A Novel Group Decision Making Approach using Pythagorean Fuzzy Preference Relation

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Abstract—Pythagorean Fuzzy Preference Relations (PFPRs) have been considered in recent literature more powerful and flexible than the popular intuitionistic fuzzy preference relation in dealing with the linguistic imprecision for decision makers in the large scale group decision making. Following on this promising trend, a novel approach based on the PFPRs is proposed for decision support. In particular, the proposed work starts with the acquisition of the optimal comparison matrices, which essentially record the pairwise comparison of the alternatives from the positive and negative opinions. The proposed consensus reaching process is then utilised to guide the decision makers to revise the provided information in order to reach the overall group consensus, before the derivation of rankings of the alternatives. Experimental studies are provided to demonstrate the workings and effectiveness of the proposed approach in comparison with two state-of-the-art methods.

Index Terms—Pythagorean fuzzy preference relations, Group decision making, Consensus reaching process, Consensus measure.

I. INTRODUCTION

Group Decision Making (GDM) has recently attracted significant attention [1] [2] for its widespread involvement in applications such as political and economic forecasting. There usually exists a number of decision makers in GDM, easily resulting in increase diversities among the group, which may in return be utilised to improve overall accuracy. The use of fuzzy sets and its extensions, which has been successfully applied in a number of scenarios under certainty [3] [4], enhances the tolerance level of representing linguistic imprecision that often arises from practical decision making. In order to allow for more expressive capability, methods on the basis of preference relations with linguistic scale have been developed in recent literature [5]. For instance, it has been popular to adopt the Intuitionistic Fuzzy Preference Relation (IFPR) to represent the decision makers’ inaccurate cognitions in terms of the positive, negative and hesitant views [6, 7].

Pythagorean Fuzzy Sets (PFS) are an extension of Intuitionistic Fuzzy Sets (IFS) [8], which have been considered in recent literature even more powerful and effective than IFS describing vagueness and uncertainties in some real-world scenarios [9]. However, the individual consistency problem is an important matter for methods based on PFS for GDM. Although there exists method that is able to automatically improve the consistency of information provided by decision maker [10], a number of the approaches in the literature typically assume decision makers to provide information to relieve the individual consistency problem by default.

In order to reach the consensus for GDM [1], the consensus measure and reaching process are indispensable components to guide decision makers to generate a final collective opinion. However, biased opinions significantly different from what the majority of decision makers hold may hinder reaching the consensus [11]. The approach in the literature to handle the consensus issue can generally be categorised as follows: First, the information originally provided by decision makers is required to revise through combining the member and non-member degrees of PFPR, which generally involves complicated computation for both the positive and negative opinions [15]: The second approach, however, revises the information from the perspective of positive opinion only, which simplifies the overall computation, but has the risk of ignoring the negative opinions altogether [16].

Following on the promising trend of Pythagorean Fuzzy Sets PFS in handling the linguistic vagueness, this paper proposes the Pythagorean Fuzzy Preference Relation (PFPR) for GDM, which is further supported by a proposed Consensus Reaching Process (CRP) to reach the collective consensus. In particular, the proposed work starts by the acquisition of the optimal comparison matrices utilising the algorithm in [10], which essentially record the pairwise comparison of the alternatives from the positive and negative opinions through the use of Linguistic Discrete Region (LDR). The proposed CRP is then utilised to guide the decision makers to revise the provided information so that the group consensus can be achieved. Although in the proposed CRP the decision makers only modify the information from positive view, the negative information can be derived on the basis of the provided positive information and persisting uncertainty. Finally, the preferences of the alternatives for all decision makers are calculated and aggregated based on the PFS aggregation operators, and the ranking of the alternatives can then be obtained through the score of the aggregated PFSs.

The remainder of this paper is organized as follows: Section II presents the details of the proposed method. Section III conducts the experimental study to demonstrate the workings and effectiveness of the proposed approach in comparison with relevant approaches. Section IV concludes the paper and outlines ideas for future work.
II. THE PROPOSED METHOD

The proposed PFPR-based approach for group decision making briefly comprise of three steps. First, the pairwise comparison of the alternatives from the positive and negative opinions are provided from decision makers in the form of LDR. The optimal comparison matrices are then generated, which serve as input to construct the PFPR. Secondly, a consensus reaching process is then developed to guide the decision makers to revise the comparison matrix, which only consists of the membership degrees of the PFPR, but with the non-membership degrees of the PFPR calculated through the persisted uncertainty and updated membership degrees accordingly. Finally, the preferences of the alternatives for all decision makers are calculated and aggregated based on the PFS aggregation operators, and the ranking of the alternatives can then be obtained through the score of the aggregated PFSs. The details of each step are introduced in the following subsections.

A. Constructing the Pythagorean Fuzzy Preference Relation

Before constructing the PFPR for sub-sequence operations, the definitions of the PFS and PFPR are given as follows.

**Definition 1.** [8] A Pythagorean Fuzzy Sets (PFS) P over a universe of discourse A is defined as

\[ p = \{(x, \mu_P(x), \nu_P(x)) | x \in A\} \]

where \( \mu_P: A \rightarrow [0, 1] \) and \( \nu_P: A \rightarrow [0, 1] \) verify

\[ \mu^2_{P}(x) + \nu^2_{P}(x) \leq 1, \forall x \in A \]  

(1)

\( \mu_P(x) \) and \( \nu_P(x) \) are the membership degree and non-membership degree of \( x \) to \( P \) accordingly.

For the PFS, \( \pi_P(x) = \sqrt{1 - \mu^2_P(x) - \nu^2_P(x)} \) denotes the hesitancy degree, and it represents the amount of lacking information in the determination of the membership and non-membership degrees of \( x \in A \). For convenience, \( \mu_P(x), \nu_P(x) > 0 \) is called a Pythagorean Fuzzy Number (PFN) denoted as \( \mu = (\mu_P, \nu_P) \).

**Definition 2.** [9] A Pythagorean Fuzzy Preference Relation PFPR M on a finite set of alternatives \( A = \{A_1, A_2, \ldots, A_n \} \) is characterized by a membership function \( \mu_M: A \times A \rightarrow [0, 1] \) and a non-membership function \( \nu_M: A \times A \rightarrow [0, 1] \) such that

\[ 0 \leq (\mu_M^2(A_i, A_j) + \nu_M^2(A_i, A_j)) \leq 1, \forall (A_i, A_j) \in A \times A \]

where \( \mu_M(A_i, A_j) = \mu_{ij} \) is interpreted as the certainty degree up to which \( A_i \) is preferred to \( A_j \), and \( \nu_M(A_i, A_j) = \nu_{ij} \) is interpreted as the certainty degree up to which \( A_i \) is non preferred to \( A_j \). A PFPR can also be represented by a matrix \( M = (\mu_{ij}, \nu_{ij})_{n \times n} \).

The procedure of constructing PFPR for the decision makers provided information is detailed as follows.

1) The decision makers denoted as \( (E = \{e_k | k \in \{1, 2, \ldots, m \}) \) provide their opinions depending on the pairwise comparisons of the alternatives \( (\bar{A} = \{A_i | i \in \{1, 2, \ldots, n \}) \) with LDR, represented as \( U_k = (u^k_{ij})_{n \times n}, k \in \{1, 2, \ldots, m \} \) from positive and \( V_k = (v^k_{ij})_{n \times n}, k \in \{1, 2, \ldots, m \} \) from negative. The LDR can be represented as follows.

**Definition 3.** [10] Let \( S = \{s_k | k = 0, 1, \ldots, g \} \) be a linguistic term set. The discrete region \( D = \{s_i, s_j | (0 \leq i < j \leq g) \) represents a finite subset of \( S \). \( D = \{s_i, s_{i+1}, \ldots, s_j \} \), where \( s_i < s_{i+1} < \ldots < s_j \).

2) The obtained information of the decision makers presented as \( U_k \) and \( V_k (k \in \{1, 2, \ldots, m \}) \) is translated into the set-matrices utilizing the numerical scale model,

\[ r_i = (\sqrt{c} \Delta^{-1}(s_i) - \frac{c}{2}) \]

(2)

where \( c = 2 \) and \( \Delta^{-1}(s_i) = i(i \in \{0, 1, \ldots, g\}) \). Then, the iterative searching algorithm is used to search the optimal matrix with higher consistency index from \( U_k \) and \( V_k \), \( k \in \{1, 2, \ldots, m \} \).

3) The PFPR for the decision maker provided information is constructed based on the obtained optimal matrix and the concept of PFS.

Let \( \alpha^k_{ij} = F(r^k_{ij}(w)) \) and \( \beta^k_{ij} = F(r^k_{ij}(o)) (i, j \in \{1, 2, \ldots, n \}) \) for \( O^U_k \) and \( O^V_k \) matrix \( M_k = ((\mu^k_{ij}, \nu^k_{ij}))_{n \times n} \), \( k \in \{1, 2, \ldots, m \} \) denote the constructed PFPR. If \( 0 = ((\alpha^k_{ij})^2 + (\beta^k_{ij})^2) \geq 0 \), \( (\alpha^k_{ij}, \beta^k_{ij}) \) is a PFS and \( (\mu^k_{ij}, \nu^k_{ij}) = (\alpha^k_{ij}, \beta^k_{ij}) \). Otherwise \( (\alpha^k_{ij} - \beta^k_{ij}, \beta^k_{ij} - \delta^k_{ij}) \) is an PFS, and \( (\mu^k_{ij}, \nu^k_{ij}) = (\alpha^k_{ij} - \delta^k_{ij}, \beta^k_{ij} - \delta^k_{ij}) \).

Where

\[ F(r_i) = \frac{1}{2} (1 + \log_{1/2}(1 - c^{-1}(s_i))) \]

and

\[ \delta^k_{ij} = \frac{1}{2} (\alpha^k_{ij} + \beta^k_{ij} - \sqrt{2 - 2 |\alpha^k_{ij} - \beta^k_{ij}|}) \]

B. Consensus Measure and Reaching Process for PFPR

1) Consensus Measure for PFPR: Once the PFPR is obtained through the above procedure, the consensus reaching process is conducted, which generally consists of two components: (i) A consensus measure that calculates the level of the agreement among decision makers and, (ii) A feedback recommendation mechanism that aims to improve the agreement level among the decision makers [12]. Various consensus models have been proposed recently [2, 5, 12–14]. Usually, the consensus measure for GDM is often calculated by measuring the difference between individual opinions and group opinions. Let \( E = \{e_1, e_2, \ldots, e_m \} \) and \( A = \{A_1, A_2, \ldots, A_n \} \) denote the decision makers and the alternatives, and the constructed PFPRs for the decision maker be presented as \( M_k = ((\mu^k_{ij}, \nu^k_{ij}))_{n \times n} \in \{1, 2, \ldots, m \} \). The Consensus Level (CL) associated with the decision maker \( e_k \) is defined as [5],

\[ CL_k = 1 - \sum_{i,j=1,i \neq j}^{n} \frac{|\mu^k_{ij} - \mu^k_{ij}|}{n(n-1)} \]

(3)
where $\mu_{ij}^k(i,j = 1,2,\ldots,n)$ is calculated utilizing the weighted average operator, $\mu_{ij}^k = \sum_{k=1}^{n} \omega_k \mu_{ij}^k$, and $W = \{w_1, w_2, \ldots, w_m\}$ is the weighting vector of the decision makers $E$.

Let $\varepsilon$ be a parameter to justify whether the consensus associated with decision maker $e_k$ is acceptable or not. If $CL_k \geq \varepsilon$, the consensus measure of the decision maker ($e_k$) is accepted, and vice versa. Thus the decision makers can be partitioned into two exclusive consensus groups, represented as $G_A = \{e_k|CL_k \geq \varepsilon, k \in \{1,2,\ldots,m\}\}$ and $G_U = \{e_k|CL_k < \varepsilon, k \in \{1,2,\ldots,m\}\}$. In particular, Li C. et al. [12] proposes a consensus measure for all decision makers as follows:

$$CL = \frac{|G_A|}{m} = \frac{|\{e_k|CL_k \geq \varepsilon\}|}{m} \quad (4)$$

where $|G_A|$ is the number of the decision makers in $G_A$. If the consensus measure obtained from Eq. (4) is acceptable, the ranking of the alternatives is computed based on the weighted arithmetic mean. In the event of the consensus not being reached, adjustments are made further to improve the consensus level. Generally speaking, a small number of decision makers who score very low consensus measures would be required to adjust their opinions following on the various consensus rules, before reaching an acceptable consensus measure among all the decision makers.

2) Consensus Reaching Process for PFPR: In order to reach the consensus level accepted by all decision makers, the followings are conducted.

1) Based on the Eq.(3), the consensus measures of the decision makers are calculated and represented as $\{CL_k|k \in \{1,2,\ldots,m\}\}$. There are the certain consensus measures which are lower than $\varepsilon$, but others are larger than $\varepsilon$, where these decision makers’ consensus measures are unacceptable when $\varepsilon$ is set as the threshold. The consensus measure for all decision makers is calculated by Eq. (4), where the aggregated results are calculated with the arithmetic mean via the membership degrees of PFPRs.

2) Let $\eta$ be the threshold to the consensus measure ($CL$) for all decision makers, if $CL \geq \eta$, which means the consensus measure for all decision makers is acceptable, thus the information provided by the decision makers is not required to modify. Otherwise, the decision makers whose consensus measures are lower than $\varepsilon$ are required to modify the provided information to improve the consensus measure ($CL$). First, the decision maker with highest consensus measure is selected from the decision makers with unacceptable consensus measures, denoted as $e_{k_0}^a$. Another decision maker with acceptable consensus measure is searched for and denoted as $e_{j_0}^a$, and $e_{k_0}^a$ and $e_{j_0}^a$ have the smallest distance or the maximized similarity. According to the membership degrees of PFPR for $e_{j_0}^a$, the membership degrees for $e_{k_0}^a$ are modified, which are close to the ones of $e_{j_0}^a$ with higher consensus measure. The consensus measures for each decision maker and all decision makers are computed again accordingly, and it can be obtained the consensus measure of all decision makers can be improved.

3) Repeating the process of the information revision, until $CL \geq \eta$. The updated membership degrees of PFPRs are denoted as $M^a_k = (\mu_{ij}^k(u))_{n \times n}$, where $l$ means the serial number of the decision maker who has modified the provided information, thus it can be obtained that

$$\nu_{ij}^l(u) = \begin{cases} \sqrt{1 - (\mu_{ij}^1(u))^2 - (\pi_{ij}^1)^2} & (\mu_{ij}^1)^2 + (\nu_{ij}^1)^2 - (\mu_{ij}^1(u))^2 \geq \varepsilon \\ \sqrt{1 - (\mu_{ij}^1(u))^2} & otherwise \end{cases} \quad (5)$$

As a result, the update PFPRs for the decision maker can be calculated as

$$M^a_k = (\mu_{ij}^k(r), \nu_{ij}^k(r))_{n \times n} = \begin{cases} (\mu_{ij}^l(u), \nu_{ij}^l(u))_{n \times n}, & k = l \\ (\mu_{ij}^l(u), \nu_{ij}^l(u))_{n \times n}, & k \neq l \end{cases} \quad (6)$$

C. Ranking of the Alternatives

Finally, the ranking of the alternatives can be obtained by aggregating the results in the form of PFPRs. In particular, some basic operators on PFS can be defined as follows.

Definition 4. [9] Let $p = (\mu_p, \nu_p)$ be a PFS. Define the score function as

$$S(p) = \nu_p^2 - \nu_p^2 \quad (7)$$

where $S(p) \in [-1,1]$, and an accuracy function as

$$H(p) = \nu_p^2 + \nu_p^2 \quad (8)$$

where $H(p) \in [0,1]$.

With respect to Definition 4, the ranking of PFSs can be obtained based on the following rules.

Definition 5. [9] Let $p_1$ and $p_2$ be two PFSs, then we have:

1) If $S(p_1) > S(p_2)$, then $p_1$ is superior to $p_2$, denoted by $p_1 \succ p_2$.
2) If $S(p_1) = S(p_2)$, then

(I) If $H(p_1) = H(p_2)$, then $p_1$ is equivalent to $p_2$, denoted by $p_1 = p_2$;

(II) If $H(p_1) > H(p_2)$, then $p_1$ is superior to $p_2$, denoted by $p_1 \succ p_2$.

Second, based on the updated PFPRs of the decision maker denoted as $M^a_k (k = 1,2,\ldots,m)$, the preference of the alternative $A_i$ for the decision maker $e_k$ can be calculated as

$$p(A_i^k(\mu)) = \prod_{j=1}^{n} (\mu_{ij}^k(r))^{\frac{1}{2}} \quad (9)$$

and

$$p(A_i^k(\nu)) = 1 - \prod_{j=1}^{n} (1 - \nu_{ij}^k(r))^{\frac{1}{2}} \quad (10)$$

where, $P(A_i^k) = (p(A_i^k(\mu)), p(A_i^k(\nu))) (k = 1,2,\ldots,m, i = 1,2,\ldots,n)$. 
Definition 6. [9] Let \( p_k = (\mu_k, \nu_k)(k = 1, 2, \ldots, m) \) be a set of PFs and \( W = (\omega_1, \omega_2, \ldots, \omega_m)^T \) be the weight vector of \( p_i \), with \( \sum_{k=1}^{m} \omega_k = 1 \), then a Pythagorean fuzzy weighted averaging (PFWA) operator is a mapping \( PA: P^m \rightarrow P \), where

\[
PA(p_1, p_2, \ldots, p_m) = \left( 1 - \prod_{k=1}^{m} (1 - \mu_k)^\omega_k, \prod_{k=1}^{m} (\nu_k)^\omega_k \right)
\]

(11)

The aggregated result of \( P(A^k_i) \) \( (k = 1, 2, \ldots, m, i = 1, 2, \ldots, n) \) can be calculated via PFWA, and presented as \( PA(A_i) = (PA(A_i(\mu)), PA(A_i(\nu))) \), where

\[
PA(A_i(\mu)) = 1 - \prod_{k=1}^{m} (1 - p(A^k_i(\mu)))^\omega_k
\]

and

\[
PA(A_i(\nu)) = \prod_{k=1}^{m} p(A^k_i(\nu))^\omega_k
\]

Thus, for \( i = 1, 2, \ldots, n \), it can be obtained that

\[
S(A_i) = PA^2(A_i(\mu)) - PA^2(A_i(\nu))
\]

and

\[
H(A_i) = PA^2(A_i(\mu)) + PA^2(A_i(\nu))
\]

The ranking of the alternatives can be obtained based on \( S(A_i) \) and \( H(A_i) \).

III. EXPERIMENTAL STUDY

A. Experimental Setup

In the experiment, a case study is conducted where 12 students are invited to evaluate the performance of a cell phone from the perspective of (After Sale Service, Brand, Price). Each index is regarded as an alternative, denoted as \( A_i(i \in \{1, 2, \ldots, 3\}) \), and the invited 12 students are the decision makers and presented as \( E = \{e_1, e_2, \ldots, e_{12}\} \). The used linguistic term set is \( S = \{ s_0 = \text{extremely impossible}, s_1 = \text{less impossible}, s_2 = \text{slight impossible}, s_3 = \text{equally possible}, s_4 = \text{possible}, s_5 = \text{high possible}, s_6 = \text{extremely possible} \} \). In order to identify ranking of the factors that affect the performance of the cell phone, the proposed approach is then applied to the evaluations made from the group of students with the final result further compared with two recent alternative approaches.

B. Case Study on Cell Phone Evaluation

Following the procedures as proposed in Section II, the PFPR is constructed first. Through the pair-wise comparison of the alternatives via LDR in terms of the positive and negative views, the following matrices with LDR are constructed based on the information provided by each decision maker, where LDRs denote the uncertainties over the pairwise comparisons of the alternatives:

\[
U_1 = \begin{bmatrix}
[s_8] & [s_1, s_2] & [s_1] \\
[s_4, s_5] & [s_3] & [s_1, s_2] \\
[s_5] & [s_4, s_5] & [s_3]
\end{bmatrix}
\]

and

\[
V_1 = \begin{bmatrix}
[s_8] & [s_3] & [s_4, s_5] \\
[s_1, s_2] & [s_3] & [s_8] \\
[s_8] & [s_3] & [s_8]
\end{bmatrix}
\]

Utilizing the numerical model \( (\sqrt{c})^{A^{-1}(s_k)} \) (where \( c = 2 \) [10], the matrix \( U_1 \) with LDR is then translated into the set-matrix \( M^U_1 \).

\[
\begin{bmatrix}
1 : [1.000] & 2 : [0.500, 0.707] & 1 : 1.000 \\
2 : [1.414, 2.000] & 1 : [0.500, 0.707] & 2 : [1.000] \\
1 : 2.000 & 2 : [1.414, 2.000] & 1 : [1.000]
\end{bmatrix}
\]

The optimal matrix with higher consistency index can then be searched from the set-matrix \( M^U_1 \); following on the iterative algorithm [10], resulting in the following matrix,

\[
A^U_1 = \begin{bmatrix}
1.000 & 0.707 & 0.500 \\
1.414 & 1.000 & 0.707 \\
2.00 & 1.414 & 1.000
\end{bmatrix}
\]

Similarly, the optimal matrix \( A^V_1 \) can be obtained according to \( V_1 \) as follows:

\[
A^V_1 = \begin{bmatrix}
1.000 & 1.000 & 1.414 \\
1.000 & 1.000 & 1.000 \\
0.707 & 1.000 & 1.000
\end{bmatrix}
\]

The PFPR denoted as \( M_1 \) can be constructed by combining the matrices \( A^U_1 \) and \( A^V_1 \) via the concept of PFs.

\[
\begin{bmatrix}
(0.5000, 0.5000) & (0.3423, 0.5000) & (0.1845, 0.6577) \\
(0.6577, 0.5000) & (0.5000, 0.5000) & (0.3423, 0.5000) \\
(0.8155, 0.3423) & (0.6577, 0.5000) & (0.5000, 0.5000)
\end{bmatrix}
\]

Once PFPR is constructed, the consensus measures for every decision maker are calculated via the membership degrees of

<table>
<thead>
<tr>
<th>Methods</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>Ranking of Alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method [15]</td>
<td>0.4221</td>
<td>0.3061</td>
<td>0.2810</td>
<td>( A_1 \succ A_2 \succ A_3 )</td>
</tr>
<tr>
<td>Method [16]</td>
<td>0.4980</td>
<td>0.2152</td>
<td>0.2869</td>
<td>( A_1 \succ A_3 \succ A_2 )</td>
</tr>
<tr>
<td>Proposed method</td>
<td>0.1598</td>
<td>-0.0921</td>
<td>-0.2031</td>
<td>( A_1 \succ A_2 \succ A_3 )</td>
</tr>
</tbody>
</table>
the PFPR, where the aggregated results are calculated with
the arithmetic mean, resulting in the consensus measures
for the decision makers as, \{0.7240, 0.9168, 0.7064, 0.8028,
0.8554, 0.7678, 0.7503, 0.7590, 0.7678, 0.8554, 0.7240, 0.8028\}.
It can be observed that, among the 12 decision makers,
the consensus measures for \(e_1, e_3, e_1\) are lower than 0.75,
whereas those from the rest of decision makers are all
larger than 0.75. That is to say, if the threshold to accept
the consensus measure is set at the level of 0.75, the
final consensus measure for all the decision makers is
\(CL = \frac{9}{12} = 0.75\).

If, however, the threshold is set as 0.8, the consensus
measure for all the decision makers is unacceptable and the
information provided by the decision makers with lower con-
sensus measures requires to be modified. Among all decision
makers with unacceptable consensus measures, the decision
maker \(e_1\) has the highest consensus measure with its associated
decision information translated into the following matrix,
\[
M_1(\mu) = \begin{bmatrix}
0.5000 & 0.3423 & 0.1845 \\
0.6577 & 0.5000 & 0.3423 \\
0.8155 & 0.6577 & 0.5000 \\
\end{bmatrix}
\]

According to the proposed method, the matrix
\[
M_3(\mu) = \begin{bmatrix}
0.5000 & 0.6577 & 0.1845 \\
0.3423 & 0.5000 & 0.3423 \\
0.8155 & 0.6577 & 0.5000 \\
\end{bmatrix}
\]
is selected as the mediator to guide \(e_1\) to adjust the matrix
\(M_1\) in order to improve its consensus measure. Based on the
consensus reaching process in proposed method, the modified
matrix of membership degrees of PFPR for \(e_1\) is,
\[
M_1^m(\mu) = \begin{bmatrix}
0.5000 & 0.5000 & 0.1845 \\
0.5000 & 0.5000 & 0.3423 \\
0.8155 & 0.6577 & 0.5000 \\
\end{bmatrix}
\]

Once the matrix \(M_1\) is refreshed, the consensus measures
for the opinions of the decision makers updated as follows,
\{0.7722, 0.9124, 0.7021, 0.8072, 0.8598, 0.7722, 0.7546,
0.7546, 0.7722, 0.8598, 0.7196, 0.8072\}. It can be observed
that the consensus measure of the modified information for
\(e_1\) has now been improved to 0.7722, leading to the overall
consensus measure for all the decision makers being 0.83,
which is now acceptable. The matrix of hesitancy degrees of
PFPR for \(e_1\) is denoted as
\[
H_1 = \begin{bmatrix}
0.5000 & 0.6329 & 0.5333 \\
0.3174 & 0.5000 & 0.6329 \\
0.2179 & 0.3174 & 0.5000 \\
\end{bmatrix}
\]

As a result, the updated matrix of non-membership degrees
of PFPR for \(e_1\) is calculated as
\[
M_1^m(\nu) = \begin{bmatrix}
0.5000 & 0.3423 & 0.6577 \\
0.6577 & 0.5000 & 0.5000 \\
0.3423 & 0.5000 & 0.5000 \\
\end{bmatrix}
\]
Based on the updated PFPRs of the decision makers,
the preferences of the alternative \(A_i\) for the decision mak-
er can be calculated and aggregated as, \(PA(A_1(\mu)) = \)
0.5570, \(PA(A_2(\mu)) = 0.4644, PA(A_3(\mu)) = 0.4421, \)
and \(PA(A_1(\nu)) = 0.3879, PA(A_2(\nu)) = 0.5548, PA(A_3(\nu)) = \)
0.6313. As a result, we can obtain that
\(S(A_1) = 0.1598, S(A_2) = -0.0921, S(A_3) = -0.2031.\)
The ranking of the alternatives is \(A_1 \succ A_2 \succ A_3\).

C. Comparative Analysis

In order to demonstrate the effectiveness of the proposed
method, a comparative analysis is carried out in comparison
with two state-of-the-art GDM methods [15] [16]. The final
rankings returned by different methods are summarised in
Table I.

It is worth noting that how to revise the information pro-
vided by the decision makers with the unacceptable consensus
measure is not advised in [15]. Thus the adaption of CRP in
[15] is consistent with the proposed method in the experiment.
It can be observed that the ranking of the alternatives based
on [15] is the same as the proposed method from the Table
I. However, the method presented in [15] is based on the
Intuitionistic Fuzzy Preference Relations (IFPR), which has
considered with less flexibility for decision makers to express
the information than PFPR.

If [16] is used instead, the final result for the ranking of
the alternatives is different from the proposed method, with
the membership degrees of the Intuitionistic Multiplicative
Preference Relations (IMPR) revised via the optimal model
to improve the consensus measure of all decision makers, but
the non-membership degrees of IMPR being completely ignored
in this method. In addition, the method [16] is also based
on the Intuitionistic Fuzzy Set (IFS), with less flexibility in
expressing the information for the decision makers compared
to the use of PFS. In a nutshell, the proposed method is able
to deliver ranking results in consistent with the one [15] recently
proposed in the literature, but has the advantage of providing
better flexibility for decision makers to express their opinions.

IV. Conclusion

Inspired by the potentials of pythagorean fuzzy preference
relation (PFPR) that enables for decision makers to simul-
taneously provide both the positive and negative evaluation,
which has been considered more flexible than the popular Intu-
itionistic Fuzzy Preference Relations (IFPR) and Intuitionistic
Multiplicative Preference Relations (IMPR), this paper has
proposed a three-step novel group decision making method
based on PFPR. A case study is conducted to demonstrate
the working and effectiveness of the proposed approach, with
the final results achieved in consistent with the one [15]
recently proposed in the literature, but has the advantage of
providing better flexibility for decision makers to expression
their opinions.

This promising research also opens up an avenue for sig-
ificant further investigation. In addition to developing further
extensions for group decision making involving various con-
sensus reaching processes, future work will apply the proposed
method to real-world problems involving uncertainties [17]
[18] for decision support.
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REFERENCES


