, which is the main motivation of this paper.

In practice, due to inaccurate measurements and variations of system parameters, parametric uncertainties widely exist in TSFDS and may destroy admissibility of the system.

Corresponding authors: Feng Xu and Bin Liang.
This work was supported by National Natural Science Foundation of China (No.U1813216), and the Science and Technology Research Foundation of Shenzhen (JCYJ20170817152701660, JCYJ20160301100921349).

978-1-7281-6932-3/20/$31.00 ©2020 IEEE

Especially, when the perturbations occur in derivative matrices, this problem becomes more involved. A flurry of research has focused on robust stabilization for uncertain TSFDS. In [13], the robust stabilization problem for linear time-invariant descriptor systems with the norm-bounded uncertainty in derivative matrix was first investigated. It proved that the problem has a solution only if the uncertainty does not increase the rank of the derivative matrix. In [14], the researchers extended methods proposed in [13] to the linear time-invariant descriptor systems with uncertainties in all matrices. Furthermore, it showed that this problem can be divided into two cases, $i.e.$, one is expressed by a constant matrix left multiplied by an invertible uncertain matrix, and the other is produced by its dual form. For descriptor systems, one common method is to normalize them with the derivative state feedback. In [15], a novel robust proportional plus derivative state feedback controller for a class of uncertain TSFDS with distinct derivative matrices was presented. However, when derivative states are not available, this method will not be applicable. Other research works, such as [16], [17], investigated the control problem for TSFDS with uncertainties in system matrices rather than derivative matrices. To the best of our knowledge, the admissibility analysis and robust stabilization via state feedback for continuous-time TSFDS with uncertainties in all matrices is still an open issue.

This paper focuses on the admissibility analysis and robust stabilization via state feedback for continuous-time TSFDS with uncertainties in all matrices. With the consideration of different derivative matrices and distinct MFs in each subsystem, our results will generalize previous results. The admissibility analysis of the nominal open-loop TSFDS is considered first based on an equivalent augmented system, then a PDC controller is designed to stabilize the nominal system. Second, the robust stabilization for TSFDS with uncertainties in all matrix is investigated. The uncertainty in each derivative matrix is equivalently reformulated as a constant matrix left multiplied by an invertible uncertain matrix, then another similar augmented system is proposed to deal with this problem. All these conditions are reformulated as strict LMIs through some useful lemmas, which can be efficiently solved.

This paper is organized as follows: Section II considers admissibility analysis and control design for the nominal system. Section III considers robust stabilization for the uncertain system. Section IV provides a simulation example to show effectiveness of the main results. Section V concludes this paper. Some useful lemmas are provided in the appendix.

Notations: Ω_n denotes the integer set $\{1, 2, ..., n\}$. $\mathbb{R}$ is the field of real numbers and $\mathbb{C}$ is the field of complex numbers. For a square matrix X, X^T denotes its transpose, and $X > 0$ ($X < 0$) indicates that X is positive (negative) definite, respectively. $\|X\|$, det(X) and rank(X) represent the Euclidean norm, determinant and rank of X, respectively. The degree of a polynomial is denoted by deg(*). I and 0 are the identity and zero matrices of appropriate dimension. The symbol * stands for the transpose elements in symmetric block matrices, $i.e.$, $A + (*) = A + A^T$, $\begin{bmatrix} A & * \\ B & C \end{bmatrix} = \begin{bmatrix} A & B^T \\ B & C \end{bmatrix}$. For brevity, we denote $\sum_{k=1}^{re} v_k(z(t))X_k$, $\sum_{i=1}^{r} h_i(z(t))X_i$, $\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))X_{ij}$ and $\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{re} h_i(z(t))h_j(z(t))v_k(z(t))X_{ijk}$ by X_v, X_h, X_{hh} and X_{hhv}, respectively. $z(t)$ is the premise vector which depends on the state vector. $h_i(z(t))$ and $v_k(z(t))$, denoted by h_i and v_k, satisfy the convex sum property. Besides, $\dot{h}_i \geq \phi_i$, $\dot{v}_k \geq \varphi_k$, where $\phi_i \leq 0$ and $\varphi_k \leq 0$ are bounds of time derivative of MFs which are prior known.

II. Admissibility Analysis and Control Design for Nominal System

A. Preliminaries

Consider the following uncertain continuous-time TSFDS:

$$\begin{aligned}
&\sum_{k=1}^{re} v_k(z(t))\,(E_k+\Delta E_k)\,\dot{x}(t) = \\
&\sum_{i=1}^{r} h_i(z(t))\,[(A_i+\Delta A_i)\,x(t) + (B_i+\Delta B_i)\,u(t)], \quad (1)\\
&y(t) = \sum_{i=1}^{r} h_i(z(t))C_i x(t)
\end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector and $y(t) \in \mathbb{R}^q$ is the output vector. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{q\times n}$ and $E_k \in \mathbb{R}^{n\times n}$ are nominal matrices which represent local models of the fuzzy system. ΔE_k, ΔA_i and ΔB_i are uncertainties which are usually written as $\Delta E_k = {H_e}^T \Delta e W_{ek}$, $\Delta A_i = {H_a}^T \Delta a W_{ai}$, $\Delta B_i = {H_b}^T \Delta b W_{bi}$, with H_e, H_a, H_b, W_{ek}, W_{ai}, W_{bi}, constant matrices and: $\Delta e^T \Delta e \leq I$, $\Delta a^T \Delta a \leq I$ and $\Delta b^T \Delta b \leq I$. In particularly, $E_k + \Delta E_k$ may be singular. The system (1) can be rewritten into a compact form as

$$\begin{aligned}
(E_v + \Delta E_v)\,\dot{x}(t) &= (A_h + \Delta A_h)\,x(t) + \\
&\quad (B_h + \Delta B_h)\,u(t), \quad (2)\\
y(t) &= C_h x(t).
\end{aligned}$$

The following assumption is made on derivative matrices of the system (1), which means we consider a special case as [11], [18].

Assumption 2.1: Rank$(E_k) = n_1 \leq n$ and there exist invertible constant matrices Q_k, such that $E_k = Q_k\bar{E}$, $k \in \Omega_{re}$.

Based on Assumption 2.1, a new equivalent augmented system of (1) without uncertainties can be given as

$$\begin{aligned}
E^*\dot{x}^*(t) &= \sum_{i=1}^{r}\sum_{k=1}^{re} h_i v_k (A^*_{ik}x^*(t) + B^*_i u(t)), \\
y(t) &= \sum_{i=1}^{r} h_i C^*_i x^*(t)
\end{aligned} \quad (3)$$

where

$$E^* = \begin{bmatrix} \bar{E} & 0 \\ 0 & 0 \end{bmatrix}, A_{ik}^* = \begin{bmatrix} 0 & I \\ A_i & -Q_k \end{bmatrix}, B_i^* = \begin{bmatrix} 0 \\ B_i \end{bmatrix},$$
$$C_i^* = \begin{bmatrix} C_i & 0 \end{bmatrix}, x^*(t) = \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}, w(t) = \bar{E}\dot{x}(t).$$

For convenience, the system (3) is rewritten as

$$\begin{aligned} E^*\dot{x}^*(t) &= A_{hv}^* x^*(t) + B_h^* u(t), \\ y(t) &= C_h^* x^*(t). \end{aligned} \tag{4}$$

The following definition of admissibility will be considered.

Definition 2.1 [11]: The unforced system (2) without uncertainties is admissible if the following conditions hold:

1) The system is regular if there exists a complex number $s \in \mathbb{C}$, satisfying $\det(sE_v - A_h) \neq 0$.

2) The system is impulse free if $\deg(\det(sE_v - A_h)) = \operatorname{rank}(E^*)$.

3) The system is stable if $\sigma(E_v, A_h) \subset \{s | s \in C, \operatorname{Re}(s) < 0\}$, and $\forall t \in [0, \infty)$, where $\sigma(E, A) = \{s | \det(sE - A) = 0\}$.

With a similar proof in [11], we have

$$\begin{aligned} \det(sE_v - A_h) &= \det\left(\begin{bmatrix} s\bar{E} & -I \\ -A_h & Q_v \end{bmatrix}\right) \\ &= \det(sE^* - A_{hv}^*), \end{aligned}$$

so systems (2) and (4) are equivalent in terms of admissibility.

B. Admissibility Analysis of Open-loop System

The following results present admissible conditions for the open-loop system (4).

Theorem 2.1: The system (4) is admissible if there exist matrices $P_{ik} \in \mathbb{R}^{2n\times 2n}$, $X_k \in \mathbb{R}^{2n\times 2n}$, $Y_i \in \mathbb{R}^{2n\times 2n}$, $H \in \mathbb{R}^{2n\times 2n}$ and $L \in \mathbb{R}^{2n\times 2n}$ such that following conditions hold:

$$P_{ik}{}^T E^{*T} = E^* P_{ik} \geq 0 \tag{5}$$

$$E^*(P_{ik} + X_k) \geq 0 \tag{6}$$

$$E^*(P_{ik} + Y_i) \geq 0 \tag{7}$$

$$G_{ik} < 0 \tag{8}$$

where $i \in \Omega_r$, $k \in \Omega_{re}$,

$$G_{ik} = \begin{bmatrix} \begin{array}{l} -E^*\Phi_k - E^*\Psi_i \\ +H^T A_{ik}^*{}^T + A_{ik}^* H \end{array} & * \\ P_{ik} - H + L^T A_{ik}^*{}^T & -L - L^T \end{bmatrix},$$

$$\Phi_k = \sum_{i=1}^{r} \phi_i (P_{ik} + X_k), \Psi_i = \sum_{k=1}^{re} \varphi_k (P_{ik} + Y_i).$$

Proof: Based on the convex summation property of MFs and (8), we obtain

$$\begin{bmatrix} \begin{array}{l} -E^*\Phi_v - E^*\Psi_h \\ +H^T A_{hv}^*{}^T + A_{hv}^* H \end{array} & * \\ P_{hv} - H + L^T A_{hv}^*{}^T & -L - L^T \end{bmatrix} < 0. \tag{9}$$

Using Lemma 3 with $T = -E^*\Phi_v - E^*\Psi_h$, we have

$$P_{hv}{}^T A_{hv}^*{}^T + A_{hv}^* P_{hv} - E^*\Phi_v - E^*\Psi_h < 0. \tag{10}$$

From (6) and (7), it is clear that $-E^*\Phi_v - E^*\Psi_h \geq 0$, then

$$P_{hv}{}^T A_{hv}^*{}^T + A_{hv}^* P_{hv} < 0. \tag{11}$$

For the matrix E^*, there exist two nonsingular matrices U and V, such that

$$UE^*V = \begin{bmatrix} I_{n1} & 0 \\ 0 & 0 \end{bmatrix}. \tag{12}$$

Accordingly, taking

$$V^{-1} P_{ik} U^T = \begin{bmatrix} P_{1ik} & P_{2ik} \\ P_{3ik} & P_{4ik} \end{bmatrix}, U A_{ik}^* V = \begin{bmatrix} A_{1ik}^* & A_{2ik}^* \\ A_{3ik}^* & A_{4ik}^* \end{bmatrix},$$

respectively. Since there exist matrices P_{ik}, such that (6) holds, we obtain that $P_{2ik} = 0$. Therefore,

$$V^{-1} P_{hv} U^T = \begin{bmatrix} P_{1hv} & 0 \\ P_{3hv} & P_{4hv} \end{bmatrix}, \tag{13}$$

then pre-multiplying and post-multiplying (11) by U and U^T, respectively, we obtain

$$\begin{bmatrix} \star & \star \\ \star & A_{4hv}^* P_{4hv} + P_{4hv}{}^T A_{4hv}^*{}^T \end{bmatrix} < 0 \tag{14}$$

where $\star$ denotes entries which will not affect the proof. It follows that $\|P_{4hv}\| \leq \sum_{i=1}^{r}\sum_{k=1}^{re} \|P_{4ik}\|$, then A_{4hv}^* is invertible and its inverse matrix is bounded based on Lemma 1. By far, we prove that the system (4) is regular and impulse free and the next step is to prove the stability. The following non-quadratic fuzzy Lyapunov function will be employed:

$$V(x^*(t)) = x^{*T}(t) E^{*T} P_{hv}{}^{-1} x^*(t) \tag{15}$$

where $E^{*T} P_{hv}{}^{-1} = P_{hv}{}^{-T} E^* \geq 0$, which is equivalent to (5). The time derivative of the previous function is

$$\begin{aligned} \dot{V} =& x^{*T}\left(A_{hv}^*{}^T P_{hv}{}^{-1} + P_{hv}{}^{-T} A_{hv}^* + E^{*T}\frac{d}{dt}\left(P_{hv}{}^{-1}\right)\right) x^* \\ =& x^{*T}\left(A_{hv}^*{}^T P_{hv}{}^{-1} + P_{hv}{}^{-T} A_{hv}^*\right) x^* - \\ & x^{*T}\left(E^{*T} P_{hv}{}^{-1}\left(\sum_{i=1}^{r}\sum_{k=1}^{re} (\dot{h}_i v_k + h_i \dot{v}_k) P_{ik}\right) P_{hv}{}^{-1}\right) x^*. \end{aligned} \tag{16}$$

Based on the convex summation property of MFs, we have

$$\sum_{i=1}^{r} \dot{h}_i X = \sum_{k=1}^{re} \dot{v}_k Y = 0. \tag{17}$$

Thus, with the introduction of slack matrices X_k and Y_i, we obtain

$$\begin{aligned} & \sum_{i=1}^{r}\sum_{k=1}^{re}\left(\dot{h}_i v_k + h_i \dot{v}_k\right) P_{ik} \\ =& \sum_{k=1}^{re} v_k \sum_{i=1}^{r} \dot{h}_i (P_{ik} + X_k) + \sum_{i=1}^{r} h_i \sum_{k=1}^{re} \dot{v}_k (P_{ik} + Y_i). \end{aligned} \tag{18}$$

Then, combining (18) with (16), we obtain

$$\begin{aligned}\Sigma \doteq& {A_{hv}^{*}}^{T} {P_{hv}}^{-1} + {P_{hv}}^{-T} A_{hv}^{*} - \\ & E^{*T} {P_{hv}}^{-1} \left(\sum_{k=1}^{re} v_k \sum_{i=1}^{r} \dot{h}_i \left(P_{ik} + X_k \right) \right) {P_{hv}}^{-1} - \\ & E^{*T} {P_{hv}}^{-1} \left(\sum_{i=1}^{r} h_i \sum_{k=1}^{re} \dot{v}_k \left(P_{ik} + Y_i \right) \right) {P_{hv}}^{-1}.\end{aligned} \tag{19}$$

If $\Sigma < 0$, then $\dot{V} < 0$. Multiplying Σ on the left by ${P_{hv}}^{T}$ and right by P_{hv}, it can be seen that

$$\begin{aligned}& {P_{hv}}^{T} {A_{hv}^{*}}^{T} + A_{hv}^{*} P_{hv} - \\ & E^{*} \left(\sum_{k=1}^{re} v_k \sum_{i=1}^{r} \dot{h}_i \left(P_{ik} + X_k \right) \right) - \\ & E^{*} \left(\sum_{i=1}^{r} h_i \sum_{k=1}^{re} \dot{v}_k \left(P_{ik} + Y_i \right) \right) \\ \leq & {P_{hv}}^{T} {A_{hv}^{*}}^{T} + A_{hv}^{*} P_{hv} - \\ & E^{*} \left(\sum_{k=1}^{re} v_k \sum_{i=1}^{r} \phi_i \left(P_{ik} + X_k \right) \right) - \\ & E^{*} \left(\sum_{i=1}^{r} h_i \sum_{k=1}^{re} \varphi_k \left(P_{ik} + Y_i \right) \right) \\ = & {P_{hv}}^{T} {A_{hv}^{*}}^{T} + A_{hv}^{*} P_{hv} - E^{*} \Phi_v - E^{*} \Psi_h,\end{aligned} \tag{20}$$

which means if LMIs (8) holds, then $\dot{V}\left(x^{*}\right) < 0$. Therefore, the system (4) is stable as well as admissible. □

Clearly, the matrix inequalities in (5), (6) and (7) are not strict LMIs, thus the following strict results can be obtained based on Lemmas 2 and 4, then the toolbox in MATLAB such as Yalmip Toolbox [19] can be directly used to solve LMIs.

Theorem 2.2: The system (4) is admissible if there exist matrices $J_{ik} \in \mathbb{R}^{2n \times 2n}$, $H \in \mathbb{R}^{2n \times 2n}$, $L \in \mathbb{R}^{2n \times 2n}$ and symmetric matrices $K_{ik} \in \mathbb{R}^{2n \times 2n}$, $X_k \in \mathbb{R}^{2n \times 2n}$, $Y_i \in \mathbb{R}^{2n \times 2n}$, such that following conditions hold:

$$E^{*} K_{ik} {E^{*}}^{T} + W > 0 \tag{21}$$

$$E^{*} K_{ik} {E^{*}}^{T} + E^{*} X_k {E^{*}}^{T} + W > 0 \tag{22}$$

$$E^{*} K_{ik} {E^{*}}^{T} + E^{*} Y_i {E^{*}}^{T} + W > 0 \tag{23}$$

$$\hat{G}_{ik} < 0 \tag{24}$$

where $i \in \Omega_r$, $k \in \Omega_{re}$,

$$\hat{G}_{ik} = \begin{bmatrix} -\hat{\Phi}_k - \hat{\Psi}_i + H^T {A_{ik}^{*}}^{T} + A_{ik}^{*} H & * \\ \hat{P}_{ik} - H + L^T {A_{ik}^{*}}^{T} & -L - L^T \end{bmatrix},$$

$$\hat{\Phi}_k = \sum_{i=1}^{r} \phi_i \left(E^{*} K_{ik} {E^{*}}^{T} + E^{*} X_k {E^{*}}^{T} \right),$$

$$\hat{\Psi}_i = \sum_{k=1}^{re} \varphi_k \left(E^{*} K_{ik} {E^{*}}^{T} + E^{*} Y_i {E^{*}}^{T} \right),$$

$$\hat{P}_{ik} = \left(I - \left(E^{*-} E^{*} \right) \right) J_{ik} + \left(E^{*-} E^{*} \right) K_{ik} {E^{*}}^{T},$$

E^{*-} and W are defined in Lemmas 2 and 4, respectively.

The proof of Theorem 2.2 is mainly based on the reformulation of (5), (6) and (7) into (21), (22) and (23). Thus, it is omitted here.

C. Control Design of Close-loop System

Consider the following PDC controller for TSFDS (2):

$$\begin{aligned} u(t) &= -\sum_{j=1}^{r} \sum_{k=1}^{re} h_j v_k F_{jk} x(t) \\ &= -\begin{bmatrix} F_{hv} & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} = -F_{hv}^{*} x^{*}(t). \end{aligned} \tag{25}$$

Then the close-loop system (4) is rewritten as

$$E^{*} \dot{x}^{*}(t) = \left(A_{hv}^{*} - B_h^{*} F_{hv}^{*} \right) x^{*}(t). \tag{26}$$

Thus, replacing (E^{*}, A_{hv}^{*}) with $(E^{*}, A_{hv}^{*} - B_h^{*} F_{hv}^{*})$ and selecting $H = \begin{bmatrix} R & 0 \\ H_3 & H_4 \end{bmatrix}$, $L = \begin{bmatrix} R & 0 \\ L_3 & L_4 \end{bmatrix}$ in Theorem 2.2, the following results can be obtained based on Lemma 5.

Theorem 2.3: The system (26) is admissible if there exist matrices $J_{ik} \in \mathbb{R}^{2n \times 2n}$, $H = \begin{bmatrix} R & 0 \\ H_3 & H_4 \end{bmatrix}$, $L = \begin{bmatrix} R & 0 \\ L_3 & L_4 \end{bmatrix}$ with $R \in \mathbb{R}^{n \times n}$, $H_3 \in \mathbb{R}^{n \times n}$, $H_4 \in \mathbb{R}^{n \times n}$, $L_3 \in \mathbb{R}^{n \times n}$, $L_4 \in \mathbb{R}^{n \times n}$, $N_{jk} \in \mathbb{R}^{m \times n}$ and symmetric matrices $K_{ik} \in \mathbb{R}^{2n \times 2n}$, $X_k \in \mathbb{R}^{2n \times 2n}$, $Y_i \in \mathbb{R}^{2n \times 2n}$ such that LMIs (21), (22), (23) and following conditions hold:

$$\begin{aligned} & \hat{G}_{iik} < 0, \\ & \frac{2}{r-1} \hat{G}_{iik} + \hat{G}_{ijk} + \hat{G}_{jik} < 0, i \neq j \end{aligned} \tag{27}$$

where $i, j \in \Omega_r$, $k \in \Omega_{re}$,

$$\hat{G}_{ijk} = \begin{bmatrix} \begin{array}{c} -\hat{\Phi}_k - \hat{\Psi}_i + A_{ik}^{*} H - B_i^{*} N_{jk}^{*} \\ + \left(A_{ik}^{*} H - B_i^{*} N_{jk}^{*} \right)^T \end{array} & * \\ \hat{P}_{ik} - H + \left(A_{ik}^{*} L - B_i^{*} N_{jk}^{*} \right)^T & -L - L^T \end{bmatrix},$$

$N_{jk}^{*} = \begin{bmatrix} N_{jk} & 0 \end{bmatrix}$, and $\hat{\Phi}_k$, $\hat{\Psi}_i$ and $\hat{P}_{ik}$ are given in Theorem 2.2. Feedback gains can be obtained as $F_{jk} = N_{jk} R^{-1}$.

III. Robust Stabilization for Uncertain System

In this section, the admissibility analysis and robust stabilization for TSFDS with uncertainties are considered. As investigated in [13], if the stabilization problem can be solved, the uncertainties must not increase the rank of derivative matrices, which proposes following assumptions for system (1).

Assumption 3.1: $\text{Rank}(E_k + \Delta E_k) = \text{rank}(E_k) = n_1 \leq n$.

Furthermore, researchers in [14] pointed out that for each subsystem that satisfies Assumption 3.1, each derivative matrix with uncertainty can be transformed into two cases, *i.e.*, one is expressed by a constant matrix left multiplied by an invertible uncertain matrix, and the other is produced by its dual form. Only the first case will be considered in this paper, so we have the following assumption.

$$\Xi_{ijk} = \left[\begin{array}{cc} \left[\begin{array}{cc} -\Phi_k - \Psi_i + A_{ik}^* H - B_i^* N_{jk}^* + \left(A_{ik}^* H - B_i^* N_{jk}^* \right)^T & * \\ P_{ik} - H + \left(A_{ik}^* L - B_i^* N_{jk}^* \right)^T & -L - L^T \end{array} \right] + \Gamma S_{ijk} \Gamma^T & * \\ \Upsilon_{ijk} & -S_{ijk} \end{array} \right] \tag{31}$$

Assumption 3.2: For each subsystem of the system (1), the derivative matrix with uncertainty can be transformed into an equivalent form expressed by a constant matrix left multiplied by an invertible uncertain matrix, $i.e.$,

$$E_k + \Delta E_k = \left(I + \Delta E_k{}' \right) E_k \tag{28}$$

where $\Delta E_k = H_e{}^T \Delta_e W_{ek}$ and $\Delta E_k{}' = H_e{}'^T \Delta_e W_{ek}{}'$. As for the algorithm of transformation (28), one can find details in [13], [14].

Based on the Assumptions 2.1, 3.1 and 3.2, a new equivalent augmented system of (1) can be given as

$$\begin{aligned} E^* \dot{x}^*(t) &= \sum_{i=1}^{r} \sum_{k=1}^{re} h_i v_k (\bar{A}_{ik}^* x^*(t) + \bar{B}_i^* u(t)), \\ y(t) &= \sum_{i=1}^{r} h_i C_i^* x^*(t) \end{aligned} \tag{29}$$

where

$$\bar{A}_{ik}^* = \left[\begin{array}{cc} 0 & I \\ A_i + \Delta A_i & -\left(I + \Delta E_k{}' \right) Q_k \end{array} \right] = A_{ik}^* + \Delta A_{ik}^*,$$

$\bar{B}_i^* = \left[\begin{array}{c} 0 \\ B_i + \Delta B_i \end{array} \right] = B_i^* + \Delta B_i^*$, E^*, B_i^*, A_{ik}^* and C_i^* are same as system (3).

Consider the same PDC controller (25) for the uncertain system (29), we have the following theorem.

Theorem 3.1: The system (29) is quadratically stable and admissible if there exist matrices $P_{ik} \in \mathbb{R}^{2n \times 2n}$, $X_k \in \mathbb{R}^{2n \times 2n}$, $Y_i \in \mathbb{R}^{2n \times 2n}$, $H = \left[\begin{array}{cc} R & 0 \\ H_3 & H_4 \end{array} \right]$, $L = \left[\begin{array}{cc} R & 0 \\ L_3 & L_4 \end{array} \right]$ with $R \in \mathbb{R}^{n \times n}$, $H_3 \in \mathbb{R}^{n \times n}$, $H_4 \in \mathbb{R}^{n \times n}$, $L_3 \in \mathbb{R}^{n \times n}$, $L_4 \in \mathbb{R}^{n \times n}$, $N_{ik} \in \mathbb{R}^{m \times n}$ and scalars $\tau_{ijk}^a, \tau_{ijk}^b, \tau_{ijk}^e$, such that conditions (5), (6), (7) and following conditions hold:

$$\begin{aligned} & \Xi_{iik} < 0, \\ & \frac{2}{r-1} \Xi_{iik} + \Xi_{ijk} + \Xi_{jik} < 0, i \neq j \end{aligned} \tag{30}$$

where $i, j \in \Omega_r$, $k \in \Omega_{re}$, Ξ_{ijk} is in the form (31), Υ_{ijk} is in the form (32), and Γ and S_{ijk} are in the following forms:

$$\Gamma = \left[\begin{array}{ccc} 0 & 0 & 0 \\ H_a{}^T & H_b{}^T & H_e{}'^T \\ & 0 & \end{array} \right],$$

$$S_{ijk} = \left[\begin{array}{ccc} \tau_{ijk}^a & 0 & 0 \\ 0 & \tau_{ijk}^b & 0 \\ 0 & 0 & \tau_{ijk}^e \end{array} \right].$$

Proof: Consider the same non-quadratic fuzzy Lyapunov function (15) for the system (29), then a similar result as (20) can be obtained by congruence property and introduction of slack matrices, $i.e.$, if

$$\begin{aligned} & P_{hv}{}^T \left(\bar{A}_{hv}^* - \bar{B}_h^* F_{hv}^* \right)^T + \left(\bar{A}_{hv}^* - \bar{B}_h^* F_{hv}^* \right) P_{hv} \\ & - E^* \Phi_v - E^* \Psi_h < 0 \end{aligned} \tag{33}$$

holds, then $\dot{V}(x^*) < 0$, which means the system (29) is quadratically stable. According to Lemma 3, it is equivalent to

$$\Sigma \doteq \left[\begin{array}{cc} \begin{array}{l} -E^* \Phi_v - E^* \Psi_h + \\ \left(\bar{A}_{hv}^* - \bar{B}_h^* F_{hv}^* \right) H \\ + H^T \left(\bar{A}_{hv}^* - \bar{B}_h^* F_{hv}^* \right)^T \end{array} & * \\ \begin{array}{l} P_{hv} - H + \\ L^T \left(\bar{A}_{hv}^* - \bar{B}_h^* F_{hv}^* \right)^T \end{array} & -L - L^T \end{array} \right] < 0. \tag{34}$$

Dividing (34) into two parts, $i.e.$, the certain part

$$\Sigma_1 \doteq \left[\begin{array}{cc} \begin{array}{c} -E^* \Phi_v - E^* \Psi_h + \\ (A_{hv}^* - B_h^* F_{hv}^*) H \\ + H^T (A_{hv}^* - B_h^* F_{hv}^*)^T \end{array} & * \\ P_{hv} - H + L^T (A_{hv}^* - B_h^* F_{hv}^*)^T & -L - L^T \end{array} \right]$$

and the uncertain part

$$\Sigma_2 \doteq \left[\begin{array}{cc} \begin{array}{c} (\Delta A_{hv}^* - \Delta B_h^* F_{hv}^*) H \\ + H^T (\Delta A_{hv}^* - \Delta B_h^* F_{hv}^*)^T \end{array} & * \\ L^T (\Delta A_{hv}^* - \Delta B_h^* F_{hv}^*)^T & 0 \end{array} \right].$$

With $H = \left[\begin{array}{cc} R & 0 \\ H_3 & H_4 \end{array} \right]$ and $L = \left[\begin{array}{cc} R & 0 \\ L_3 & L_4 \end{array} \right]$, the uncertain part can be rewritten as

$$\Sigma_2 = \Gamma \left[\begin{array}{ccc} \Delta_a & 0 & 0 \\ 0 & \Delta_b & 0 \\ 0 & 0 & \Delta_e \end{array} \right] \Upsilon_{hhv} + (*). \tag{35}$$

Using Lemma 6 with S_{hhv}, we obtain

$$\Sigma_2 \leq \Gamma S_{hhv} \Gamma^T + \Upsilon_{hhv}{}^T S_{hhv}^{-1} \Upsilon_{hhv}, \tag{36}$$

which means if

$$\Sigma_1 + \Gamma S_{hhv} \Gamma^T + \Upsilon_{hhv}{}^T S_{hhv}^{-1} \Upsilon_{hhv} < 0, \tag{37}$$

then $\dot{V}(x^*) < 0$. With the Shur complement to (37), we obtain $\Xi_{hhv} < 0$ and condition (30) can be further obtained based on Lemma 5. A similar process in Theorem 2.1 can be done to prove the system (29) is regular and impulse free, so it is omitted here. □

Furthermore, strict LMI conditions can be obtained as follows:

$$\Upsilon_{ijk} = \begin{bmatrix} W_{ai}R & 0 & W_{ai}R & 0 \\ -W_{bi}N_{jk} & 0 & -W_{bi}N_{jk} & 0 \\ -W_{ek}{}'Q_kH_3 & -W_{ek}{}'Q_kH_4 & -W_{ek}{}'Q_kL_3 & -W_{ek}{}'Q_kL_4 \end{bmatrix} \tag{32}$$

$$\hat{\Xi}_{ijk} = \begin{bmatrix} \begin{bmatrix} -\hat{\Phi}_k - \hat{\Psi}_i + A^*_{ik}H - B^*_iN^*_{jk} + \left(A^*_{ik}H - B^*_iN^*_{jk}\right)^T & * \\ \hat{P}_{ik} - H + \left(A^*_{ik}L - B^*_iN^*_{jk}\right)^T & -L - L^T \end{bmatrix} + \Gamma S_{ijk}\Gamma^T & * \\ \Upsilon_{ijk} & -S_{ijk} \end{bmatrix} \tag{39}$$

Theorem 3.2: The system (29) is quadratically stable and admissible if there exist matrices $J_{ik} \in \mathbb{R}^{2n\times 2n}$, $H=\begin{bmatrix} R & 0 \\ H_3 & H_4 \end{bmatrix}$, $L=\begin{bmatrix} R & 0 \\ L_3 & L_4 \end{bmatrix}$ with $R \in \mathbb{R}^{n\times n}$, $H_3 \in \mathbb{R}^{n\times n}$, $H_4 \in \mathbb{R}^{n\times n}$, $L_3 \in \mathbb{R}^{n\times n}$, $L_4 \in \mathbb{R}^{n\times n}$, $N_{ik} \in \mathbb{R}^{m\times n}$, symmetric matrices $K_{ik} \in \mathbb{R}^{2n\times 2n}$, $X_k \in \mathbb{R}^{2n\times 2n}$, $Y_i \in \mathbb{R}^{2n\times 2n}$ and scalars $\tau^a_{ijk}, \tau^b_{ijk}, \tau^e_{ijk}$, such that conditions (21), (22), (23) and following conditions hold:

$$\begin{aligned} &\hat{\Xi}_{iik} < 0, \\ &\frac{2}{r-1}\hat{\Xi}_{iik} + \hat{\Xi}_{ijk} + \hat{\Xi}_{jik} < 0, i \neq j \end{aligned} \tag{38}$$

where $i, j \in \Omega_r$, $k \in \Omega_{re}$, $\hat{\Xi}_{ijk}$ is in the form (39), and $\hat{\Phi}_k$, $\hat{\Psi}_i$ and $\hat{P}_{ik}$ are given in Theorem 2.2.

IV. Illustrative Examples

In this section, a numerical example is provided to illustrate effectiveness of the main results in this paper. Consider a TSFDS with uncertainties given by

$$\begin{aligned}
&E_1 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\
&A_1 = \begin{bmatrix} -20 & -9 \\ 2 & -5 \end{bmatrix}, A_2 = \begin{bmatrix} -10 & -15 \\ 1 & -5 \end{bmatrix}, \\
&W_{e1} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}^{\mathrm{T}}, W_{e2} = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}^{\mathrm{T}}, H_e{}^T = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \\
&W_{a1} = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}^{\mathrm{T}}, W_{a2} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}^T, H_a{}^T = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}, \\
&B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, W_b = 0.4, H_b{}^T = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}, \\
&v_1(t) = (1 - \cos x_1(t))/2, v_2(t) = (1 + \cos x_1(t))/2, \\
&h_1(t) = (1 + \sin x_1(t))/2, h_2(t) = (1 - \sin x_1(t))/2.
\end{aligned}$$

The following two cases will be considered.

Case 1: A singular TSFDS without uncertainties aims to show effectiveness of the results in Section III.

Computing the characteristic equation of each subsystems, we find that subsystems (E_1, A_1) and (E_1, A_2) are not stable. Choosing $Q_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $Q_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\bar{E} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and solving LMIs in Theorem 2.2, no solutions are found, which means the open-loop system is not admissible. With the PDC controller (25), solving LMIs in Theorem 2.3, we obtain feedback gains as $F_{11} = \begin{bmatrix} 16.5367 & 27.3916 \end{bmatrix}$, $F_{12} = \begin{bmatrix} 0.6372 & 13.2012 \end{bmatrix}$, $F_{21} = \begin{bmatrix} 9.5671 & 35.1767 \end{bmatrix}$ and $F_{22} = \begin{bmatrix} -0.3695 & 15.4277 \end{bmatrix}$. Consider initial states as $x(t) = \begin{bmatrix} 3.0 & 1.0 \end{bmatrix}^T$, Figure 1 shows state responses of the open-loop and close-loop systems. It is clear that our controller can stabilize the system, which demonstrates the effectiveness of Theorem 2.3.

Case 2: A singular TSFDS with uncertainties aims to show effectiveness of the results in Section IV.

Using the algorithm proposed in [13], we obtain $H_e{}' = H_e$, $W_{e1}' = \begin{bmatrix} 0.05 & 0.05 \end{bmatrix}$ and $W_{e2}' = \begin{bmatrix} 0.2 & 0 \end{bmatrix}$ for Assumption 3.2. With uncertainties in all matrices and $\Delta e = \Delta a = \Delta b = sin(0.1t)$, solving LMIs in Theorem 3.2, we obtain robust feedback gains as $F_{11} = \begin{bmatrix} 13.4471 & 14.6660 \end{bmatrix}$, $F_{12} = \begin{bmatrix} -1.4826 & 5.0571 \end{bmatrix}$, $F_{21} = \begin{bmatrix} 4.2746 & 17.4241 \end{bmatrix}$ and $F_{22} = \begin{bmatrix} -0.7969 & 3.7396 \end{bmatrix}$. Figure 2 shows state responses of the open-loop and close-loop systems, it is clear that our controller can make the uncertain system admissible, which demonstrates the effectiveness of Theorem 3.2.

V. Conclusion

This paper investigates the admissibility analysis and robust stabilization via state feedback for TSFDS with uncertainties. The system contains different derivative matrices and different MFs, which presents a wider class of systems. An augmented system equivalent to the original system is presented to deal with derivative matrices and uncertainties are reformulated into a new equivalent form which can be conveniently handled. Based on a non-quadratic fuzzy Lyapunov function, the admissibility analysis and robust stabilization conditions via state feedback are derived. All the conditions are expressed as strict LMIs, which can be easily solved with available solvers. Since only one specific uncertainty for TSFDS is considered in this paper, $i.e.$, Assumptions 3.2, our future work will mainly focus on other forms of uncertainties and various controller design problems.

Appendix

The following results are used in this paper:

Lemma 1 [8]: Suppose that for a given piecewise continuous matrix $A(t) \in \mathbb{R}^{n\times n}$. If there exist a bounded time varying matrix $P(t) \in \mathbb{R}^{n\times n}$ and a scalar $\alpha > 0$ satisfying

$$A^T(t)P(t) + P^T(t)A(t) \leq -\alpha I \tag{40}$$

for all t, then $A(t)$ is invertible and $A^{-1}(t)$ is bounded.

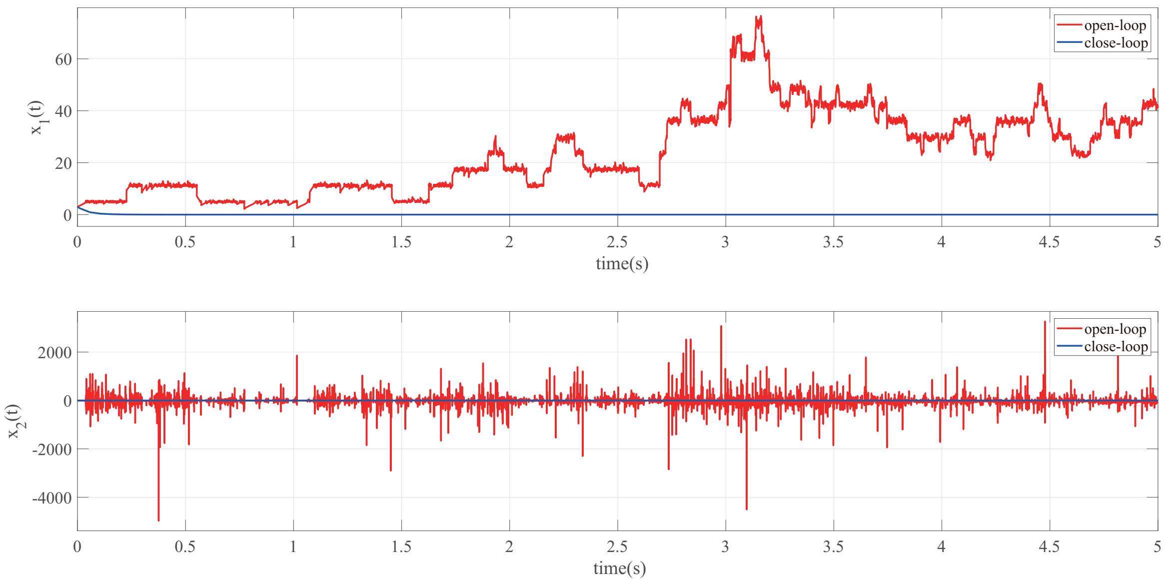

Fig. 1. Trajectories of nominal system.

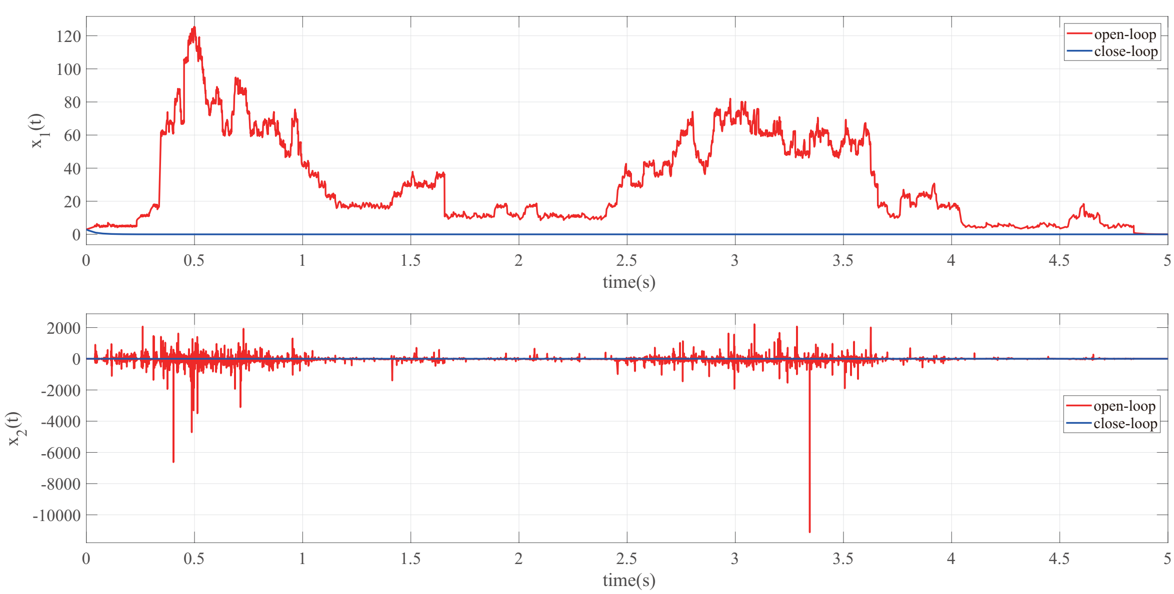

Fig. 2. Trajectories of uncertain system.

Lemma 2 [20]: For a given matrix $\hat{E} \in \mathbb{R}^{n\times n}$, the solution X of the matrix equation $\hat{E}X = X^T \hat{E}^T$ is

$$X = \left(I - \left(\hat{E}^{-}\hat{E}\right)\right) J + \left(\hat{E}^{-}\hat{E}\right) K \hat{E}^{T} \tag{41}$$

where $\hat{E}^{-}$ is the generalized inverse of $\hat{E}$, $J \in \mathbb{R}^{n\times n}$ is an arbitrary matrix and $K \in \mathbb{R}^{n\times n}$ is a symmetric matrix.

Lemma 3 [9]: Given a matrix $A \in \mathbb{R}^{n\times n}$ and a symmetric matrix $T \in \mathbb{R}^{n\times n}$, there exists an invertible matrix $P \in \mathbb{R}^{n\times n}$ such that

$$T + P^T A^T + AP < 0 \tag{42}$$

if and only if there exist matrices $H \in \mathbb{R}^{n\times n}$ and $L \in \mathbb{R}^{n\times n}$ such that

$$\begin{bmatrix} T + H^T A^T + AH & * \\ P - H + L^T A^T & -L - L^T \end{bmatrix} < 0. \tag{43}$$

Lemma 4 [11]: For a given matrix $\hat{E} \in \mathbb{R}^{n\times n}$ with $\mathrm{rank}(\hat{E}) = q \leq n$, if there exist matrices $P \in \mathbb{R}^{n\times n}$ and $X \in \mathbb{R}^{n\times n}$ satisfying

$$\hat{E}P = P^{\mathrm{T}} \hat{E}^{\mathrm{T}} \tag{44}$$

$$\hat{E}P + W > 0 \tag{45}$$

$$\hat{E}X = X^{\mathrm{T}}\hat{E}^{\mathrm{T}} \tag{46}$$

$$\hat{E}P + \hat{E}X + W > 0 \tag{47}$$

where

$$W = U^{-1}\begin{bmatrix} 0 & 0 \\ 0 & I_{n-q} \end{bmatrix} U^{-\mathrm{T}}$$

and $U, V \in \mathbb{R}^{n\times n}$ are invertible matrices satisfying

$$U\hat{E}V = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}.$$

Then P and X satisfy

$$P^{\mathrm{T}}\hat{E}^{\mathrm{T}} = \hat{E}P \geq 0 \tag{48}$$

$$\hat{E}P + \hat{E}X \geq 0. \tag{49}$$

Lemma 5 [21]: Let G_{ij} be matrices of proper dimensions. Then

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j G_{ij} < 0 \tag{50}$$

holds if the following conditions hold:

$$G_{ii} < 0, i \in \Omega_r \tag{51}$$

$$\frac{2}{r-1}G_{ii} + G_{ij} + G_{ji} < 0,\ i,j \in \Omega_r,\ i \neq j. \tag{52}$$

Lemma 6 [22]: Given any real matrices X, Y and positive definite matrix $S = S^T \geq 0$, we have

$$X^TY + Y^TX \leq X^TSX + Y^TS^{-1}Y. \tag{53}$$

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