A Novel Group Decision Making Approach using Pythagorean Fuzzy Preference Relation

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***Abstract*—Pythagorean Fuzzy Preference Relations (*PFPRs*) have been considered in recent literature more powerful and flexible than the popular intuitionistic fuzzy preference relation in dealing with the linguistic imprecision for decision makers in the large scale group decision making. Following on this promising trend, a novel approach based on the *PFPRs* is proposed for decision support. In particular, the proposed work starts with the acquisition of the optimal comparison matrices, which essentially record the pairwise comparison of the alternatives from the positive and negative opinions. The proposed consensus reaching process is then utilised to guide the decision makers to revise the provided information in order to reach the overall group consensus, before the derivation of rankings of the alternatives. Experimental studies are provided to demonstrate the workings and effectiveness of the proposed approach in comparison with two state-of-the-art methods.**

***Index Terms*—Pythagorean fuzzy preference relations, Group decision making, Consensus reaching process, Consensus measure.**

I. Introduction

Group Decision Making (GDM) has recently attracted significant attention [1] [2] for its widespread involvement in applications such as political and economic forecasting. There usually exists a number of decision makers in *GDM*, easily resulting in increase diversities among the group, which may in return be utilised to improve overall accuracy. The use of fuzzy sets and its extensions, which has been successfully applied in a number of scenarios under certainty [3] [4], enhances the tolerance level of representing linguistic imprecision that often arises from practical decision making. In order to allow for more expressive capability, methods on the basis of preference relations with linguistic scale have been developed in recent literature [5]. For instance, it has been popular to adopt the Intuitionistic Fuzzy Preference Relation (*IFPR*) to represent the decision makers' inaccurate cognitions in terms of the positive, negative and hesitant views [6, 7].

Pythagorean Fuzzy Sets (*PFS*) are an extension of Intuitionistic Fuzzy Sets (*IFS*) [8], which have been considered in recent literature even more powerful and effective than *IFS* describing vagueness and uncertainties in some real-world scenarios [9]. However, the individual consistency problem is an important matter for methods based on *PFS* for *GDM*. Although there exists method that is able to automatically improve the consistency of information provided by decision maker [10], a number of the approaches in the literature typically assume decision makers to provide information to relieve the individual consistency problem by default.

In order to reach the consensus for *GDM* [1], the consensus measure and reaching process are indispensable components to guide decision makers to generate a final collective opinion. However, biased opinions significantly different from what the majority of decision makers hold may hinder reaching the consensus [11]. The approach in the literature to handle the consensus issue can generally be categorised as follows: First, the information originally provided by decision makers is required to revise through combining the member and non-member degrees of *PFPR*, which generally involves complicated computation for both the positive and negative opinions [15]; The second approach, however, revises the information from the perspective of positive opinion only, which simplifies the overall computation, but has the risk of ignoring the negative opinions altogether [16].

Following on the promising trend of Pythagorean Fuzzy Sets *PFS* in handling the linguistic vagueness, this paper proposes the Pythagorean Fuzzy Preference Relation (*PFPR*) for *GDM*, which is further supported by a proposed Consensus Reaching Process (*CRP*) to reach the collective consensus. In particular, the proposed work starts by the acquisition of the optimal comparison matrices utilising the algorithm in [10], which essentially record the pairwise comparison of the alternatives from the positive and negative opinions through the use of Linguistic Discrete Region (*LDR*). The proposed *CRP* is then utilised to guide the decision makers to revise the provided information so that the group consensus can be achieved. Although in the proposed *CRP* the decision makers only modify the information from positive view, the negative information can be derived on the basis of the provided positive information and persisting uncertainty. Finally, the preferences of the alternatives for all decision makers are calculated and aggregated based on the *PFS* aggregation operators, and the ranking of the alternatives can then be obtained through the score of the aggregated *PFS*s.

The remainder of this paper is organized as follows: Section II presents the details of the proposed method. Section III conducts the experimental study to demonstrate the workings and effectiveness of the proposed approach in comparison with relevant approaches. Section IV concludes the paper and outlines ideas for future work.

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II. THE PROPOSED METHOD

The proposed *PFPR*-based approach for group decision making briefly comprise of three steps. First, the pairwise comparison of the alternatives from the positive and negative opinions are provided from decision makers in the form of *LDR*. The optimal comparison matrices are then generated, which serve as input to construct the *PFPR*. Secondly, a consensus reaching process is then developed to guide the decision makers to revise the comparison matrix, which only consists of the membership degrees of the *PFPR*, but with the non-membership degrees of the *PFPR* calculated through the persisted uncertainty and updated membership degrees accordingly. Finally, the preferences of the alternatives for all decision makers are calculated and aggregated based on the *PFS* aggregation operators, and the ranking of the alternatives can then be obtained through the score of the aggregated *PFS*s. The details of each step are introduced in the following subsections.

A. Constructing the Pythagorean Fuzzy Preference Relation

Before constructing the *PFPR* for sub-sequence operations, the definitions of the *PFS* and *PFPR* are given as follows.

Definition 1. [8] A Pythagorean Fuzzy Sets (*PFS*) P over a universe of discourse A is defined as

$$p = \{(x, < \mu_p(x), \nu_p(x) > | x \in A\}$$

where $\mu_p : A \to [0,1]$, and $\nu_p : A \to [0,1]$ verify

$$\mu_p^2(x) + \nu_p^2(x) \leq 1, \ \forall x \in A \tag{1}$$

$\mu_p(x)$ and $\nu_p(x)$ are the membership degree and non-membership degree of x to P accordingly.

For the *PFS*, $\pi_p(x) = \sqrt{1 - \mu_p^2(x) - \nu_p^2(x)}$ denotes the hesitancy degree, and it represents the amount of lacking information in the determination of the membership and non-membership degrees of $x \in A$. For convenience, $< \mu_p(x), \nu_p(x) >$ is called a Pythagorean Fuzzy Number (*PFN*) denoted as $p = (\mu_p, \nu_p)$.

Definition 2. [9] A Pythagorean Fuzzy Preference Relation *PFPR M* on a finite set of alternatives $A = \{A_1, A_2, \ldots, A_n\}$ is characterized by a membership function $\mu_M : A \times A \to [0,1]$ and a non-membership function $\nu_M : A \times A \to [0,1]$ such that

$$0 \leq (\mu_M^2(A_i, A_j) + \nu_M^2(A_i, A_j)) \leq 1, \forall (A_i, A_j) \in A \times A$$

where $\mu_M(A_i, A_j) = \mu_{ij}$ is interpreted as the certainty degree up to which A_i is preferred to A_j, and $\nu_M(A_i, A_j) = \nu_{ij}$ is interpreted as the certainty degree up to which A_i is non preferred to x_j. A *PFPR* can also be represented by a matrix $M = (\mu_{ij}, \nu_{ij})_{n \times n}$.

The procedure of constructing *PFPR* for the decision makers provided information is detailed as follows.

1) The decision makers denoted as ($E = \{e_k | k \in \{1, 2, \ldots, m\}\}$) provide their opinions depending on the pairwise comparisons of the alternatives ($\bar{A} = \{A_i | i \in \{1, 2, \ldots, n\}\}$) with *LDR*, represented as $U_k = (\bar{u}_{ij}^k)_{n \times n}, k \in \{1, 2, \ldots, m\}$ from positive and $V_k = (\bar{v}_{ij}^k)_{n \times n}, k \in \{1, 2, \ldots, m\}$ from negative. The *LDR* cab be represented as follows.

Definition 3. [10] Let $S = \{s_k | k = 0, 1, \ldots, g\}$ be a linguistic term set. The discrete region $D = [s_i, s_j\ ](0 \leq i < j \leq g)$ represents a finite subset of S. $D = \{s_i, s_{i+1}, \ldots, s_j\}$, where $s_i < s_{i+1} < \ldots < s_j$.

2) The obtained information of the decision makers presented as U_k and $V_k (k \in \{1, 2, \ldots, m\})$ is translated into the set-matrices utilizing the numerical scale model,

$$r_i = (\sqrt{c})^{\Delta^{-1}(s_i) - \frac{g}{2}} \tag{2}$$

where $c = 2$, and $\Delta^{-1}(s_i) = i (i \in \{0, 1, \ldots, g\})$. Then, the iterative searching algorithm is used to search the optimal matrix with higher consistency index from U_k and $V_k, k \in \{1, 2, \ldots, m\}$. Let $O_k^U = (r_{ij(u)}^k)_{n \times n}$ and $O_k^V = (r_{ij(v)}^k)_{n \times n}$ denote the obtained optimal matrices from U_k and V_k, $k \in \{1, 2, \ldots, m\}$.

3) The *PFPR* for the decision maker provided information is construct based on the obtained optimal matrix and the concept of *PFS*.

Let $\alpha_{ij}^k = F(r_{ij(u)}^k)$ and $\beta_{ij}^k = F(r_{ij(v)}^k)$ $(i, j \in \{1, 2, \ldots, n\})$ for O_k^U and O_k^V, matrix $M_k = \left((\mu_{ij}^k, \nu_{ij}^k)\right)_{n \times n}$, $(k \in \{1, 2, \ldots, m\})$ denote the constructed *PFPR*. If $1 - ((\alpha_{ij}^k)^2 + (\beta_{ij}^k)^2) \geq 0$, $(\alpha_{ij}^k, \beta_{ij}^k)$ is an *PFS* and $(\mu_{ij}^k, \nu_{ij}^k) = (\alpha_{ij}^k, \beta_{ij}^k)$. Otherwise $(\alpha_{ij}^k - \delta_{ij}^k, \beta_{ij}^k - \delta_{ij}^k)$ is an *PFS*, and $(\mu_{ij}^k, \nu_{ij}^k) = (\alpha_{ij}^k - \delta_{ij}^k, \beta_{ij}^k - \delta_{ij}^k)$. Where

$$F(r_i) = \frac{1}{2}(1 + \log_{g/2}^{r_i})$$

and

$$\delta_{ij}^k = \frac{1}{2}\left(\alpha_{ij}^k + \beta_{ij}^k - \sqrt{2 - |\alpha_{ij}^k - \beta_{ij}^k|}\right)$$

B. Consensus Measure and Reaching Process for PFPR

1) Consensus Measure for PFPR: Once the PFPR is obtained through the above procedure, the consensus reaching process is conducted, which generally consists of two components: (i) A consensus measure that calculates the level of the agreement among decision makers and, (ii) A feedback recommendation mechanism that aims to improve the agreement level among the decision makers [12]. Various consensus models have been proposed recently [2, 5, 12–14]. Usually, the consensus measure for *GDM* is often calculated by measuring the difference between individual opinions and group opinions. Let $E = \{e_1, e_2, \ldots, e_m\}$ and $\bar{A} = \{A_1, A_2, \ldots, A_n\}$ denote the decision makers and the alternatives, and the constructed *PFPR*s for the decision maker be presented as $M_k = \left((\mu_{ij}^k, \nu_{ij}^k)\right)_{n \times n} (k \in \{1, 2, \ldots, m\})$. The Consensus Level (*CL*) associated with the decision maker e_k is defined as [5],

$$CL_k = 1 - \sum_{i,j=1; i \neq j}^{n} \frac{|\mu_{ij}^k - \mu_{ij}^c|}{n(n-1)} \tag{3}$$

where $\mu_{ij}^c(i,j = 1,2,\ldots,n)$ is calculated utilizing the weighted average operator, $\mu_{ij}^c = \sum_{k=1}^{n}\omega_k\mu_{ij}^k$, and $W = \{w_1, w_2, \ldots, w_m\}$ is the weighting vector of the decision makers *E*.

Let ε be a parameter to justify whether the consensus associated with decision maker e_k is acceptable or not. If $CL_k \geq \varepsilon$, the consensus measure of the decision maker (e_k) is accepted, and vice verse. Thus the decision makers can be partitioned into two exclusive consensus groups, represented as $G_A = \{e_k|CL_k \geq \varepsilon, k \in \{1,2,\ldots,m\}\}$ and $G_U = \{e_k|CL_k < \varepsilon, k \in \{1,2,\ldots,m\}\}$. In particular, Li C. et al. [12] proposes a consensus measure for all decision makers as follows:

$$CL = \frac{|G_A|}{m} = \frac{|(\{e_k|CL_k \geq \varepsilon\})|}{m} \tag{4}$$

where $|G_A|$ is the number of the decision makers in G_A. If the consensus measure obtained from Eq. (4) is acceptable, the ranking of the alternatives is computed based on the weighted arithmetic mean. In the event of the consensus not being reached, adjustments are made further to improve the consensus level. Generally speaking, a small number of decision makers who score very low consensus measures would be required to adjust their opinions following on the various consensus rules, before reaching an acceptable consensus measure among all the decision makers.

2) Consensus Reaching Process for PFPR: In order to reach the consensus level accepted by all decision makers, the followings are conducted.

1) Based on the Eq.(3), the consensus measures of the decision makers are calculated and represented as, $\{CL_k|k \in \{1,2,\ldots,m\}\}$. There are the certain consensus measures which are lower than ε, but others are larger than ε, where these decision makers' consensus measures are unacceptable when ε is set as the threshold. The consensus measure for all decision makers is calculated by Eq. (4), where the aggregated results are calculated with the arithmetic mean via the membership degrees of *PFPR*s.

2) Let η be the threshold to the consensus measure (*CL*) for all decision makers, if *CL* $\geq \eta$, which means the consensus measure for all decision makers is acceptable, thus the information provided by the decision makers is not required to modify. Otherwise, the decision makers whose consensus measures are lower than ε are required to modify the provided information to improve the consensus measure (*CL*). First, the decision maker with highest consensus measure is selected from the decision makers with unacceptable consensus measures, denoted as $e_{k_0}^u$. Another decision maker with acceptable consensus measure is searched for and denoted as $e_{j_0}^a$, and $e_{k_0}^u$ and $e_{j_0}^a$ have the smallest distance or the maximized similarity. According to the membership degrees of *PFPR* for $e_{j_0}^a$, the membership degrees for $e_{k_0}^u$ are modified, which are close to the ones of $e_{j_0}^a$ with higher consensus measure. The consensus measures for each decision maker and all decision makers are computed again accordingly, and it can be obtained the consensus measure of all decision makers can be improved.

3) Repeating the process of the information revision, until *CL* $\geq \eta$. The updated membership degrees of *PFPR*s are denoted as $M_l^u = (\mu_{ij}^l(u))_{n\times n}$, where l means the serial number of the decision maker who has modified the provided information, thus it can be obtained that

$$\nu_{ij}^l(\mathrm{u}) = \begin{cases} \sqrt{1-\left(\mu_{ij}^l(u)\right)^2 - \left(\pi_{ij}^l\right)^2} \\ \qquad\qquad (\mu_{ij}^l)^2 + (\nu_{ij}^l)^2 - (\mu_{ij}^l(u))^2 \geq o \\ \sqrt{1-\left(\mu_{ij}^l(u)\right)^2} \qquad otherwise \end{cases} \tag{5}$$

As a result, the update *PFPR*s for the decision maker can be calculated as

$$\begin{aligned} M_k^r &= (\mu_{ij}^k(r), \nu_{ij}^k(r))_{n\times n} \\ &= \begin{cases} (\mu_{ij}^k(u), \nu_{ij}^k(u))_{n\times n}, & k = l \\ (\mu_{ij}^k, \nu_{ij}^k)_{n\times n}, & k \neq l \end{cases} \end{aligned} \tag{6}$$

C. Ranking of the Alternatives

Finally, the ranking of the alternatives can be obtained by aggregating the results in the form of *PFPR*s. In particular, some basic operators on *PFS* can be defined as follows.

Definition 4. [9] Let $p = (\mu_p, \nu_p)$ be a *PFS*. Define the score function as

$$S(p) = \mu_p^2 - \nu_p^2 \tag{7}$$

where $S(p) \in [-1,1]$, and an accuracy function as

$$H(p) = \mu_p^2 + \nu_p^2 \tag{8}$$

where $H(p) \in [0,1]$.

With respect to Definition 4, the ranking of *PFS*s can be obtained based on the following rules.

Definition 5. [9] Let p_1 and p_2 be two *PFS*s, then we have:

1) If $S(p_1) > S(p_2)$, then p_1 is superior to p_2, denoted by $p_1 \succ p_2$.
2) If $S(p_1) = S(p_2)$, then
   (I) If $H(p_1) = H(p_2)$, then p_1 is equivalent to p_2, denoted by $p_1 = p_2$;
   (II) If $H(p_1) > H(p_2)$, then p_1 is superior to p_2, denoted by $p_1 > p_2$.

Second, based on the updated *PFPR*s of the decision maker denoted as M_k^r $(k = 1,2,\ldots,m)$, the preference of the alternative A_i for the decision maker e_k can be calculated as

$$p(A_i^k(\mu)) = \prod_{j=1}^{n}\left(\mu_{ij}^k(r)\right)^{\frac{1}{n}} \tag{9}$$

and

$$p(A_i^k(\nu)) = 1 - \prod_{j=1}^{n}\left(1-\nu_{ij}^k(r)\right)^{\frac{1}{n}} \tag{10}$$

where, $P(A_i^k) = (p(A_i^k(\mu)), p(A_i^k(\nu)))$ $(k = 1,2,\ldots,m, i = 1,2,\ldots,n)$.

TABLE I
DECISION-MAKING RESULTS WITH RESPECT TO DIFFERENT *GDM* METHODS

Methods	Ranking Values			Ranking of Alternatives
	A_1	A_2	A_3	
Method [15]	0.4221	0.3061	0.2810	$A_1 \succ A_2 \succ A_3$
Method [16]	0.4980	0.2152	0.2869	$A_1 \succ A_3 \succ A_2$
Proposed method	0.1598	-0.0921	-0.2031	$A_1 \succ A_2 \succ A_3$

Definition 6. [9] Let $p_k = (\mu_k, \nu_k)(k = 1, 2, \ldots, m)$ be a set of *PFS*s and $\mathbf{W} = (\omega_1, \omega_2, \ldots, \omega_m)^T$ be the weight vector of p_i, with $\sum_{k=1}^{m} \omega_k = 1$, then a Pythagorean fuzzy weighted averaging (*PFWA*) operator is a mapping *PA*: $P^m \to P$, where

$$PA(p_1, p_2, \ldots, p_m) = \left(1 - \prod_{k=1}^{m} (1 - \mu_k)^{\omega_k}, \prod_{k=1}^{m} (\nu_k)^{\omega_k}\right) \tag{11}$$

The aggregated result of $P(A_i^k)$ $(k = 1, 2, \ldots, m, i = 1, 2, \ldots, n)$ can be calculated via *PFWA*, and presented as $PA(A_i) = (PA(A_i(\mu)), PA(A_i(\nu)))$, where

$$PA(A_i(\mu)) = 1 - \prod_{k=1}^{m} (1 - p(A_i^k(\mu)))^{\omega_k}$$

and

$$PA(A_i(\nu)) = \prod_{k=1}^{m} p(A_i^k(\nu))^{\omega_k}$$

Thus, for $i = 1, 2, \ldots, n$, it can be obtained that

$$S(A_i) = PA^2(A_i(\mu)) - PA^2(A_i(\nu))$$

and

$$H(A_i) = PA^2(A_i(\mu)) + PA^2(A_i(\nu))$$

The ranking of the alternatives can be obtained based on $S(A_i)$ and $H(A_i)$.

III. EXPERIMENTAL STUDY

A. Experimental Setup

In the experiment, a case study is conducted where 12 students are invited to evaluate the performance of a cell phone from the perspective of {*After Sale Service*, *Brand*, *Price*}. Each index is regarded as an alternative, denoted as $A_i(i \in \{1, 2, \ldots, 3\})$, and the invited 12 students are the decision makers and presented as $E = \{e_1, e_2, \ldots, e_{12}\}$. The used linguistic term set is S = { s_0 = *extremely impossible*, s_1 = *less impossible*, s_2 = *slight impossible*, s_3 = *equally possible*, s_4 = *possible*, s_5 = *high possible*, s_6 = *extremely possible* }. In order to identify ranking of the factors that affect the performance of the cell phone, the proposed approach is then applied to the evaluations made from the group of students with the final result further compared with two recent alternative approaches.

B. Case Study on Cell Phone Evaluation

Following the procedures as proposed in Section II, the *PFPR* is constructed first. Through the pair-wise comparison of the alternatives via *LDR* in terms of the positive and negative views, the following matrices with *LDR* are constructed based on the information provided by each decision maker, where *LDR*s denote the uncertainties over the pairwise comparisons of the alternatives:

$$U_1 = \begin{bmatrix} [s_3] & [s_1, s_2] & [s_1] \\ [s_4, s_5] & [s_3] & [s_1, s_2] \\ [s_5] & [s_4, s_5] & [s_3] \end{bmatrix}$$

and

$$V_1 = \begin{bmatrix} [s_3] & [s_3] & [s_4, s_5] \\ [s_3] & [s_3] & [s_3] \\ [s_1, s_2] & [s_3] & [s_3] \end{bmatrix}$$

Utilizing the numerical model $(\sqrt{c})^{\Delta^{-1}(s_k)-3}$ (where $c = 2$) [10], the matrix U_1 with *LDR* is then translated into the set-matrix M_s^U,

$$\begin{bmatrix} 1:[1.000] & 2:[0.500, 0.707] & 1:0.500 \\ 2:[1.414, 2.000] & 1:[1.000] & 2:[0.500, 0.707] \\ 1:2.000 & 2:[1.414, 2.000] & 1:[1.000] \end{bmatrix}$$

The optimal matrix with higher consistency index can then be searched from the set-matrix M_s^U, following on the iterative algorithm [10], resulting in the following matrix,

$$A_U^* = \begin{bmatrix} 1.000 & 0.707 & 0.500 \\ 1.414 & 1.000 & 0.707 \\ 2.00 & 1.414 & 1.000 \end{bmatrix}$$

Similarly, the optimal matrix A_V^* can be obtained according to V_1 as follows:

$$A_V^* = \begin{bmatrix} 1.000 & 1.000 & 1.414 \\ 1.000 & 1.000 & 1.000 \\ 0.707 & 1.000 & 1.000 \end{bmatrix}$$

The *PFPR* denoted as M_1 can be constructed by combining the matrices A_U^* and A_V^* via the concept of *PFS*.

$$\begin{bmatrix} (0.5000, 0.5000) & (0.3423, 0.5000) & (0.1845, 0.6577) \\ (0.6577, 0.5000) & (0.5000, 0.5000) & (0.3423, 0.5000) \\ (0.8155, 0.3423) & (0.6577, 0.5000) & (0.5000, 0.5000) \end{bmatrix}$$

Once PFPR is constructed, the consensus measures for every decision maker are calculated via the membership degrees of the *PFPR*, where the aggregated results are calculated with the arithmetic mean, resulting in the consensus measures for the decision makers as, $\{0.7240, 0.9168, 0.7064, 0.8028, 0.8554, 0.7678, 0.7503, 0.7590, 0.7678, 0.8554, 0.7240, 0.8028\}$. It can be observed that, among the 12 decision makers, the consensus measures for e_1, e_3, e_{11} are lower than 0.75, whereas those from the rest of decision makers are all larger than 0.75. That is to say, if the threshold to accept the consensus measure is set at the level of 0.75, the final consensus measure for all the decision makers is $CL = \frac{9}{12} = 0.75$.

If, however, the threshold is set as 0.8, the consensus measure for all the decision makers is unacceptable and the information provided by the decision makers with lower consensus measures requires to be modified. Among all decision makers with unacceptable consensus measures, the decision maker e_1 has the highest consensus measure with its associated decision information translated into the following matrix,

$$M_1(\mu) = \begin{bmatrix} 0.5000 & 0.3423 & 0.1845 \\ 0.6577 & 0.5000 & 0.3423 \\ 0.8155 & 0.6577 & 0.5000 \end{bmatrix}$$

According to the proposed method, the matrix

$$M_9(\mu) = \begin{bmatrix} 0.5000 & 0.6577 & 0.1845 \\ 0.3423 & 0.5000 & 0.3423 \\ 0.8155 & 0.6577 & 0.5000 \end{bmatrix}$$

is selected as the mediator to guide e_1 to adjust the matrix M_1 in order to improve its consensus measure. Based on the consensus reaching process in proposed method, the modified matrix of membership degrees of *PFPR* for e_1 is,

$$M_1^m(\mu) = \begin{bmatrix} 0.5000 & 0.5000 & 0.1845 \\ 0.5000 & 0.5000 & 0.3423 \\ 0.8155 & 0.6577 & 0.5000 \end{bmatrix}$$

Once the matrix M_1 is refreshed, the consensus measures for the opinions of the decision makers updated as follows, $\{0.7722, 0.9124, 0.7021, 0.8072, 0.8598, 0.7722, 0.7546, 0.7546, 0.7722, 0.8598, 0.7196, 0.8072\}$. It can be observed that the consensus measure of the modified information for e_1 has now been improved to 0.7722, leading to the overall consensus measure for all the decision makers being 0.83, which is now acceptable. The matrix of hesitancy degrees of *PFPR* for e_1 is denoted as

$$H_1 = \begin{bmatrix} 0.5000 & 0.6329 & 0.5333 \\ 0.3174 & 0.5000 & 0.6329 \\ 0.2179 & 0.3174 & 0.5000 \end{bmatrix}$$

As a result, the updated matrix of non-membership degrees of *PFPR* for e_1 is calculated as

$$M_1^m(\nu) = \begin{bmatrix} 0.5000 & 0.3423 & 0.6577 \\ 0.6577 & 0.5000 & 0.5000 \\ 0.3423 & 0.5000 & 0.5000 \end{bmatrix}$$

Based on the updated *PFPR*s of the decision makers, the preferences of the alternative A_i for the decision makers can be calculated and aggregated as, $PA(A_1(\mu)) = 0.5570$, $PA(A_2(\mu)) = 0.4644$, $PA(A_3(\mu)) = 0.4421$, and $PA(A_1(\nu)) = 0.3879$, $PA(A_2(\nu)) = 0.5548$, $PA(A_3(\nu)) = 0.6313$. As a result, we can obtain that

$S(A_1) = 0.1598$, $S(A_2) = -0.0921$, $S(A_3) = -0.2031$. The ranking of the alternatives is $A_1 \succ A_2 \succ A_3$.

C. Comparative Analysis

In order to demonstrate the effectiveness of the proposed method, a comparative analysis is carried out in comparison with two state-of-the-art *GDM* methods [15] [16]. The final rankings returned by different methods are summarised in Table I.

It is worth noting that how to revise the information provided by the decision makers with the unacceptable consensus measure is not advised in [15]. Thus the adaption of *CRP* in [15] is consistent with the proposed method in the experiment. It can be observed that the ranking of the alternatives based on [15] is the same as the proposed method from the Table I. However, the method presented in [15] is based on the Intuitionistic Fuzzy Preference Relations (*IFPR*), which has considered with less flexibility for decision makers to express the information than *PFPR*.

If [16] is used instead, the final result for the ranking of the alternatives is different from the proposed method, with the membership degrees of the Intuitionistic Multiplicative Preference Relations (*IMPR*) revised via the optimal model to improve the consensus measure of all decision makers, but the non-membership degrees of *IMPR* being completely ignored in this method. In addition, the method [16] is also based on the Intuitionistic Fuzzy Set (*IFS*), with less flexibility in expressing the information for the decision makers compared to the use of *PFS*. In a nutshell, the proposed method is able to deliver ranking results in consistent with the one [15] recently proposed in the literature, but has the advantage of providing better flexibility for decision makers to express their opinions.

IV. Conclusion

Inspired by the potentials of pythagorean fuzzy preference relation (*PFPR*) that enables for decision makers to simultaneously provide both the positive and negative evaluation, which has been considered more flexible than the popular Intuitionistic Fuzzy Preference Relations (*IFPR*) and Intuitionistic Multiplicative Preference Relations (*IMPR*), this paper has proposed a three-step novel group decision making method based on *PFPR*. A case study is conducted to demonstrate the working and effectiveness of the proposed approach, with the final results achieved in consistent with the one [15] recently proposed in the literature, but has the advantage of providing better flexibility for decision makers to expression their opinions.

This promising research also opens up an avenue for significant further investigation. In addition to developing further extensions for group decision making involving various consensus reaching processes, future work will apply the proposed method to real-world problems involving uncertainties [17] [18] for decision support.

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