A New Aggregation Operator For Intuitionistic Fuzzy Sets With Applications In The Risk Estimation And Decision Making Problem

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***Abstract*—In all aspects of life, we have to face with uncertainty involved feeling, cognition, evaluation and decision making (DM). In the last decades, a great number of such theories as fuzzy sets, interval-valued fuzzy sets, type-2 fuzzy sets, hesitant sets, grey sets, rough sets and intuitionistic fuzzy sets (IFSs) have been proposed to deal with uncertainty in the real-life imperfect information. They have drawn more and more attention of scholars and been adopted in many various applications. Failure Modes and Effects Analysis (FMEA) is a reliability analysis technique that plays a prominent role in improving the reliability and safety of systems, products, and/or services. It can be used to identify and eliminate known or potential failure modes to enhance reliability and safety of complex systems. Although commonly used, the conventional FMEA has been heavily criticized in the literature for such various limitations as failure mode evaluations, risk factor weights, risk priority number (RPN) computation and dealing with uncertain information. In this paper, we propose a new aggregation operator for IFSs, that proves to be more adequate for fusing information. It is employed in the intuitionistic fuzzy FMEA method in the risk estimation of the system failures with uncertain information. It is validated also in solving a common DM problem, compared with the other existing methods.**

Keywords—intuitionistic fuzzy sets, aggregation operator, failure modes and effects analysis, system risk estimation

I. INTRODUCTION

In many practical situations, the risk assessment made by conventional FMEA is limited due to ambiguousness and uncertainty of available data. Among the methods recently proposed, the fuzzy approach is a powerful tool for dealing with the uncertain information, making FMEA more flexible in solving the real-life problems. Braglia et al. [1] used the fuzzy technique for order preference by similarity to ideal solution (TOPSIS) approach for prioritizing failure modes in FMEA, in which the ranking for failure causes is determined based on the measurement of the Euclidean distance of an alternative from an ideal solution. Based on the TOPSIS, Sachdeva et al. [2] presented an alternative FMEA approach for prioritizing failure modes, which considers the risk factors for failure occurrence, non-detection, maintainability, spare parts, economic safety and economic cost and employs the Shannon entropy concept to compute the objective weights of the six risk factors. Song et al. [3] developed a failure evaluation structure based on fuzzy TOPSIS and comprehensive weighting method to improve the effectiveness of FMEA technique, and proposed an FMEA approach using rough set theory and group TOPSIS method for ranking the risk of failure modes under subjective and uncertain environment. Liu et al. [4] introduced a new modified TOPSIS method, namely the intuitionistic fuzzy hybrid TOPSIS approach, to determine the risk priorities of the failure modes identified in FMEA. Chang [5] proposed a risk assessment method based on soft TOPSIS approach to solve the risk assessment problem in FMEA under a linguistic environment.

Some drawbacks have been noticed in practical implementation of FMEA, mainly risk priority evaluations, complexity and intricacy of use. A great number of methods have been proposed in the literature to overcome this issue, improving the criticality analysis process of the conventional FMEA. Wang et al. [6] have showed drawbacks and given significant criticisms for the traditional FMEA. Then, the authors proposed a new fuzzy FMEA which allows the risk factors and their relative weights to be evaluated in linguistic forms. Liu et al. [7] presented a risk evaluation methodology for FMEA based on combination weighting and fuzzy VIKOR method, in which integration of fuzzy analytical hierarchical process (AHP) and entropy method is utilized to deal with the uncertainty and vagueness from human subjective perception and experience with risk evaluation process. To demonstrate potential applications, the authors adopted the new fuzzy FMEA for analyzing the risk of general anesthesia process. Besides, Zhou and Thai [8] applied grey theory and fuzzy theory in FMEA of the oil tanker equipment failure to show that the evaluation of failure modes by both fuzzy theory and grey theory are quite similar. The fuzzy set theory has been also applied to the DM problems [9], [10] and preference relations [11]. Since

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Atanassov [12] extended Zadeh's fuzzy set to the IFS, it has been intensively studied and applied to many different fields, such as DM [11], intuitionistic fuzzy cognitive maps [13] and pattern recognition [14]. Xu and Liao [15] extended the classical AHP and the fuzzy AHP to the intuitionistic fuzzy AHP procedure for handling comprehensive multi-criteria decision making (MCDM) problems.

In order to get a decision result, an important step is the aggregation of information expressed in terms of the IFSs. It is highly important to gain maximum knowledge conveyed by information using a proper aggregation method. Up to now, many aggregation operators have been put forward in the literature, such as the family of intuitionistic fuzzy averaging aggregation (IFAA) operators [15], the family of intuitionistic fuzzy ordered weighted averaging (IFOWA) operators [16], the family of induced intuitionistic fuzzy ordered weighted averaging (I-IFOWA) operators [17]. However, as shown below, they provide unreasonable results in some cases. In this paper, a new combined aggregation operator based on novel operations for IFSs is proposed, which provides balanced results making it more reasonable and suitable for outranking and pairwise comparison in the MCDM procedures. To demonstrate the applicability and practicality compared with other methods, the proposed method is applied in the FMEA-based system risk estimation under uncertainty based on the expert judgments and in solving a common MCDM problem as well.

The rest of the paper is organized as follows: Section II briefly recalls some basic notions of IFSs and existing aggregation operators with the critical remarks. In Section III, a definition of operations and aggregation operator on IFVs is proposed with some proven properties. Section IV shows an application of the developed technique in the risk estimation of the tanker system failures. Section V provides and analyzes results of the developed aggregation operator compared with the existing ones in solving a common DM problem. In Section VI, the concluding remarks are given.

II. Preliminaries

A. Intuitionistic Fuzzy Values (IFVs)

The IFS was introduced by Atanassov [12] as an object of the universe of discourse *X:*

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}, \quad (1)$$

where $\mu_A(x)$ denotes a degree of membership and $\nu_A(x)$ denotes a degree of non-membership of x to A, $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ such that $0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X$. The relation between any two IFSs A and B in X is defined as: $A \subseteq B$ iff $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for $\forall x \in X$. For simplicity, hereinafter we denote a single element IFS as IFV $A = \langle \mu_A, \nu_A \rangle$.

Let $A = \langle \mu_A, \nu_A \rangle$ and $B = \langle \mu_B, \nu_B \rangle$ be two IFVs. The operations of addition $\oplus$ and multiplication $\otimes$ on IFVs were defined in [12] as follows:

$$A \oplus B = \langle \mu_A + \mu_B - \mu_A\mu_B, \nu_A\nu_B \rangle, \quad (2)$$

$$A \otimes B = \langle \mu_A\mu_B, \nu_A + \nu_B - \nu_A\nu_B \rangle, \quad (3)$$

$$\lambda A = \langle 1 - (1 - \mu_A)^\lambda, {\nu_A}^\lambda \rangle, (\lambda > 0), \quad (4)$$

$$A^\lambda = \langle {\mu_A}^\lambda, 1 - (1 - \upsilon_A)^\lambda \rangle, (\lambda > 0). \quad (5)$$

In order to compare and get an order relation between any pair of IFVs A and B, the score S(A) and accuracy H(A) functions were introduced in [15] as follows:

$$S(A) = \mu_A - \nu_A, \quad H(A) = \mu_A + \nu_A \quad (6)$$

If (S(A) > S(B)) , then A is greater than B and is denoted by $A \succ B$;

If (S(A) = S(B)) , then

a) If (H(A) = H(B)) , then A = B ;

b) If (H(A) < H(B)), then $B \succ A$.

B. Aggregation Operator For IFVs

The operations (2) – (5) are used to aggregate local criteria for solving MCDM problems in the IFSs. Let $A_1, \dots, A_m$ be IFVs representing the values of local criteria and $w_1, \dots, w_m$; $\sum_{j=1}^m w_j = 1$ be their weights. Then intuitionistic fuzzy weighted arithmetic mean (IFWAM) can be obtained using operations (2) and (4) as follows:

$$IFWAM_w(A_1, \dots, A_m) = w_1A_1 \oplus \dots \oplus w_mA_m = \langle 1 - \prod_{j=1}^m (1 - \mu_A)^{w_j}, \ \prod_{j=1}^m (\nu_A)^{w_j} \rangle. \quad (7)$$

Intuitionistic fuzzy weighted geometric mean (IFWGM) can be obtained using operations (3) and (5) as follows:

$$\mathrm{IFWGM}_w(A_1, \dots, A_m) = w_1A_1 \otimes \dots \otimes w_mA_m = \langle \prod_{j=1}^m (\mu_A)^{w_j}, 1 - \prod_{j=1}^m (1 - \nu_A)^{w_j} \rangle. \quad (8)$$

These aggregation operators are most popular in the solution of multiple criteria decision-making problems in the IFSs. The weighted arithmetic average operator emphasizes the group's influence, whereas the weighted geometric average operator emphasizes the individual influence. However, there are some unreasonable results like non-monotonicity provided by these operators as shown below.

Example 1. Let $A = \langle 0.15, 0.3 \rangle$, $B = \langle 0.4, 0.5 \rangle$ and $C = \langle 0.15, 0.1 \rangle$ be three IFVs. Since the score function $S(A) = -0.15$ and $S(B) = -0.1$, so we get $B \succ A$. With the weights $w_1 = w_2 = 0.5$, we aggregate B with C and A with C using (8). Respectively, we get $\mathrm{IFWGM}_w(B, C) = \langle 0.24, 0.33 \rangle$, $\mathrm{IFWGM}_w(A, C) = \langle 0.15, 0.21 \rangle$. Calculating their score function, we get $S(\mathrm{IFWGM}_w(B, C)) = -0.08$, $S(\mathrm{IFWGM}_w(A, C)) = -0.06$, then we have $\mathrm{IFWGM}_w(B, C) \prec \mathrm{IFWGM}_w(A, C)$ whereas $B \succ A$ showing that the operator IFWGM is non-monotonic on the IFVs.

To overcome these flaws, various aggregation operators have been proposed recently such as the family of intuitionistic fuzzy interaction Maclaurin symmetric means [18], the family of intuitionistic fuzzy power Heronian aggregation (IFPHA) operators [19], the family of intuitionistic fuzzy power Muirhead mean (IFPMM) operators [20], the family of intuitionistic fuzzy Bonferroni mean (IFBM) aggregation operators [21] and intuitionistic fuzzy hybrid weighted arithmetic and geometric aggregation operators (IFHWAGA) [22]. These operators have the common feature that the interrelationship between arguments could be adjusted by a combination of the arithmetic and geometric aggregation operators with the parameter vector p. However, they all have the same counterintuitive cases as considered below. Let us consider the called intuitionistic fuzzy Muirhead mean (IFWMM) aggregation operator [20], which is regarded as a generalization of such operators as power Heronian mean, Bonferroni mean, Maclaurin mean or hybrid arithmetic and geometric mean as follows.

Definition 1 [20]. Let $A_i = \langle \mu_i, \nu_i \rangle$ (i = 1,2,. . .,n) be a collection of IFVs, and $p = (p_1, p_2, \dots, p_n) \in R^n$ be a vector of parameters. The intuitionistic fuzzy MM (IFMM) is defined as follows:

$$IFMM^p(A_1, A_2, \dots, A_n) = \left(\frac{1}{n!}\sum_{\vartheta \in S_n}\prod_{j=1}^n A_{\vartheta(j)}^{p_j}\right)^{\frac{1}{\sum_{j=1}^n p_j}} = \langle\left(1-\left(\prod_{\vartheta\in S_n}\left(1-\prod_{j=1}^n \mu_{\vartheta(j)}^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}}, 1-\left(1-\left(\prod_{\vartheta\in S_n}\left(1-\prod_{j=1}^n\left(1-\nu_{\vartheta(j)}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^n p_j}}\rangle, \quad (9)$$

where ϑ(j), (j = 1,2,…,n) is any permutation of (1,2,…,n), and S_n is the collection of all permutations of (1,2,…,n). According to the parameter vector p, the defined IFMM operator reduces to the following cases:

If $p = (1, 0, \dots, 0)$, the IFMM reduces to intuitionistic fuzzy arithmetic mean (IFAM) operator

$$IFMM^{(1,0,\dots,0)}(A_1, A_2, \dots, A_n) = \frac{1}{n}\sum_{i=1}^n A_i = \langle 1-\prod_{i=1}^n\left(1-\mu_{A_i}\right)^{1/n}, \prod_{i=1}^n \nu_{A_i}{}^{1/n}\rangle \quad (9a)$$

If $p = (1, 1, \dots, 1)$, the IFMM reduces to intuitionistic fuzzy geometric mean (IFGM) operator

$$IFMM^{(1,1,\dots,1)}(A_1, A_2, \dots, A_n) = \left(\prod_{i=1}^n A_i\right)^{1/n} = \langle \prod_{i=1}^n \mu_{A_i}{}^{1/n}, 1-\prod_{i=1}^n\left(1-\nu_{A_i}\right)^{1/n}\rangle \quad (9b)$$

If $p = (1, 1, 0 \dots, 0)$, the IFMM reduces to intuitionistic fuzzy Bonferroni mean (IFBM) operator

$$IFMM^{(1,1,0\dots,0)}(A_1, A_2, \dots, A_n) = \left(\frac{1}{n(n-1)}\sum_{\substack{i,j=1\\ i\neq j}}^n A_i A_j\right)^{1/2} \quad (9c)$$

If $p = \left(\overbrace{1 \dots, 1}^{k}, \overbrace{0 \dots, 0}^{n-k}\right)$, the IFMM reduces to intuitionistic fuzzy Bonferroni mean (IFBM) operator:

$$IFMM^{\left(\overbrace{1\dots,1}^{k},\overbrace{0\dots,0}^{n-k}\right)}(A_1, A_2, \dots, A_n) = \left(\frac{\sum_{1\le i_1<\dots<i_k\le n}\prod_{j=1}^k A_{ij}}{C_n^k}\right)^{1/k} \quad (9d)$$

where C_n^k is the binomial coefficient, $(i_1, i_2, \dots, i_k)$ traverses all the k-tuple combinations of (1,2,…,n).

However, it is easily seen that if any of the arguments in the IFMM operator (9a), i.e. $A_i = \langle 1, 0\rangle$, then resulting IFV of aggregation equals to $\langle 1, 0\rangle$, which is counterintuitive. Analogously, it is unreasonable if any of the arguments in the IFMM operator (9b), i.e. $A_i = \langle 0, 1\rangle$ leads the resulting IFV of aggregation equals to $\langle 0, 1\rangle$. These cases pertain also to such aggregation operators, as a geometric mean, Bonferroni mean, Heronian mean, Maclaurin mean or hybrid arithmetic and geometric mean as shown in the following example.

Example 2. Let $A_1 = \langle 0, 1\rangle$, $A_2 = \langle 0.7, 0.1\rangle$ and $A_3 = \langle 0.8, 0.2\rangle$ be three IFVs, and P = (1,1,1), then according to (9b) we have $IFMM^p(A_1, A_2, A_3) = \langle \prod_{i=1}^3 \mu_{A_i}{}^{1/3}, 1-\prod_{i=1}^3\left(1-\nu_{A_i}\right)^{1/3}\rangle = \langle 0, 1\rangle$.

Moreover, the IFWAM and IFWGM operators are the special cases of the IFMM operator. So the drawbacks of the IFWAM and IFWGM operators imply the drawbacks of the IFMM operator as it is shown in the Example 1.

Recently, to overcome the drawbacks of the previous operators the improved q-rung orthopair fuzzy weighted averaging (q-IROFWA) aggregation operator has been proposed in [23] as follows.

$$q-IROFWA_w(A_1, A_2, \dots, A_n) = \langle \frac{\left(1-\prod_{j=1}^n\left(1-\mu_j^q\right)^{w_j}\right)^{\frac{1}{q}},}{\left(\prod_{j=1}^n\left(1-\mu_j^q\right)^{w_j}-\prod_{j=1}^n\left(1-\mu_j^q-\vartheta_j^q\right)^{w_j}\right)^{1/q}}\rangle. \quad (9e)$$

However, the proposed q-IROFWA operator is also non-monotonic on the IFVs as in the Example 1. Indeed, using (9ε) for q=1, we calculate $q-IROFWA_w(A, C) = \langle 0.15, 0.21\rangle$ and $q-IROFWA_w(B, C) = \langle 0.29, 0.44\rangle$, respectively. Since, from (6) we obtain the score functions $S\left(q-IROFWA_w(A, C)\right) = -0.06$ and $S\left(q-IROFWA_w(B, C)\right) = -0.15$ showing $q-IROFWA_w(A, C) > q-IROFWA_w(B, C)$, which is inconsistent with A<B.

III. Intuitionistic Fuzzy Weighted Combined Mean (IFWCM) Operator

In order to overcome the aforementioned shortcomings of the popular IFWAM and IFWGM operators in ranking the IFVs, we propose the operations on IFVs as follows.

Definition 2. For any two IFVs $A = \langle \mu_A, \nu_A\rangle$ and $B = \langle \mu_B, \nu_B\rangle$ the following operations are proposed:

$$A \oplus B = \langle \frac{\mu_A+\mu_B}{2}, 1-\frac{(1-\nu_A)+(1-\nu_B)}{2}\rangle, \quad (10)$$

$$A \otimes B = \langle 1-(1-\mu_A)(1-\mu_B), \nu_A \nu_B \rangle, \tag{11}$$

$$\lambda A = \langle \lambda \mu_A, 1-\lambda(1-\nu_A) \rangle, \lambda \in [0,1], \tag{12}$$

$$A^\lambda = \langle 1-(1-\mu_A)^\lambda, \nu_A{}^\lambda \rangle, \lambda \in [0,1]. \tag{13}$$

It is easily noticed that they provide also IFVs. Based on these operations, the intuitionistic fuzzy weighted average mean operator (IFWAM) and geometric mean operator (IFWGM) are as follows:

$$IFWAM(A_1, A_2, \dots, A_n) = \langle \sum_{i=1}^n w_i \mu_{A_i}, \sum_{i=1}^n w_i \nu_{A_i} \rangle, \tag{14}$$

$$IFWGM(A_1, A_2, \dots, A_n) = \langle 1-\prod_{i=1}^n (1-\mu_{A_i})^{w_i}, \prod_{i=1}^n (\nu_{A_i})^{w_i} \rangle \tag{15}$$

Then we propose the intuitionistic fuzzy weighted combined mean operator (IFWCM), which combines the IFWAM and IFWGM aggregation operators and provides the more intuitively resulting values in fusing information.

Definition 3. Let $A_i = \langle \mu_{A_i}, \nu_{A_i} \rangle, (i = 1, 2, \dots, n)$ be a collection of IFVs with the weights $w_i \in [0,1], \sum_{i=1}^n w_i = 1$, the IFWCM is defined as:

$$IFWCM(A_1, A_2, \dots, A_n) = (\sum_{i=1}^n w_i A_i)^\theta (\prod_{i=1}^n A_i{}^{w_i})^{(1-\theta)} = \langle \left(\sum_{i=1}^n w_i \mu_{A_i}\right)^\theta \left(1-\prod_{i=1}^n (1-\mu_{A_i})^{w_i}\right)^{(1-\theta)}, \left(\sum_{i=1}^n w_i \nu_{A_i}\right)^\theta \left(\prod_{i=1}^n \nu_{A_i}{}^{w_i}\right)^{(1-\theta)} \rangle, \tag{16}$$

where $\theta \in [0,1]$.

For different values of the parameter θ we get different effects of aggregation between arithmetic and geometric means. Particularly, for $\theta = 1$ the IFWCM operator reduces to the IFWAM operator and for $\theta = 0$ the IFWCM operator reduces to the IFWGM operator. Taking $\theta = 0.5$ we derive an aggregated value as a mean of the IFWAM and IFWGM operators.

Theorem 1. Let $\{A_1, A_2, \dots, A_n\}$ be a collection of IFVs and $\{w_1, w_2, \dots, w_n\}$ be a weighting vector, such that $w_i > 0, \sum_{i=1}^n w_i = 1$, the aggregation operator $IFWCM$ fulfills the following properties:

a) Idempotency:

If $A_1 = A_2 = \dots = A_n = A = \langle \mu_A, \nu_A \rangle$ then

$$IFWCM(A_1, A_2, \dots, A_n) = A, \tag{16a}$$

Proof: If $A_1 = A_2 = \dots = A_n = A = \langle \mu_A, \nu_A \rangle$ then $IFWCM(A, A, \dots, A) = \langle (\sum_{i=1}^n w_i \mu_A)^\theta (1-\prod_{i=1}^n (1-\mu_A)^{w_i})^{(1-\theta)}, (\sum_{i=1}^n w_i \nu_A)^\theta (\prod_{i=1}^n \nu_A{}^{w_i})^{(1-\theta)} \rangle = \langle (\mu_A)^\theta (1-1+\mu_A)^{(1-\theta)}, (\nu_A)^\theta (\nu_A)^{(1-\theta)} \rangle = \langle \mu_A, \nu_A \rangle = A$. This ends the proof.

b) Commutativity:

If $\{\hat{A}_1, \hat{A}_2, \dots, \hat{A}_n\}$ is any permutation of $\{A_1, A_2, \dots, A_n\}$, then

$$IFWCM(A_1, A_2, \dots, A_n) = IFWCM(\hat{A}_1, \hat{A}_2, \dots, \hat{A}_n). \tag{16b}$$

The proof is straightforward from (16).

c) Monotonicity:

Let $A_i = \langle \mu_{A_i}, \nu_{A_i} \rangle$ and $B_i = \langle \mu_{B_i}, \nu_{B_i} \rangle$, $(i = 1, 2, \dots, n)$ be two collections of IFVs and $\{w_1, w_2, \dots, w_n\}$ be a weight vector, such that $w_i > 0, \sum_{i=1}^n w_i$. If $A_i \succcurlyeq B_i$ for all $i = 1, 2, \dots, n$, then

$$IFWCM(A_1, A_2, \dots, A_n) \succcurlyeq IFWCM(B_1, B_2, \dots, B_n). \tag{16c}$$

Proof:

For $w_i > 0, \sum_{i=1}^n w_i, i = 1, 2, \dots, n$, we get $A_i \succcurlyeq B_i \Leftrightarrow \mu_{A_i} \geq \mu_{B_i}, \nu_{A_i} \leq \nu_{B_i} \Leftrightarrow w_i \mu_{A_i} \geq w_i \mu_{B_i}, w_i \nu_{A_i} \leq w_i \nu_{B_i} \Leftrightarrow \sum_{i=1}^n w_i \mu_{A_i} \geq \sum_{i=1}^n w_i \mu_{B_i}$ and $\sum_{i=1}^n w_i \nu_{A_i} \leq \sum_{i=1}^n w_i \nu_{B_i}$.

We also have $A_i \succcurlyeq B_i \Leftrightarrow \mu_{A_i} \geq \mu_{B_i}, \nu_{A_i} \leq \nu_{B_i} \Leftrightarrow \left(1-\mu_{A_i}\right)^{w_i} \leq \left(1-\mu_{B_i}\right)^{w_i}, \left(\nu_{A_i}\right)^{w_i} \leq \left(\nu_{B_i}\right)^{w_i} \Leftrightarrow 1-\prod_{i=1}^n \left(1-\mu_{A_i}\right)^{w_i} \geq 1-\prod_{i=1}^n \left(1-\mu_{B_i}\right)^{w_i}$ *and* $\prod_{i=1}^n \nu_{A_i}{}^{w_i} \leq \prod_{i=1}^n \nu_{A_i}{}^{w_i}$.

Combining these two formulas we get

$\left(\sum_{i=1}^n w_i \mu_{A_i}\right)^\theta \left(1-\prod_{i=1}^n \left(1-\mu_{A_i}\right)^{w_i}\right)^{(1-\theta)} \geq \left(\sum_{i=1}^n w_i \mu_{B_i}\right)^\theta \left(1-\prod_{i=1}^n \left(1-\mu_{B_i}\right)^{w_i}\right)^{(1-\theta)}$ *and* $\left(\sum_{i=1}^n w_i \nu_{A_i}\right)^\theta \left(\prod_{i=1}^n \nu_{A_i}{}^{w_i}\right)^{(1-\theta)} \leq \left(\sum_{i=1}^n w_i \nu_{B_i}\right)^\theta \left(\prod_{i=1}^n \nu_{B_i}{}^{w_i}\right)^{(1-\theta)} \Leftrightarrow (\sum_{i=1}^n w_i A_i)^\theta (\prod_{i=1}^n A_i{}^{w_i})^{(1-\theta)} \geq (\sum_{i=1}^n w_i A_i)^\theta (\prod_{i=1}^n A_i{}^{w_i})^{(1-\theta)}$, that implies the proof.

d) Boundedness:

Let $A_L = min\{A_1, A_2, \dots, A_n\}$ and $A^U = max\{A_1, A_2, \dots, A_n\}$, then

$$A_L \preccurlyeq IFWCM(A_1, A_2, \dots, A_n) \preccurlyeq A^U. \tag{16d}$$

The proof is straightforward from the monotonic property.

Example 3. In order to show the superiority of the method we consider again the case in Example 1 using the IFWCM with $\theta = 0.5$. We obtain $\mathrm{IFWCM_w(B, C)} = \langle 0.28, 0.26 \rangle$, $\mathrm{IFWCM_w(A, C)} = \langle 0.15, 0.19 \rangle$ and $S\left(\mathrm{IFWCM_w(B, C)}\right) = 0.02$, $S\left(\mathrm{IFWCM_w(A, C)}\right) = -0.04$. Therefore, we have $\mathrm{IFWGM_w(B, C)} \succ \mathrm{IFHWM_w(A, C)}$ whereas $\mathrm{B} \succ \mathrm{A}$ showing that operator $\mathrm{IFWGM_w}$ is monotonic on the IFVs.

IV. Application In Risk Estimation

A. Intuitionistic Fuzzy Risk Factors

The traditional FMEA determines the risk priorities of failure modes using the RPNs as a simple product of probabilities of the occurrence (O), severity (S) and detection (D) of the failure mode. Usually, the risk factors O, S and D are evaluated by experts in linguistic terms. The linguistic terms of the probability of these risk factors and their related IFVs are assumed from the set as shown in Table I.

Suppose there are n failure modes $F_i, (i = 1, \dots, n)$ of the system, and m experts $E_j, (j = 1, \dots, m)$. Let $R_{ij}^o = \langle \mu_{ij}^o, \nu_{ij}^o \rangle$, $R_{ij}^s = \langle \mu_{ij}^s, \nu_{ij}^s \rangle$ and $R_{ij}^d = \langle \mu_{ij}^d, \nu_{ij}^d \rangle$ be the intuitionistic fuzzy ratings of F_i on the risk factors O, S and D; ω_o, ω_s and ω_d be the weights of the three risk factors, $\lambda_j, (j = 1, \ldots, m)$ be the relative importance weights of the experts, $\sum_{j=1}^{m} \lambda_j = 1$. Without loss of generality, using the IFWCM operator (16) with $\theta = 0.5$, we aggregate the intuitionistic fuzzy ratings of all experts on failure modes F_i with respect to the risk factors O, S and D, respectively:

$$R_i^o = IFWCM_\lambda(R_{i1}^o, R_{i2}^o, \ldots, R_{im}^o) = \langle \left(\sum_{j=1}^{m} \lambda_j \mu_{ij}^o\right)^\theta \left(1 - \prod_{j=1}^{m}\left(1-\mu_{ij}^o\right)^{\lambda_j}\right)^{(1-\theta)}, \left(\sum_{j=1}^{m} \lambda_j \nu_{ij}^o\right)^\theta \left(\prod_{j=1}^{m} \nu_{ij}^{o\,\lambda_j}\right)^{(1-\theta)} \rangle, (17)$$

$$R_i^s = IFWCM_\lambda(R_{i1}^s, R_{i2}^s, \ldots, R_{im}^s) = \langle \left(\sum_{j=1}^{m} \lambda_j \mu_{ij}^s\right)^\theta \left(1 - \prod_{j=1}^{m}\left(1-\mu_{ij}^s\right)^{\lambda_j}\right)^{(1-\theta)}, \left(\sum_{j=1}^{m} \lambda_j \nu_{ij}^s\right)^\theta \left(\prod_{j=1}^{m} \nu_{ij}^{s\,\lambda_j}\right)^{(1-\theta)} \rangle, (18)$$

$$\mathrm{R_i^d} = \mathrm{IFWCM_\lambda}(\mathrm{R_{i1}^d}, \mathrm{R_{i2}^d}, \ldots, \mathrm{R_{im}^d}) = \langle \left(\sum_{j=1}^{m} \lambda_j \mu_{ij}^d\right)^\theta \left(1 - \prod_{j=1}^{m}\left(1-\mu_{ij}^d\right)^{\lambda_j}\right)^{(1-\theta)}, \left(\sum_{j=1}^{m} \lambda_j \nu_{ij}^d\right)^\theta \left(\prod_{j=1}^{m} \nu_{ij}^{d\,\lambda_j}\right)^{(1-\theta)} \rangle. (19)$$

TABLE I. INTUITIONISTIC FUZZY RATINGS OF THE RISK FACTORS

Rating	*Probability*	*IFV*
Very remote (VR)	Very remote chance	<0.1, 0.8>
Very low (VL)	Very low chance	<0.2, 0.7>
Low (L)	Low chance	<0.3, 0.6>
Moderately low (ML)	Moderately low chance	<0.4, 0.5>
Moderate (M)	Moderate chance	<0.5, 0.4>
Moderately high (MH)	Moderately high chance	<0.6, 0.3>
High (H)	High chance	<0.7, 0.2>
Very high (VH)	Very high chance	<0.8, 0.1>
Almost certain (AC)	Almost certainty	<0.9, 0.0>

B. *Intuitionistic Fuzzy Risk Priority Number (IFRPN)*

The IFRPN of the failure mode F_i can be aggregated using the IFWCM operator (16) as follows:

$$\mathrm{IFRPN_i} = \mathrm{IFWCM_\omega}(\mathrm{R_i^o}, \mathrm{R_i^s}, \mathrm{R_i^d}) = \langle \left(\frac{\omega_o \mu_i^o + \omega_s \mu_i^s + \omega_d \mu_i^d}{3}\right)^\theta \left(1 - (1-\mu_i^o)^{\omega_o}(1-\mu_i^s)^{\omega_s}\left(1-\mu_i^d\right)^{\omega_d}\right)^{1-\theta}, \left(\frac{\omega_o \nu_i^o + \omega_s \nu_i^s + \omega_d \nu_i^d}{3}\right)^\theta \left(\nu_i^{o\,\omega_o} \nu_i^{s\,\omega_s} \nu_i^{d\,\omega_d}\right)^{1-\theta} \rangle. \quad (20)$$

The score functions of the IFRPNs of failure modes F_i are calculated using (6). The ranking order of the intuitionistic fuzzy membership knowledge measures represents the risk priority of potential causes. In the system failure analysis, failure mode with the biggest score function should be given the top priority.

C. *Numerical Example*

To demonstrate the applicability of the proposed method, an example about tanker system failure of a global tanker ship management company is adapted from [8]. Assume that a FMEA team consisting of five experts identifies 17 potential system failure modes on tankers (Table II) and needs to prioritize them in terms of their failure risks so that high risky failure modes can be corrected with top priorities. Experts evaluate risk factors of failure modes as the probability of their occurrence, severity and detect ability, respectively, using the linguistic terms defined in Table I. The five experts are assigned with the following relative weights: 0.15, 0.25, 0.25, 0.20 and 0.15. Aggregated intuitionistic fuzzy ratings of all experts on failure mode F_i with respect to the risk factors O, S and D denoted as R_i^o, R_i^s and R_i^d, respectively, are computed from (17), (18) and (19). The weights of the risk factors O, S and D are assumed to be 0.40, 0.35 and 0.25. Based on (20), IFRPNs of the 17 failure modes can be calculated as shown in Table II. For example, assuming $\theta = 0.5$, aggregated intuitionistic fuzzy ratings of all experts on failure mode F_1 (auxiliary engine) with respect to the risk factors O, S and D, respectively are obtained as: $R_1^o = \langle 0.424, 0.475 \rangle$, $R_1^s = \langle 0.455, 0.444 \rangle$, $R_1^d = \langle 0.518, 0.381 \rangle$, and the IFRPN of the failure mode F_1 is computed as $IFRPN_1 = \langle 0.459, 0.440 \rangle$. Finally, the priority order of the ship system failure modes is established in Table II, according to the values of their score function.

TABLE II. RESULTS OF THE RISK PRIORITY ORDER

Failure modes	*Ranking by [8]*	*Ranking by proposed IFFMEA*		
		$\theta = 0.5$	$\theta = 1$	$\theta = 0$
Auxiliary engine	8	8	7	9
Auxiliary machinery	6	7	8	7
Boiler	7	10	10	10
Cargo pump	14	13	13	13
Cargo system	17	17	17	17
Deck machinery	3	3	3	3
Electrical system	10	9	9	8
Emergency system	12	14	14	14
Hull part	15	15	15	15
Hydraulic system	13	12	12	12
Inert gas system	11	11	11	11
Main engine	1	1	1	1
Monitoring system	4	5	5	6
Mooring	9	4	4	4
Navigation system	2	2	2	2
Piping system	5	6	6	5
Steering Gear	16	16	16	16

To investigate the robustness of the proposed aggregation operator, the ranking order was made for various attitudes from pessimistic to optimistic, of the DM holders, assuming various values of the parameter θ from the interval [0, 1] in the given example. The results are summarized in the last three columns of the Table II. In the pessimistic case, i.e. $\theta = 1$, the rankings of the tanker system failure modes made by both approaches are consistent in the four most risky failures and seven least risky ones. In the optimistic case, i.e. $\theta = 0$, the rankings of the tanker system failure modes made by both approaches are consistent in up to the fifth riskiest failure. It indicates some differences in the middle places of the ranking order between approaches due to different methods used. In comparison with the ranking order made by [8], the results showed convergence of the two approaches in the three most risky failures and three least risky ones. It indicates the method used in [8] is less reliable in outranking with uncertain information like expert judgments and can be justified as follows. In [8], the aggregation of information was performed in two steps. In the first step, subjective opinions in term of triangular fuzzy values provided by experts were aggregated using the weighted arithmetic mean operator, into collective rates of alternatives on the risk factors O, S and D, respectively. Next, they were aggregated by using the weighted geometric mean operator into overall rates of alternatives. In this paper, the aggregation of information was performed by the combined operator of these two operators, providing more reliable results compared to existing techniques (mentioned in Section II) at any aggregation stage independently on how many times they were aggregated. This overcomes the drawback in the existing fuzzy FMEA that using the aggregation operators many times leads to the same resulting values. Moreover, the proposed method allows stakeholders choose a proper value of θ related to the attitudinal (pessimistic or optimistic) character of decision makers.

V. Application In Decision Making Problem

In this section, to show the effectiveness and practicality of the proposed aggregation operator in DM approach, an example is adapted from [24] as follows. Let x_i $(i = 1, 2, \dots, 4)$ is the set of four alternatives of the air quality in Quangzhou city in the years 2006, 2007, 2008 and 2009, respectively, which are evaluated by three experts e_k(k=1, 2, 3) with respect to three attributes: SO_2 (a_1), NO_2 (a_2) and PM_{10} (a_3). Let $\lambda = (0.4, 0.2, 0.4)$ be the weight vector of the three experts and $w = (0.314, 0.355, 0.331)$ be the weight vector of the attributes. The evaluation values $r_{ij}^{(k)} (k = 1,2,3)$ of the air quality under the three attributes are provided in the form of IFVs, which are given in Tables III, IV and V.

TABLE III. THE DECISION MATRIX $r_{ij}^{(1)}$ OF THE EXPERT e_1

	a_1	a_2	a_3
x_1	<0.265, 0.350>	<0.330, 0.390>	<0.245, 0.275>
x_2	<0.345, 0.245>	<0.430, 0.290>	<0.245, 0.375>
x_3	<0.365, 0.300>	<0.480, 0.315>	<0.340, 0.370>
x_4	<0.430, 0.300>	<0.460, 0.245>	<0.310, 0.520>

TABLE IV. THE DECISION MATRIX $r_{ij}^{(2)}$ OF THE EXPERT e_2

	a_1	a_2	a_3
x_1	<0.125, 0.470>	<0.220, 0.420>	<0.345, 0.490>
x_2	<0.335, 0.335>	<0.300, 0.370>	<0.205, 0.630>
x_3	<0.250, 0.445>	<0.310, 0.585>	<0.240, 0.580>
x_4	<0.365, 0.365>	<0.355, 0.320>	<0.325, 0.485>

TABLE V. THE DECISION MATRIX $r_{ij}^{(3)}$ OF THE EXPERT e_3

	a_1	a_2	a_3
x_1	<0.260, 0.425>	<0.220, 0.450>	<0.255, 0.500>
x_2	<0.270, 0.370>	<0.320, 0.215>	<0.135, 0.575>
x_3	<0.510, 0.220>	<0.450, 0.370>	<0.490, 0.350>
x_4	<0.390, 0.340>	<0.305, 0.475>	<0.465, 0.485>

The whole DM process is developed in the following steps:

Step 1. Since all the attributes are of the same type, so there is no need to normalize it.

Step 2. Aggregate all individual decision matrices with their weights λ_k, using the IFWCM operator (16) with $\theta = 0.5$, into a group collective decision matrix denoted by z_{ij} (Table VI).

Step 3. Aggregate all attribute values in the group collective decision matrix z_{ij} with the attribute weighting vector w using the IFWCM operator (16), into overall evaluation values denoted by $z_i = IFWCM_w(z_{i1}, z_{i2}, z_{i3})$:
$z_1 = \langle 0.257, 0.407 \rangle$, $z_2 = \langle 0.292, 0.353 \rangle$, $z_3 = \langle 0.408, 0.359 \rangle$ and $z_4 = \langle 0.385, 0.387 \rangle$.

TABLE VI. THE COLLECTIVE MATRIX z_{ij} OF THE EXPERT GROUP.

	a_1	a_2	a_3
x_1	<0.236, 0.403>	<0.265, 0.419>	<0.269, 0.399>
x_2	<0.313, 0.310>	<0.361, 0.273>	<0.193, 0.499>
x_3	<0.404, 0.291>	<0.435, 0.385>	<0.383, 0.399>
x_4	<0.401, 0.328>	<0.379, 0.344>	<0.377, 0.498>

Step 4. Compute the score functions (6) of alternatives from their overall evaluation values z_i: $S(z_1) = -0.15$, $S(z_2) = -0.06$, $S(z_3) = 0.04$ and $S(z_4) = -0.001$, and rank the alternatives in the descending order of their score functions. The most desirable alternative is the one with the biggest score value. We obtain $S(z_3) > S(z_4) > S(z_2) > S(z_1)$. Thus, the ranking order of alternatives is $x_3 \succ x_4 \succ x_2 \succ x_1$ and the best alternative is x_3, which coincides with that obtained in [24].

In order to highlight the reliability of the proposed method, we investigate the effect of the parameter θ on the obtained results as shown in Table VII. It can be seen that, the score functions of alternatives are changed along with the changing value of parameter θ. Nevertheless, the ranking order remains the same indicating the reliability of the proposed aggregation operator.

TABLE VII. EFFECT OF THE PARAMETER Θ ON THE DM RESULTS

Parameter θ	Score function	Ranking order
0	$S(z_1) = -0.146, S(z_2) = -0.048,$ $S(z_3) = 0.059$ and $S(z_4) = 0.007$	$x_3 \succ x_2 \succ x_4 \succ x_1$
0.1	$S(z_1) = -0.147, S(z_2) = -0.051,$ $S(z_3) = 0.057$ and $S(z_4) = 0.005$	$x_3 \succ x_2 \succ x_4 \succ x_1$
0.3	$S(z_1) = -0.146, S(z_2) = -0.056,$ $S(z_3) = 0.053$ and $S(z_4) = -0.002$	$x_3 \succ x_2 \succ x_4 \succ x_1$
0.5	$S(z_1) = -0.15, S(z_2) = -0.06,$ $S(z_3) = 0.04$ and $S(z_4) = -0.001$	$x_3 \succ x_2 \succ x_4 \succ x_1$
0.7	$S(z_1) = -0.152, S(z_2) = -0.066,$ $S(z_3) = 0.046$ and $S(z_4) = -0.005$	$x_3 \succ x_2 \succ x_4 \succ x_1$
0.9	$S(z_1) = -0.154, S(z_2) = -0.071,$ $S(z_3) = 0.042$ and $S(z_4) = -0.008$	$x_3 \succ x_2 \succ x_4 \succ x_1$
1.0	$S(z_1) = -0.154, S(z_2) = -0.074,$ $S(z_3) = 0.040$ and $S(z_4) = -0.010$	$x_3 \succ x_2 \succ x_4 \succ x_1$

We utilize some existing methods to solve the same MCDM problem to verify the proposed method. A comparative analysis of the results obtained shows the convergence and divergence between them. We compare our developed method with that developed by Liu et al. [24] based on the weighted t-spherical fuzzy power Muirhead mean (WT-SPFPMM) operator and the t-spherical fuzzy power dual Muirhead mean (WT-SPFPDMM) operator, by Wei [25] based on picture fuzzy weighted averaging (PFWA) operator and by Xu and Yager [26] based on extended picture fuzzy Bonferroni mean (PFBM) operator. The ranking orders obtained by these methods are summarized in Table VIII.

The method developed in [25] is based on basic weighted averaging operator for PFNs and T-SPFNs and is regarded that it cannot consider the interrelationships among PFNs and T-SPFNs. The WT-SPFPMM and WT-SPFPDMM operators are based on the power Muirhead mean operator, which can consider the interrelationship among input arguments. However, as it is shown in Section II, they provide some counterintuitive cases. In fact, for the same parameter p, they infer the two different ranking orders. The extended PFBM operator [26] is also a particular case of the Muirhead mean operator when parameter p=(1,1,0) as shown in Section II, so it produces the same results as WT-SPFPMM operator [24]. Instead, our proposed method provides the ranking order results consistent with that of the WT-SPFPDMM operator. Moreover, the proposed operator is much more simple and practical in DM problems.

TABLE VIII. COMPARATIVE ANALYSIS WITH OTHER OPERATORS

Aggregation operator	Ranking order
WT-SPFPMM [24] with parameter p=(1,1,0)	$x_4 \succ x_3 \succ x_2 \succ x_1$
WT-SPFPDMM [24] with parameter p=(1,1,0)	$x_3 \succ x_4 \succ x_2 \succ x_1$
PFWA [25]	$x_3 \succ x_2 \succ x_4 \succ x_1$
Extended PFBM [26]	$x_4 \succ x_3 \succ x_2 \succ x_1$
The proposed IFWCM	$x_3 \succ x_4 \succ x_2 \succ x_1$

VI. CONCLUSION

In this paper, a novel aggregation operator for IFVs has been proposed to overcome the drawbacks of the existing methods. The applicability of the proposed method is demonstrated in the application of a risk evaluation problem of the tanker system failures. An improved intuitionistic fuzzy FMEA method under uncertainty has been proposed and the obtained results have shown the effectiveness of the proposed method in dealing with uncertainty of available information. Compared with the existing FMEA, the proposed method shows more intuitive and flexible in describing the real-life problems, particularly in the expert investigations. It can be used in the decision-making of managers, arranging the period inspections and maintenances of the equipment properly, which improve the system reliability and safety. The practicality of the proposed method is demonstrated in the application of a DM problem. The comparative analysis with other methods in solving the same problem is conducted to validate the effectiveness of the new proposed method.

REFERENCES

[1] M. Braglia, M. Frosolini and R. Montanari, "Fuzzy TOPSIS approach for failure mode, effects and criticality analysis," Qual. Reliab. Eng. Int. 19(5), pp. 425–443, 2003.

[2] A. Sachdeva, D. Kumar and P. Kumar, "Multi-factor failure mode critically analysis using TOPSIS," J. Indus. Eng. Int. 5(8), pp. 1–9, 2009.

[3] W. Song, X. Ming, Z. Wu and B. Zhu, "A rough TOPSIS approach for failure mode and effects analysis in uncertain environments," Qual. Reliab. Eng. Int. 30(4), pp. 473–486, 2014.

[4] H.C. Liu, J.X. You, M.M. Shan and L.N. Shao, "Failure mode and effects analysis using intuitionistic fuzzy hybrid TOPSIS approach," Soft Comput. 19(4), pp. 1085–1098, 2015.

[5] K.H. Chang, "A novel general risk assessment method using the soft TOPSIS approach," J. Ind. Prod. Eng. 32(6), pp. 408–421, 2015.

[6] Y.M. Wang, K.S. Chin, G.K. Poon and J.B. Yang, "Risk evaluation in failure mode and effects analysis using fuzzy weighted geometric mean," Expert Syst. Appl. 36 (2), pp. 1195–1207, 2009.

[7] H.C. Liu, J.X. You, X.Y. You and M.M. Shan, "A novel approach for failure mode and effects analysis using combination weighting and fuzzy VIKOR method," Applied Soft Computing 28, pp. 579-588, 2015.

[8] Q. Zhou and V.V. Thai, "Fuzzy and grey theories in failure mode and effect analysis for tanker equipment failure prediction," Safety Science 83, pp. 74–79, 2016.

[9] S.M. Chen, S.H. Cheng and T.C. Lan, "Multicriteria decision making based on the TOPSIS method and similarity measures between intuitionistic fuzzy values," Information Sciences, 367-368, pp. 279-295, 2016.

[10] E. Herrera-Viedma, F. Chiclana, F. Herrera and S. Alonso, "A group decision-making model with incomplete fuzzy preference relations based on additive consistency," IEEE Trans. Syst., Man, Cybern., vol. 37, no. 1, pp. 176–189, 2007.

[11] Z.S. Xu, "Intuitionistic preference relations and their application in group decision making," Information Sciences, 177, pp. 2363-2379, 2007.

[12] K.T. Atanassov, "Intuitionistic fuzzy sets," Fuzzy Sets and System 20, pp. 87-96, 1986.

[13] E.I. Papageorgiou and D.K. Iakovidis, "Intuitionistic fuzzy cognitive maps," IEEE Trans. Fuzzy Syst., vol. 21, no. 2, pp. 342–354, 2013.

[14] H. Nguyen, “A novel similarity/dissimilarity measure for intuitionistic fuzzy sets and its application in pattern recognition,” Expert Systems with Applications, 45, pp. 97-107, 2016.

[15] Z.S. Xu and H.C. Liao, “Intuitionistic fuzzy aggregation operators,” IEEE Transactions on Fuzzy Systems, vol. 22, no. 4, pp. 749-761, 2014.

[16] C. Kahraman, S. Ç. Onar and B. Öztaysi, “B2C marketplace prioritization using hesitant fuzzy linguistic AHP,” Int. J. Fuzzy Syst., vol. 20, no. 7, pp. 2202-2215, 2018.

[17] S. Zeng, C. Llopis-Albert and Y. Zhang, “A novel induced aggregation method for intuitionistic fuzzy set and its application in multiple attribute group decision making,” International Journal of Intelligent Systems 33(11), DOI: 10.1002/int.22009, 2018.

[18] P. Liu and W. Liu, “Intuitionistic fuzzy interaction Maclaurin symmetric means and their application to multiple-attribute decision-making,” Technological and Economic Development of Economy 24, 4, pp. 1533-1559, 2018.

[19] P. Liu and S-M. Chen, “Heronian aggregation operators of intuitionistic fuzzy numbers based on the Archimedean t-norm and t-conorm,” International Conference on Machine Learning and Cybernetics , DOI: 10.1109/ICMLC.2016.7872971, 2016.

[20] P. Liu and D. Li, “Some Muirhead Mean Operators for Intuitionistic Fuzzy Numbers and Their Applications to Group Decision Making,” PLoS ONE 12(1), DOI:10.1371/journal. pone.0168767, 2017.

[21] Z-S. Chen, K-S. Chin and K-L. Tsui, “Constructing the geometric Bonferroni mean from the generalized Bonferroni mean with several extensions to linguistic 2-tuples for decision-making,” Appl. Soft Comput., vol. 78, pp. 595-613, 2019.

[22] J. Ye, “Intuitionistic fuzzy hybrid arithmetic and geometric aggregation operators for the decision-making of mechanical design schemes,” Appl. Intell., 47(2), DOI: 10.1007/s10489-017-0930-3, 2017.

[23] L. Xu, Y. Liu and H. Liu, “Some improved q -rung orthopair fuzzy aggregation operators and their applications to multiattribute group decision-making,” Mathematical Problems in Engineering 2019(2), DOI: 10.1155/2019/2036728, 2019.

[24] P. Liu, Q. Khan, T. Mahmood and N. Hassan, “T-Spherical fuzzy power muirhead meanoperator based on novel operational lawsand their application in multi-attributegroup decision making,” IEEE Access, vol. 7, DOI: 10.1109/ACCESS.2019.2896107, 2019.

[25] G. Wei, “Picture fuzzy aggregation operators and their application to multiple attribute decision making,” J. Intell. Fuzzy Syst., vol. 33, no. 2, pp. 713-724, 2017.

[26] Z. Xu and R. Yager, “Intuitionistic fuzzy Bonferroni means," IEEE Trans. Syst., Man, Cybern. C, Appl. Rev., vol. 41, no. 2, pp. 568-578, 2011.