Modeling User Feedback: Fuzzy sampling, Portability, and Degree of Annoyance

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***Abstract*—User feedback is an important aspect of any social platform. The platform needs to adapt based on user preferences, eliminating the need for user input over time. This paper describes the experimentation with feedback received from users on their comfort level with temperatures in their work environment. The building energy management system will adapt itself so that users do not need to adjust the temperatures. This study addresses a number of issues with the user feedback modeling including managing a data imbalance with the help of fuzzy clustering, ability to transfer the user models between rooms and across different buildings, as well as, predicting a degree of discomfort. The experiments are based on user feedback collected from two commercial buildings.**

I. Introduction

Considerable effort is spent on understanding user needs while designing a system. This is important for systems that are software based or are physical facilities such as a commercial building. The real test of the design comes after the system is put into use. Ideally, all the users should be completely comfortable with the operations of the system from day one. However, in practice, many are irritated by certain aspects. There should be a mechanism for users to provide indirect feedback by manually adjusting the conditions to their liking. The system should record these behaviors and fine-tune itself so that the users do not need to make further adjustments. This paper demonstrates one such attempt using room temperatures in commercial buildings as an example.

One of the primary purposes of a building energy management system is energy conservation. However, the energy savings is rarely the primary goal. The productivity of occupants is essential. Improvement of the productivity of the occupants can be achieved by ensuring comfortable indoor environmental conditions. This paper describes the results of modeling the occupants' response to a system that centrally maintains comfortable temperatures in two buildings while optimizing HVAC (heating, ventilation, and air conditioning) related energy consumption. Individuals can provide feedback in order to raise or lower temperatures of their rooms. The objective of the study is to use the initial feedback to predict the individual preferences using machine learning techniques thereby eliminating the need for future feedback.

The study addresses different issues with user feedback modeling. The feedback data is imbalanced (i.e., there are many more data points in the time series with no feedback than the number of feedbacks). A novel fuzzy clustering based downsampling technique is shown to outperform conventional sampling. The paper also studies if preference modeling of one occupant can benefit from the preferences of other occupants in a building. Additionally, the work explores the possibility of transferring feedback predictions from one building to a different building. The data collection used in this study was conducted over two years with different collection techniques. In the first building, the occupants could only report whether they feel comfortable, cold, or hot. While in the second building the feedback is recorded by the degree of movement of the thermostat. These two different types of feedback mechanisms allow comparing the ability to predict the degree of discomfort. Finally, this report compares the ability of the system to recall all the feedback versus the accuracy of predictions.

II. Review of Literature

A. User feedback modeling

User feedback is an important aspect of system evolution. A system should improve its actions and recommendations by learning from user's reaction and feedback. User feedback modelling has been used extensively in information retrieval/search engines [1], recommender systems [2], ontology matching [3], and positioning [4]. There are various approaches proposed to utilize user feedback in a building energy management system including a solution based on using continuous user feedback without any machine learning models and sensors [5]. Another effort is focused on adjusting temperature setpoints based on the needs of individual users [6]. This study concentrates on evaluating machine learning models that use different weather and environmental conditions to maximize user feedback recall in a building energy management system.

B. Sampling and Clustering

Sampling: when models are trained to solve classification problems, it is desirable to have a balanced dataset so that a machine learning model could provide better fitting. This means that each of the classes in the dataset should have approximately the same number of observations. However, real-world cases are seldom that ideal and class imbalance

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issues have to be addressed before the training process. There are two major techniques that can be utilized to avoid class imbalance side effects: upsampling and downsampling.

Upsampling works by adding additional data points that are clones of the underrepresented classes until all classes have a nearly equal number of samples [7]. Upsampling can be done using the `preProcess` function from the Caret library[1] in R [8]. Caret implements upsampling by randomly replicating observations from the minority class until the number of samples in the minority class is roughly equivalent to the number of samples in the majority class. Since upsampling may introduce inaccuracies in a model, repeated cross-validation can be used to provide consistency of the resulting model [9].

On the other hand, downsampling solves the class imbalance issue by taking a subset of observations from the overrepresented class instead of using all observations from that class. The size of the subset of samples from the overrepresented class should approximately be equal to the number of samples in the underrepresented class [7]. Caret library in R also provides functionality to randomly downsample a dataset using `preProcess` function [8]. It is likewise necessary to perform repeated cross-validation in order to assure correctness of the resulting model.

Clustering: one of the most widely used clustering techniques is k-means clustering [10], [11]. The algorithm distributes a set of objects among k clusters. Crisp k-means clustering assigns an object to exactly one cluster. Fuzzy c-means clustering algorithm is a fuzzy generalization of the clustering that uses a fuzzy membership. The fuzzy membership function describes the degree of membership of an object to a given cluster ranging from 0 to 1 with the constraint that the sum of the fuzzy memberships of an object to all the clusters must be equal to 1 [12], [13].

Unsupervised Fuzzy Competitive Learning (UFCL) is a modification on the fuzzy c-means algorithm based on the unsupervised stochastic approximation that closely resembles fuzzy c-means but is capable of providing better results in some circumstances [14].

C. Evaluation metrics

The analysis of Receiver Operator Characteristic (ROC) curve is one of the most appropriate metrics of a models performance when solving classification problems [15]. The ROC curve is built based on the computation of the following performance measures of a particular model:

- The Recall (Sensitivity) of the model: the ratio of true positive predictions obtained to the total number of real positive events. This measurement is also known as the true positive rate of predictions [15].
- The Specificity: the ratio of true negative predictions obtained to the total number of real negative events. This measurement is also known as the true negative rate of predictions [15].

[1]a software package that provides various functions for solving classification and regression problems

The ROC curve is created by plotting the true positive rate of predictions versus false positive rate which is calculated using (1 − true negative rate). Ideally, the model would provide both high Recall and high Specificity, so the Area Under the Curve (AUC) is maximized. When the AUC value approaches 1, it means that the quality of the classification is nearly perfect. If it is close to 0.5, the model generally cannot distinguish between negative and positive classes. Values of AUC less than 0.5 indicate that the model is likely to treat positive classes as negative ones and vice versa [15]. Since AUC usually calculated for two classes and the paper works with multiclass classification, AUC results computed as the arithmetic mean of AUC collected for each class using one-vs.-rest approach.

III. Study Data

Observations of various conditions are recorded with fifteen-minute intervals in two buildings. In the first building, the data was collected from eighteen rooms on three floors and there are three types of feedback: "Comfortable", "Too Cold", "Too Warm". A total of 133 feedbacks was received: (Comfortable, 3), (Too cold, 79), (Too warm, 51), from 218,629 observations. When users do not provide any feedback, the "Comfortable" feedback is assumed. This means that 99.94% of the observations correspond to the "Comfortable" class. Table I shows the distribution of feedback from each room in the first building.

TABLE I
Feedback summary by room.

Room	Comfortable	Too Cold	Too Warm
118	0	6	0
119	0	5	0
120	0	1	4
206	0	2	0
207	0	2	0
226	0	3	0
228	0	0	22
232	0	0	1
234	0	3	0
235	0	11	5
300	1	11	0
301	0	0	15
306	1	11	1
320	1	2	0
326	0	1	0
328	0	0	2
332	0	20	0
334	0	1	0

In the second building, the data was collected from ten rooms where temperature adjustments made by the users were recorded by thermostats. In this case, feedback is a real number ranging from -4.2 to 4.0. A total of 124 feedbacks was received from 152,630 observations. Similarly, it is assumed that feedback is "Comfortable" or 0 if the users do not make any temperature adjustments. This means that 99.92% of the observations correspond to the "Comfortable" class. For both buildings the independent variables are:

- Date
- Time
- Weather variables (temperature, humidity, solar radiation, etc.)
- Room and building characteristics (e.g., if it has a north-facing window)
- Room state variables (e.g., if the room is occupied)

IV. SAMPLING USING CLUSTERING

While it would be beneficial to use all observations for analysis, it may not be feasible with existing computational resources. Furthermore, it can be hard to collect all observations. In most cases, the main purpose of creating machine learning models is to get high precision or accuracy of predictions. But 99.94% of the observations are classified as "Comfortable", which means that predicting everything as Comfortable will give us the accuracy of 99.94%. Practically, it would be desirable to cover as many feedbacks that belong to either "Too cold" or "Too warm" class as possible. Thus, maximizing the recall of the classes "Too cold" and "Too warm" is more important in this case than maximizing accuracy. In order to balance the dataset, downsampling is used to decrease the original 218,499 observations from the "Comfortable" class to approximately 80. Another option is to expand the number of "Too cold" and "Too warm" observations to 218,499. Nevertheless, downsampling may be more suitable for maximizing recall [16]. Downsampling with a random selection of observations from the "Comfortable" class may not work well with respect to covering the whole set of observations from the class. Therefore, clustering is proposed to divide the observation space for the "Comfortable" class. The specimen closest to the center of each cluster will be treated as an observation for the "Comfortable" class.

The clustering-based downsampling can be implemented by creating clusters using a k-means clustering algorithm. The number of clusters is defined by the desired number of observations from the "Comfortable" class. Fuzzy c-means clustering may potentially identify outliers slightly better than k-means [9]. Therefore, a fuzzy c-means algorithm will be employed to create clusters. Then, the observation with the highest fuzzy membership for each cluster will be used as an observation from the "Comfortable" class.

V. PREDICTING COMFORT LEVEL FEEDBACK

To evaluate the performance of machine learning models built with the clustering based downsampling approach, various machine learning techniques were attempted [9]. The trained models identify whether feedback for a given observation belongs to "Comfortable", "Too Cold" or "Too Warm" class. Random forest models showed the most favorable results among other types of machine learning techniques used in the experiments:

- sl: Linear Support Vector Machine
- rf: Random Forest
- nn: Neural Network
- rp: Decision Tree

Fig. 1 illustrates how each of the models with various sampling techniques perform in terms of AUC.

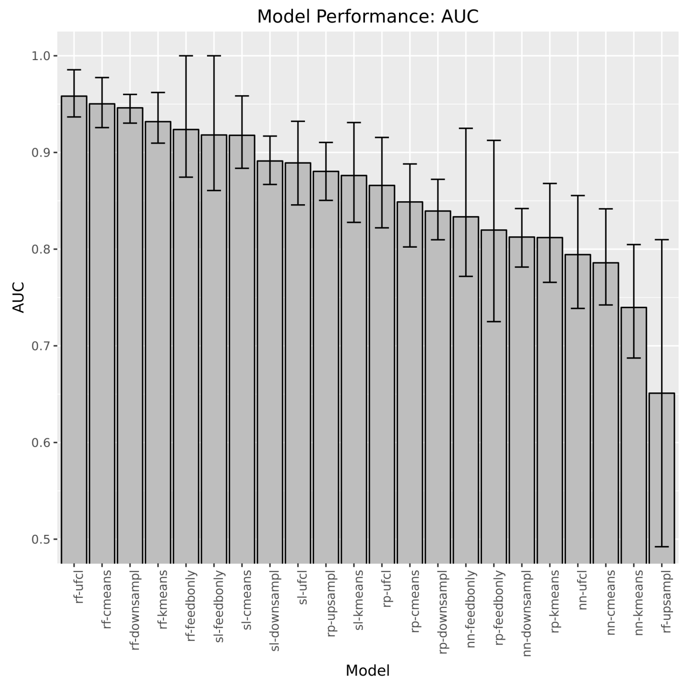

Fig. 1. AUC Performance Comparison Across Each Analysis.

This portrays that clustering-based downsampling with c-means and UFCL algorithms give better AUC than other sampling techniques. The UFCL fuzzy clustering model shows the best performance with an AUC of 95.8%. On the other hand, upsampling demonstrated relatively poor results.

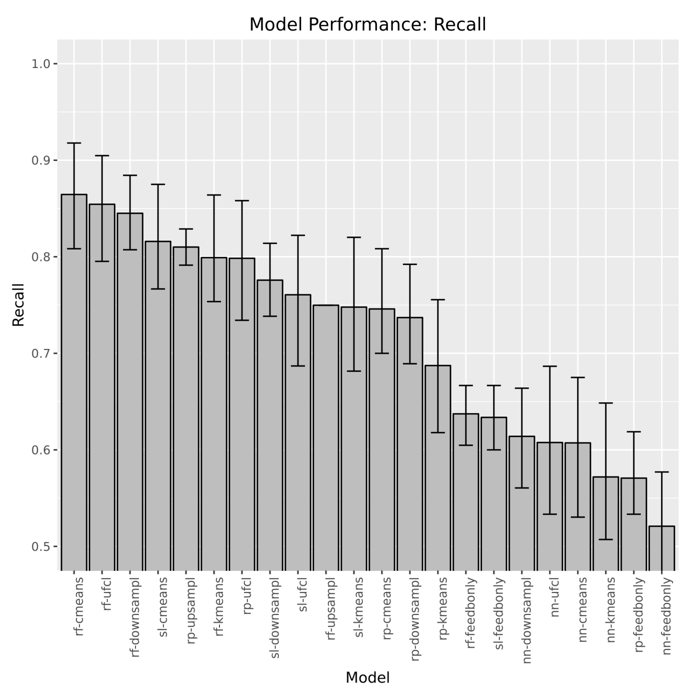

Fig. 2. Recall Performance Comparison Across Each Analysis.

Similarly, Fig. 2 shows the impact of employing various sampling techniques on Recall.

Again, downsampling based on c-means and UFCL clustering methods helped to train models that demonstrate the highest recall values among all other approaches used in the experiments. Meanwhile, the c-means fuzzy clustering model gives a recall of 86.5%, which is slightly higher than the UFCL model.

VI. Maximizing recall while maintaining accuracy of predictions

When solving classification problems, there are various kinds of model performance metrics available during the training process: Accuracy, Kappa, AUC, Mean Recall calculated for each of the classes, etc. Since the goal of the experiments is to maximize recall, a custom metric that optimizes mean recall for classes "Too cold" and "Too Warm" was created. Ross et al. compared the performance of the recall maximized models versus AUC maximized models [16]. The models were created for separate rooms as well as for the whole building (agglomerated models).

Table II shows the AUC and the recall for the entire building and for individual rooms when AUC maximized models are used.

TABLE II
Comparison of Models based on Agglomerated and Individual Datasets - Maximizing AUC.

Room	Rm. AUC	Aggl. AUC	Rm. Recall	Aggl. Recall
118	0.9998	0.9395	1	1
119	0.9995	0.9633	1	1
120	1.0000	0.6648	1	0
206	1.0000	0.8648	1	1
226	1.0000	0.8472	1	1
228	0.9965	0.9074	1.00	0.9333
232	1.0000	0.8649	1	1
235	0.9946	0.7990	1.00	0.8125
300	0.9998	0.9555	1	1
301	0.9962	0.9658	1	1
306	0.9949	0.8833	1	1
328	1.0000	0.9896	1	1
332	0.9992	0.8264	1	1
building	0.9918	0.9918	0.98	0.9800

Where models trained for individual rooms perform slightly better than the agglomerated model, the model trained for the whole building still provides reasonably good results for most of the rooms.

Likewise, Table III shows the AUC and the recall for the entire building and for individual rooms when recall maximized models are employed. Similarly, the composite model provides overall good AUC and recall and even outperforms individual room models in some cases.

Comparison of the results in Table II and Table III show that models built with maximizing AUC perform slightly better than the recall maximized models. While creating models with maximizing recall does not help to improve actual recall, it negatively impacts on AUC in most cases.

TABLE III
Comparison of Models based on Agglomerated and Individual Datasets - Maximizing Recall.

Room	Rm. AUC	Aggl. AUC	Rm. Recall	Aggl. Recall
118	0.9824	0.9335	1	1
119	0.9982	0.9592	0.75	1.0000
120	N/A	0.6745	N/A	0
206	1.0000	0.8346	1	1
226	1.0000	0.8444	1	1
228	0.9903	0.9023	1.00	0.9333
232	N/A	0.8526	N/A	1
235	N/A	0.7895	N/A	0.8125
300	0.9989	0.9564	1	1
301	0.9979	0.9645	1	1
306	0.9932	0.8820	1	1
328	N/A	0.9806	N/A	1
332	0.9990	0.8241	1	1
building	0.9905	0.9905	0.98	0.9800

Moreover, it was impracticable to get recall maximized models for some of the rooms (N/A cells in Table III) because there was not enough feedback to use for recall maximization. Therefore, AUC maximized models are chosen for further experiments.

VII. Can users from one room benefit from feedback of others in the building?

In order to check if users from one room can benefit from the feedback of others in the building, the performance of the models trained for individual rooms was evaluated on the whole dataset. Table IV shows how models trained for individual rooms are performing with datasets from other rooms.

TABLE IV
Individual Room based models performance.

Model	Mean_AUC	Mean_Recall
118	0.4855	0.1346
119	0.4546	0.1231
120	0.6184	0.0769
206	0.3892	0.1026
226	0.4745	0.1282
228	0.3244	0.0769
232	0.1351	0.0769
235	0.3862	0.1340
300	0.2704	0.0865
301	0.3399	0.0974
306	0.5017	0.1282
328	0.2986	0.0769
332	0.4210	0.1026

Table IV illustrates that the majority of the models trained for individual rooms failed to provide both acceptable AUC and the recall mostly because there is not enough feedback for each of the rooms. Also, most of the rooms represent only one or two of the three feedback classes, which means that they inherently cannot classify feedback that is not in the set of classes they were trained with. In conclusion, users from one room generally cannot benefit from feedback from other individual rooms.

VIII. Using Experience from One Building for New Building Energy Management System

The results described so far were based on the dataset collected from a building where the occupants used a web-based system that provided feedback as to whether they were comfortable, hot, or cold. The study was extended to another building where smart thermostats recorded the temperature adjustments made by the users. This new dataset provided the opportunity to test how well a model from one building can be transferred to a new building. Since the feedback for the second building was more refined in terms of the degree of changes to the thermostat, it was reduced it to "Comfortable", "Too cold" and "Too Warm" to match the first building.

Although the earlier experiments showed that random forest is preferable over other models [9], it was decided that it would be compared with neural networks for this particular case to check if neural networks can provide more portable models. Fig. 3 provides a comparison of random forest and neural network models performance for the Building 1 dataset.

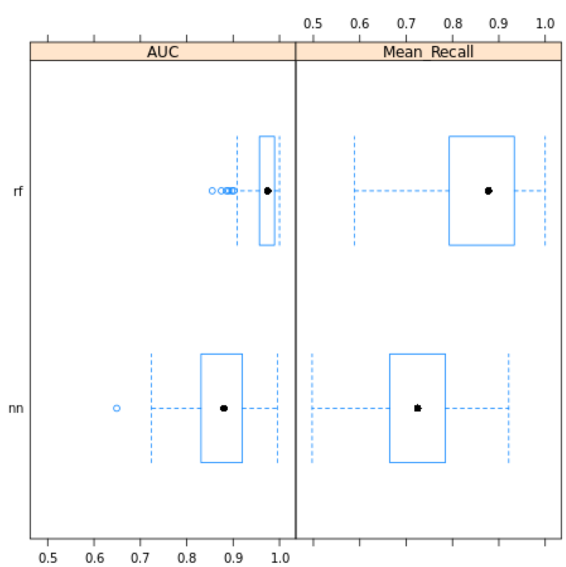

Fig. 3. Model Performance Comparison: Building 1.

Likewise, Fig. 4 provides a comparison of random forest and neural network models performance for the Building 2 dataset.

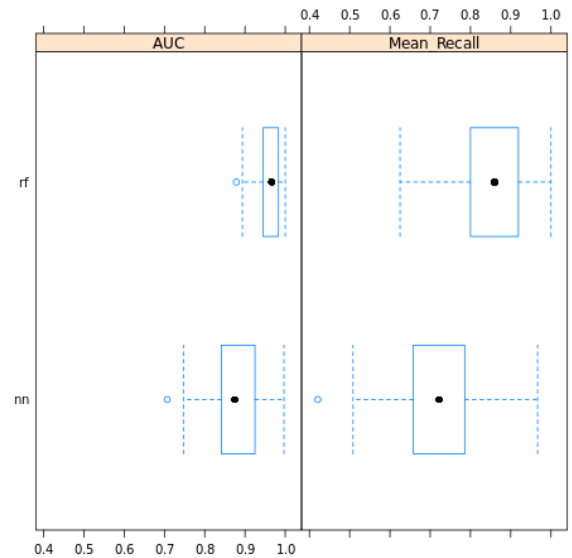

Fig. 4. Model Performance Comparison: Building 2.

Random forest models provided better AUC and recall then neural networks for both Building 1 and Building 2 datasets.

The first two rows in Table V show how well the models from Building 1 performed on Building 1, as well as on Building 2. The second pair of rows in Table V describes the performance of the models from Building 2 on Building 1, as well as, on Building 2. Finally, the third two rows use models developed for combined feedback from both the buildings on individual buildings.

TABLE V
Testing transferability of models between buildings.

Model	Building 1		Building 2	
	AUC	Recall	AUC	Recall
Building 1 model: rf	0.9918	0.9800	0.7738	0.7944
Building 1 model: nn	0.9117	0.9447	0.7150	0.4574
Building 2 model: rf	0.7880	0.2911	0.9882	1.0000
Building 2 model: nn	0.6756	0.2443	0.8421	0.8180
Combined model for two buildings: rf	0.9949	0.9800	0.9884	1.0000
Combined model for two buildings: nn	0.9148	0.8414	0.8613	0.7788

Random forest models demonstrated better transferability between the two buildings. The model from Building 1 still shows satisfactory results when applied to Building 2: 77.38% AUC and 79.44% recall. However, the model from Building 2 performs not as good when applied to Building 1: 78.80% AUC and 29.11% recall. The combined model outperforms all other models, even the ones trained for the same building in terms of AUC. The AUC of the combined model applied to Building 1 is 99.49% whereas the AUC of the model trained for the Building 1 is 99.18%. Similarly, the AUC of the combined model applied to Building 2 is 98.84% while AUC of the model trained for Building 2 is 98.82%. Overall, using experience from one building for new building energy management system may be a practical option in some cases.

IX. Modeling discrete versus continuous user feedback

Predicting whether an occupant is comfortable, cold or hot will be easier than predicting the degree of the annoyance with the room temperature. Here, the extent of the adjustment of the thermostat is treaded as the degree of annoyance. Since the extent of the thermostat adjustment is represented by real numbers, binning is employed to map real values to one of nine resulting classes. Fig. 5 provides a comparison of random forest and neural network model recall for the Building 2 dataset treating extent of the thermostat adjustment as one of the nine nominal classes.

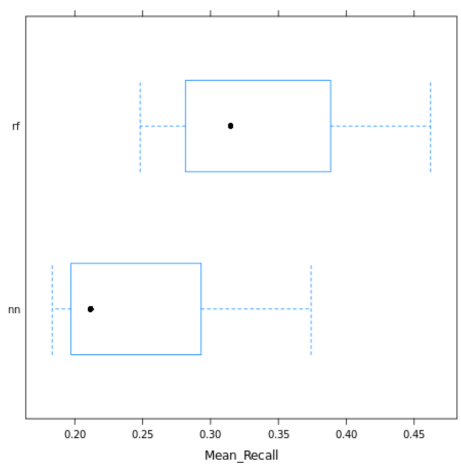

Fig. 5. Model Recall Comparison: Building 2 - Nominal outcome.

However, the extent of the thermostat adjustment can also be treated as an ordinal variable. In this case, the extent of the thermostat adjustment can be expressed as an integer varying from -4 to 4.

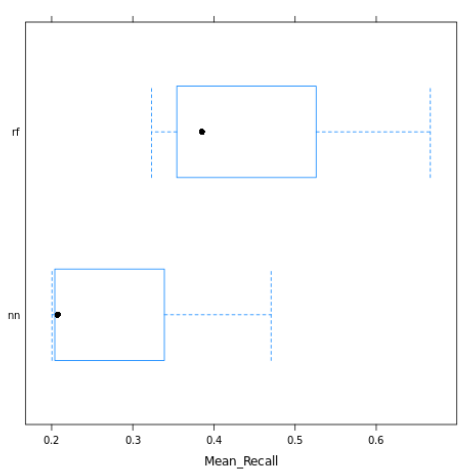

Fig. 6. Model Recall Comparison: Building 2 - Ordinal outcome.

Fig. 6 provides a comparison of random forest and neural network model recall for the Building 2 dataset treating extent of the thermostat adjustment as an integer. Random forest models provided better recall in both cases, so the random forest models will be used for further comparisons in this paper. Fig. 7 shows a comparison of model performance when nominal versus an ordinal variable is used to express the extent of the thermostat adjustment.

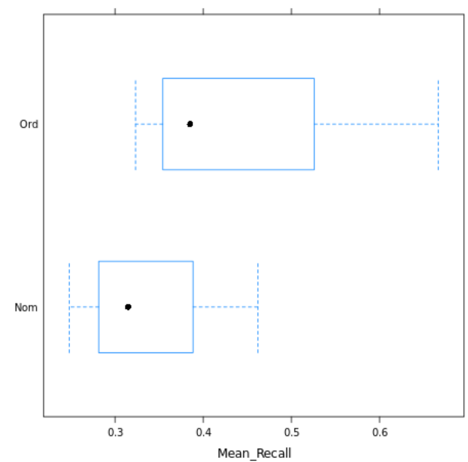

Fig. 7. Recall Comparison

Fig. 7 portrays that better model performance with respect to recall can be achieved when the ordinal nature of the extent of the thermostat adjustment is taken into account.

Table VI compares the quality of predicting discrete values such as “Comfortable”, “Too cold” and “Too Warm” versus predicting the extent of temperature adjustment.

TABLE VI
Predicting discrete versus continuous feedback.

Model	Discrete predictions		Continuous predictions	
	AUC	Recall	AUC	Recall
Discrete feedback	0.9881	1.0000	N/A	N/A
Continuous feedback - nominal	0.9585	1.0000	0.9980	0.9763
Continuous feedback - ordinal	0.9549	1.0000	0.9850	1.0000

The model built with discrete feedback (“Comfortable”, “Too cold” and “Too Warm”) shows slightly better performance in discrete predictions in terms of AUC compared to the other two models (0.9881 versus 0.9585 and 0.9549). All models provide 100% recall for the Building 2 dataset with discrete predictions. While it is easy to map continuous feedback to discrete feedback by treating positive thermostat adjustments as “Too Cold”, negative thermostat adjustments as “Too Warm” and no adjustments as “Comfortable”, it is not possible to do it vice versa. Thus, models built with discrete feedback cannot be utilized for continuous predictions.

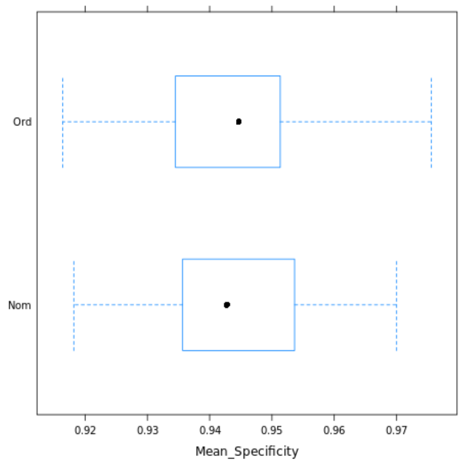

Fig. 8. Specificity Comparison.

The continuous feedback model which takes into account the ordinal nature of the extent of the thermostat adjustment gives better recall for continuous predictions than the model that uses nominal classes (1.0000 versus 0.9763). Nonetheless, it demonstrates slightly worse performance in terms of AUC both for discrete predictions (0.9549 versus 0.9585) and continuous predictions (0.9850 versus 0.9980). Since AUC is a composite metric, the model which takes into account the ordinal nature of the extent of the thermostat adjustment gives a little worse specificity. Fig. 8 shows that working with the ordinal feedback gives better mean specificity, but using the nominal feedback provides slightly higher lower and upper quartiles for specificity.

X. Conclusion

This paper describes the problem of modeling user preference to provide a comfortable work environment without a continuing need for user intervention. The study uses data collected from two buildings. The first building used a web-based system that allowed users to specify if they were comfortable, cold, or hot. The second building used a smart thermostat that registered the temperature adjustments made by the occupants. One of the major issues with user feedback is that it is limited in size, creating an imbalance between feedback points and non-feedback points on a time continuum. The study experimented with a number of sampling techniques and found that a downsampling technique based on fuzzy clustering was the best option. The report recognized the importance of recalling all the feedback over the accuracy of predictions. Experimentation with different supervised learning techniques showed that random forest classifiers performed the best in maximizing recall while providing high accuracy. This paper reports the effect of agglomerating feedback from different rooms to create a dataset to increase the number of feedback points in the dataset as opposed to modeling individual rooms with a smaller number of feedback points. Modeling individual rooms is shown to be a better option. However, the models for the entire building provide a reasonable performance in case there is no feedback for a given room. This conclusion was further tested by using user preference model for one building on a different building in the same region. Again, a model developed for a particular room is the best option. However, transferring the model from one building to a new building is shown to be a viable option when a new building energy management system is installed. The study also concluded that predicting whether an occupant was comfortable, cold or hot was easier than predicting the degree of changing the thermostat temperature. However, the accuracy of prediction of thermostat adjustment was acceptable. Finally, the experiments showed that models could provide better recall if the ordinal nature of the degree of the thermostat adjustment is taken into account.

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