Milk-run Routing and Scheduling Subject to Fuzzy Pickup and Delivery Time Constraints: An Ordered Fuzzy Numbers Approach

Abstract— The design of logistic trains fleet oriented distributed and scalability-robust control policies that ensure deadlock-free operations is of crucial importance for efficient material handling systems. This study considers a multi-item assembly system where in-plant transport operations are organized in milk-run loops. A solution to a milk-run routing and scheduling problem subject to fuzzy pickup and delivery time constraints is developed. This type of problem can be treated in terms of a fuzzy constraint satisfaction problem, therefore, the main objective is to provide a reference model analytical formulas of which enable one to obtain solutions that do not require time-consuming computer simulations. Two versions of the model were parameterized assuming independent implementation of convex and ordered fuzzy numbers. The accuracy of both models was experimentally verified according to the results of multiple simulations. Results from this study provide an approach to avoid congestions while concurrently maintaining throughput at maximal achievable level.

Keywords— Routing, cyclic scheduling, logistic trains, fuzzy constraint satisfaction problem, ordered fuzzy number (OFN)

I. INTRODUCTION

The milk-run driven delivery concept [2], [11], [17] boiling down to NP-hard problems of logistic trains routing and scheduling problems attempts to design plans to inform whom to serve, how much to deliver, and what regularly repeated routes to travel on by what fleet of vehicles. This can be viewed as belonging to the class of Vehicle Routing Problem [4], [18], [20]. Given this context, the objective of the present study is to develop a method, derived from the reference model, in support of using decision-support-system-like software. We employ the declarative modeling framework because of its fast prototyping capability. Importantly, declarative models focus primarily on developing the solution [2], [30]. The present study is a continuation of previous work that explored methods of fast prototyping of solutions associated with problems related to routing, batching and scheduling of tasks typically performed in batch flow production systems, as well as problems related to the planning and control of production flow in departments of automotive companies [2], [3]. This paper has three main contributions. First, representing a departure from the commonly accepted assumption of the deterministic nature of transport processes, this study incorporates human uncertainties (i.e., the logistic trains are driven by operators). Taking into account the related distribution of delivery moments enables the construction of more realistic and accurate models for assessing the effectiveness of prototyped route variants. Second, the declarative model-driven approach to assess alternative milk-run routing and scheduling variants is formulated in detail. The obtained ordered fuzzy number-driven model enables searching for logistic trains congestion-free routes in terms of fuzzy constraint satisfaction problem. Third, the proposed approach allows the replacement of computer simulation methods of routes prototyping with an analytical method based on an ordered fuzzy number formalism. In that context it can be recognized as an outperforming solution approach for in-plant milk-run driven delivery problems. This paper is organized as follows. Section 2 presents a review of selected literature. This section also provides basic information about Ordered Fuzzy Numbers (OFNs). A motivating example to highlight the problem under consideration is discussed in Section 3. Section 4 details a declarative model dedicated to the prototyping of milk-run traffic systems. Consequently, the Fuzzy Constraint Satisfaction Problem aimed at logistic trains fleet delivery mission planning is formulated in Section 5. Next, Section 6 depicts how the model can be used in supply cycles prototyping tasks, and section 7 provides reports key conclusions and makes suggestions for future research.

II. LITERATURE REVIEW

A. Milk-run routing and time scheduling

In a milk-run system, routes, time schedules, type and number of parts to be transported are assigned to different...
logistics trains so that they can collect orders from different suppliers [11]. In other words, the trains perform several pick-ups/deliveries in a round trip to meet customer demands. The benefits of using a system of this type include improved efficiency of the overall logistics system and substantial potential savings in environmental and human resources, as well as remarkable cost advantages related to inventory and transportation costs [8], [19], [29]. The Milk-run Vehicle Routing Problem [18] is viewed as a special case of the Vehicle Routing Problem (VRP) [4], [20], which in turn is a generalization of the Traveling Salesman Problem seeking to find the optimal set of routes for a fleet of vehicles delivering goods or services to various locations. For this reason, there is a large body of (OR) literature on vehicle scheduling, routing, and dispatching that address relevant aspects of transportation of goods. However, there are only a few papers devoted specifically to in-plant milk-run traffic problems. In this respect, the most relevant are areas subject to critical and often unpredictable traffic congestions. This typically occurs when logistic operators allocate too many collecting tasks to available vehicles, thus generating unperformed activities due to assumed just-in-time constraints and violating contractual obligations assumed with their clients [21]. Most of the research in the field of distribution logistics is devoted to the analysis of the methods of organizing transport processes in ways that minimize the size of the fleet, the distance traveled (energy consumed), or the space occupied by a distribution system. By focusing on the search for optimal solutions, these studies implicitly assume that there exist admissible solutions; in other words, solutions that ensure collision-free and/or deadlock-free (congestion-free) flow of concurrent transport processes. In practice, this requires either on-line updating (revision) of the routing policies used, or prior (offline) planning of congestion-free vehicle routes and schedules. Studies on generating dynamic routing policies are conducted sporadically [4]; even less frequent are investigations of robust routing and scheduling of milk-run traffic, which are, by and large, limited to AGV systems. The congestion avoidance problem, which conditions the existence of admissible solutions, is considered an NP-hard problem [13], [32]. Because the necessary and sufficient conditions for deadlock-free execution of concurrent processes are not known, system analysis (i.e. analysis of the states potentially leading to system deadlocks) is most frequently performed using laborious and time-consuming computer simulations [4], [10]. In practical applications, congestion avoidance methods implement the sufficient conditions for collision-free execution of processes. This implies that the time-consuming method of analyzing distribution networks that detect deadlocks between concurrent transport flows can be replaced by searching for a synchronization mechanism that would guarantee cyclic execution of these flows. Methods that are most commonly used for such purposes include the formalism of max-plus algebra [22], [28], graph theory [31], simulations [9] and constraint programming [2], [30]. It should be noted that the possibility of fast implementation of the process-synchronization mechanism comes at the expense of omitting some of the potentially possible scenarios for deadlock-free execution of the processes.

The shortcomings of the above-described methods to generate admissible solutions can restrict their implementation in DSS systems, in particular those supporting planning in milk-run traffic systems. To address this challenge, our contribution assesses the use of declarative modeling methods (e.g. [3]) in solutions that provide interactive decision support for prototyping in-plant milk-run traffic systems.

B. Ordered fuzzy numbers

In the proposed declarative modeling method, the imprecise variables adopt values in the form of OFNs. The concept of OFNs, defined by Kosiński, Prokopowicz and Ślęzak [14], was proposed as a response to problems related to the application of fuzzy numbers [5], [15]. The OFN are defined as follows [15]:

**Definition 1.** An OFN is defined as a pair of continuous real functions specified on an interval $[0, 1]$, i.e.: $$\tilde{A} = (f_A, g_A), \text{ where: } f_A, g_A : [0, 1] \rightarrow \mathbb{R}. \quad (1)$$

The functions $f_A$ and $g_A$ are called the up and down part of ordered fuzzy number $\tilde{A}$, respectively. They are also called as branches of fuzzy number $\tilde{A}$. The values of these continuous functions are limited ranges, which can be defined as the following bounded intervals: $UP_\tilde{A} = \{l_A, u_A\} \text{ and } DOWN_\tilde{A} = \{p_{A1}, p_{A2}\}$. Assuming that: $f_A$ is increasing and $g_A$ is decreasing as well as that $f_A \leq g_A$, the membership function $\mu_\tilde{A}$ of the ordered fuzzy number $\tilde{A}$ can be seen in Fig 1a and 1b:

$$\mu_\tilde{A}(x) = \begin{cases} f_A^{-1}(x) & \text{when } x \in UP_\tilde{A} \\ g_A^{-1}(x) & \text{when } x \in DOWN_\tilde{A} \\ 0 & \text{in the remaining cases} \end{cases} \quad (2)$$

![Fig. 1. a) OFN $\tilde{A}$ in terms of convex fuzzy numbers, b) functions $f_A, g_A$ determining $\tilde{A}$ (positive orientation), c) discrete representation of $\tilde{A}$ (dx = 0.25) (based on: [15])](image-url)

A large class of OFNs represents the whole class of convex fuzzy numbers with continuous membership functions. An additional property called orientation (direction) is defined for OFN. There are two types of orientation: positive when $\tilde{A} = (f_A, g_A)$ (direction $\tilde{A}$ matches the direction of the $OX$ axis) and negative when $\tilde{A} = (g_A, f_A)$ (direction $\tilde{A}$ matches the direction opposite to the $OX$ axis). Assuming that the values of all fuzzy variables have different orientations, this allows us to define algebraic operations that meet the listed conditions of the Ring. Due to the fact that the OFN is defined as the pair of...
functions \( f_A \), \( g_A \) the all algebraic operations performed on OFNs are also defined as relevant mathematical operations executed on these functions. The adopted definitions of relations and algebraic operations used in the developed model are as follows:

**Definition 2.** Let \( \hat{A} = (f_A, g_A) \) and \( \hat{B} = (f_B, g_B) \) be ordered fuzzy numbers. \( \hat{A} \) is equal to \( \hat{B} \) (\( \hat{A} = \hat{B} \)), \( \hat{A} \) is greater or equal or greater \( \hat{B} \) (\( \hat{A} \geq \hat{B} \)), \( \hat{A} \) is less or equal or less \( \hat{B} \) (\( \hat{A} \leq \hat{B} \)) if: \( \forall x \in [0,1] \) \( f_A(x) \geq f_B(x) \) and \( g_A(x) \leq g_B(x) \), where: the symbol \( \geq \) replaces suitably: \( =, \geq, \leq, < \).

**Definition 3.** [15]. Let \( \hat{A} = (f_A, g_A), \hat{B} = (f_B, g_B) \) and \( \hat{C} = (f_C, g_C) \) be ordered fuzzy numbers. The operations of adding \( \hat{C} = \hat{A} + \hat{B} \), subtraction \( \hat{C} = \hat{A} - \hat{B} \), multiplication \( \hat{C} = \hat{A} \times \hat{B} \) and division \( \hat{C} = \hat{A} / \hat{B} \) are defined as follows: \( \forall x \in [0,1] \) \( f_C(x) = f_A(x) \times f_B(x) \) \( g_C(x) = g_A(x) \times g_B(x) \), where: the symbol \( \ast \) replaces suitably: \( +, -, \times, \div \); The division operation is defined for \( \hat{B} \) such that \( |f_B| > 0 \) and \( |g_B| > 0 \) for each \( x \in [0,1] \).

In recent years, the concept of OFNs has been constantly evolving and applied to various practical applications. Many publications are devoted to the analysis of the OFN model and its confrontation with convex fuzzy sets [5], [7], [23]. After performing algebraic calculations on convex fuzzy numbers, the numbers’ support usually becomes extremely broad, so the information represented by this number has limited practicality. The OFNs do not have this drawback.

The OFN model has additional advantages beyond algebraic operations. The orientation of OFNs gives additional options in applications to represent imprecise values. The idea of a property of processing data called sensitivity to the direction was proposed by Prokopowicz [24], Zarzycki et al. [33] presents an efficient use of the OFN model in the description of processes undergoing dynamic changes. The trend of imprecise value as an OFN was also used in a critical path analysis [5]. In [7] a practical algorithm of OFN arithmetic in a crisis control center monitoring was presented. The trend modeling in the evaluation of medical data was presented in [18]. Lastly, Marszałek and Burczyński [16] proposed a Ordered fuzzy GARCH model for volatility forecasting.

Another branch of OFN applications development involves the use of multi-criteria decision making (MCDM) methods, as a development of popular approaches [26]. In MCDM methods, the OFN orientation differentiates the type of the criterion (cost and profit). According to our review of the literature and to the best of our knowledge, the approach proposed in this paper represents the first attempt to use OFNs for milk-run routing and scheduling.

III. ILLUSTRATIVE EXAMPLE

Here we consider a multi-item batch flow production system in which the in-plant transport operations of a set parts supply are organized in a milk-run loop passing through 14 work stations \( SN_1, ..., SN_{14} \) while servicing two assembly lines shown in Fig. 2. Consequently, two types of products \( W_1 \), \( W_2 \) are manufactured in the system, by batches of each kind of product are moved between the neighboring assembly stations by dedicated gantry robots. In subsequent steps, i.e., at different work stations, \( SN_i (i = 1, ..., 14) \), particular products are assembled from the parts delivered in containers to the work station buffers \( B_i - B_{14} \). Some buffers are shared by several stations e.g., buffer \( B_5 \) is shared by stations \( SN_5 \) and \( SN_6 \). The six types of parts packed in containers \( CT_1 - CT_6 \) are delivered to the buffers by the two logistic train \( LT_1 \) and \( LT_2 \) following the routes marked with a green and orange lines (see Fig. 2). Assembly stations and the buffers associated with them must be supplied with containers due to the schedule presented in Fig. 3. On the presented schedule, the moments of collecting containers from buffers are marked with points with colors that correspond to the containers \( CT_1 - CT_6 \). The grey ranges specify the periods in which the buffer stocks should be replenished, i.e., equal to \( 900 \)s. Exceeding the deadlines \( dx \) (see Fig. 3) may result in the lack of the required deliveries, which may further lead to production suspension. The schedule demonstrates that deliveries must be made within cyclically repeated time windows (with size: \( T = 2970 \)s). The logistic trains (\( LT_1 \) and \( LT_2 \)) traveling along the fixed routes are used as in-plant means of transport to deliver the required quantity of parts to buffers within the given time windows. In general, the objective of this study is to find a method that allows for the congestion-free travel of the logistic trains in order to guarantee the timely delivery of the ordered part sets. In other words, the answer to the following question is researched: Does there exist, in the given system, a set of routes of logistic trains and the associated delivery schedules that guarantee timely delivery of the materials necessary for the production process to be completed? Many approaches reported in the literature enable answering this question for systems of scale encountered in practice (number of serviced points \( \leq 15 \), logistic fleet size \( \leq 5 \)) [2], [3]. An example of a route that guarantees timely delivery and the resulting schedule is presented in Figs. 2 and 3. The routes received (s sequences visited sequentially by \( LT_1 \) and \( LT_2 \) buffers) are in the form: \( \pi_1 = (B_1, B_2, B_6, B_4, B_9, B_{11}, B_3), \pi_2 = (B_1, B_2, B_{10}, B_9, B_3, B_5) \).

These routes guarantee a collision-free and deadlock-free delivery. It is worth emphasizing, however, that the techniques used to determine them (Constraints Programming CP and Mixed Integer Programming MIP) require precisely defined values of system parameters, and the exact value of transport times between buffers is required, as well as loading/unloading times. Transport operations in milk-run systems are usually carried out by people - the logistic train is run by an employee. This means that planned times assumed for loading/unloading operations, as well as transportation operations, can be quite uncertain. Uncertainty of the duration of the operation results in uncertain moments of occupation and release of stops. As a result, the actual implementation of the schedule may differ significantly from the planned one. Therefore, even minor deviations from the plan can result in serious consequences such as blocking.

Figure 4 illustrates the situation where the 90s delay (relative to the deadline resulting from the planned schedule in Fig. 3) of train \( LT_1 \) coupled the simultaneous acceleration of train \( LT_2 \) by 60s results in a blockade in the sector \( \oplus - \odot \) (the moment of blockage 1230s).
A sector in which blockage may occur (Fig. 4) in the event of failure to meet the deadlines set out in the schedule of Fig. 3.

Fig. 2. Layout of considered Milk-Run System

Cyclically repeated “time window”

Legend:
- 1st buffer
- transportation sector
- work station SN
- production flow for product
- logistic trains LT₁ and LT₂

Fig. 3. Gantt chart of delivery schedule

Legend:
- waiting operation of the first logistic train
- delivery operation executed by the first logistic train
- transport operation executed by the first logistic train
- container CT₁
- container CT₂
- container CT₃
- container CT₄
- container CT₅
- container CT₆

Moment when container CT₁ is taken from B₉

Period of time in which the containers should be delivered to B₁
The developed model uses decision variables that adopt values such as addition and multiplication to ensure it can be solved. Assumptions the existence of a neutral element (zero) for operations makes it complex to implement due to the abovementioned drawbacks [1], [12], [27], but they are quite restrictive. Therefore, there is a need to synthesize such routes when assuming a certain range of data uncertainty, will guarantee a collision-free and deadlock-free cyclical implementation of operations. As previously mentioned, available approaches [2], [3] to determine such routes (implementing declarative programing techniques) are limited only to those situations with precise data. Accounting for data uncertainty through adaptations in these fuzzy variable models is difficult because of the imperfections of classical fuzzy numbers algebraic [1]. Relations describing the relationships between fuzzy variables (variables assuming fuzzy values) on algebraic operations (in particular, addition and multiplication) do not meet the conditions of the Ring (among others if the condition $\forall a \in \mathbb{F} \quad A + 0 = A$ is met then condition $\forall a \in \mathbb{F} \exists ! b \in \mathbb{F} \quad A + B = 0$ is not achieved). In addition, algebraic operations based on standard fuzzy numbers follow Zadeh’s extension principle. In practice, means that no matter what algebraic operations we deploy, the support of the fuzzy number expands (the imprecision of information increases). Consequently, it is impossible to solve algebraic equations on fuzzy variables. In particular, it means that for any fuzzy numbers $a, b, c$ the following implication $(a + b = c) \Rightarrow [(c - b = a) \land (c - a = b)]$ does not hold. This makes it impossible to solve a simple equation $A + X = C$. This fact dramatically hinders the use of approaches based on declarative models, in which most of the relationships between decision variables are described in the form of linear/nonlinear equations and/or algebraic inequalities. There are various approaches in the literature that circumvent the abovementioned drawbacks [1], [12], [27], but they are quite complex to implement.

In this paper we propose a declarative model of milk-run system implementing formalism of OFN algebra, which assumes the existence of a neutral element (zero) for operations such as addition and multiplication to ensure it can be solved using algebraic equations.

IV. REFERENCE MODEL OF MILK-RUN DESIGN

The developed model uses decision variables that adopt values in the form of OFNs defined as Definition 1. For the needs of the developed model, an ordered fuzzy number $\hat{A}$ is specified by sequences $f_A$ and $g_A$ containing values of functions $f_A$ and $g_A$ obtained as a result of interval $[0, 1]$ discretization, i.e.

$$f_A' = (f_A(0), f_A(dx), \ldots, f_A((M - 1)dx), f_A(1)),$$

$$g_A' = (g_A(1), g_A((M - 1)dx), \ldots, g_A(1dx), g_A(0)),$$

where $(M + 1)$ is the number of discrete points (Fig. 1c).

An example of discretization of OFN is displayed in Fig. 1c. In the case of positive orientation the support of $\hat{A}$ is $\sup A = (f_A(0), g_A(0))$ in the negative orientation $\inf A = (g_A(0), f_A(0))$. In the considered model, the input data has a positive orientation and the decision variables are positive / negative. Adopting such an OFN representation allows the implementation of the above defined operations, see Def. 2-3.

Declarative model

Previously introduced terminology and designations concerning OFN as well as the following notation are used in the course of the Milk-Run model development.

Symbols:

- $B_i$: $i$-th buffer.
- $LT_{ij}$: $i$-th logistic train.
- $o_f$: operation of delivery/loading/unloading of materials to/at buffer $B_i$.

Parameters:

- $\omega$: a number of buffers in the milk-run system considered.
- $n$: a number of logistic trains.
- $X_{e,f-\lambda}$: a binary variable used to indicate the crossed paths:

$$X_{e,f-\lambda} = \begin{cases} 1 & \text{if the path } B_e - B_f \text{ crosses the path } B_1 - B_\lambda \\ 0 & \text{in the remaining cases} \end{cases}$$

Imprecise parameters: (defined as positive oriented OFN and marked by the symbol "^~"):

- $d_{g,A}$: execution time of a transport operation (the same for each logistic train) between buffers $B_i$ and $B_j$.
- $\xi$: time of operation $o_f$.
- $d_{x,A}$: delivery deadline of containers to buffer $B_1$ (see example in Fig. 3).
- $\rho$: delivery margin, specifying the time period within which the delivery should be made (see Fig. 3).
- $\Gamma$: a window width understood as a periodically repeated period of time in which deliveries must be made to all buffers (see Fig. 3).

Variables:

Crisp variables:

- $r_b$: an index of the operation that precedes $o_f$ (operations $o_{rb}$ and $o_f$ are executed by the same logistic train), $r_b = 0$ means that $o_{rb}$ is the first operation on the route.
- $r_f$: an index of the operation that follows $o_f$, (the delivery operations $o_f$ and $o_r$ are carried out by the same logistic train).

Imprecise variables (positive/negative oriented OFN):

- $x^1$: moment of commencement of the delivery operation $o_f$ on the buffer $B_1$.
- $y_i$: moment of completion of the operation $o_f$ on the buffer $B_i$.
- $x^2_i$: moment of buffer $B_i$ releasing by the operation $o_f$.
Sets and sequences:

- $LT$: set of logistic trains $LT_v$
- $B$: set of buffers $B_k$
- $O$: set of delivery operations $O_k$
- $RB$: sequence of predecessor indices of delivery operations, $RB = (rb_1, \ldots, rb_{n+1}, \ldots, rb_{m+1}(m-1))$, $rb_a \in \{0, \ldots, \omega\}$
- $RF$: sequence of successor indices of delivery operations, $RF = (rf_1, \ldots, rf_{n+1}, \ldots, rf_{m+1}(m-1))$, $rf_a \in \{1, \ldots, \omega\}$

In the context of the above proposed model, the problem under consideration may be defined as follows:

Assuming that:

- there is a known collection of $B$ buffers in which a supermarket stop and warehouse buffers are distinguished,
- given the set of delivery operations $O$,
- given the fleet size of logistic trains $LT$,
- given are fuzzy values of transport operations $d_{\beta,\lambda}^T$ (they are assumed to be the same for each train),
- given are fuzzy values of delivery/loading/unloading operation times $t_{\lambda}$
- given are fuzzy values of containers delivery deadline $d_{\lambda}^T$ and delivery margin $d_{\lambda}$
- given a fuzzy value of time period $T$, in which deliveries to all buffers are to be made the following question can be considered:

Does there exist a set of routes $\Pi$ operated by the given fleet $LT$, which ensures that a fuzzy cyclic schedule $\bar{X}$ that guarantee timely delivery (with given deadlines $d_{\lambda}^T$ and delivery margin $d_{\lambda}$) of the materials necessary for the production process to be completed? The above problem can be viewed as a Fuzzy Constraint Satisfaction Problem (FCS) defined by (16):

$$FCS = \left( (\bar{\phi}, \bar{B}), \bar{\lambda}, \bar{\beta} \right)$$

where: $\bar{\phi} = \{ \bar{\lambda}, \bar{\Pi} \}$ – a set of decision variables including: $\bar{\lambda}$ – a fuzzy cyclic schedule: $\bar{X} = (\bar{X}, \bar{Y}, \bar{S}', \bar{S})$; $\bar{\Pi}$ – the set of routes determined by sequences $RB, RF$. $\bar{\phi} –$ a finite set of decision variable functions: $\bar{X}_i, \bar{Y}_i, \bar{S}_i \in \mathcal{F}$ ($\mathcal{F}$ is a set of OFNs (1)), $rb_2 \in \{0, \ldots, \omega\}$, $rf_2 \in \{1, \ldots, \omega\}$.

VI. COMPUTATIONAL EXPERIMENTS

Consider the milk-run system layout from Fig. 2. The goal is to find congestion-free routes of a given fleet of logistic trains (i.e. the set $\Pi$). It requires a cyclic supply of containers $CT_1$-$CT_9$ to buffers $B_1$-$B_{11}$ in time windows with a width of $T = 2970$ s (in considered case $T$ is defined as a singleton – the OFN with strict neutral direction). The amount of delivered containers is collected in tab. 1, and the expected delivery deadlines (fuzzy values of $d_{\lambda}^T$ and $d_{\lambda}$) reported in Fig. 5a.

Table 1. Number of containers $nc$ delivered to buffers $B_i$ - $B_{11}$

<table>
<thead>
<tr>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$B_5$</th>
<th>$B_6$</th>
<th>$B_7$</th>
<th>$B_8$</th>
<th>$B_9$</th>
<th>$B_{10}$</th>
<th>$B_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>nc</td>
<td>17</td>
<td>11</td>
<td>7</td>
<td>10</td>
<td>12</td>
<td>7</td>
<td>6</td>
<td>17</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

* it is assumed that buffers identical to supermarket $B_1$ and warehouse $B_{11}$ are not described by the appropriate number of containers.

In the considered version of the system, it is assumed that the available vehicle fleet consists of two logistic trains $LT_1$ and $LT_2$. It is also assumed that the fuzzy times of delivery operation ($d_{\lambda}$) follow those presented in Fig. 5b and the admissible fuzzy travel times ($d_{\lambda}^T$) presented in Fig. 5c.
In this case, the answer to the following question is sought: Does there exist a set of routes $\Pi$ operated by the given two logistic trains $L_T^1$ and $L_T^2$, which ensures that a fuzzy cyclic schedule $\hat{X}$ that guarantees timely delivery (according to Fig. 5) of the materials necessary for the production process to be completed in the system shown in Fig. 2? To search for the answer to the above question, the problem $FCS$ (16) was formulated, and then implemented in the constraint.
programming environment OzMozart (Windows 10, Intel Core Duo 2 3.00 GHz, 4 GB RAM). Its solution was obtained after 2 s of computation. The results are shown in graphical form in Figs. 6 and 7. The obtained values of sequences \( RB, \) \( RF \) lead to the following routes (Fig. 6): \( \pi_1 = (B_1, B_9, B_6, B_5, B_3), \) \( \pi_2 = (B_1, B_9, B_6, B_4, B_1, B_{11}), \) According to Tab. 1, the capacity of logistic trains equals: 30 and 68 containers, respectively. The obtained a fuzzy value of decision variables \( \hat{x} \) and determined by them the cyclic schedule to guarantee the timely delivery of the containers (see Fig. 7). In the presented schedule, the execution of each operation is illustrated in the form of ribbon-like arterial road which increasing width determines time period of train movement resulting in the growing uncertainty of the moments of attachment and release of buffers. For example, the moment when buffer \( B_{11} \) can be occupied is determined by a fuzzy variable \( \hat{x}_{11} \) (Fig. 7a), whose support is an interval [1473s, 1750s] (i.e., the interval width equals to 277s), in turn the buffer release moment following \( y_{11} \) has the support [1573s, 1880s] (i.e., with the interval width 307s). It is noteworthy that the width of the ribbon-like arterial roads increases until the next time window begins. The uncertainty of decision variables is, however, reduced at the end of each time window as a result of the operation of waiting trains on buffers \( B_1, B_6, \) Consequently, increasing uncertainty is not transferred to subsequent cycles of the system. Uncertainty reduction is obtained as a result of implementation of OFNs formalism. Fuzzy variables describing the waiting time of trains on buffers \( B_1, B_6, \) have a negative orientation (see Fig. 8 – laytimes \( \bar{w}_1 \) and \( \bar{w}_6 \), which makes that the results of algebraic operations \( \hat{x}_{11} = y_{11} + \bar{w}_1 \) and \( \hat{x}_5 = y_{11} + \bar{w}_6 \) those using these variables will have less uncertainty. In practice, this means that the train operator shortens the buffer time in the event of long delays and extends it in case of earlier arrival. Achieving a similar phenomenon of reducing uncertainty is not possible in case of standard fuzzy numbers usage. According to the Zadeh’s extension principle, uncertainty of variables should increase with subsequent cycles of system operation until the information about their value ceases to be useful. An example illustrating this situation is shown in Fig. 9 where the level of uncertainty increases with successive delivery time windows. Of course, it is apparent that the supply of containers to each of the buffers operates within a given period of time (see the time windows depicted in grey in Fig. 7b). It is worth noting that the adoption of such a schedule guarantees congestion-free movement of the logistic trains in context to uncertainty of parameters specified in Fig. 5. In order to verify the results obtained, a simulation of delivery performance was carried out in the system shown on Fig. 2. Two logistic trains operate along routes \( \pi_1 \) and \( \pi_2 \) (see Fig. 6). Travel times of trains between distinguished buffers \( \delta_{B_{11}} \) and delivery times \( \hat{\delta}_1 \) are assumed to be random variables given by triangular distributions of probability density whose parameters correspond to the variation ranges from Fig. 5. The results of the simulation are reported in Fig. 8. For each of the buffers \( B_1, B_{11} \), OFN of the starting moments \( (\hat{x}_1) \), termination \( (\hat{y}_1) \) of the delivery operation \( (\hat{t}_1) \), and the corresponding histograms are determined. The frames used to distinguish operations carried out on buffers \( B_7, B_8, B_{11} \) are shown in Fig. 7a. The green color charts are marked corresponding to the route operations \( \pi_1 \), and orange depicts route \( \pi_2 \). The charts were connected by arches illustrating algebraic relationships between individual variables. For instance, considering the train \( LT_2 \) route \( (\pi_2) \), the relations between variables describing operations performed on buffers \( B_8, B_9, B_3, B_1 \) are as follows:

\[
\begin{align*}
\hat{x}_9 &= \hat{y}_9 + d_{9,8} \quad \text{(seizure of } B_8 \text{ is possible after releasing } B_9), \\
\hat{x}_2 &= \hat{y}_2 + d_{2,9} \quad \text{(seizure of } B_2 \text{ is possible after releasing } B_9), \\
\hat{x}_3 &= \hat{y}_3 + d_{2,5} \quad \text{(seizure of } B_5 \text{ is possible after releasing } B_2), \\
\hat{x}_3 &= \hat{y}_3 + d_{3,5} \quad \text{(seizure of } B_3 \text{ is possible after releasing } B_5), \\
\hat{y}_6 &= \hat{x}_3 + d_{1,6} \hat{t} \quad \text{(seizure of } B_3 \text{ is possible after releasing } B_1).
\end{align*}
\]

The presented relations above were used during the simulation and determined the histograms reflecting the distributions of random variables. All of the received histograms are included within calculated OFN values (see Fig. 8). In some cases, e.g. \( x_8 \), the obtained OFN value exceeds those reported in the histogram, implying that the results of the calculations carried out (being the solution of \( FCS \) (16) problem) should be regarded as the upper limit of the values obtained. It should also be noted that none of the simulated variants (over 1 000 000 instances highlighted by bar charts on Fig. 8) included instances of no collision/blockage between trains and deliveries were carried out in given time windows.

VII. CONCLUDING REMARKS

This study demonstrated that the proposed reference model enabling FCSP formulation is a useful tool to state and resolve both routing and scheduling problems subject to constraints assumed by a given speed distribution of logistic trains. Adopting the travel times of logistic trains described by fuzzy numbers with known membership functions allows one to deliver a more realistic model of the movement of human-driven vehicles in milk-run systems. In summary, the proposed approach can replicate the typical computer simulation methods of route prototyping with an analytical one employing an OFN formalism. As a consequence, it can be recognized as an outperforming solution approach for in-plant milk-run driven delivery problems. Looking to the future, we recognize two possible improvements. First, models could be extended to enable the design of proactive logistic trains fleet schedule that are robust to vehicle damages and/or production orders change. Second, there should be the development of sufficient conditions that would allow planners to reschedule milk-run flows while guaranteeing the smooth transition between two successive cyclic steady states corresponding to the current and rescheduled logistic train fleet flows.

REFERENCES


Fig. 7. Graphic illustrations of sample fuzzy variables a), Gantt’s chart like of obtained cyclic fuzzy schedule b)

Fig. 8. Illustration of graphic summary of simulation results
Fig. 9. An example illustrating growing uncertainty following successive delivery time windows, i.e. the results obtained from model employing standard fuzzy numbers for the data from Fig. 5.


