Combining Consensus and Tracking Errors in Sliding Mode Control of High Order Uncertain Stochastic Multi-Agent Systems

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***Abstract*—Controlling stochastic multi-agent systems (MAS) is a promising branch of systems engineering owing to its ability to solve complex practical problems by taking inspiration from the social behavior of living organisms. Consensus on more than one state is a less considered problem in the framework of leader-following control of high-order uncertain nonlinear stochastic multi-agent systems with unknown dynamics. To this end, in this paper, a sliding surface integrates a tracking error polynomial with consensus error between states of the connected agents. Accordingly, a new distributed fuzzy sliding mode controller is designed for a class of stochastic multi-agent systems with partially unknown deterministic dynamics and unknown bounded stochastic dynamics. The proposed control protocol steers agents to reach agreement on states of the leader and concurrently approximates the unknown part of their deterministic dynamics by using fuzzy systems with adaptive parameters. In the sense of Lyapunov, it is proved that consensus error and all of the closed-loop signals of each agent are semi-globally uniformly ultimately bounded (SGUUB) in mean square. Under the designed controller, nonlinear closed-loop experimental dynamics behave well in both tracking and reaching state consensus.**

Keywords—Leader-following consensus control; Stochastic multi-agent systems; Adaptive fuzzy controller; Sliding surface.

I. INTRODUCTION

Multi-Agent Systems (MAS) are a group of homogeneous/heterogeneous agents in which each system has restricted insight into their surrounding environment and low capabilities to change it. However, the synergistic behavior of the agents helps them to observe the environment better and make complex decisions. Different properties can be considered to categorize a MAS. Depending on the problem at hand, agents may connect directly to each other in a decentralized manner or through the base unit. Robustness of the system against agent failure and avoidance of information accumulation in a specific location are some of the main advantages of using a decentralized architecture. The availability of the leader in the group of agents is another essential issue. In leaderless consensus control, agents reach consensus on a point or path according to their initial conditions. While, in a leader-following case, agents track the dynamics of the leader or a reference signal [1].

Leader-following tracking control is an essential problem in distributed control, and so far, it has been used in several applications. To this aim, followers should agree on a signal observed from their leader or converge to the convex hull of a polygon whose vertices are the state of the leaders. Authors in [2] discuss the problem of consensus on quantized information between agents with second-order nonlinear dynamics. In [3], both asymptotic and finite-time consensus is obtained using different sliding mode controllers for second-order nonlinear agents. This coordinated behavior is not always the only objective of the group. In some problems, agents should also preserve the relative distance with each other and then arrange themselves around the leader. In [4], mid-level agents called formation-leaders are also defined to track the reference path of the primary leader and concurrently maintain time-varying formation. Moreover, according to the containment control, followers are settled in the convex hull spanned by the formation-leaders. In practice, due to the inaccuracy in modeling, the time-varying behavior of the systems, and disturbance in the environment, the exact dynamics of systems rarely exist. This restricts the applicability of the control protocols mentioned above, assuming the existence of agents model.

Fuzzy logic and neural network are two pillars of artificial intelligence having universal approximation capability [5]. Combining such a paradigm with adaptive control is a common approach to deal with model uncertainties caused by unknown dynamics. In [6], fuzzy logic systems are used to estimate the unknown dynamics of followers described by strict-feedback systems. In [7], within the framework of a distributed reinforcement learning algorithm, each agent is modeled by three fuzzy logic systems playing the roles of the critic, actor, and system identifier units. Approximating the unknown dynamics of pure-feedback nonlinear MAS with saturated input and time delay are accomplished in [8] with the aid of a radial basis function neural network.

In the real world, agents are also subjected to several sources of stochastic uncertainties such as measurement noise, noise in communication between agents, stochastic time delay, and stochastic disturbance [9]. Using Itô dynamic is a way to model these types of uncertainties. For example, adaptive fuzzy controllers for strict-feedback stochastic uncertain systems with unmeasured states are introduced by

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dynamic surface control [10] and combining the backstepping technique with small gain theorem [11]. Both of them use a fuzzy observer to estimate unmeasured states. Such approaches are also developed to deal with large-scale nonlinear systems with dead-zone input [12], systems in the pure-feedback form [13], and non-strict feedback multi-input multi-output systems[14]. Sliding mode control is a beneficial method to deal with full-state feedback systems being affine in the control input. It defines a sliding surface combining the tracking error and its derivatives and leads to a robust controller to maintain the states of the systems on the sliding surface. In [15], Leader-following tracking control for a linear stochastic system is addressed by a distributed sliding mode controller. To design sliding surfaces, First, the dynamics of each agent are split into two parts by a linear transformation. Then, each sliding surface integrates the leader-following consensus error of two subsystems. The authors then extend their works to a second-order nonlinear stochastic multi-agent system in [16] and propose a new approach to force followers' states reaching consensus on states of leaders in mean square.

Most of the literature in the leader-following control of stochastic MAS only focuses on output consensus problem (e.g. [17]) or only considers consensus between agents with first or second-order dynamics (e.g. [18]). However, at practice, it may be necessary to deal with agents having higher-order dynamics and aggregation on more that one state. In this paper, a distributed adaptive fuzzy sliding mode controller is proposed to establish leader-following consensus on $n-1$ dimensions of followers with n-order uncertain stochastic controllable dynamics. To do so, we put together a conventional tracking error polynomial with consensus error between agents. Under the proposed controller, it is proved that consensus error and all closed-loop signals of agents stay SGUUB in mean square. In comparison with [15], [16], our approach can handle the unknown deterministic nonlinear dynamics of followers by applying fuzzy systems tuned online by robust adaptation rules and also preserve the stability of the system in the presence of unknown stochastic dynamics. The effectiveness of the proposed algorithm is investigated on five identical stochastic inverted pendulums.

The rest of the paper is organized as follows: Section II formulates the problem and describes the controller design. Brief preliminaries about fuzzy systems are stated in Section III. The stability analysis of our proposed method is explained in Section IV. Section V is dedicated to the simulation results. Finally, conclusions are drawn in Section VI.

II. Problem Formulation and Controller Design

A. System Description

Consider the following stochastic nonlinear dynamic for each agent:

$$x_i^{(n)} = f_i(X_i) + g_i(X_i)u_i + d_i(X_i,t) + h_i(X_i)\dot{B}(t), \tag{1}$$

Alternatively, in more conventional form for stochastic systems, it can be rewritten by differential operators as

$$\begin{cases} dx_{i1} = x_{i2}dt \\ dx_{i2} = x_{i3}dt \\ \vdots \\ dx_{in} = [f_i(X_i) + g_i(X_i)u_i + d_i(X_i,t)]dt \\ \qquad +h_i(X_i)dB(t), \end{cases} \tag{2}$$

where $X_i = \left[x_i, \dot{x}_i, \dots, x_i^{(n-1)}\right]^T = [x_{i1}, x_{i2}, \dots, x_{in}]^T \in \mathbb{R}^n$ is the state vector and $u_i \in \mathbb{R}$ is the control input for ith agent within a group consisting of N homogenous agents. $g_i: \mathbb{R}^n \to \mathbb{R}$ is locally Lipschitz known nonlinear functions. Also, $f_i: \mathbb{R}^n \to \mathbb{R}$ and $h_i: \mathbb{R}^n \to \mathbb{R}$ are locally Lipschitz unknown nonlinear functions satisfying $f(0) = 0$ and $h(0) = 0$. $d_i(X_i,t)$ denotes external disturbance with unknown upper bound D, that is $|d_i(X_i,t)| \le D$. $B(t)$ is a scalar standard Wiener process and $dB(t)/dt$ is a Gaussian white noise process with the following properties [19], [20]:

$$E\big(dB(t)\big) = 0, \tag{3}$$

$$E\big(dB(t_1)dB(t_2)\big) = \begin{cases} dt & dt_1 = dt_2 \\ 0 & dt_1 \neq dt_2 \end{cases}. \tag{4}$$

B. Control Objectives

The first control objective is to drive the states of each agent to follow the desired trajectory $X_d = \left[x_d, \dot{x}_d, \dots, x_d^{(n-1)}\right]^T = [x_{d1}, x_{d2}, \dots, x_{dn}]^T$ such that the error $E_i = [e_{i1}, e_{i2}, \dots, e_{in}]^T = X_i - X_d$, becomes bounded in a small region around zero. As a second objective, all agents must reach an agreement on their states. Third, all the closed-loop signals must stay bounded in mean square.

C. Controller Design

The proposed sliding surface is designed as below

$$s_i(t) = \sum_{j=1}^{n-1} \lambda_{ij} e_{ij} + e_{in} + \sum_{k=1}^{N} a_{ik} \sum_{j=1}^{n-1} \big(x_{ij} - x_{kj}\big). \tag{5}$$

The two first terms in (5) correspond to the tracking error polynomial. Where positive constants λ_{ij} should be chosen in such a way that $p^n + \lambda_{n-1}p^{n-1} + \dots + \lambda_1$, be Hurwitz. The last term defines the consensus error between $n-1$ dimensions of the agents. Where a_{ik} denotes the non-zero adjacency weight between agent i and k in the communication graph of agents. Since the existence of the stochastic term in the state equation n increases the instability of the system, the consensus in dimension n is out of consideration of our work.

To steer the system to the sliding surface and keep it there from time τ, the nominal controller should cause $ds_i(t)/dt$=0, for all time after τ. Using state equations (2), $ds_i(t)$ is computed as

$$ds_i(t) = \sum_{j=1}^{n-1} \lambda_{ij} de_{ij} + de_{in}$$
$$+ \sum_{k=1}^{N} a_{ik} \sum_{j=1}^{n-1} \left(dx_{ij} - dx_{kj} \right)$$
$$= [\sum_{j=2}^{n} \lambda_{ij} e_{ij} + f_i(X_i) + g_i(X_i) u_i + d_i(X_i, t)$$
$$- \dot{x}_{dn} + \sum_{k=1}^{N} a_{ik} \sum_{j=2}^{n} (x_{ij} - x_{kj})] dt$$
$$+ h_i(X_i) dB(t), \tag{6}$$

Since $f_i(.)$ is assumed to be unknown and approximated by adaptive fuzzy systems, the proposed controller is chosen as (7) based on feedback linearization technique.

$$u_i = g_i{}^{-1}(X_i)(-\sum_{j=2}^{n} \lambda_{ij} e_{ij} - \hat{f}_i(X_i) + \dot{x}_{dn}$$
$$-\sum_{k=1}^{N} a_{ik} \sum_{j=2}^{n} \left(x_{ij} - x_{kj} \right) - k_i s_i), \tag{7}$$

where $k_i s_i$ with positive constant k_i is a switching term giving the system the capability of handling uncertainty [19].

III. Preliminaries on Fuzzy Systems

Fuzzy system by virtues of universal approximation and managing deterministic uncertainty is a useful tool for modeling unknown system dynamics. A fuzzy system comprises four main parts: fuzzifier, fuzzy rule base, fuzzy inference engine, and defuzzifier. The rule base contains m if-then rules describes as

$$Rule\ l{:}\ if\ x_{i1}\ is\ A_{i1}^{l}\ and\ \ldots\ x_{ij}\ is\ A_{ij}^{l}\ and\ \ldots$$
$$and\ x_{in}\ is\ A_{in}^{l}\ then\ f_i(X_i)\ is\ W_i^{l}\ , \tag{8}$$

where $X_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T$ is the n-dimensional input of the fuzzy system, A_{ij}^{l} is the fuzzy set of jth dimension in the antecedent of rule l; and W_i^{l} is the fuzzy set in the consequent of lth rule.

Here, we use the singleton fuzzifier, product inference engine, and center average defuzzifier. Then the output of the fuzzy system is computed as

$$\hat{f}_i(X_i|W_i) = \frac{\sum_{l=1}^{m} \overline{w}_i^{l} \prod_{j=1}^{n} \mu_{A_{ij}^{l}}\left(x_{ij}\right)}{\sum_{l=1}^{m} \prod_{j=1}^{n} \mu_{A_{ij}^{l}}\left(x_{ij}\right)} = W_i^T \varphi_i(X_i), \tag{9}$$

where $\mu_{A_{ij}^{l}}\left(x_{ij}\right)$ denotes the membership degree for input j of agent i to the set A_{ij}^{l}. $\varphi_i(X_i) = [\varphi_{i1}(X_i), \ldots, \varphi_{im}(X_i)]^T$ is the

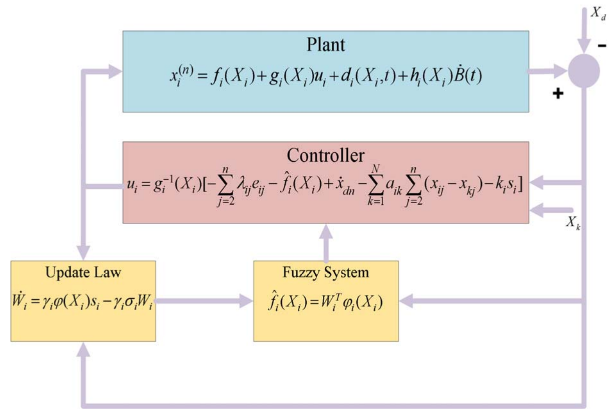


Fig. 1. A diagram of the proposed controller for agent i.

vector of fuzzy basis functions with $\varphi_{il}(X_i) = \frac{\prod_{j=1}^{n} \mu_{A_{ij}^{l}}(x_{ij})}{\sum_{l=1}^{m} \prod_{j=1}^{n} \mu_{A_{ij}^{l}}(x_{ij})}$ as it's lth element. $\overline{w}_i^{l}$ is the center of W_i^{l}. These adjustable parameters are aggregated in a vector $W_i = [\overline{w}_i^{1}, \overline{w}_i^{2}, \ldots, \overline{w}_i^{m}]^T$. The optimal value of W_i is given by

$$W_i^* = \underset{W_i \in \Omega_i}{\operatorname{argmin}} \left[\sup_{X_i \in U_i} \left| f_i(X_i) - \hat{f}_i(X_i|W_i) \right| \right], \tag{10}$$

where Ω_i and U_i are compact sets for W_i and X_i, respectively. The minimum approximation error for the above fuzzy system is defined by [21], [22]

$$\begin{aligned} \varepsilon_i &= f_i(X_i) - \hat{f}_i(X_i|W_i^*) = f_i(X_i) - W_i^{*T} \varphi_i(X_i) \\ &= f_i(X_i) - \widetilde{W}_i^T \varphi_i(X_i) - W_i^T \varphi_i(X_i), \end{aligned} \tag{11}$$

where $\widetilde{W}_i = W_i^* - W_i$.

IV. Stability Analysis

The proposed controller drives the follower to track the leader based on the information; it receives about approximated dynamics and states of leader and other agents, as depicted in Fig.1. In the sequel, we introduce some lemmas and two assumptions about the dynamics of systems to prove the boundness of consensus error in Theorem1.

Assumption 1: $h_i(X_i)$ can be chosen such $|h_i(X_i)| \leq H$, for all $X_i \in \mathbb{R}^n$. Where H is a positive unknown constant.

Assumption 2: The desired trajectory x_d and its derivatives are available, continuous, and bounded.

Lemma 1 [23]: Consider the stochastic system $dx = f(X) + h(X)dB$, Where $X \in \mathbb{R}^n$ is the state vector. $f{:}\, \mathbb{R}^n \to \mathbb{R}$ and $h{:}\, \mathbb{R}^n \to \mathbb{R}$ are locally Lipschitz nonlinear functions satisfying $f(0) = 0$ and $h(0) = 0$. Suppose $V{:}\, R^n \to R^+$ is positive definite, second-order differentiable function and there exists positive constants α and β, and class K_∞ functions δ_1 and δ_2 such that $V(X)$ and its differential, $\mathcal{L}V$, pass inequalities (12) and (13), respectively.

$$\delta_1(\|X\|) \leq V(X) \leq \delta_2(\|X\|), \tag{12}$$

$$\mathcal{L}V = \frac{\partial V}{\partial X} f(X) + \frac{1}{2} Tr\left(h(X)^T \frac{\partial^2 V}{\partial X^2} h(X) \right) \leq -\alpha V + \beta, \tag{13}$$

Then the system has a unique solution for each $X_0 \in R^n$ that satisfies the following:

$$E\big(V(X)\big) \leq V(X_0)e^{-\alpha t} + \frac{\beta}{\alpha} \qquad \forall t \geq t_0, \tag{14}$$

Furthermore, if inequality (14) is satisfied, then the states of the system are semi-globally uniformly ultimately bounded (SGUUB) in mean square.

Lemma 2 [18] (Young inequality): The next inequality holds for vectors $X, Y \in R^n$, if two constants $p > 1, q > 1$ satisfy in $\frac{1}{p} + \frac{1}{q} = 1$.

$$X^T Y \leq \frac{\varepsilon^p}{p} \|X\|^p + \frac{1}{q\varepsilon^q} \|Y\|^q, \tag{15}$$

with $\varepsilon > 0$.

Theorem 1: Consider N homogeneous agents described by nonlinear stochastic differential equations (2), where the controller u_i is designed according to (7) and W_i is adjusted by the adaptation law (16). Then, the overall Lyapunov function $\sum_{i=1}^{N} V_i$ is bounded and also the closed-loop system is bounded in probability [18], [19], [24] and [23].

$$\dot{W}_i = \gamma_i \varphi(X_i) s_i - \gamma_i \sigma_i W_i, \tag{16}$$

Proof: Set the following Lyapunov function candidate for each agent

$$V_i = \frac{1}{2} s_i^2 + \frac{1}{2\gamma_i} \widetilde{W}_i^T \widetilde{W}_i, \tag{17}$$

The differential of (17) is computed as bellow using Lemma1,

$$\begin{aligned} \mathcal{L}V_i = [s_i(&\sum_{j=2}^{n} \lambda_{ij} e_{ij} + f_i(X_i) + g_i(X_i) u_i \\ &+ d_i(X_i, t) - \dot{x}_{dn} + \sum_{k=1}^{N} a_{ik} \sum_{j=2}^{n} (x_{ij} - x_{kj})) \\ &+ \frac{1}{2} h_i^2(X_i)] - \frac{1}{\gamma_i} \widetilde{W}_i^T \dot{W}_i, \end{aligned} \tag{18}$$

Replacing (7) into u_i leads to the following equation

$$\begin{aligned} \mathcal{L}V_i = (s_i(&-k_i s_i + f_i(X_i) - \hat{f}_i(X_i|W_i) \\ &+ g_i(X_i) u_i + d_i(X_i, t) - \dot{x}_{dn} \\ &+ \frac{1}{2} h_i^2(X_i)) - \frac{1}{\gamma_i} \widetilde{W}_i^T \dot{W}_i\,, \end{aligned} \tag{19}$$

By substituting $f_i(X_i)$ and $\hat{f}_i(X_i|W_i)$ from (11) and (9) respectively, one has

$$\begin{aligned} \mathcal{L}V_i = &-k_i s_i^2 + \varepsilon_i s_i + d_i(X_i, t) s_i + \frac{1}{2} h_i^2(X_i) \\ &+ \widetilde{W}_i^T \left(-\frac{1}{\gamma_i} \dot{W}_i + s_i \varphi_i(X_i) \right), \end{aligned} \tag{20}$$

From $\widetilde{W}_i = W_i^* - W_i$ and after applying the adaptation rule (16), $\mathcal{L}V_i$ can be obtained as

$$\begin{aligned} \mathcal{L}V_i &= -k_i s_i^2 + \varepsilon_i s_i + d_i(X_i, t) s_i + \sigma_i \widetilde{W}_i^T W_i \\ &\quad + \frac{1}{2} h_i^2(X_i) \\ &= -k_i s_i^2 + \varepsilon_i s_i + d_i(X_i, t) s_i - \sigma_i \widetilde{W}_i^T \widetilde{W}_i \\ &\quad + \sigma_i \widetilde{W}_i^T W_i^* + \frac{1}{2} h_i^2(X_i), \end{aligned} \tag{21}$$

Replacing upper limit for $h(X_i)$, $d_i(X_i, t)$ and other terms with unknown sign (Lemma2) yields

$$\begin{aligned} \mathcal{L}V_i \leq &-(k_i - 1) s_i^2 - \frac{\sigma_i}{2} \widetilde{W}_i^T \widetilde{W}_i + \frac{1}{2} \varepsilon_i^2 + \frac{1}{2} D^2 \\ &+ \frac{\sigma_i}{2} W_i^{*T} W_i^* + \frac{1}{2} H^2, \end{aligned} \tag{22}$$

By (22), the differential $\mathcal{L}V$ for the overall Lyapunov function $V = \sum_{i=1}^{N} V_i$ is obtained as follow

$$\begin{aligned} \mathcal{L}V \leq \sum_{i=1}^{N} [&-(k_i - 1) s_i^2 - \frac{\sigma_i}{2} \widetilde{W}_i^T \widetilde{W}_i + \frac{1}{2} \varepsilon_i^2 + \frac{1}{2} D^2 \\ &+ \frac{\sigma_i}{2} W_i^{*T} W_i^* + \frac{1}{2} H^2]. \end{aligned} \tag{23}$$

By choosing $\alpha = \min_i(\min((k_i - 1), \sigma_i/2))$ with $k_i > 1$ $(i = 1, \ldots, N)$ and $\beta = \sum_{i=1}^{N} (\frac{1}{2} \varepsilon_i^2 + \frac{\sigma_i}{2} W_i^{*T} W_i^*) + \frac{N}{2}(H^2 + D^2)$, the inequality (13) is obtained.

Computing the expectation of (13) and utilizing the relation $\mathrm{E}(\mathcal{L}V) = dE(V)/dt$, results in

$$\mathrm{E}(\mathcal{L}V) = \frac{dE(V)}{dt} \leq -\alpha E(V) + \beta, \tag{24}$$

Now, Let $\alpha \geq \beta/C$, then $dE(V)/dt \leq -(\beta/C)E(V) + \beta$. It implies $dE(V)/dt \leq 0$, on $E(V) = C$. So, $V \leq C$ is an invariant set for the system (1). That is if $E(V(0)) \leq 0$ then

TABLE I. AVERAGE MSE OF TRACKING AFTER TEN INDEPENDENT RUNS FOR THREE DIFFERENT COMMUNICATION TOPOLOGIES. BOLDED NUMBERS IN EACH COLUMN INDICATE THE LEAST CONSENSUS ERROR e_{i1} OR e_{i2} FOR I[TH] AGENT AMONG 3 CASES

Case		*Agent1*	*Agent2*	*Agent3*	*Agent4*	*Agent5*
Case(i): Complete graph	e_{i1}	**1.2E-7**	**1.2E-7**	**1.1E-7**	1.2E-7	8.4E-8
	e_{i2}	3.6E-6	3.4E-6	3.3E-6	**3.9E-6**	**5.2E-6**
Case(ii): Graph of Fig.2	e_{i1}	1.3E-7	1.4E-7	1.3E-7	**1.1E-7**	7.3E-8
	e_{i2}	3.5E-6	**3E-6**	**2.8E-6**	**3.9E-6**	5.5E-6
Case(iii): Unconnected graph	e_{i1}	1.4E-7	1.5E-7	1.4E-7	1.2E-7	**5.9E-8**
	e_{i2}	**3.4E-6**	3.2E-6	2.9E-6	4.2E-6	5.3E-6

$E(V(t)) \leq C$ for all $t > 0$. Furthermore, applying Lemma1 on equation (13) leads to the upper bound β/α for $E(V(t))$. So, all signals of the closed-loop system are SGUUB in mean square. Hence, the boundedness of the sliding mode variable and the leader-following consensus for the system are obtained.

V. SIMULATION RESULTS

In this section, the proposed fuzzy sliding mode controller is applied to the problem of distributed agreement between five identical stochastic inverted pendulums. We investigate the effectiveness of our approach under two scenarios, including different external disturbance signals and various topologies for followers. The dynamic of agent i $(i = 1, \dots, 5)$ is modeled as

$$\begin{aligned} dx_{i1} &= x_{i2}dt \\ dx_{i2} &= \left(\frac{g\sin(x_{i1}) - \frac{1}{2}amlx_{i2}^2 \sin(2x_{i1})}{\frac{4l}{3} - aml\cos^2(x_{i1})} + \frac{a\cos(x_{i1})}{\frac{4l}{3} - aml\cos^2(x_{i1})} u_i + d_i(x_{i1}, x_{i2}, t) \right) dt + b\sin(x_{i1}) dB(t), \end{aligned} \tag{25}$$

where x_{i1} is the angle of the pendulum from the vertical axis, x_{i2} is the angular velocity, and $g = 9.8\,m/s^2$ is the gravitational acceleration; m and M are the mass of pendulum and mass of the cart, respectively; $a = 1/(m + M)$, $2l$ is the length of pendulum in meter, u_i is the scalar control signal, and b is the magnitude of the stochastic terms. The parameters in (25) are valued as $M = 8kg$, $m = 2kg$, $l = 0.5m$, and $b = 0.5$. Indeed, parameters of the designed controller are fixed at $k_i = 10$, $\lambda_{i2} = 10$, $\sigma_i = 0.002$ and $\gamma_i = 20$ $(i = 1, \dots, 5)$. The initial states of the system are set to $[x_{11}(0), x_{21}(0), x_{31}(0), x_{41}(0), x_{51}(0)] = [-0.05, -0.025, 0, 0.25, 0.05]$ and $x_{i2}(0) = 0$, $(i = 1, \dots, 5)$. Our control objective is to make angles of all agents x_{i1} reaching consensus on the desired angle $x_{d1} = 0.1\sin(\pi t)$. Agents communicate with a connected undirected graph (Fig. 2) in which $a_{23} = 1$, $a_{35} = 1$, $a_{15} = 1$, $a_{45} = 1$. The membership functions for state variables are defined as

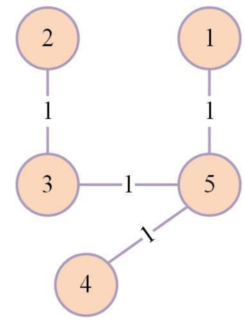


Fig. 2. Communication graph of the agents.

$\mu_{A_j^l}(x_{ij}) = \exp\left(-\left(x_{ij} - c_j\right)^2 / 2\sigma_i^2\right), (l = 1, \dots, 25;\ j = 1, 2)$ where $c_j \in \rho_j * \{-3, -1.5, 0, 1.5, 3\}$ and $\sigma_j = \rho_j$. $\rho_1 = 0.2$, $\rho_2 = 0.01$ are scaling factors fitting the range of membership function to the range of the corresponding state variable [6], [25].

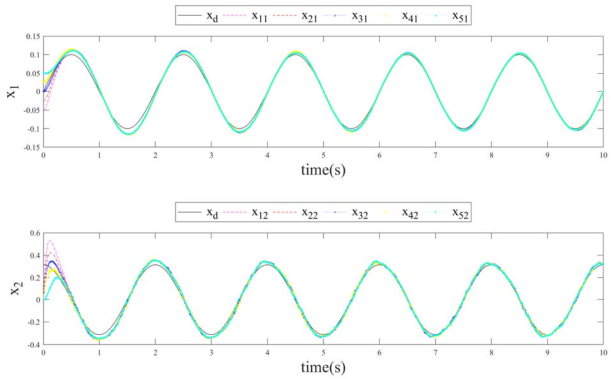
The total time of each simulation is set to $T = 10s$ with sampling time $t = 0.01s$. Simulation is performed in Matlab R2019a environment by using a tablet PC with 8 GB RAM and Intel 7[th] generation core i7-7660U CPU.

A. *Studying disturbance attenuation property of the system*

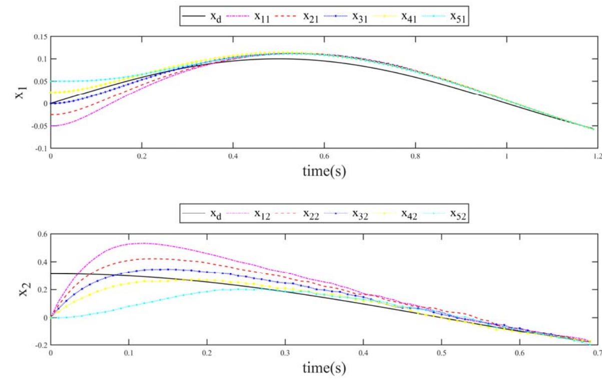
Concerning external disturbance attenuation property of the closed-loop system, three different cases are studied: (i) absence of disturbance, (ii) presence of sinusoidal disturbance $d(t) = 5\sin(\pi t/2)$ and (iii) presence of rectangular pulse disturbance $d(t) = \Pi(t) = \begin{cases} 5 & 2 \leq t \leq 4 \\ 0 & otherwise \end{cases}$. Fig. 3 includes states of all agents in case (i) to (iii), respectively. Either in the presence of disturbance or without it, except for some changes in transient behavior of the system, agents can reach consensus on the desired angle in a few seconds and track it with a small error during simulation time. Concerning the angular velocities $x_{i2} (i = 1, \dots, 5)$, the signals are kept bounded and track the reference with small error. Fig. 4. depicts the control inputs u_i and the tracking errors e_{i1} which are limited in a narrow bound, as proved in Section IV.

B. *Studying the effect of different graph topologies of followers on the performance of MAS*

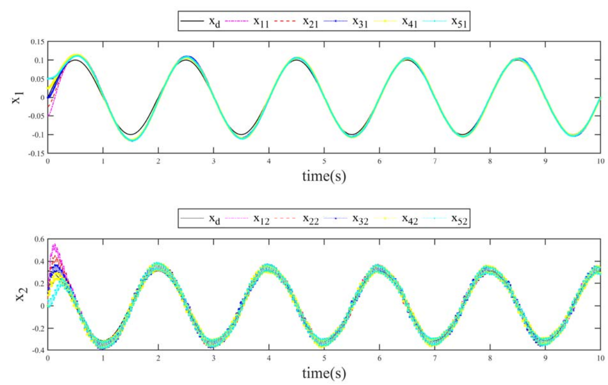
Now, we investigate the effectiveness of combining the tracking error polynomial with consensus error in the sliding surface. Three different communication topologies are considered for the experiment. Complete communication graph is the first case, the graph of Fig. 2 is the second case, and independent agents without connections between them is the third one. Table 1 shows the average value of the mean square error of tracking on ten consecutive runs. According to the Table 1, most of the agents perform better in the case that they are more connected to the others. Especially about the first state, as in the proposed method, consensus error is only computed for the first state of a second-order system.



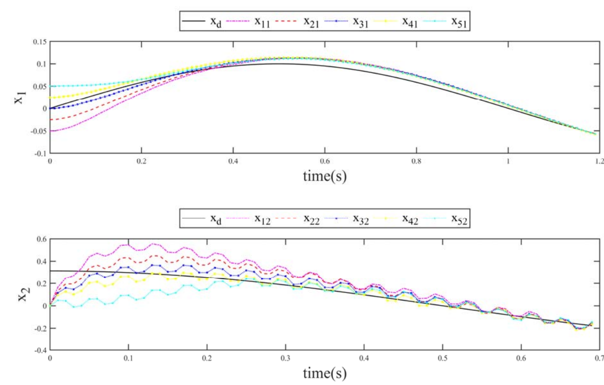
(a)



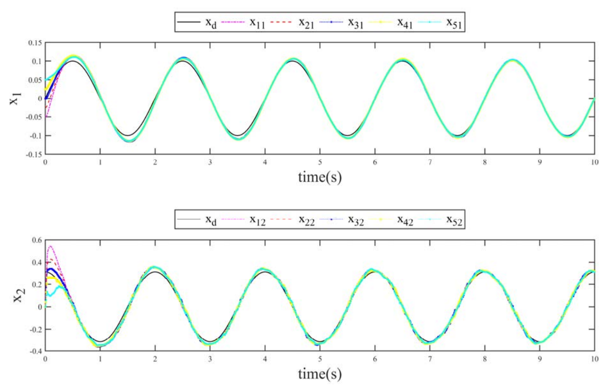
(b)



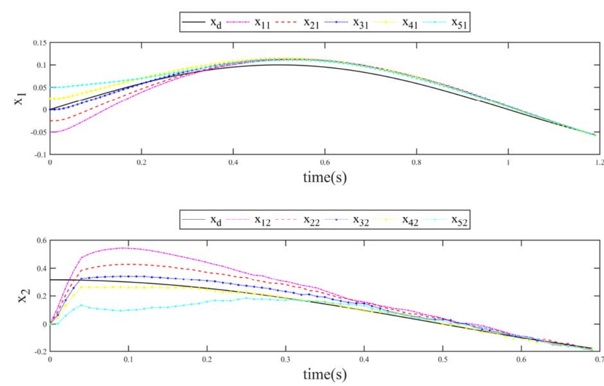
(c)



(d)

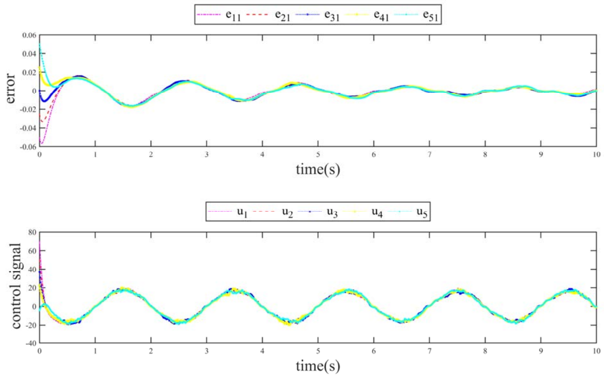


(e)

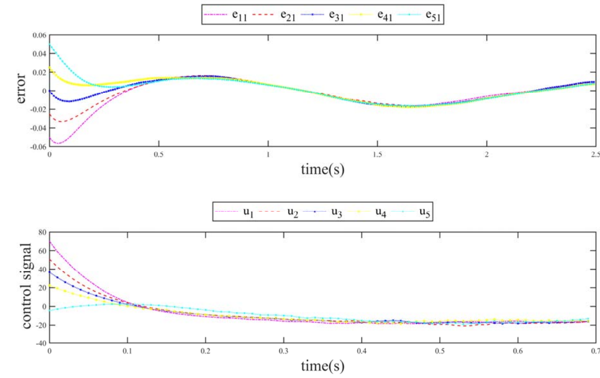


(f)

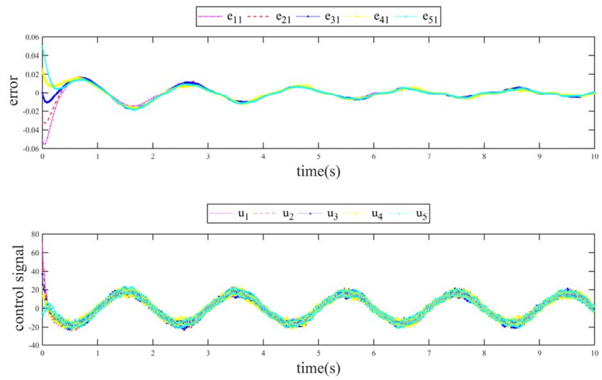
Fig. 3. The trajectories of x_{i1} and x_{i2} for all agents ($i = 1, \ldots, 5$) in case (i) - (iii) ((a), (c) and (e)). Zoomed view of transient time in case (i) - (iii) ((b), (d) and (f)).



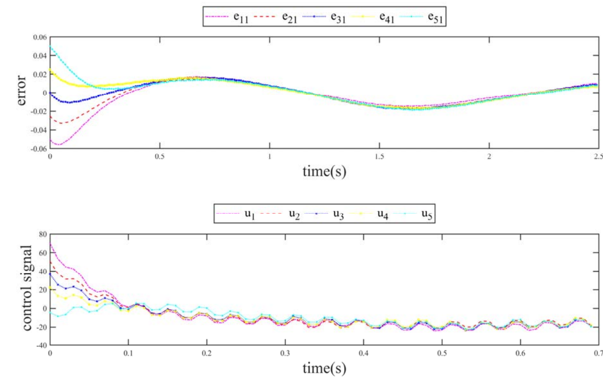
(a)



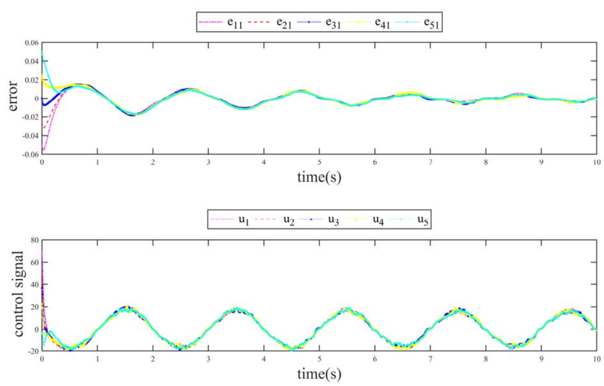
(b)



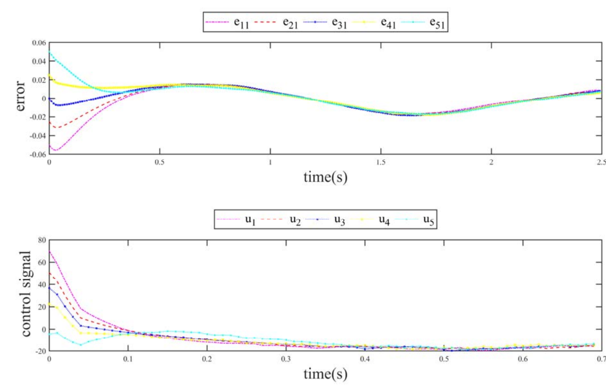
(c)



(d)



(e)



(f)

Fig. 4. Tracking error (e_{i1}) and control input (u_i) for all agents ($i = 1, \ldots, 5$) in case (i) - (iii) ((a), (c) and (e)). Zoomed view of transient time in case (i) - (iii) ((b), (d) and (f)).

VI. CONCLUSIONS

The paper proposes a distributed fuzzy sliding mode controller for high-order stochastic MAS with input affine Itô dynamics to drive agents to a time-varying consensus states of the leader. To this end, the sliding surface combines the tracking error polynomial with consensus error between states of the connected agents. In this way, agents communicate with each other to handle stochastic uncertainties in tracking error signals. Simulations show that this approach can be more successful when agents have more connections with each other. Estimating the dynamic of the agents is another important aspect of the paper. Part of the deterministic dynamics of each agent (drift dynamic) is estimated using a fuzzy system with adaptive parameters. Furthermore, it is assumed that the stochastic dynamics are unknown with an unknown upper bound. In the sense of Lyapunov, we prove that states of the agents stay bounded in mean squared using the proposed controller. Simulation of closed-loop systems with stochastic inverted pendulums as its followers confirm the theoretical result. In the future, we hope to consider some other practical constraints imposed on the system in designing controllers such as partial observability of systems, restricted access to states of the leader.

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