Image Super Resolution with Sparse Data Using ANFIS Interpolation

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***Abstract*—Image super resolution is one of the most popular topics in the field of image processing. However, most of the existing super resolution algorithms are designed for the situation where sufficient training data is available. This paper proposes a new image super resolution approach that is able to handle the situation with sparse training data, using the recently developed ANFIS (Adaptive Network based Fuzzy Inference System) interpolation technique. In particular, the training image data set is divided into different subsets. For subsets with sufficient training data, the ANFIS models are trained using standard ANFIS learning procedure, while for those with insufficient data, the models are obtained through ANFIS interpolation. In the literature, little work exists for image super resolution on sparse data. Therefore, in the experimental evaluations of this paper, the proposed approach is compared with existing super resolution methods with full data, demonstrating that this work is able to produce highly promising results.**

***Index Terms*—ANFIS Interpolation, Image Super Resolution, Sparse Training Data**

I. Introduction

Image Super Resolution (SR) techniques are used to transform any Low Resolution (LR) image into High Resolution (HR) image. LR images are very common in real world applications due to various reasons amongst them budget constraints to purchasing high definition camera is the most common one. In order to improve the resolution of such images, various SR techniques have been proposed in the literature [1]–[6]. Along with the development of advanced machine learning technqiues, the learning based approach has become one of the most popular SR methods.

Learning based SR aims to produce trained mappings that simulate the underlying relationship between the LR and HR image, through the use of a large amount of training data. Particularly, linear mappings are firstly proposed to describe the relationship of LR and HR images [2]. Whilst it is very simple to implement such methods often suffer from inaccurate results. This raises the requirement of developing more accurate, non-linear mappings. For instance, a deep convolutional neural network (CNN) is used in [3] to generate nonlinear mappings between LR and HR images, representing the state of the art techniques in SR. However, such a strategy requires a huge amount of training data to work, whilst the resulting mappings are not easy to interpret despite their accuracy. A possible alternative is to explore fuzzy rule based approaches [5] which can produce non-linear mappings through the use of a set of fuzzy rules, while making both the training process and the application of learned mappings interpretable.

Following this motivation, to ensure the accuracy of learned fuzzy rules that depict the non-linear mappings between LR and HR images, ANFIS [7] (Adaptive Network based Fuzzy Inference System) has recently been employed [8]. As with most existing methods in the literature, it also assumes that the image SR problems are addressed with sufficient training data. Unfortunately, in many real world situations, certain images or certain parts of an image are very difficult to obtain. Thus, the SR task over such insufficient image training data is still a challenging problem. The most recently introduced ANFIS interpolation [9] technique may provide a possible solution to this type of problem, being capable of constructing ANFIS models with only a small number of training data (termed sparse training data) in certain problem space through exploiting well trained ANFIS models in the neighbouring areas.

Inspired by the above observation this paper proposes a new image SR approach with sparse training data, using the ANFIS interpolation technique. In particular, the training data is divided into a number of sub-datasets. Then for those subsets with sufficient training data, the corresponding mappings are learned using the standard ANFIS learning method, whilst for those subsets with sparse training data, the corresponding mappings are interpolated using the proposed ANFIS interpolation approach.

The rest of this paper is organised as follows. Section II presents an overview of the ANFIS interpolation technique. Section III details the proposed SR approach, including both the training and testing phases. Experimental investigation is reported and discussed in Section IV, including the SR experiments with full data as well as sparse data for comparison. Finally, Section V concludes the paper.

II. ANFIS Interpolation

ANFIS interpolation [9] is an extension of classical Fuzzy Rule Interpolation (FRI) methodology [10], [11], aiming to construct an effective target ANFIS $\mathcal{A}_t$ under the situations of data shortage, by interpolating two neighbouring source

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ANFISs $\mathcal{A}_{s1}$ and $\mathcal{A}_{s2}$. The general ANFIS interpolation process can be summarized in the following 3 steps.

A. Rule Dictionary Generation

The underlying fuzzy rules of the learned ANFISs $\mathcal{A}_{s1}$ and $\mathcal{A}_{s2}$ (named source ANFISs) are firstly extracted, which are to be subsequently used for generating a rule dictionary. For the present application problem of image SR, it is reasonable to assume that the ith extracted rule R_i of the TSK type [12] can be expressed in the following format:

$$R_i : if\ x\ is\ A_i,\ then\ z_i = p_i x + r_i \tag{1}$$

where the input variable x stands for a certain LR grey value, A_i denotes a fuzzy set value of x, z_i represents the output of the ith rule or the HR grey value given the fuzzy value of x, and r_i is a constant coefficient within the linear combination of the rule consequent. The SR problem is assumed to be a regression from the LR image to the HR image, and p_i and r_i are the regression coefficients.

A rule dictionary $D = \{D_a, D_c\}$ can then be generated by reorganising the above extracted fuzzy rules, with an antecedent part D_a and a consequent part D_c, each collecting all the antecedents and consequents of those rules. Suppose that two source ANFISs consist of n_1 and n_2 rules, respectively, then there will be totally $N = (n_1 + n_2)$ rules in the rule dictionary. That is, D_a consists of the antecedent parts of all the rules:

$$D_a = \{A_1\ A_2\ \cdots\ A_N\} \tag{2}$$

and the consequent part D_c consists of the consequents of the rules:

$$D_c = \begin{bmatrix} p_1 & p_2 & \cdots & p_N \\ r_1 & r_2 & \cdots & r_N \end{bmatrix} \tag{3}$$

where each column denotes the linear coefficients in the consequent part of a certain rule.

B. Intermediate ANFIS Generation

Having obtained the above rule dictionary, the next step of ANFIS interpolation is to interpolate a group of fuzzy rules to equivalently form an intermediate ANFIS. Firstly, the sparse training data (expressed by $\{(x, z)\}$) are divided into C clusters using the K-means algorithm (although if preferred, any other numeric value-based clustering method may be used as the alternative to perform clustering). For the centre of each cluster, a new fuzzy rule is interpolated. Then, by aggregating all interpolated rules, an intermediate ANFIS results.

More concretely, for each cluster $C_k, k \in \{1, \ldots, C\}$, compute its centre, resulting in $c^{(k)}$. Given the previously obtained antecedent part rule dictionary D_a, the first subroutine to create the intermediate ANFIS is to select L closest rule antecedents $\{A_i \in D_a, i = 1, \ldots, L\}$ with respect to $c^{(k)}$. This is done on the basis of a distance metric, say for simplicity, $d^i = d(A_i, c^{(k)}) = |Rep(A_i) - c^{(k)}|$, where $Rep(A_i)$ stands for the representative value of the fuzzy set A_i [13]. The L rule antecedents $\{A_i\}$ with the smallest distances d^i are chosen, whose index set is denoted by $\mathcal{L}$. For computational simplicity, L is usually taken to be just two [14].

From this, the next subroutine is set to find the best reconstruction weights for the chosen closest rules. This is achieved by solving the following optimisation problem under the constraint that the sum of all the weights equals to 1:

$$w^{(k)} = \min_{w^{(k)}} ||c^{(k)} - \sum_{i \in \mathcal{L}} Rep(A_i) w_i^{(k)}||^2,\ s.t.\ \sum_{i \in \mathcal{L}} w_i^{(k)} = 1 \tag{4}$$

where $w_i^{(k)}$ denotes the relative weighting of R_i. The solution of this constrained least square problem is as follows:

$$w^{(k)} = \frac{G^{-1}\mathbf{1}}{\mathbf{1}^T G^{-1} \mathbf{1}} \tag{5}$$

where $G = (c^{(k)}\mathbf{1}^T - Y)^T (c^{(k)}\mathbf{1}^T - Y)$ is a defined Gram matrix, $\mathbf{1}$ is a column vector of ones, and the columns of Y are the selected rule antecedents.

Following conventional FRI approaches as per [11], [15], the weights $w^{(k)}$ are applied onto both the antecedent part and the consequent part in interpolating a new rule to summarise the kth cluster:

$$R_k : if\ x\ is\ A_k,\ then\ z_k = p_k x + r_k \tag{6}$$

where the parameters are generated by:

$$A_k = \sum_{i \in \mathcal{L}} w_i^{(k)} A_i,\ p_k = \sum_{i \in \mathcal{L}} w_i^{(k)} p_i,\ r_k = \sum_{i \in \mathcal{L}} w_i^{(k)} r_i \tag{7}$$

with $k = 1, 2, \cdots, C$.

C. ANFIS Fine-turning

The interpolated intermediate ANFIS is then used as the initial network to be fine-tuned to construct the final ANFIS $\mathcal{A}_t$ using the standard ANFIS training algorithm as described in [7]. This fine-tuning process involving limited training data is now possible because the intermediate ANFIS provides an initial setup for the expected network. The entire process of ANFIS interpolation is summarized as Alg. 1.

III. Proposed Approach

This section presents the proposed SR algorithm dealing with sparse training data, including a training phase using the ANFIS interpolation technique, and a testing phase. The flowchart of the proposed approach is shown in Fig. 1.

A. Training Phase with ANFIS Interpolation

The training image data set is constructed by the following steps (assuming the application domain is for natural image analysis): 1) Collect 75 Bitmap images as the training images, including all kinds of sharp natural images such as people, buildings, plants, and animals. 2) Down sample the HR images with a scale factor of s; 3) Up scale the down sampled LR images to a certain desired size using bicubic interpolation, so that the LR and the desired HR images are of the same size but the former are of lower resolution.

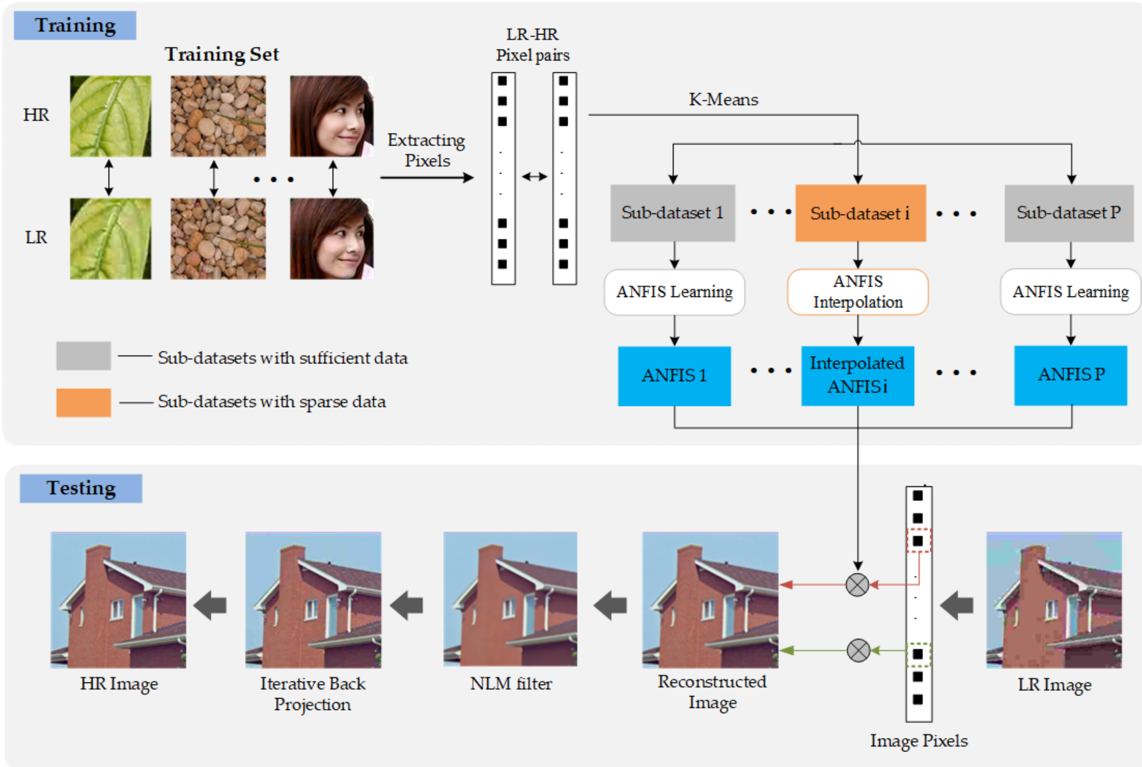

Fig. 1. Flowchart of proposed approach

Algorithm 1: ANFIS Interpolation

Input:

1. Two source ANFISs: $\mathcal{A}_{s1}, \mathcal{A}_{s2}$
2. Sparse training data

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1) Rule Dictionary Generation

1. Extract fuzzy rules $\{R_i\}$ from $\mathcal{A}_{s1}$ and $\mathcal{A}_{s2}$;
2. Construct antecedent part D_a by Eqn. (2);
3. Construct consequent part D_c by Eqn. (3);

2) Intermediate ANFIS Generation

1. Divide sparse training data into C clusters;
2. For each cluster centre $c^{(k)}$, interpolate rule R_k:
   (a) Select L closest atoms in D_a;
   (b) Compute weights $w^{(k)}$ for chosen atoms;
   (c) Generate new rule R_k using weights $w^{(k)}$:
3. Integrate all interpolated rules.

3) ANFIS Fine-tuning

Output:

Interpolated ANFIS: $\mathcal{A}_t$

The resulting LR-HR image pixel pairs form the training data set. They are partitioned into a number of sub-datasets forming the source domains which each contain sufficient training data and the target domains which each contain limited training data. For those containing sufficient training data, it is straightforward to generate an effective ANFIS per subset using the standard ANFIS training procedure as described in [7]. However, for those subsets that lack sufficient training data, i.e., those only being associated with sparse data, the accuracy of the ANFIS directly learned using just the standard ANFIS learning procedure may be rather poor. Therefore, ANFIS interpolation is employed here in an effort to improve the performance of such ANFIS mappings with sparse training data.

For each of sparse data subsets, two neighbouring well trained ANFISs are chosen as the source ANFISs. Here, the neighbourhood relationships are decided on the basis of topological locations of the relevant image pixel subsets (which may be simply implemented using the Euclidean distance metric between the centres of any two subsets). Then the ANFIS mapping of a target subset (that has limited training data contained) can be interpolated using the ANFIS interpolation technique previously described in Section II. Unless otherwise stated, the number of closest antecedents to be selected from D_a is set to 2, which is a common practice in the literature and which is indeed sufficient if an advanced weighted FRI mechanism such as that of [16] is utilised to perform individual rule interpolation [14].

Note that to decide on whether a subset is sparse it might be simple to manually set a threshold. However, this would reduce the level of the algorithm's automation. In this work, the following method is utilized instead. First, calculate the number of data points in each sub-datasets. Then, calculate the average number n_a, if the number of data points in i^{th} sub-dataset $n_i < \alpha n_a$ then the i^{th} sub-dataset is judged as a sparse dataset, where α is a pre-set small co-efficient.

Algorithm 2: Image SR with Sparse Data
A. Training Phase
Input:
Training image data set $\{Z\}$
1. Extract LR-HR pixel pairs from training set;
2. Divide pixel pairs into P subsets: $\{P_i \mid \sum_i P_i = P\}$ using K-Means clustering algorithm;
3. For each subset P_i with sufficient data: Train ANFIS $\mathcal{A}_\mathbf{i}$ with standard learning method;
4. For each P_i with sparse data: Choose 2 closest ANFISs as source ANFISs; Interpolate ANFIS using ANFIS interpolation.
Output:
Multiple learned ANFIS models $\{\mathcal{A}_i\}$
B. Testing Phase
Input:
Testing LR image X
1. Extract pixels from X;
2. Divide pixels into P subsets as per training phase;
3. For each pixel $x_i \in P_i$: Choose relevant ANFIS model $\mathcal{A}_i$; Inference using $\mathcal{A}_i$;
4. Integrate HR pixels to form HR image Y;
5. Post-processing: Suppress noise using NLM filter; Refine resulting image using IBP.
Output:
HR image Y

B. Testing Phase

In the above training phase, the training dataset is grouped into several clusters, these cluster centers are recorded, and for each cluster, an ANFIS is trained or interpolated. After obtaining the multiple learned ANFIS mapping models $\{\mathcal{A}_i\}$, they are subsequently employed in the testing phase, which is implemented according to the following principle: if x_i (given input LR pixel) is close to v_k (the k_{th} cluster center), then the relationship between y_i (estimated HR pixel) and x_i is expressed as:

$$y_i = A_k(xi)$$

In particular, given a testing image X, and the learned ANFIS mappings $\{\mathcal{A}_i\}$, running the proposed SR algorithm involves the following implementation details. Firstly, extract pixels from the LR image and divide them into subsets as done in the training phase. Euclid distances between the input LR pixel and the cluster centers are calculated with the ANFIS corresponding to the cluster with the smallest distance chosen for inference. Then feed each input pixel to the selected ANFIS mapping to compute the output that is to be part of the computed raw SR image. The algorithm determines whether a directly trained ANFIS or an interpolated ANFIS is to be used according to whether a testing sample falls within a subset that corresponds to an area of dense training data or not. This process is repeated for all image pixels, and after that, a raw reconstructed SR image results. Thirdly, a Non Local Means (NLM) filter [17] is employed for suppressing the noise caused by the inference process. Finally, the Iterative Back Projection (IBP) method is used as a post processing technique to refine the resulting SR image. The entire process of the proposed approach is outlined in Alg. 2.

IV. Experimental Studies

This section presents experimental evaluations of the proposed approach, with respect to two commonly used performance criteria.

A. Performance Criteria

Peak Signal to Noise Ratio (PSNR) and Structure SIMilarty (SSIM) are the two most commonly used image SR metrics to evaluate the SR performance in most cases and hence, are adopted here as well. Generally speaking, the larger the values of both, PSNR and SSIM, the better the results.

1) PSNR: It estimates the ratio of signal to noise by computing the following Mean Square Error (MSE) between the original (ground truth) HR image $\mathbf{Y}$ and the calculated HR image $\hat{\mathbf{Y}}$:

$$MSE = \frac{\| \mathbf{Y} - \hat{\mathbf{Y}} \|_F^2}{MN} \tag{8}$$

where M and N are the respective length and width of an image, and $\| \ . \|_F$ represents the Frobenius norm of a matrix. Considering the nature of PSNR which has a wide and dynamic range of values, the following logarithmic decibel scale is used to measure the performance:

$$PSNR = 10 \log_{10}(\frac{255^2}{MSE}) \tag{9}$$

where 255 is the maximum possible pixel values of an image.

2) SSIM: It computes the similarities between the original (ground truth) HR image $\mathbf{Y}$ and the estimated HR image $\hat{\mathbf{Y}}$, and is defined by

$$SSIM = \frac{4\mu_{\mathbf{Y}}\mu_{\hat{\mathbf{Y}}}\sigma_{\mathbf{Y},\hat{\mathbf{Y}}}}{(\mu_{\mathbf{Y}}^2 + \mu_{\hat{\mathbf{Y}}}^2)(\sigma_{\mathbf{Y}}^2 + \sigma_{\hat{\mathbf{Y}}}^2)} \tag{10}$$

where $\mu_{\mathbf{Y}}$ and $\mu_{\hat{\mathbf{Y}}}$ are the mean, and $\sigma_{\mathbf{Y}}$ and $\sigma_{\hat{\mathbf{Y}}}$ are the corresponding standard deviation values of $\mathbf{Y}$ and $\hat{\mathbf{Y}}$ respectively. The SSIM values vary in the range between 0 and 1.

B. Experimental Results

Ten commonly used test images in the problem domain are chosen for testing. Both experimental results from the use of full data and sparse data are considered. The main goal of the proposed algorithm is to deal with the sparse dataset. However, experiments with full-dataset (all sub-datasets are with sufficient data) are also conducted in order to indicate that the proposed algorithm can give competitive performance when compared with existing SR algorithms. This leads to two sets of separate experiments. Note that K-means clustering algorithm is employed for the partition of the training data set, and that the sparse sub-dataset is constructed by deleting a large portion of dataset in the simulation experiments. Common parameters used in the experiments are: the number of image subsets $P = 3$ (for easy illustration); the size of ground truth images ($\mathbf{Y}$) = 256×256 each; and the scale factor for all experimental cases = 2.

1) Experiments with Full Data: This subsection concerns with the experiments that involve the use of full data (i.e., all subsets are covered with sufficient training data). This is in order to provide the best possible solution for the problem at hand for comparison with the case where only sparse data is available for the target region that is to be presented later. Here, the results of running ANFIS models that are trained with sufficient data are compared against bicubic interpolation and two other popular image SR methods: 1) sparse representation based SR [1], and 2) fuzzy rule based SR [5].

Quantitative measurements are listed in Table I (where SD stands for Standard Deviation), whilst visual results are shown in Fig. 2 with detailed patches on the up right corner. In nutshell, it can be observed from Table I that the average PSNR and SSIM values over all ten images using ANFIS outperform the existing methods. If comparing the performances (again, in terms of both PSNR and SSIM) over individual images then ANFIS beats the others in half of the total images, namely child, couple, baboon, Girl 2 and RS. If only considering SSIM value then ANFIS performs better on eight out of the ten images, with slightly less well performance for the butterfly and hat images only. In this case, the SD is the smallest also, indicating the robustness of the approach. These results clearly show the strengths of utilising ANFIS to implement image SR when sufficient training data is available.

2) Experiments with Sparse Data: The purpose of the present work is however to deal with situations where only sparse training data is available for certain areas of the images. Yet, the above results of using full data are very useful still, as the ANFIS trained under that condition is taken as the reference model given its superior performance over the existing alternative methods. To conduct experimental evaluation, the proposed SR approach that utilises ANFIS interpolation while working with just sparse training data is herein compared with such a reference model and also, with the ANFIS that is trained by employing the original ANFIS learning method without interpolation, using the same sparse data.

For illustrative simplicity, given the number of image subsets P being 3, suppose that there are sufficient training data for subsets 1 and 3, but the training data over subset 2 is sparse. In order to simulate the situation in which only sparse training data is in subset 2, a large portion (98%) of the data is deliberately removed in this set of experiments.

The visual results are shown in Fig. 3 with a detailed patch on the up right corner, while the PSNR and SSIM values are listed in Table II. It can be observed from this table that the average PSNR and SSIM values of both the proposed and the reference model are extremely close. Apart from this, the proposed approach significantly outperforms the ANFIS trained with the original learning method without interpolation. Results of Table II demonstrate that the proposed ANFIS interpolation technique offers a powerful means for image super resolution with sparse training data.

Note that the post-processing technique is important to improve the SR performance, which is also been widely used in SR algorithms. In this work, such post-processing is used in all 3 compared methods. So the improved performance is related to the interpolation technique. Note also that the computation complexity of the ANFIS interpolation has been analyzed in paper [9]. The contribution of the proposed method is that it can deal with SR problems with sparse training data, while most existing methods only consider the situation with sufficient training data.

V. Conclusion

This paper has proposed a novel ANFIS interpolation based method for single frame image super resolution, particularly in cases where the available training data is sparse. Comparative experimental investigations have been carried out. The results have shown that the use of ANFIS for SR is at least, on a par with popular existing approaches when sufficient training data is available throughout the problem domain. More significantly, the use of ANFIS interpolation leads to superior SR outcomes than the original ANFIS model when only sparse training data is provided in certain areas of the problem space, almost the same as those achievable using an ANFIS trained with full data. However, the present approach performs ANFIS interpolation in a static manner, that is, all training data is provided at the start of the learning process. How to develop a dynamic ANFIS interpolation method (e.g., following the idea of [18]) for image SR is of great importance in the applied field. Also, how such work may be adapted to help support more effective SR techniques that are implemented with deep learning (which nonetheless require substantial training data [19]) remains active research.

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TABLE I
RESULTS OF FULL DATA EXPERIMENTS WITH SCALE FACTOR BEING 2

Image	**Index**	**Method**			
		Bicubic Interpolation	**Sparse Representation [1]**	**Fuzzy Rule [5]**	**Proposed**
Child	PSNR(dB)/SSIM	33.964/0.933	35.380/0.949	35.438/0.943	**36.180/0.960**
Butterfly	PSNR(dB)/SSIM	27.457/0.915	**30.461/0.950**	30.356/0.949	29.939/0.942
Hat	PSNR(dB)/SSIM	31.727/0.897	**33.628/0.926**	33.127/0.910	33.346/0.922
Couple	PSNR(dB)/SSIM	33.797/0.924	34.725/0.939	35.061/0.934	**35.194/0.942**
Plane	PSNR(dB)/SSIM	29.879/0.927	31.284/0.946	**31.764**/0.942	31.602/**0.949**
Girl 1	PSNR(dB)/SSIM	33.872/0.938	34.721/0.949	**35.372**/0.941	34.916/**0.951**
Baboon	PSNR(dB)/SSIM	26.984/0.751	27.516/0.804	27.441/0.774	**27.727/0.810**
Peppers	PSNR(dB)/SSIM	32.348/0.950	34.193/0.962	**34.898**/0.960	34.256/**0.964**
Girl 2	PSNR(dB)/SSIM	35.424/0.938	36.049/0.946	36.089/0.941	**36.579/0.951**
RS	PSNR(dB)/SSIM	28.355/0.855	29.821/0.898	29.654/0.881	**30.095/0.902**
Average	PSNR(dB) $\pm$ SD / SSIM $\pm$ SD	30.931 $\pm$ 2.8305 / 0.903 $\pm$ 0.0598	32.778 $\pm$ 2.8244 / 0.927 $\pm$ 0.0466	32.920 $\pm$ 2.9727 / 0.918 $\pm$ 0.0552	**32.984** $\pm$ 2.9938 / **0.930** $\pm$ 0.0457

Fig. 2. Results of full data experiments with scale factor being 2

Fig. 3. Results of sparse data experiments with scale factor being 2

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TABLE II
RESULTS OF SPARSE DATA EXPERIMENTS WITH SCALE FACTOR BEING 2

Image	Index	Method		
		Original ANFIS with full data	**Original ANFIS with sparse data**	**Interpolated ANFIS with sparse data**
Child	PSNR(dB)/SSIM	36.180/**0.957**	34.921/0.945	**36.186/0.957**
Butterfly	PSNR(dB)/SSIM	29.939/**0.942**	28.770/0.924	29.899/**0.942**
Hat	PSNR(dB)/SSIM	33.346/**0.922**	32.525/0.899	33.338/**0.922**
Couple	PSNR(dB)/SSIM	**35.194/0.942**	34.755/0.938	35.189/**0.942**
Plane	PSNR(dB)/SSIM	31.602/**0.949**	30.968/0.941	**31.604/0.949**
Girl 1	PSNR(dB)/SSIM	34.916/**0.951**	34.655/0.949	**34.919/0.951**
Baboon	PSNR(dB)/SSIM	27.727/**0.810**	27.414/0.794	27.724/**0.810**
Peppers	PSNR(dB)/SSIM	34.254/**0.964**	32.999/0.942	34.223/**0.964**
Girl 2	PSNR(dB)/SSIM	36.579/**0.951**	35.648/0.938	**36.580/0.951**
RS	PSNR(dB)/SSIM	30.095/**0.902**	29.328/0.875	30.091/**0.902**
Average	PSNR(dB) $\pm$ SD / SSIM $\pm$ SD	**32.983** $\pm$ 2.9937 / **0.929** $\pm$ 0.0455	32.198 $\pm$ 2.9254/ 0.915 $\pm$ 0.0485	32.975 $\pm$ 2.9983 / **0.929** $\pm$ 0.0454

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