Milk-run Routing and Scheduling Subject to Fuzzy Pickup and Delivery Time Constraints: An Ordered Fuzzy Numbers Approach

1st Grzegorz Bocewicz
Faculty of Electronics and Computer Science
Koszalin University of Technology
Koszalin, Poland
ORCID: 0000-0002-5181-2872

2nd Zbigniew Banaszak
Faculty of Electronics and Computer Science
Koszalin University of Technology
Koszalin, Poland
ORCID: 0000-0001-7219-3903

3rd Katarzyna Rudnik
Faculty of Production Engineering and Logistics
Opole University of Technology
Opole, Poland
ORCID: 0000-0002-0653-6610

4th Marcin Witczak
Institute of Control and Computation Engineering
University of Zielona Gora
Zielona Gora, Poland
ORCID: 0000-0002-0031-0004

5th Czesław Smutnicki
Faculty of Electronics,
Wroclaw University of Science and Technology
Wroclaw, Poland
ORCID: 0000-0003-4640-5364

6th Jarosław Wikarek
Department of Information Systems,
Kielce University of Technology,
Kielce, Poland
ORCID: 0000-0003-4758-1490

***Abstract*— The design of logistic trains fleet oriented distributed and scalability-robust control policies that ensure deadlock-free operations is of crucial importance for efficient material handling systems. This study considers a multi-item assembly system where in-plant transport operations are organized in milk-run loops. A solution to a milk-run routing and scheduling problem subject to fuzzy pickup and delivery time constraints is developed. This type of problem can be treated in terms of a fuzzy constraint satisfaction problem, therefore, the main objective is to provide a reference model analytical formulas of which enable one to obtain solutions that do not require time-consuming computer simulations. Two versions of the model were parameterized assuming independent implementation of convex and ordered fuzzy numbers. The accuracy of both models was experimentally verified according to the results of multiple simulations. Results from this study provide an approach to avoid time consuming computer simulation-based calculations of logistic trains fleet schedules avoiding congestions while concurrently maintaining throughput at maximal achievable level.**

***Keywords*— Routing, cyclic scheduling, logistic trains, fuzzy constraint satisfaction problem, ordered fuzzy number (OFN)**

I. Introduction

The milk-run driven delivery concept [2], [11], [17] boiling down to NP-hard problems of logistic trains routing and scheduling problems attempts to design plans to inform whom to serve, how much to deliver, and what regularly repeated routes to travel on by what fleet of vehicles. This can be viewed as belonging to the class of Vehicle Routing Problem [4], [18], [20]. Given this context, the objective of the present study is to develop a method, derived from the reference model, in support of using decision-support-system-like software. We employ the declarative modeling framework because of its fast prototyping capability. Importantly, declarative models focus primarily on developing the solution [2], [30]. The present study is a continuation of previous work that explored methods of fast prototyping of solutions associated with problems related to routing, batching and scheduling of tasks typically performed in batch flow production systems, as well as problems related to the planning and control of production flow in departments of automotive companies [2], [3]. This paper has three main contributions. First, representing a departure from the commonly accepted assumption of the deterministic nature of transport processes, this study incorporates human uncertainties (i.e., the logistic trains are driven by operators). Taking into account the related distribution of delivery moments enables the construction of more realistic and accurate models for assessing the effectiveness of prototyped route variants. Second, the declarative model-driven approach to assess alternative milk-run routing and scheduling variants is formulated in detail. The obtained ordered fuzzy number-driven model enables searching for logistic trains congestion-free routes in terms of fuzzy constraint satisfaction problem. Third, the proposed approach allows the replacement of computer simulation methods of routes prototyping with an analytical method based on an ordered fuzzy number formalism. In that context it can be recognized as an outperforming solution approach for in-plant Milk-run driven delivery problems. This paper is organized as follows. Section 2 presents a review of selected literature. This section also provides basic information about Ordered Fuzzy Numbers (OFNs). A motivating example to highlight the problem under consideration is discussed in Section 3. Section 4 details a declarative model dedicated to the prototyping of milk-run traffic systems. Consequently, the Fuzzy Constraint Satisfaction Problem aimed at logistic trains fleet delivery mission planning is formulated in Section 5. Next, Section 6 depicts how the model can be used in supply cycles prototyping tasks, and section 7 provides reports key conclusions and makes suggestions for future research.

II. Literature Review

A. Milk-run routing and time scheduling

In a milk-run system, routes, time schedules, type and number of parts to be transported are assigned to different

978-1-7281-6932-3/20/$31.00 ©2020 IEEE

logistics trains so that they can collect orders from different suppliers [11]. In other words, the trains perform several pick-ups/deliveries in a round trip to meet customer demands. The benefits of using a system of this type include improved efficiency of the overall logistics system and substantial potential savings in environmental and human resources, as well as remarkable cost advantages related to inventory and transportation costs [8], [19], [29]. The Milk-run Vehicle Routing Problem [18] is viewed as a special case of the Vehicle Routing Problem (VRP) [4], [20], which in turn is a generalization of the Traveling Salesman Problem seeking to find the optimal set of routes for a fleet of vehicles delivering goods or services to various locations. For this reason, there is a large body of (OR) literature on vehicle scheduling, routing, and dispatching that address relevant aspects of transportation of goods. However, there are only a few papers devoted specifically to in-plant milk-run traffic problems. In this respect, the most relevant are areas subject to critical and often unpredictable traffic congestions. This typically occurs when logistic operators allocate too many collecting tasks to available vehicles, thus generating unperformed activities due to assumed just-in-time constraints and violating contractual obligations assumed with their clients [21]. Most of the research in the field of distribution logistics is devoted to the analysis of the methods of organizing transport processes in ways that minimize the size of the fleet, the distance traveled (energy consumed), or the space occupied by a distribution system. By focusing on the search for optimal solutions, these studies implicitly assume that there exist admissible solutions; in other words, solutions that ensure collision-free and/or deadlock-free (congestion-free) flow of concurrent transport processes. In practice, this requires either on-line updating (revision) of the routing policies used, or prior (offline) planning of congestion-free vehicle routes and schedules. Studies on generating dynamic routing policies are conducted sporadically [4]; even less frequent are investigations of robust routing and scheduling of milk-run traffic, which are, by and large, limited to AGV systems. The congestion avoidance problem, which conditions the existence of admissible solutions, is considered an NP-hard problem [13], [32]. Because the necessary and sufficient conditions for deadlock-free execution of concurrent processes are not known, system analysis (i.e. analysis of the states potentially leading to system deadlocks) is most frequently performed using laborious and time-consuming computer simulations [4], [10]. In practical applications, congestion avoidance methods implement the sufficient conditions for collision-free execution of processes. This implies that the time-consuming method of analyzing distribution networks that detect deadlocks between concurrent transport flows can be replaced by searching for a synchronization mechanism that would guarantee cyclic execution of these flows. Methods that are most commonly used for such purposes include the formalism of max-plus algebra [22], [28], graph theory [31], simulations [9] and constraint programming [2], [30]. It should be noted that the possibility of fast implementation of the process-synchronization mechanism comes at the expense of omitting some of the potentially possible scenarios for deadlock-free execution of the processes.

The shortcomings of the above-described methods to generate admissible solutions can restrict their implementation in DSS systems, in particular those supporting planning in milk-run traffic systems. To address this challenge, our contribution assesses the use of declarative modeling methods (e.g. [3]) in solutions that provide interactive decision support for prototyping in-plant milk-run traffic systems.

B. Ordered fuzzy numbers

In the proposed declarative modeling method, the imprecise variables adopt values in the form of OFNs. The concept of OFNs, defined by Kosiński, Prokopowicz and Ślęzak [14], was proposed as a response to problems related to the application of fuzzy numbers [5], [15]. The OFN are defined as follows [15]:

Definition 1. An OFN is defined as a pair of continuous real functions specified on an interval [0, 1], i.e.:

$$\hat{A} = (f_A, g_A), \text{ where: } f_A, g_A: [0, 1] \rightarrow \mathbb{R}. \quad (1)$$

■

The functions f_A and g_A are called the *up* and *down* part of ordered fuzzy number $\hat{A}$, respectively. They are also called as branches of fuzzy number $\hat{A}$. The values of these continuous functions are limited ranges, which can be defined as the following bounded intervals: $UP_A = (l_{A0}, l_{A1})$ and $DOWN_A = (p_{A1}, p_{A0})$. Assuming that: f_A is increasing and g_A is decreasing as well as that $f_A \leq g_A$, the membership function μ_A of the ordered fuzzy number $\hat{A}$ can be seen in Fig 1a and 1b:

$$\mu_A(x) = \begin{cases} f_A^{-1}(x) & when\ x \in UP_A \\ g_A^{-1}(x) & when\ x \in DOWN_A \\ 1 & when\ x \in [l_{A1}, p_{A1}] \\ 0 & \text{in the remaining cases} \end{cases} . \quad (2)$$

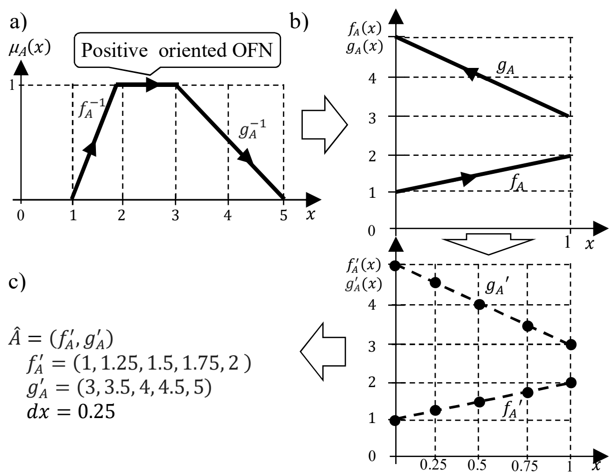

Fig. 1. a) OFN $\hat{A}$ in terms of convex fuzzy numbers, b) functions f_A, g_A determining $\hat{A}$ (positive orientation), c) discrete representation of $\hat{A}$ ($dx = 0.25$) (based on: [15])

A large class of OFNs represents the whole class of convex fuzzy numbers with continuous membership functions. An additional property called orientation (direction) is defined for OFN. There are two types of orientation: positive when $\hat{A} = (f_A, g_A)$ (direction $\hat{A}$ matches the direction of the *OX axis*) and negative when $\hat{A} = (g_A, f_A)$ (direction $\hat{A}$ matches the direction opposite to the *OX* axis). Assuming that the values of all fuzzy variables have different orientations, this allows us to define algebraic operations that meet the listed conditions of the Ring. Due to the fact that the OFN is defined as the pair of functions f_A, g_A the all algebraic operations performed on OFNs are also defined as relevant mathematical operations executed on these functions. The adopted definitions of relations and algebraic operations used in the developed model are as follows:

Definition 2. Let $\hat{A} = (f_A, g_A)$ and $\hat{B} = (f_B, g_B)$ be ordered fuzzy numbers. $\hat{A}$ is equal to $\hat{B}$ ($\hat{A} = \hat{B}$), $\hat{A}$ is greater or equal or greater $\hat{B}$ ($\hat{A} > \hat{B}$; $\hat{A} \geq \hat{B}$), $\hat{A}$ is less or equal or less $\hat{B}$ ($\hat{A} < \hat{B}$, $\hat{A} \leq \hat{B}$) if: $\forall_{x\in[0,1]} \quad f_A(x) * f_B(x) \ \wedge \ g_A(x) * g_B(x)$, where: the symbol $*$ replaces suitably: $=, >, \geq, <, \leq$. ■

Definition 3. [15]. Let $\hat{A} = (f_A, g_A)$, $\hat{B} = (f_B, g_B)$ and $\hat{C} = (f_C, g_C)$ be ordered fuzzy numbers. The operations of adding $\hat{C} = \hat{A} + \hat{B}$, subtraction $\hat{C} = \hat{A} - \hat{B}$, multiplication $\hat{C} = \hat{A} \times \hat{B}$ and division $\hat{C} = \hat{A}/\hat{B}$ are defined as follows: $\forall_{x\in[0,1]} \ f_C(x) = f_A(x) * f_B(x) \ \wedge \ g_C(x) = g_A(x) * g_B(x)$ where: the symbol $*$ replaces suitably: $+, -, \times, \div$; The division operation is defined for $\hat{B}$ such that $|f_B| > 0$ and $|g_B| > 0$ for each $x \in [0, 1]$. ■

In recent years, the concept of OFNs has been constantly evolving and applied to various practical applications. Many publications are devoted to the analysis of the OFN model and its confrontation with convex fuzzy sets [5], [7], [23]. After performing algebraic calculations on convex fuzzy numbers, the numbers' support usually becomes extremely broad, so the information represented by this number has limited practicality. The OFNs do not have this drawback.

The OFN model has additional advantages beyond algebraic operations. The orientation of OFNs gives additional options in applications to represent imprecise values. The idea of a property of processing data called sensitivity to the direction was proposed by Prokopowicz [24]. Zarzycki et al. [33] presents an efficient use of the OFN model in the description of processes undergoing dynamic changes. The trend of imprecise value as an OFN was also used in a critical path analysis [5]. In [7] a practical application of OFN arithmetics in a crisis control center monitoring was presented. The trend modeling in the evaluation of medical data was presented in [18]. Lastly, Marszałek and Burczyński [16] proposed a Ordered fuzzy GARCH model for volatility forecasting.

Another branch of OFN applications development involves the use of multi-criteria decision making (MCDM) methods, as a development of popular approaches [26]. In MCDM methods, the OFN orientation differentiates the type of the criterion (cost and profit). According to our review of the literature and to the best of our knowledge, the approach proposed in this paper represents the first attempt to use OFNs for milk-run routing and scheduling.

III. ILLUSTRATIVE EXAMPLE

Here we consider a multi-item batch flow production system in which the in-plant transport operations of a set parts supply are organized in a milk-run loop passing through 14 work stations $SN = \{SN_1, \ldots, SN_{14}\}$ while servicing two assembly lines shown in Fig. 2. Consequently, two types of products W_1, W_2 are manufactured in the system, where batches of each kind of product are moved between the neighboring assembly stations by dedicated gantry robots. In subsequent steps, i.e., at different work stations, SN_i ($i = 1, \ldots, 14$), particular products are assembled from the parts delivered in containers to the work station buffers B_1–B_{11}. Some buffers are shared by several stations e.g., buffer B_3 is shared by stations SN_1 and SN_2. The six types of parts packed in containers CT_1-CT_6 are delivered to the buffers by the two logistic train LT_1 and LT_2 following the routes marked with a green and orange lines (see Fig. 2). Assembly stations and the buffers associated with them must be supplied with containers due to the schedule presented in Fig. 3. On the presented schedule, the moments of collecting containers from buffers are marked with points with colors that correspond to the containers CT_1-CT_6. The grey ranges specify the periods in which the buffer stocks should be replenished, i.e., equal to τ_λ =900s. Exceeding the deadlines dx_λ (see Fig. 3) may result in the lack of the required deliveries, which may further lead to production suspension. The schedule demonstrates that deliveries must be made within cyclically repeated time windows (with size: $T = 2970$s). The logistic trains (LT_1 and LT_2) traveling along the fixed routes are used as in-plant means of transport to deliver the required quantity of parts to buffers within the given time windows. In general, the objective of this study is to find a method that allows for the congestion-free travel of the logistic trains in order to guarantee the timely delivery of the ordered part sets. In other words, the answer to the following question is researched: *Does there exist, in the given system, a set of routes of logistic trains and the associated delivery schedules that guarantee timely delivery of the materials necessary for the production process to be completed?* Many approaches reported in the literature enable answering this question for systems of scale encountered in practice (number of serviced points ≤15, logistic fleet size ≤5) [2], [3]. An example of a route that guarantees timely delivery and the resulting schedule is presented in Figs. 2 and 3. The routes received (s sequences visited sequentially by LT_1 and LT_2 buffers) are in the form: $\pi_1 = (B_1, B_7, B_6, B_4, B_8, B_{11}, B_5)$, $\pi_2 = (B_1, B_2, B_{10}, B_9, B_3, B_5)$.

These routes guarantee a collision-free and deadlock-free delivery. It is worth emphasizing, however, that the techniques used to determine them (Constraints Programming CP and Mixed Integer Programming MIP) require precisely defined values of system parameters, and the exact value of transport times between buffers is required, as well as loading / unloading times. Transport operations in milk-run systems are usually carried out by people - the logistic train is run by an employee. This means that planned times assumed for loading/unloading operations, as well as transportation operations, can be quite uncertain. Uncertainty of the duration of the operation results in uncertain moments of occupation and release of stops. As a result, the actual implementation of the schedule may differ significantly from the planned one. Therefore, even minor deviations from the plan can result in serious consequences such as blocking.

Figure 4 illustrates the situation where the 90s delay (relative to the deadline resulting from the planned schedule in Fig. 3) of train LT_1 coupled the simultaneous acceleration of train LT_2 by 60s results in a blockade in the sector ⑦−⑧ (the moment of blockage 1230s).

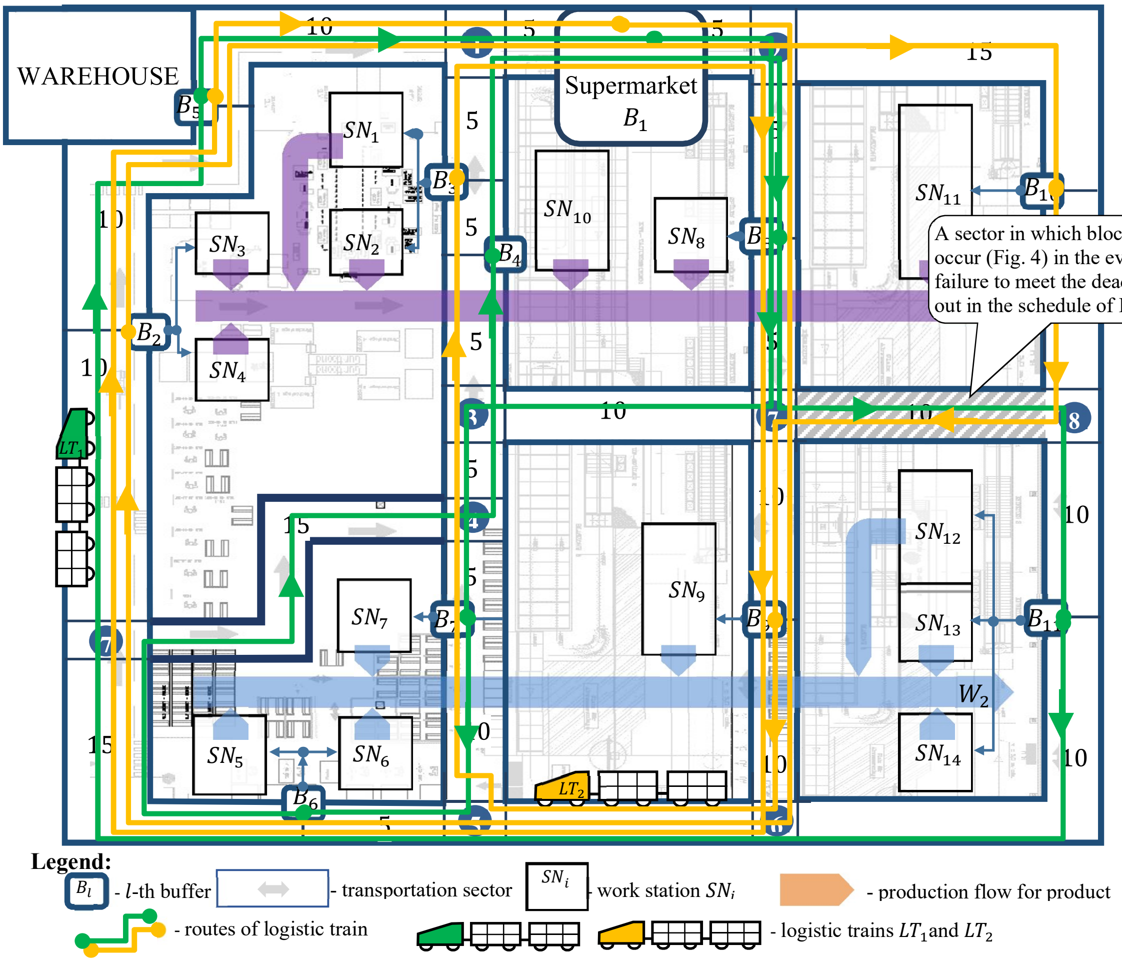

Fig. 2. Layout of considered Milk-Run System

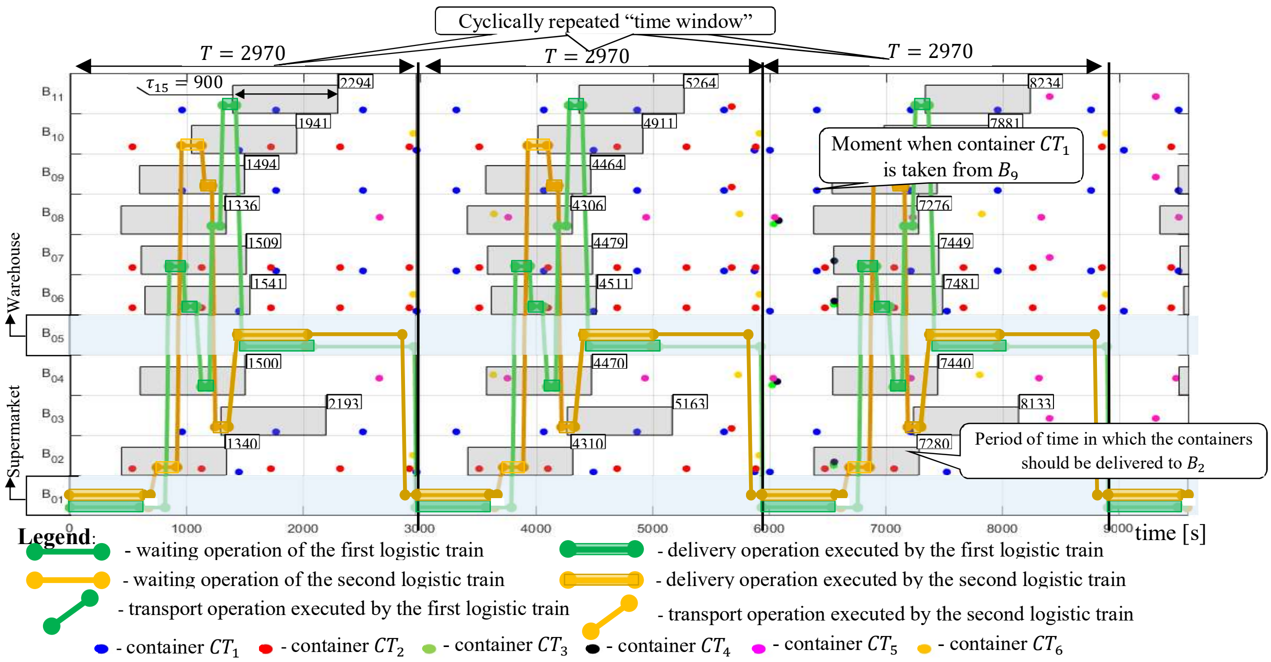

Fig. 3. Gantt chart of delivery schedule

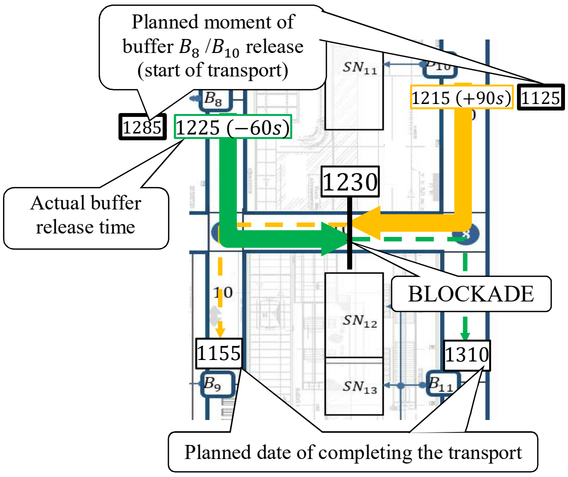

Fig. 4. Blockade resulting from deviations in the implementation of the planned schedule

Therefore, there is a need to synthesize such routes, when assuming a certain range of data uncertainty, will guarantee a collision-free and deadlock-free cyclical implementation of operations. As previously mentioned, available approaches [2],[3] to determine such routes (implementing declarative programing techniques) are limited only to those situations with precise data. Accounting for data uncertainty through adaptations in these fuzzy variable models is difficult because of imperfections of classical fuzzy numbers algebraic [1]. Relations describing the relationships between fuzzy variables (variables assuming fuzzy values) on algebraic operations (in particular, addition and multiplication) do not meet the conditions of the Ring (among others if the condition $\forall_{A \in \mathcal{F}}\ A + 0 = A$ is met then condition $\forall_{A \in \mathcal{F}}\ \exists!_{B \in \mathcal{F}}\ A + B = 0$ is not achieved). In addition, algebraic operations based on standard fuzzy numbers follow Zadeh's extension principle. In practice, this means that no matter what algebraic operations we deploy, the support of the fuzzy number expands (the imprecision of information increases). Consequently, it is impossible to solve algebraic equations on fuzzy variables. In particular, it means that for any fuzzy numbers a, b, c the following implication $(a + b = c) \Rightarrow [(c - b = a) \wedge (c - a = b)]$ does not hold. This makes it impossible to solve a simple equation $A + X = C$. This fact dramatically hinders the use of approaches based on declarative models, in which most of the relationships between decision variables are described in the form of linear/nonlinear equations and/or algebraic inequalities. There are various approaches in the literature that circumvent the abovementioned drawbacks [1], [12], [27], but they are quite complex to implement.

In this paper we propose a declarative model of milk-run system implementing formalism of OFN algebra, which assumes the existence of a neutral element (zero) for operations such as addition and multiplication to ensure it can be solved using algebraic equations.

IV. REFERENCE MODEL OF MILK-RUN DESIGN

The developed model uses decision variables that adopt values in the form of OFNs defined as Definition 1. For the needs of the developed model, an ordered fuzzy number $\hat{A}$ is specified by sequences $f_A{}'$ and $g_A{}'$ containing values of functions f_A and g_A obtained as a result of interval $[0,1]$ discretization, i.e.

$$f_A{}' = (f_A(0), f_A(dx), \dots, f_A((M-1)\,dx), f_A(1)), \quad (3)$$

$$g_A' = (g_A(1), g_A((M-1)dx), \dots, g_A(1dx), g_A(0)), dx = \frac{1}{M}, \quad (4)$$

where $(M+1)$ is the number of discrete points (Fig. 1c). An example of discretization of OFN is displayed in Fig. 1c. In the case of positive orientation the support of $\hat{A}$ is $sup_{\hat{A}} = (f_A(0), g_A(0))$ in the negative orientation $sup_{\hat{A}} = (g_A(0), f_A(0))$. In the considered model, the input data has a positive orientation and the decision variables are positive / negative. Adopting such an OFN representation allows the implementation of the above defined operations, see Def. 2-3.

Declarative model

Previously introduced terminology and designations concerning OFN as well as the following notation are used in the course of the Milk-Run model development:

Symbols:

- B_λ: λ-th buffer.
- LT_v: v-th logistic train.
- o_λ: operation of delivery/loading/unloading of materials to/at buffer B_λ.

Parameters:

Crisp parameters:

- ω: a number of buffers in the milk-run system considered.
- ln: a number of logistic trains.
- $\mathcal{K}_{\varepsilon\beta-\lambda\gamma}$: a binary variable used to indicate the crossed paths:

$$\mathcal{K}_{\varepsilon\beta-\lambda\gamma} = \begin{cases} 1 & \text{if the path } B_\varepsilon - B_\beta \text{ crosses the path } B_\lambda - B_\gamma \\ 0 & \text{in the remaining cases} \end{cases}$$

Imprecise parameters: (defined as positive oriented OFN and marked by the symbol "^"):

- $\widehat{d_{\beta,\lambda}}$: execution time of a transport operation (the same for each logistic train) between buffers B_β and B_λ.
- $\widehat{t_\lambda}$: time of operation o_λ.
- $\widehat{dx_\lambda}$: delivery deadline of containers to buffer B_λ (see example in Fig. 3).
- $\widehat{\tau_\lambda}$: delivery margin, specifying the time period within which the delivery should be made (see Fig. 3).
- $\hat{T}$: a window width understood as a periodically repeated period of time in which deliveries must be made to all buffers (see Fig. 3).

Variables:

Crisp variables:

- rb_λ: an index of the operation that precedes o_λ (operations o_{rb_λ} and o_λ are executed by the same logistic train), $rb_\lambda = 0$ means that o_λ is the first operation on the route.
- rf_λ: an index of the operation that follows o_λ, (the delivery operations o_λ and o_{rf_λ} are carried out by the same logistic train).

Imprecise variables (positive/negative oriented OFN):

- $\widehat{x_\lambda}$: moment of commencement of the delivery operation o_λ on the buffer B_λ.
- $\widehat{y_\lambda}$: moment of completion of the operation o_λ on the buffer B_λ.
- $\widehat{xs_\lambda}$: moment of buffer B_λ releasing by the operation o_λ

Sets and sequences:

LT: set of logistic trains LT_v,

$\mathcal{B}$: set of buffers B_λ,

$\mathcal{O}$: set of delivery operations o_λ,

RB: sequence of predecessor indices of delivery operations, $RB = (rb_1, \dots, rb_\alpha, \dots, rb_{\omega+2(\ln -1)})$, $rb_\alpha \in \{0, \dots, \omega\}$,

RF: sequence of successor indices of delivery operations, $RF = (rf_1, \dots, rf_\alpha, \dots, rf_{\omega+2(\ln -1)})$, $rf_\alpha \in \{1, \dots, \omega\}$. For instance, sequences RB and RF that determine routes π_1 and π_2 are shown in Fig. 2, and take the following form:

	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	$B_1{}'$	$B_5{}'$
$RB =$ (	0,	1',	9,	6,	11,	7,	1,	4,	10,	2,	8,	0,	3)
$RF =$ (	7,	10,	5',	8,	1,	4,	6,	11,	3,	9,	5,	2,	1')

Values marked with the symbol ' refer to buffers associated with the supermarket and warehouse visited by the train LT_2

π_v: route of train LT_v, sequence of docking stations serviced by train LT_v, $\pi_v = \left(B_{v_1}, \dots, B_{v_i}, B_{v_{i+1}}, \dots, B_{v_\mu}\right)$, where: $v_{i+1} = rf_{v_i}$ for $i = 1..\mu - 1$ and $v_1 = rf_{v_\mu}$,

Π: set of routes π_v,

$\widehat{X'}$: sequence of moments $\widehat{x_\lambda}$: $\widehat{X'} = (\widehat{x_1}, \dots, \widehat{x_\lambda}, \dots, \widehat{x_\omega})$,

$\widehat{Y'}$: sequence of moments $\widehat{y_\lambda}$: $\widehat{Y'} = (\widehat{y_1}, \dots, \widehat{y_\lambda}, \dots, \widehat{y_\omega})$,

$\widehat{Xs'}$: sequence of moments $\widehat{xs_\lambda}$: $\widehat{Xs'} = (\widehat{xs_1}, \dots, \widehat{xs_\lambda}, \dots, \widehat{xs_\omega})$,

$\hat{X}$: a fuzzy cyclic schedule of the operations executed in milk-run cycles: $\hat{X} = \left(\widehat{X'}, \widehat{Y'}, \widehat{Xs'}\right)$.

Constraints:

- constraints describing the orders of operations depending on the logistic train routes (specified by RB and RF):

$$\widehat{y_\lambda} = \widehat{x_\lambda} + \widehat{t_\lambda}, \lambda = 1,2, \dots, \omega, \tag{5}$$

$$rb_\lambda = 0, \forall \lambda \in BS \subseteq BI = \{1, \dots, \omega\}, |BS| = ln, \tag{6}$$

$$b_\lambda \neq rb_\beta,\ \forall \lambda, \beta \in BI \backslash BS,\ \lambda \neq \beta, \tag{7}$$

$$rf_\lambda \neq rf_\beta,\ \ \forall \lambda, \beta \in BI,\ \lambda \neq \beta, \tag{8}$$

$$(rb_\lambda = \beta) \Rightarrow (rf_\beta = \lambda), \forall b_\lambda \neq 0, \tag{9}$$

$$\widehat{xs_\lambda} \geq \widehat{y_\lambda}, \lambda = 1,2, \dots, \omega, \tag{10}$$

$$\left[(f_\lambda = \beta) \wedge (b_\beta = 0)\right] \Rightarrow \left(\widehat{xs_\lambda} = \widehat{x_\beta} + \hat{T} - \widehat{d_{\lambda,\beta}}\right), \tag{11}$$

$$\left[(f_\lambda = \beta) \wedge (b_\beta \neq 0)\right] \Rightarrow \left(\widehat{xs_\lambda} = \widehat{x_\beta} - \widehat{d_{\lambda,\beta}}\right), \tag{12}$$

$$\lambda, \beta = 1,2, \dots, \omega,$$

- if the path connecting buffers B_ε, B_β crosses the path that connects the buffers B_λ, B_γ ($\mathcal{K}_{\varepsilon\beta-\lambda\gamma} = 1$) then:

$$\left(\mathcal{K}_{\varepsilon\beta-\lambda\gamma} = 1\right) \Rightarrow \left[\left(\widehat{x_\beta} \leq \widehat{xs_\lambda}\right) \vee \left(\widehat{x_\gamma} \leq \widehat{xs_\varepsilon}\right)\right], \tag{13}$$

$$\varepsilon, \lambda, \beta, \gamma = 1,2, \dots, \omega$$

- the delivery operation o_λ should be completed before the given delivery deadline dx_λ (with a margin $\widehat{\tau_\lambda}$) resulting from production flows of an individual product:

$$\widehat{y_\lambda} + c \times \hat{T}\ \leq\ \widehat{dx_\lambda},\ \lambda = 1,2, \dots, \omega, \tag{14}$$

$$\widehat{y_\lambda} + c \times \hat{T}\ \geq\ \widehat{dx_\lambda} + \widehat{\tau_\lambda},\ \lambda = 1,2, \dots, \omega; c \in \mathbb{N}. \tag{15}$$

V. PROBLEM FORMULATION

In the context of the above proposed model, the problem under consideration may be defined as follows:

Assuming that:

- there is a known collection of $\mathcal{B}$ buffers in which a supermarket stop and warehouse buffers are distinguished,
- given the set of delivery operations $\mathcal{O}$,
- given the fleet size of logistic trains LT,
- given are fuzzy values of transport operations $\widehat{d_{\beta,\lambda}}$ (they are assumed to be the same for each train),
- given are fuzzy values of delivery/loading/unloading operation times $\widehat{t_\lambda}$
- given are fuzzy values of containers delivery deadline $\widehat{dx_\lambda}$ and delivery margin $\widehat{\tau_\lambda}$,
- given a fuzzy value of time period $\hat{T}$, in which deliveries to all buffers are to be made the following question can be considered:

Does there exist a set of routes Π operated by the given fleet LT, which ensures that a fuzzy cyclic schedule $\hat{X}$ that guarantee timely delivery (with given deadlines $\widehat{dx_\lambda}$ and delivery margin $\widehat{\tau_\lambda}$) of the materials necessary for the production process to be completed? The above problem can be viewed as a Fuzzy Constraint Satisfaction Problem (FCSP) defined by (16):

$$\widehat{FCS} = \left(\left(\hat{\mathcal{V}}, \hat{\mathcal{D}}\right), \hat{\mathcal{C}}\right), \tag{16}$$

where: $\hat{\mathcal{V}} = \{\hat{X}, \Pi\}$ – a set of decision variables including: $\hat{X}$ – a fuzzy cyclic schedule: $\hat{X} = \left(\widehat{X'}, \widehat{Y'}, \widehat{Xs'}\right)$. Π – the set of routes determined by sequences RB, FR. $\hat{\mathcal{D}}$ – a finite set of decision variable domains: $\widehat{x_\lambda}$, $\widehat{y_\lambda}$, $\widehat{xs_\lambda} \in \mathcal{F}$ ($\mathcal{F}$ is a set of OFNs (1)), $rb_\lambda \in \{0, \dots \omega\}$, $rf_\lambda \in \{1, \dots \omega\}$, $\mathcal{C}_{RE}$ – a set of constraints specifying the relationships between the operations implemented in milk-run cycles (5)-(15).

To solve $\widehat{FCS}$ (16), the values of the decision variables from the adopted set of domains for which the given constraints are satisfied must be determined. Implementation of $\widehat{FCS}$ in a constraint programming environment such as OzMozart, enables the answer to be determined. The implementation of the presented problem assumes that OFNs are described by linear functions f_A, g_A represented in discrete form as $f_A{}'$ and $g_A{}'$.

VI. COMPUTATIONAL EXPERIMENTS

Consider the milk-run system layout from Fig. 2. The goal is to find congestion-free routes of a given fleet of logistic trains (i.e. the set Π). It requires a cyclic supply of containers CT_1-CT_6 to buffers B_1-B_{11} in time windows with a width of $\hat{T} = 2970$ [s] (in considered case $\hat{T}$ is defined as a singleton – the OFN with strict neutral direction). The amount of delivered containers is collected in tab. 1, and the expected delivery deadlines (fuzzy values of $\widehat{dx_\lambda}$ and $\widehat{\tau_\lambda}$) reported in Fig. 5a.

Table. 1. Number of containers nc delivered to buffers B_1-B_{11}

	B_1*	B_2	B_3	B_4	B_5*	B_6	B_7	B_8	B_9	B_{10}	B_{11}
nc	-	17	11	7	-	10	12	7	6	17	11

* – it is assumed that buffers identical to supermarket B_1 and warehouse B_5 are not described by the appropriate number of containers

In the considered version of the system, it is assumed that the available vehicle fleet consists of two logistic trains LT_1 and LT_2. It is also assumed that the fuzzy times of delivery operation ($\widehat{t_\lambda}$) follow those presented in Fig. 5b and the admissible fuzzy travel times ($\widehat{d_{\beta,\lambda}}$) presented in Fig 5c.

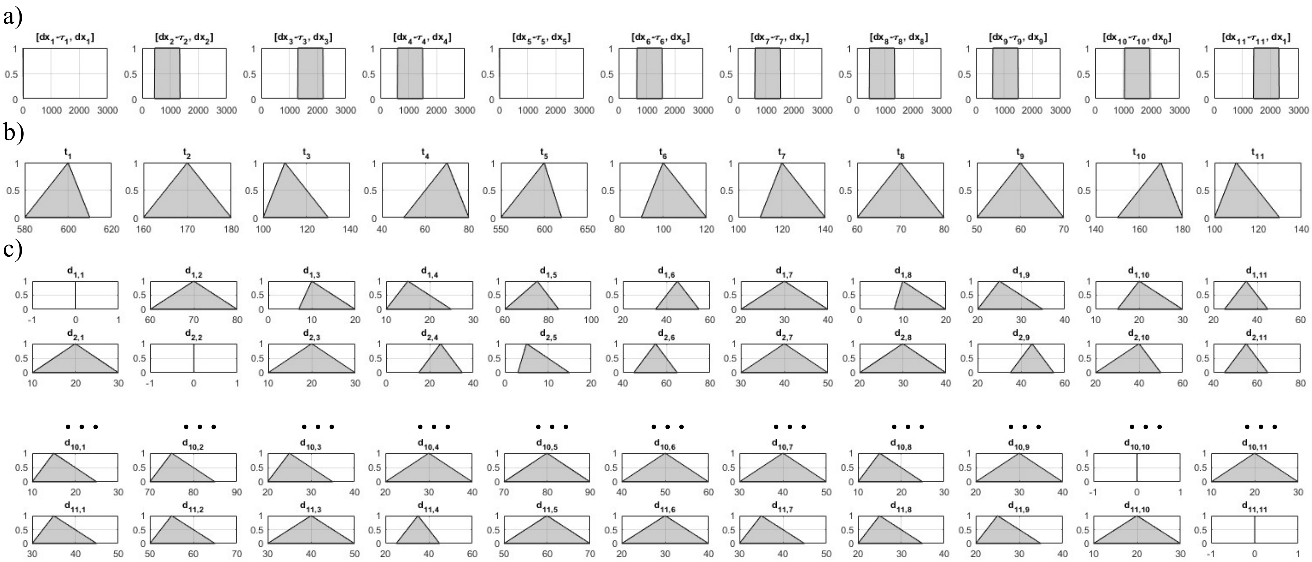

Fig. 5. Input data specifying: delivery time windows a), loading times b), periods of train moving between a pair of buffers c).

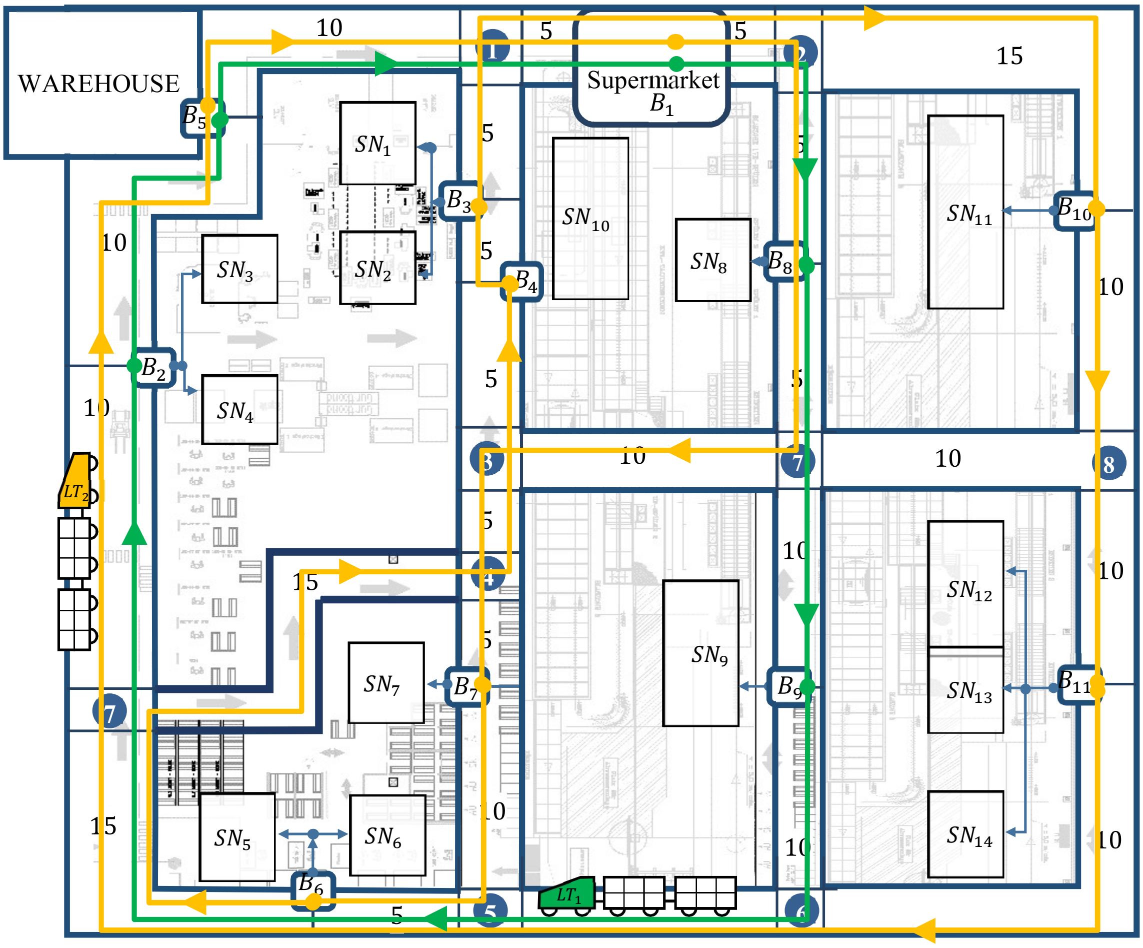

Fig. 6 Routes guaranteeing timely delivery

In this case, the answer to the following question is sought: *Does there exist a set of routes Π operated by the given two logistic trains LT_1 and LT_2, which ensures that a fuzzy cyclic schedule $\hat{X}$ that guarantee timely delivery (according to Fig. 5) of the materials necessary for the production process to be completed in the system shown in Fig.2?* To search for the answer to the above question, the problem $\widehat{FCS}$ (16) was formulated, and then implemented in the constraint programming environment OzMozart (Windows 10, Intel Core Duo2 3.00 GHz, 4 GB RAM). Its solution was obtained after 2 [s] of computation. The results are shown in graphical form in Figs. 6 and 7. The obtained values of sequences RB, RF lead to the following routes (Fig. 6): $\pi_1 = (B_1, B_8, B_9, B_2, B_5)$, $\pi_2 = (B_1, B_7, B_6, B_4, B_{10}, B_{11}, B_5)$. According to Tab. 1, the capacity of logistic trains equals: 30 and 68 containers, respectively. The obtained a fuzzy value of decision variables $\hat{X}$ and determined by them the cyclic schedule to guarantee the timely delivery of the containers (see Fig. 7). In the presented schedule, the execution of each operation is illustrated in the form of ribbon-like arterial road which increasing width determines time period of train movement resulting in the growing uncertainty of the moments of attachment and release of buffers. For example, the moment when buffer B_{11} can be occupied is determined by a fuzzy variable $\widehat{x_{11}}$ (Fig. 7a), whose support is an interval [1473s, 1750s] (i.e., the interval width equals to 277s), in turn the buffer release moment following $\widehat{y_{11}}$ has the support [1573s, 1880s] (i.e., with the interval width 307s). It is noteworth that the width of the ribbon-like arterial roads increases until the next time window begins. The uncertainty of decision variables is, however, reduced at the end of each time window as a result of the operation of waiting trains on buffers B_1, B_5. Consequently, increasing uncertainty is not transferred to subsequent cycles of the system. Uncertainty reduction is obtained as a result of implementation of OFNs formalism. Fuzzy variables describing the waiting time of trains on buffers B_1, B_5 have a negative orientation (see Fig. 8 – laytimes $\widehat{w_1}$ and $\widehat{w_5}$), which makes that the results of algebraic operations ($\widehat{xs_1} = \widehat{y_1} + \widehat{w_1}$ and $\widehat{xs_5} = \widehat{y_5} + \widehat{w_5}$) those using these variables will have less uncertainty. In practice, this means that the train operator shortens the buffer time in the event of long delays and extends it in case of earlier arrival. Achieving a similar phenomenon of reducing uncertainty is not possible in case of standard fuzzy numbers usage. According to the Zadeh's extension principle, uncertainty of variables should increase with subsequent cycles of system operation until the information about their value ceases to be useful. An example illustrating this situation is shown in Fig. 9 where the level of uncertainty increases with successive delivery time windows. Of course, it is apparent that the supply of containers to each of the buffers operates within a given period of time (see the time windows depicted in grey in Fig. 7b). It is worth noting that the adoption of such a schedule guarantees congestion-free movement of the logistic trains in context to uncertainty of parameters specified in Fig. 5. In order to verify the results obtained, a simulation of delivery performance was carried out in the system shown on Fig. 2. Two logistic trains operate along routes π_1 and π_2 (see Fig. 6). Travel times of trains between distinguished buffers ($\widehat{d_{\beta,\lambda}}$) and delivery times ($\widehat{t_\lambda}$) are assumed to be random variables given by triangular distributions of probability density whose parameters correspond to the variation ranges from Fig. 5.

The results of the simulation are reported in Fig. 8. For each of the buffers B_1-B_{11}, OFN of the starting moments ($\widehat{x_\lambda}$), termination ($\widehat{y_\lambda}$) of the delivery operation (o_λ), and the corresponding histograms are determined. The frames used to distinguish operations carried out on buffers B_7, B_9, B_{11} are shown in Fig. 7a. The green color charts are marked corresponding to the route operations π_1, and orange depicts route π_2. The charts were connected by arches illustrating algebraic relationships between individual variables. For instance, considering the train LT_2 route (π_2), the relations between variables describing operations performed on buffers B_8, B_9, B_2, B_5, B_1 are as follows:

$\widehat{x_9} = \widehat{y_8} + \widehat{d_{8,9}}$ (seizure of B_9 is possible after releasing B_8),
$\widehat{x_2} = \widehat{y_9} + \widehat{d_{9,2}}$ (seizure of B_2 is possible after releasing B_9),
$\widehat{x_5} = \widehat{y_2} + \widehat{d_{2,5}}$ (seizure of B_5 is possible after releasing B_2),
$\widehat{x_1} = \widehat{y_5} + \widehat{d_{5,1}}$ (seizure of B_1 is possible after releasing B_5),
$\widehat{x_8} = \widehat{xs_1} + \widehat{d_{1,8}} - \hat{T}$ (seizure of B_8 is possible after releasing B_1).

The presented relations above were used during the simulation and determined the histograms reflecting the distributions of random variables. All of the received histograms are included within calculated OFN values (see Fig. 8). In some cases, e.g. x_5, the obtained OFN value exceeds those reported in the histogram, implying that the results of the calculations carried out (being the solution of $\widehat{FCS}$ (16) problem) should be regarded as the upper limit of the values obtained. It should also be noted that none of the simulated variants (over 1 000 000 instances highlighted by bar charts on Fig. 8) included instances of no collision/blockage between trains and deliveries were carried out in given time windows.

VII. CONCLUDING REMARKS

This study demonstrated that the proposed reference model enabling FCSP formulation is a useful tool to state and resolve both routing and scheduling problems subject to constraints assumed by a given speed distribution of logistic trains. Adopting the travel times of logistic trains described by fuzzy numbers with known membership functions allows one to deliver a more realistic model of the movement of human-driven vehicles in milk-run systems. In summary, the proposed approach can replace the typical computer simulation methods of route prototyping with an analytical one employing an OFN formalism. As a consequence, it can be recognized as an outperforming solution approach for in-plant milk-run driven delivery problems. Looking to the future, we recognize two possible improvements. First, models could be extended to enable the design of proactive logistic trains fleet schedule that are robust to vehicle damages and/or production orders change. Second, there should be the development of sufficient conditions that would allow planners to reschedule milk-run flows while guaranteeing the smooth transition between two successive cyclic steady states corresponding to the current and rescheduled logistic train fleet flows.

REFERENCES

[1] Bocewicz G., Nielsen I., Banaszak Z. Production flows scheduling subject to fuzzy processing time constraints, International Journal of Computer Integrated Manufacturing, vol. 29(10), 2016, 1105-1127

[2] Bocewicz G., Nielsen P., Banaszak Z. Declarative Modeling of Milk-run Vehicle Routing Problem for Split and Merge Supply Streams Scheduling. Advances in Intelligent Systems and Computing (AISC). Springer, vol. 853, 2019, 157-172

[3] Bocewicz G., Bożejko W., Wójcik R., Banaszak Z. Milk-run routing and scheduling subject to a trade-off between vehicle fleet size and storage capacity, Management and Production Engineering Review, vol. 10(3), 2019, 41–53 DOI: 10.24425/mper.2019.12959

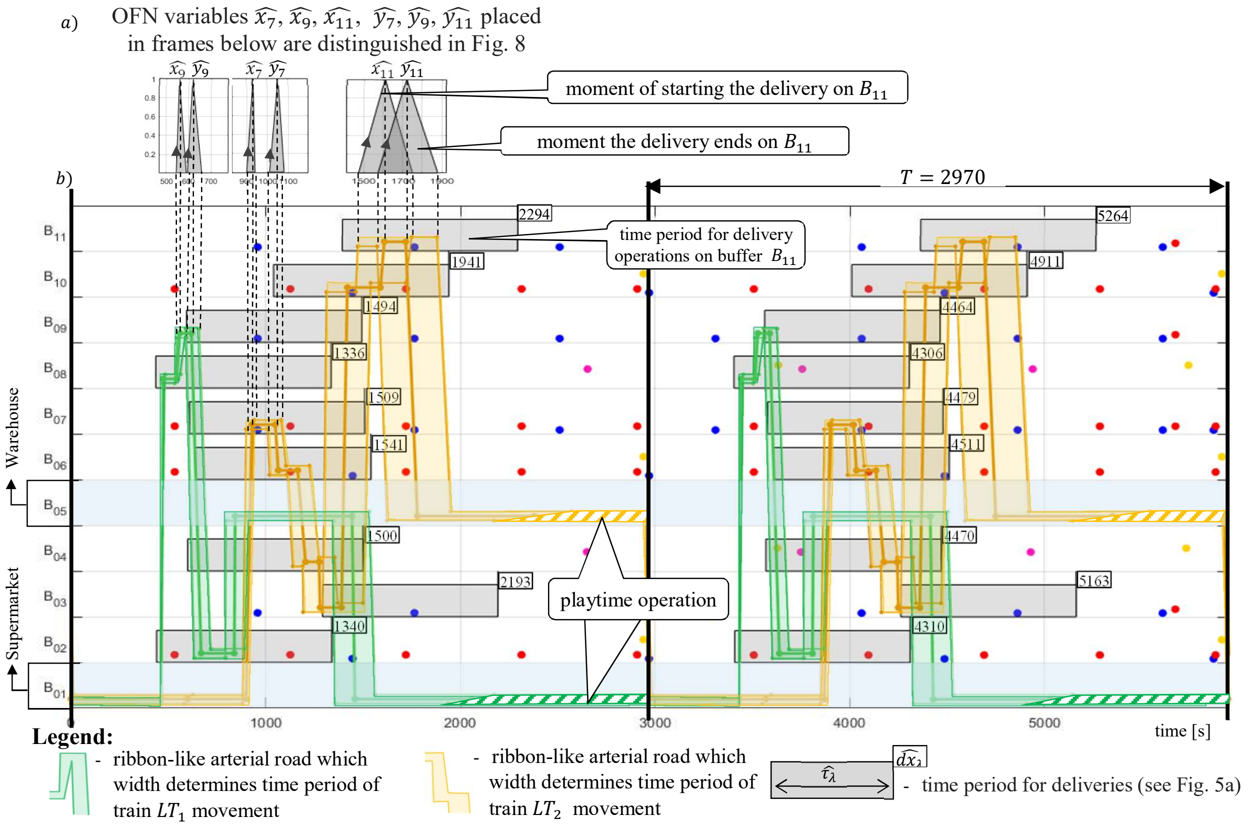

Fig. 7. Graphic illustrations of sample fuzzy variables a), Gantt's chart like of obtained cyclic fuzzy schedule b)

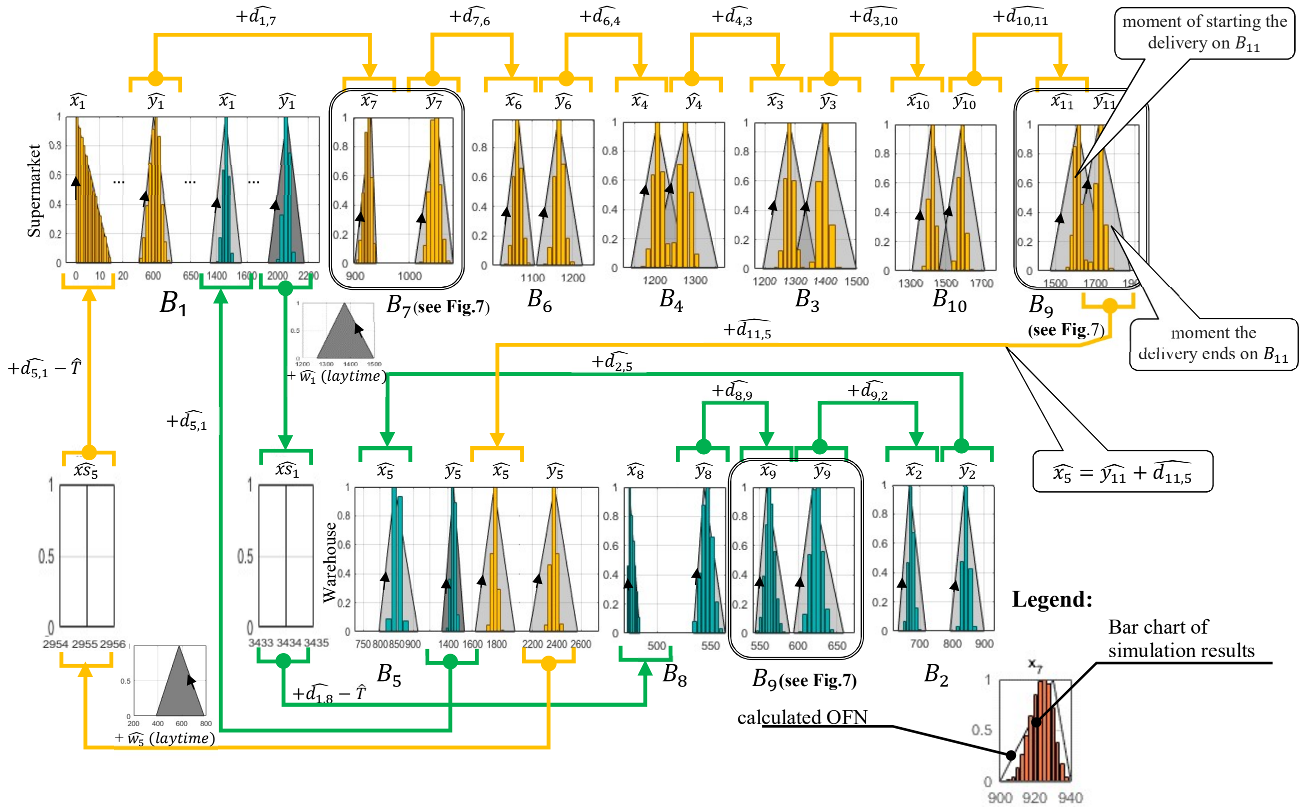

Fig. 8. Illustration of graphic summary of simulation results

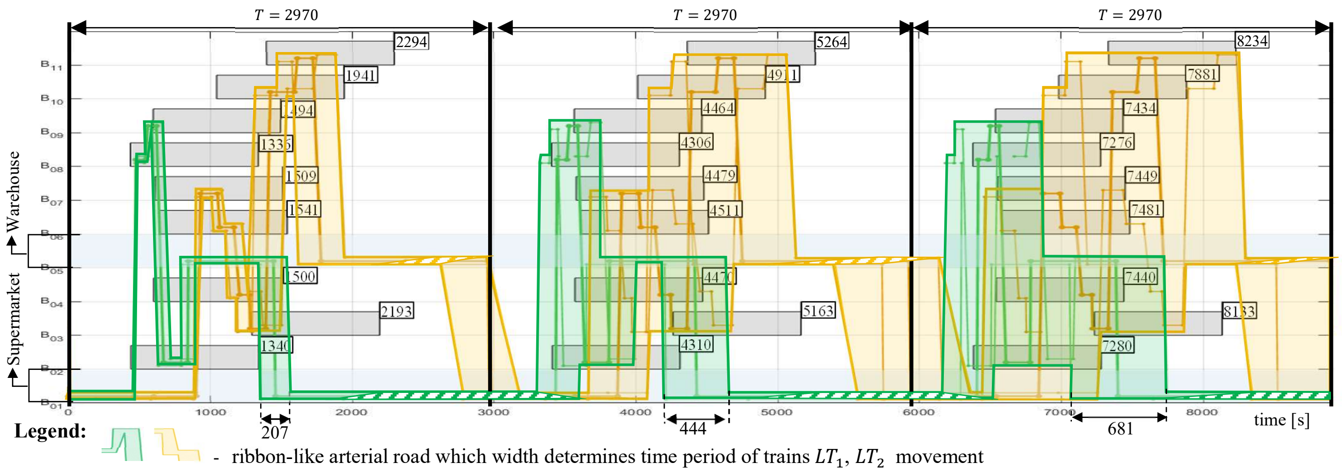

Fig. 9. An example illustrating growing uncertainty following successive delivery time windows, i.e. the results obtained from model employing standard fuzzy numbers for the data from Fig. 5.

[4] Carić T., Galić A., Fosin J., Gold H., Reinholz A. A Modelling and Optimization Framework for Real-World Vehicle Routing Problems Vehicle Routing Problem, In-The Croatia, 2008, 15-34

[5] Chwastyk A., Kosiński W. Fuzzy calculus with applications, Mathematica Applicanda, vol. 41 (1), 2013, 47-96

[6] Chwastyk A., Pisz I. Critical path analysis with imprecise activities times, Proceedings of the 32nd International Business Information Management Association Conference, 2018, 2004-2013

[7] Czerniak J.M., Dobrosielski W.T., Apiecionek Ł., Ewald D., Paprzycki M. Practical application of OFN arithmetics in a crisis control center monitoring, Studies in Computational Intelligence, 655, 2016, 51-64

[8] De Moura D.A., Botter R.C. Delivery and pick-up problem transportation – milk run or conventional systems. Independent Journal of Management & Productionm vol. 7 (3), 2016, 746-770

[9] Fedorko G., Vasil M., Bartosova M. Use of simulation model for measurement of MilkRun system performance, Open Eng. vol. 9, 2019, 600-605, https://doi.org/10.1515/eng-2019-0067

[10] Güner A.R., Chinnam R.B. (2017) Dynamic routing for milk-run tours with time windows in stochastic time-dependent networks. Transportation Research Part E: Logistics and Transportation Review, vol. 97 (C), 2019, 251-267

[11] Kilic H.S., Durmusoglu M.B., Baskak M. Classification and modeling for in-plant milk-run distribution systems. Int. Journal of Advanced Manufacturing Technology. vol. 62(9-12), 2012, 1135–1146

[12] Klir G. J.: Fuzzy arithmetic with requisite constraints. Fuzzy Sets and Systems 91, 1997, 165–175,

[13] Kok A.L., Hans E.W., Schutten J.M.J. Optimizing departure times in vehicle routes, European Journal of Operational Research,vol. 210(3), 2011, 579-587

[14] Kosiński W., Prokopowicz P., Ślęzak D., Fuzzy numbers with algebrai coperations: algorithmic approach. Proc. IIS'2002, Sopot, June 3-6, PhysicaVerlag, 2002, 311-320

[15] Kosinski W., Prokopowicz P., Slezak D. On algebraic operations on fuzzy numbers. Proc. Int. Symp. IIS: IIPWM'03, Zakopane, Poland, Heidelberg: Physica Verlag, 2003, 353-362

[16] Marszałek A., Burczyński T. Ordered fuzzy GARCH model for volatility forecasting, Advances in Intelligent Systems and Computing, vol. 642, 2018,480-492

[17] Mácsay V., Bányai T. Toyota production system in milk-run based in-plant supply. Journal of Production Engineering. vol. 20, no 1, 2017, 141-146, http://doi.org/10.24867/JPE-2017-01-141

[18] Mei H., Jingshuai Y., Teng M., Xiuli L., Ting W. The Modeling of Milk-run Vehicle Routing Problem Based on Improved C-W Algorithm that Joined Time Window. Transportation Research Procedia. Elsevier, vol. 25, 2017,716–728

[19] Meyer A. , Milk Run Design (Definitions, Concepts and Solution Approaches), PhD thesis, Institute of Technology. Fakultät für Maschinenbau, KIT Scientific Publishing, Karlsruhe, 2015

[20] Nguyen P.K, Crainic T.G., Toulouse M., Multi-trip pickup and delivery problem with time windows and synchronization, Annals of Operations Research, Springer, vol. 253(2), 2017,899-934

[21] Novaes A.G.N, Bez E.T., Burin P.J., Aragão D.P. Dynamic milk-run OEM operations in over-congested traffic conditions Computers & Industrial Engineering, vol. 88, 2015,326-340

[22] Polak M., Majdzik P., Banaszak Z., Wójcik R. The performance evaluation tool for automated prototyping of concurrent cyclic processes. Fundamenta Informaticae. vol. 60, (1-4), 2004, 269-289

[23] Prokopowicz P., Ślęzak D. Ordered fuzzy numbers: Sources and intuitions, Studies in Fuzziness and Soft Computing, 356, 2017, 47-56

[24] Prokopowicz P. Processing the Direction with Ordered Fuzzy Numbers. Theory and Applications of Ordered Fuzzy Numbers, Studies in Fuzziness and Soft Computing, vol. 356, 2017, 81-98

[25] Prokopowicz P., Mikołajewski D., Mikołajewska E., Tyburek K. Modeling trends in the hierarchical fuzzy system for multi-criteria evaluation of medical data. Advances in Intelligent Systems and Computing, vol. 643, 2018,207-219

[26] Rudnik K., Kacprzak D. Fuzzy TOPSIS method with ordered fuzzy numbers for flow control in a manufacturing system, Applied Soft Computing Journal, vol. 52, 2017, 1020-1041

[27] Sanchez E.: Solutions of fuzzy equations with extended operations. Fuzzy Sets and Systems 12, 1984, 237–248,

[28] Seybold L., Witczak M., Majdzik P., Stetter, R. Towards robust predictive fault–tolerant control for a battery assembly system. International, J. of App. Math. and Comp. Sci., vol. 25(4), 2015, 849-862

[29] Schmidt T., Meinhardt I., Schulze F. New Design Guidelines for in-plant Milk-run Systems, 14th IMHRC Proceedings, Karlsruhe, Germany, 25, 2016,

[30] Sitek P., Wikarek J. Capacitated Vehicle Routing Problem with Pick-up and Alternative Delivery (CVRPPAD) – model and implementation using hybrid approach, Annals of Operations Research, 2017, DOI: https://doi.org/10.1007/s10479-017-2722-x

[31] Smutnicki C. Minimizing cycle time in manufacturing systems with additional technological constraints. Proc. of 22nd International Conference on Methods and Models in Automation & Robotics 463-470, 2017,https://doi.org/10.1109/MMAR.2017.8046872

[32] Toth P., Vigo D. The Vehicle Routing Problem. Siam, Philadelphia. 2002, http://dx.doi.org/10.1137/1.9780898718515

[33] Zarzycki H., Czerniak J.M. Dobrosielski, W.T. Detecting nasdaq composite index trends with OFNs, Studies in Fuzziness and Soft Computing, vol. 356, 2017, 195-205