# Reachable set boundedness and fuzzy sliding mode control of MPPT for nonlinear photovoltaic systems

Zhixiong Zhong[1], *Member, IEEE*, Xingyi Wang[1,*], Rathinasamy Sakthivel[2], and Chih-Min Lin[3], *Fellow, IEEE*
*Key Laboratory of Information Processing and Intelligent Control, Minjiang University, Fuzhou 350121, China.*
*Department of Applied Mathematics, Bharathiar University, Coimbatore 641046, India.*
*Department of Electrical Engineering, Yuan Ze University, Taoyuan 320, Taiwan.*
Email: xingyiwang2019@126.com; *Corresponding author: Xingyi Wang

***Abstract*—This paper develops a novel maximum power point tracking (MPPT) control strategy for nonlinear photovoltaic (PV) systems. The MPPT control problem of the considered PV systems is first formulated in the framework of fuzzy descriptor systems. Then, based on the new formulation, a fuzzy sliding mode control (FSMC) law is constructed to drive the state trajectories into a desired sliding surface within the finite-time $T^*$ with $T^* < T$. Moreover, sufficient conditions are derived to ensure the reachable set boundings of the closed-loop PV control systems in the finite-time intervals $[0, T^*]$ and $[T^*, T]$. A simulation example is given to show the effectiveness of the proposed design method.**

***Index Terms*—Reachable set boundedness, maximum power point tracking (MPPT), photovoltaic**

## I. Introduction

Solar power has experienced rapid growth over the past few decades. It is predicted that, by 2020, the total capacity of solar power will reach 980 GW [1]. Solar power is a source of electricity, and it is either solar thermal systems or PV systems. The basic device of a solar PV system is the PV cell, and cells can be integrated into panels or arrays. A solar PV system directly converts sun light into electricity, and feeds small loads, such as lighting systems, alarm, monitoring, and DC motors, or implements more sophisticated applications under grid-connection configurations [2]. However, compared with other clean energy systems, the PV systems suffer from low efficiency, high initial cost. Moreover, their performance is very sensitive to changes in environment conditions, such as solar irradiance and working temperature. In this case, it is very important to maintain the PV operation at its maximum efficiency under parameter uncertainties and external disturbances [3].

For a specific operating condition, PV systems usually have a unique maximum power point (MPP). It is noted that some intrinsic characteristics and disturbances on PV systems, such as aging of the device, irradiance intensity, and temperature conditions, generally lead to inefficient implementations on maximum power point tracking (MPPT) control [4]. Traditional MPPT control of PV systems aims at locating the MPP for online operation in the steady state condition, such as the perturbation and observer-based methods [5], and the hill climbing method [6]. Unfortunately, the above-mentioned methods often result in a slower convergence. Recently, dynamic MPPT control methods have been proposed to improve the transient performance, such as cuckoo search control [7], neural-network-based control [8], and online system identification [9]. However, those methods generally lack a strict convergence analysis.

In the last three decades, DC-DC converters have been widely used in PV systems. The buck, boost and buck-boost circuits are three basic configurations for DC-DC converters [10]. The duty ratio determines the switching action via pulsewidth modulation, which implements control of the DC-DC converters, and exhibits a nonlinear dynamic behavior. Moreover, in most cases the approximated linear model based on a single operating point are subjected to no minimum phase type for the DC-DC converters. Although the linear controller is easier to design and implement, it is difficult to ensure MPPT performance in all the operating conditions. Recently, it has been shown that nonlinear systems can be described by several local linear systems blending IF-THEN fuzzy rules [11]. More recently, T-S fuzzy-model-based approach has been developed for the MPPT control of PV systems with DC-DC converters [12].

Sliding mode control (SMC), as an effective robust control strategy, has been successfully applied to a wide variety of complex systems [13]. More recently, the SMC has been developed for MPPT control of PV systems [14]. Nevertheless, it is worth noting that, in all aforementioned work on SMC, the system dynamic behaviors were considered within a sufficiently long time interval. Most recently, the work of [15] introduced a novel partitioning strategy to ensure finite-time boundedness of system states by using SMC. To the best of the authors' knowledge, fast convergence and strict performance analysis on transient dynamics have not been taken into account for MPPT control of nonlinear PV systems. This motivates our present research.

In this paper, a novel MPPT method is developed for nonlinear PV systems via the descriptor system approach. First, the MPPT control problem of PV systems is reformulated into the

The work described in this paper was supported in part by the Central Government Directs Special Funds for Scientific and Technological Development, China under Grant 2019L3009, Natural Science Foundation of Fujian Province under Grant No. 2016J01267, the Scientific and Technological Projects of Fuzhou City under Grant No. 2019-G-49, the Scientific and Technological Projects of Xiamen City under Grant No. 3502Z20179033, and the Scientific Research Items of MJU under Grant No. MJY18003, the Advanced Research Program of Minjiang University under Project K-30404307.

978-1-7281-6932-3/20/$31.00 ©2020 IEEE

framework of descriptor systems, and the nonlinear dynamics of PV systems are represented by T-S fuzzy model. Based on the new model, a fuzzy sliding mode control (FSMC) law is constructed to drive the state trajectories into the specified sliding surface within the finite-time $T^*$ with $T^* < T$. Moreover, sufficient conditions are derived to ensure the reachable set boundings of the closed-loop PV control systems in the finite-time intervals $[0, T^*]$ and $[T^*, T]$. A simulation example is given to show the effectiveness of the proposed design method.

## II. Problem Formulation and Preliminaries

Consider a solar PV power system using a DC/DC boost converter as shown in Fig.1, which consists of a solar PV array, an inductor $L$, a capacitor $C_0$, and a load. Its dynamic model can be represented by the following differential equations [16],

$$\begin{cases} \dot{\phi}_{pv} = -\frac{1}{L}(1-u)\, v_{dc} + \frac{1}{L} v_{pv}, \\ \dot{v}_{dc} = \frac{1}{C_0}(1-u)\,\phi_{pv} - \frac{1}{C_0}\phi_0, \end{cases} \tag{1}$$

where $u \in [0, 1]$ represents the duty ratio; $\phi_{pv}$ and $v_{dc}$ stand for the inductor current and the capacitor voltage, respectively; $\phi_0$ and $v_0$ denote the load current and the load voltage, respectively. It should be noted that the duty ratio $u$ carries out the switching action by using PWM.

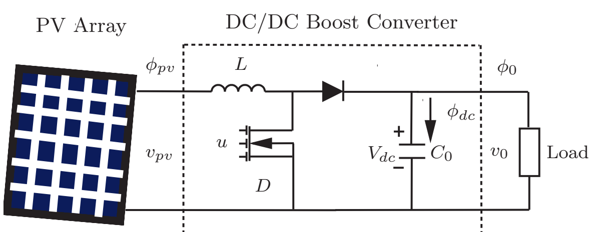


Fig. 1. A solar PV power with DC/DC boost converter.

In order to maximize the efficiency of PV power-generation systems, we will propose an MPPT technique based on the descriptor system approach. First, the electric characteristic of PV arrays is considered as follows [12]:

$$\begin{cases} \phi_{pv} = n_p I_{ph} - n_p I_{rs}\left(e^{\gamma v_{pv}} - 1\right), \\ P_{pv} = \phi_{pv} v_{pv}, \end{cases} \tag{2}$$

where $n_p$ and $n_s$ are the number of the parallel and series cells, respectively; $\gamma = q/(n_s \phi K T)$ with the electronic charge $q = 1.6\times10^{-19}$ C, Boltzmann's constant $K = 1.3805\times10^{-23}$J/°K, cell temperature $T$; $I_{ph}$ and $I_{rs}$ are the light-generated current and the reverse saturation current, respectively. Here, series resistances and their intrinsic shunt are neglected.

According to the representation of array power in (2) and by taking the partial derivative of $P_{pv}$ with respect to the PV voltage $v_{pv}$, one gets [12]

$$\frac{dP_{pv}}{dv_{pv}} = \phi_{pv} - n_p \gamma I_{rs} v_{pv} e^{\gamma v_{pv}}. \tag{3}$$

In order to obtain the MPPT performance, we let $\frac{dP_{pv}}{dv_{pv}} = 0$. The proposed descriptor system approach is shown in Fig. 2. First, we measure the PV array current $\phi_{pv}$, and solve the equation $\phi_{pv} - n_p \gamma I_{rs} v_{pv}^* e^{\gamma v_{pv}^*} = 0$ to obtain the reference PV array voltage $v_{pv}^*$. When the condition $v_{pv} \to v_{pv}^*$ holds, the closed-loop PV power control system achieves the MPPT performance. Now, by introducing the virtual state variable $\varepsilon_{pv} = v_{pv} - v_{pv}^*$, and it follows from (1)-(3) that the considered PV system with the MPPT control problem is reformulated into the following descriptor system:

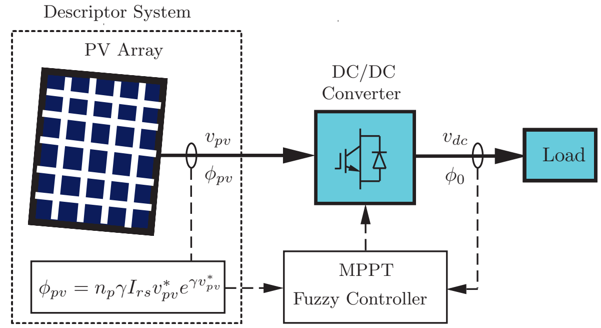


Fig. 2. MPPT fuzzy control for PV system.

$$\begin{cases} \dot{\phi}_{pv} = -\frac{1}{L}(1-u)\, v_{dc} + \frac{1}{L} v_{pv}, \\ \dot{v}_{dc} = \frac{1}{C_0}(1-u)\,\phi_{pv} - \frac{1}{C_0}\phi_0, \\ 0 \cdot \dot{\varepsilon}_{pv} = \phi_{pv} - n_p \gamma I_{rs} e^{\gamma v_{pv}^*} \varepsilon_{pv} - n_p \gamma I_{rs} v_{pv} e^{\gamma v_{pv}^*}. \end{cases} \tag{4}$$

Define $x(t) = \begin{bmatrix} \phi_{pv} & v_{dc} & \varepsilon_{pv} \end{bmatrix}^T$, $z_1 = \frac{v_{pv}}{\phi_{pv}}$, $z_2 = \frac{\phi_0}{v_{dc}}$, $z_3 = e^{\gamma v_{pv}^*}$, $z_4 = \frac{v_{pv}}{\phi_{pv}} e^{\gamma v_{pv}^*}$, $z_5 = v_{dc}$, and $z_6 = \phi_{pv}$. The nonlinear PV system in (4) is represented by the following descriptor T-S model [17]:

**Plant Rule** $\mathcal{R}^l$: **IF** $z_1$ is $\mathcal{F}_1^l$, and $z_2$ is $\mathcal{F}_2^l$, and, $\cdots$, and $z_6$ is $\mathcal{F}_6^l$, **THEN**

$$E\dot{x}(t) = A_l x(t) + B_l u(t), l \in \mathcal{L} := \{1, 2, \ldots, r\} \tag{5}$$

where

$$A_l = \begin{bmatrix} \frac{\mathcal{F}_1^l}{L} & -\frac{1}{L} & 0 \\ \frac{1}{C_0} & -\frac{\mathcal{F}_2^l}{C_0} & 0 \\ 1 - n_p \gamma I_{rs} \mathcal{F}_4^l & 0 & -n_p \gamma I_{rs} \mathcal{F}_3^l \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_l = \begin{bmatrix} \frac{\mathcal{F}_5^l}{L} \\ -\frac{\mathcal{F}_6^l}{C_0} \\ 0 \end{bmatrix}. \tag{6}$$

Then, we consider, without loss of generality, only the class of norm-bounded square integrable disturbance that acts on the output voltage $v_{dc}$, which is defined as below:

$$\omega^T(t)\,\omega(t) \le \delta, \tag{7}$$

where $\delta$ is a positive scalar.

By fuzzy blending, the global T-S fuzzy dynamic model is obtained by

$$E\dot{x}(t) = A(\mu)x(t) + B(\mu)u(t) + D\omega(t), \tag{8}$$

where $A(\mu) := \sum_{l=1}^{r} \mu_l A_l$, $B(\mu) := \sum_{l=1}^{r} \mu_l B_l$, $D = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$.

The aim is to design an FSMC law such that the state trajectories of fuzzy PV system can be driven into the sliding surface in the finite-time $T^*$ with $T^* \leq T$, and system states are bounded respectively in the time intervals $[0, T^*]$ and $[T^*, T]$ by the following reachable set:

$$\mathbb{S} \triangleq \{x(t) \in \Re^{n_x} \,|x(t) \text{ and } \omega(t) \text{ satisfy} \\ \text{(5) and (7), respectively, } t \geq 0\}. \tag{9}$$

## III. Design of FSMC Based On Reachable Set Bounding

This section first designs an FSMC law such that the state trajectories can be driven into the sliding surface in the finite-time $T^*$ with $T^* \leq T$. And then, the reachable set boundings of MPPT error are given in the time intervals $[0, T^*]$ and $[T^*, T]$, respectively.

### A. Design of FSMC law

Firstly, based on the fuzzy descriptor system (8), we construct the integral-type sliding surface function as below [19]:

$$s(t) = GEx(t) - \int_0^t G\left[A(\mu) + B(\mu)K(\mu)\right]x(s)ds, \tag{10}$$

where the matrix $G$ is choosen such that $GB_l$ is a positive definite matrix.

In the following, based on the sliding surface function (10), we will design an FSMC law $u(t)$, which can drive the trajectories of the fuzzy system (8) into the specified sliding surface $s(t) = 0$ in the finite time $T^*$.

**Theorem 3.1.** Consider the fuzzy descriptor system (8) representing the nonlinear PV system with MPPT control problem. The reachability of the specified sliding surface (10) in the finite time $T^*$ can be ensured by the following FSMC law:

$$u(t) = u_b(t) + u_c(t), \tag{11}$$

with

$$u_b(t) = \sum_{l=1}^{r} \mu_l K_l x(t), \\ u_c(t) = -\sum_{l=1}^{r} \mu_l \left[GB_l\right]^{-1} \rho(t) \operatorname{sgn}(s(t)), \tag{12}$$

where $K_l$ denotes fuzzy controller gain, $\rho(t) = \frac{\varrho + \|GD\|\|\omega(t)\|}{\lambda_{\min}\left(GB_l[GB_p]^{-1}\right)}$, $\varrho \geq \frac{1}{T}\|GEx(0)\|$, $(l,p) \in \mathcal{L}$, and sgn$(\star)$ is a switching sign function defined as

$$\operatorname{sgn}(s(t)) = \begin{cases} -1, & \text{for } s(t) < 0, \\ 0, & \text{for } s(t) = 0, \\ 1, & \text{for } s(t) > 0. \end{cases} \tag{13}$$

**Proof.** It follows from (10)-(12) that

$$\begin{aligned} & s^T(t)\dot{s}(t) \\ & = -s^T(t)GB(\mu)\sum_{l=1}^{r}\mu_l\left[GB_l\right]^{-1}\rho(t)\operatorname{sgn}(s(t)) \\ & \quad + s^T(t)GD\omega(t) \\ & \leq -\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\lambda_{\min}\left(GB_l\left[GB_p\right]^{-1}\right)\rho(t)\|s(t)\| \\ & \quad + \|GD\|\|\omega(t)\|\|s(t)\| \\ & = -\varrho\|s(t)\|. \end{aligned} \tag{14}$$

In addition, let us define

$$V_1(t) = \frac{1}{2}s^T(t)s(t). \tag{15}$$

We have

$$\begin{aligned} \dot{V}_1(t) & \leq -\varrho\|s(t)\| \\ & = -\sqrt{2}\varrho\sqrt{V_1(t)}. \end{aligned} \tag{16}$$

Based on [20], it yields

$$T^* \leq \frac{\sqrt{2}}{\varrho}\sqrt{V_1(0)}. \tag{17}$$

Besides, it follows from (15) that

$$V_1(0) = \frac{1}{2}\|s(0)\|^2. \tag{18}$$

Substituting (18) into (17), one gets

$$T^* \leq \frac{1}{\varrho}\|GEx(0)\|. \tag{19}$$

It follows from $\bar{\varrho} \geq \frac{1}{T}\left\|\bar{G}\bar{E}\bar{x}(0)\right\|$ in (14) that $T^* \leq T$, which implies that, the FSMC law (11) can drive the system trajectories of the fuzzy descriptor model (8) into the sliding surface function $s(t) = 0$ within the finite time $T^* \leq T$, thus completing this proof.

**Remark 3.1.** It is noted that the proposed switching sign function sgn$(\star)$ is discontinuous. The characteristic exhibits a high frequency oscillation, which is undesirable in practical applications. In order to eliminate chattering phenomenons, an alternative approach is to employ the following switching function [21]:

$$\operatorname{sgn}(s(t)) = \begin{cases} -1, & \text{for } s(t) < -\rho, \\ \frac{1}{\rho}s, & \text{for } |s(t)| \leq \rho, \\ 1, & \text{for } s(t) > \rho. \end{cases}$$

It is easy to see that the proposed switching sign function becomes continuous and its value converges to the interval $[-\rho, \rho]$ instead of zero. In this case, chattering conditions are eliminated.

*B. Reachable set bounding of MPPT error within $[0, T^*]$*

During the reaching phase in $[0, T^*]$, the motion of system states is outside of the sliding surface (10), i.e., $s(t) \neq 0$. By substituting the FSMC law (11) into (8), we obtain the resulting closed-loop control system as below:

$$\begin{aligned} E\dot{x}(t) &= \sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\bar{A}_{lp} + D\omega(t) \\ &\quad + \sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p B_l\left[GB_p\right]^{-1}\rho\left(t\right)\mathrm{sgn}\left(s(t)\right), \end{aligned} \tag{20}$$

where $\bar{A}_{lp} = A_l + B_l K_p$.

Define $\bar{\rho}(t) = \rho(t)\mathrm{sgn}(s(t))$, $\bar{\varrho} = \frac{\varrho}{\lambda_{\min}\left(GB_l[GB_p]^{-1}\right)}$, and $\varepsilon = \frac{\|GD\|}{\lambda_{\min}\left(GB_l[GB_p]^{-1}\right)}$. It follows from $s(t) \neq 0$ that

$$\begin{aligned} \bar{\rho}^2(t) &= \rho^2\left(t\right) \\ &= \left[\bar{\varrho} + \varepsilon\left\|\omega(t)\right\|\right]^2 \\ &= \bar{\varrho}^2 + 2\bar{\varrho}\varepsilon\left\|\omega(t)\right\| + \varepsilon^2\left\|\omega(t)\right\|^2 \\ &\leq \left(1+\varepsilon^2\right)\bar{\varrho}^2 + \left(1+\varepsilon^2\right)\left\|\omega(t)\right\|^2 \\ &\leq \left(1+\varepsilon^2\right)\bar{\varrho}^2 + \left(1+\varepsilon^2\right)\delta \\ &= \tilde{\rho}. \end{aligned} \tag{21}$$

In the following theorem, we derive a sufficient condition for the reachable set bounding of closed-loop control system (20) in the time interval $[0, T^*]$.

**Theorem 3.2.** Consider the FSMC law (11), the reachable set of the resulting closed-loop PV control system in (20) is bounded for time interval $[0, T^*]$, if there exist matrices $X = \begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix}$, $0 < X_1^T = X_1 \in \Re^{n_2\times n_2}$, $X_2 \in \Re^{n_1\times n_2}$ and $X_3$ that is a scalar, and the control gain $\bar{K}_l$, and positive scalars $\{\delta, \tilde{\rho}, \bar{\varphi}, \eta\}$, such that the following conditions hold:

$$\bar{\Phi}_{ll} < 0, 1 \leq l \leq r \tag{22}$$

$$\bar{\Phi}_{lp} + \bar{\Phi}_{pl} < 0, 1 \leq l < p \leq r \tag{23}$$

where

$$\begin{aligned} \bar{\Phi}_{lp} &= \begin{bmatrix} \bar{\Phi}_{lp(1)} & D & B_l\left[GB_p\right]^{-1} \\ \star & -\frac{\eta}{\delta}\mathbf{I} & 0 \\ \star & \star & -\frac{\eta}{\tilde{\rho}}\mathbf{I} \end{bmatrix}, \\ \bar{\Phi}_{lp(1)} &= \mathrm{Sym}\left\{X^T A_l^T + \bar{K}_p^T B_l^T\right\} + \eta X^T E^T. \end{aligned} \tag{24}$$

Furthermore, the smallest possible bounding is given as below:

$$\textbf{Minimize } \bar{\varphi}, \text{ subject to } \begin{bmatrix} \bar{\varphi}\mathbf{I} & X_1 \\ \star & \bar{\mathcal{X}}_1 \end{bmatrix} > 0, (22) \text{ and } (23),$$

where $\bar{\varphi} = \varphi^{-1}$, $\bar{\mathcal{X}}_1 = \frac{X_1}{e^{-\eta T^*}x_{1,2}^T(0)P_1 x_{1,2}(0) + 2\left(1-e^{-\eta T^*}\right)}$, and the fuzzy controller gain is calculated as below:

$$K_p = \bar{K}_p X^{-1}. \tag{25}$$

**Proof.** Consider the following Lyapunov functional

$$V_2\left(t\right) = x^T(t)E^T Px(t), \forall t \in \left[0, T^*\right]. \tag{26}$$

It is easy to see from Theorem 3.2 that $E^T P = P^T E \geq 0$.

Along the trajectory of system (20), one gets

$$\begin{aligned} \dot{V}_2\left(t\right) &= 2\left[E\dot{x}(t)\right]^T Px(t) \\ &= 2\left[\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\bar{A}_{lp}x(t)\right]^T Px(t) \\ &\quad + 2\left[\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p B_l\left[GB_p\right]^{-1}\rho\left(t\right)\mathrm{sgn}\left(s(t)\right)\right]^T Px(t) \\ &\quad + 2\left[D\omega(t)\right]^T Px(t). \end{aligned} \tag{27}$$

An auxiliary function is introduced as below:

$$J\left(t\right) = \dot{V}_2\left(t\right) + \eta V_2\left(t\right) - \frac{\eta}{\delta}\omega^2(t) - \frac{\eta}{\tilde{\rho}}\bar{\rho}^2(t), \tag{28}$$

where $\eta$ is a positive scalar.

It follows from (27) and (28) that

$$\begin{aligned} J\left(t\right) &= 2\left[\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\bar{A}_{lp}x(t)\right]^T Px(t) \\ &\quad + 2\left[\sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p B_l\left[GB_p\right]^{-1}\bar{\rho}(t)\right]^T Px(t) \\ &\quad + 2\left[D\omega(t)\right]^T Px(t) \\ &\quad + \eta x^T(t)E^T Px(t) - \frac{\eta}{\delta}\omega^2(t) - \frac{\eta}{\tilde{\rho}}\bar{\rho}^2(t) \\ &= \sum_{l=1}^{r}\sum_{p=1}^{r}\mu_l\mu_p\chi^T\left(t\right)\Phi_{lp}\chi\left(t\right), \end{aligned} \tag{29}$$

where $\chi\left(t\right) = \begin{bmatrix} x^T(t) & \omega^T(t) & \bar{\rho}^T(t) \end{bmatrix}^T$, and

$$\begin{aligned} \Phi_{lp} &= \begin{bmatrix} \Phi_{lp(1)} & P^T D & P^T B_l\left[GB_p\right]^{-1} \\ \star & -\frac{\eta}{\delta}\mathbf{I} & 0 \\ \star & \star & -\frac{\eta}{\tilde{\rho}}\mathbf{I} \end{bmatrix}, \\ \Phi_{lp(1)} &= \mathrm{Sym}\left\{A_l^T P + K_p^T B_l^T P\right\} + \eta E^T P. \end{aligned} \tag{30}$$

To cast the inequality $\sum\limits_{l=1}^{r}\sum\limits_{p=1}^{r}\mu_l\mu_p\Phi_{lp} < 0$ into LMIs, we have

$$\begin{aligned} P^{-1} &= X \\ &= \begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix}, \end{aligned} \tag{31}$$

where $0 < X_1 = X_1^T \in \Re^{n_2\times n_2}, X_2 \in \Re^{n_1\times n_2}, X_3$ is a scalar.

Now, define $\Gamma$ =diag$\{X, \mathbf{I}, \mathbf{I}\}$, and use a congruent transformation to $\sum\limits_{l=1}^{r}\sum\limits_{p=1}^{r}\mu_l\mu_p\Phi_{lp} < 0$ by $\Gamma$. After extracting the fuzzy premise variables, the inequalities in (22) and (23) can be directly obtained.

Because of (22) and (23) that $J\left(t\right) < 0$, which implies that

$$\dot{V}_2\left(t\right) + \eta V_2\left(t\right) < \frac{\eta}{\delta}\omega^2(t) + \frac{\eta}{\tilde{\rho}}\bar{\rho}^2(t). \tag{32}$$

Multiplying both sides of (32) by $e^{\eta t}$ and integrating the resulting inequality from 0 to $T^*$. It is easy to see that

$$e^{\eta T^*}V_2(T^*) < V_2(0) + \frac{\eta}{\tilde{\rho}}\int_0^{T^*} e^{\eta s}\bar{\rho}^2(s)ds + \frac{\eta}{\delta}\int_0^{T^*} e^{\eta s}\omega^2(s)ds \le x^T(0)E^TPx(0) + 2\left(e^{\eta T^*}-1\right), \quad (33)$$

which implies that

$$V_2(T^*) < e^{-\eta T^*}x^T(0)E^TPx(0) + 2\left(1-e^{-\eta T^*}\right). \quad (34)$$

Further, we specify the matrix $P$ as below:

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \quad (35)$$

where $0 < P_1^T = P_1 \in \Re^{n_2\times n_2}$, $P_2 \in \Re^{n_1\times n_2}$ and $P_3$ is a scalar, and it is easy to see that $E^TP = P^TE \ge 0$.

Now, we partition $x(t)$ as

$$x(t) = \begin{bmatrix} x_{1,2}(t) \\ x_3(t) \end{bmatrix}, x_{1,2}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \quad (36)$$

It follows from (34)-(36) that

$$x_{1,2}^T(t)P_1x_{1,2}(t) < e^{-\eta T^*}x_{1,2}^T(0)P_1x_{1,2}(0) + 2\left(1-e^{-\eta T^*}\right), t \in [0, T^*]. \quad (37)$$

Here, the aim is to design the FSMC controller in the form of (11) such that the smallest bound for the reachable set in (9) can be obtained. To do so, a simple optimization algorithm is pointed out in [23], i.e. maximize $\varphi$ subject to $\varphi\mathbf{I} < \frac{P_1}{e^{-\eta T^*}x_{1,2}^T(0)P_1x_{1,2}(0)+2\left(1-e^{-\eta T^*}\right)}$. By using Schur complement, and use a congruent transformation by $\Gamma = \text{diag}\{\mathbf{I}, X_1\}$, one can easily solve the optimization problem as shown in Theorem 3.2. This completes the proof.

Recalling the fast dynamic subsystem in (5) as below:

$$\sum_{l=1}^{r}\mu_l\left(1-n_p\gamma I_{rs}\mathcal{F}_4^l\right)\phi_{pv} - \sum_{l=1}^{r}\mu_l n_p\gamma I_{rs}\mathcal{F}_3^l\varepsilon_{pv} = 0. \quad (38)$$

It is easy to see from (4) and Theorem 3.2 that $x_1(t) < \sqrt{\bar{\varphi}}$, and we can calculate the tracking error of maximum power point $\varepsilon_{pv}$ as below:

$$\varepsilon_{pv} < \left|\frac{\sum_{l=1}^{r}\mu_l\left(1-n_p\gamma I_{rs}\mathcal{F}_4^l\right)}{\sum_{l=1}^{r}\mu_l n_p\gamma I_{rs}\mathcal{F}_3^l}\right|\sqrt{\bar{\varphi}}. \quad (39)$$

*C. Reachable set bounding of MPPT error within $[T^*, T]$*

During the time interval $[T^*, T]$ of the sliding phase, we will derive a sufficient condition to ensure the reachable set bounding of the resulting closed-loop FSMC system. When the system trajectories arrive at the sliding surface, it has $\dot{s}(t) = 0$. Thus, the equivalent controller $u_{eq}(t)$ is obtained as below:

$$GB(\mu)u_{eq}(t) = GB(\mu)K(\mu)x(t) - GD\omega(t), \quad (40)$$

where $G$ is a given matrix so that $GB(\mu)$ is nonsingular.

Motivated by [17], [18], by augmenting the the system (8) and controller (40), it yields

$$\bar{E}\dot{\bar{x}}(t) = \bar{A}(\mu)\bar{x}(t) + \bar{D}\omega(t), \quad (41)$$

where

$$\bar{E} = \begin{bmatrix} E & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \bar{x}(t) = \begin{bmatrix} x(t) \\ x(t) \\ u_{eq}(t) \end{bmatrix}, \bar{D} = \begin{bmatrix} D \\ 0 \\ -GD \end{bmatrix},$$

$$\bar{A}(\mu) = \begin{bmatrix} 0 & A(\mu) & B(\mu) \\ \mathbf{I} & -\mathbf{I} & 0 \\ GB(\mu)K(\mu) & 0 & -GB(\mu) \end{bmatrix}. \quad (42)$$

In the following, we will derive a sufficient condition to ensure the reachable set bounding of closed-loop system (41) in the time interval $[T^*, T]$.

**Theorem 3.3.** Consider the fuzzy PV system in (8) and the sliding surface function in (10). For the specified time interval $[T^*, T]$, the reachable set of the resulting closed-loop control system in (41) is bounded, if there exist the matrices $0 < X_1 = X_1^T \in \Re^{n_2\times n_2}, X_2 \in \Re^{n_1\times n_2}, \left\{X_{(21)}, X_{(22)}\right\} \in \Re^{n_3\times n_3}$, $\left\{X_{(23)}, X_{(24)}\right\} \in \Re^{n_1\times n_3}$, and the scalars $\left\{X_3, X_{(25)}\right\}$, and the controller gain $\bar{K}_l$, and the positive scalars $\left\{\delta, \tilde{\rho}, \bar{\psi}, \eta\right\}$, such that the following LMIs hold:

$$\bar{\Psi}_{ll} < 0, 1 \le l \le r, \quad (43)$$

$$\bar{\Psi}_{lp} + \bar{\Psi}_{pl} < 0, 1 \le l < p \le r \quad (44)$$

where

$$\bar{\Psi}_{lp} = \begin{bmatrix} \text{Sym}\left(\bar{\Psi}_{lp(1)}\right) + \eta\bar{E}\bar{X} & \bar{D} \\ \star & -\frac{\eta}{\tilde{\rho}}\mathbf{I} \end{bmatrix},$$

$$\bar{X} = \begin{bmatrix} X & 0 & 0 \\ X_{(21)} & X_{(22)} & 0 \\ X_{(23)} & X_{(24)} & X_{(25)} \end{bmatrix}, X = \begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix},$$

$$\bar{\Psi}_{lp(1)} = \begin{bmatrix} \bar{\Psi}_{lp(11)} & \bar{\Psi}_{lp(12)} & B_lX_{(25)} \\ X - X_{(21)} & -X_{(21)} & 0 \\ \Phi_{lp(31)} & -GB_lX_{(24)} & -GB_lX_{(25)} \end{bmatrix},$$

$$\bar{\Psi}_{lp(11)} = A_lX_{(21)} + B_lX_{(23)}, \bar{\Psi}_{lp(12)} = A_lX_{(22)} + B_lX_{(24)},$$

$$\bar{\Psi}_{lp(31)} = GB_l\bar{K}_p - GB_lX_{(23)}. \quad (45)$$

Furthermore, the smallest possible bounding is given by the following algorithm:

**Minimize** $\bar{\psi}$, subject to $\begin{bmatrix} \bar{\psi}\mathbf{I} & X_1 \\ \star & \bar{\mathcal{X}}_2 \end{bmatrix} > 0$, (43) and (44),

where $\bar{\mathcal{X}}_2 = \frac{X_1}{e^{-\eta T}x_{1,2}^T(0)P_1x_{1,2}(0)+e^{\eta(T^*-T)}-2e^{-\eta T}+1}$, $\bar{\psi} = \xi^{-1}$, and the fuzzy controller gain is calculated as below:

$$K_l = \bar{K}_lX^{-1}. \quad (46)$$

**Proof.** Consider the following Lyapunov function

$$V_3(t) = \bar{x}^T(t)\bar{E}^T\bar{P}\bar{x}(t), \forall t \in [T^*, T] \quad (47)$$

where $\bar{E}^T\bar{P} = \bar{P}^T\bar{E} \ge 0$.

Along the trajectory of system (41), we have

$$\dot{V}_3(t) = 2\left[\bar{A}(\mu)\bar{x}(t) + \bar{D}\omega(t)\right]^T\bar{P}\bar{x}(t). \quad (48)$$

An index function is introduced as follows:

$$J_2(t) = \dot{V}_3(t) + \eta V_3(t) - \frac{\eta}{\delta}\omega^T(t)\omega(t), \tag{49}$$

where $\eta$ is a positive scalar.

It follows from (47)-(49) that

$$\begin{aligned} J_2(t) &= 2\left[\bar{A}(\mu)\bar{x}(t) + \bar{D}\omega(t)\right]^T \bar{P}\bar{x}(t) \\ &\quad + \eta\bar{x}^T(t)\bar{E}^T\bar{P}\bar{x}(t) - \frac{\eta}{\delta}\omega^T(t)\omega(t) \\ &= \bar{\chi}^T(t)\Psi(\mu)\bar{\chi}(t), \end{aligned} \tag{50}$$

where $\bar{\chi}(t) = \left[\begin{array}{cc} \bar{x}^T(t) & \omega^T(t) \end{array}\right]^T$, and $\Psi(\mu) = \left[\begin{array}{cc} \text{Sym}\left(\bar{A}^T(\mu)\bar{P}\right) + \eta\bar{E}^T\bar{P} & \bar{P}^T\bar{D} \\ \star & -\frac{\eta}{\delta}\mathbf{I} \end{array}\right]$.

It is easy to see that $\Psi(\mu) < 0$, which implies that $J_2(t) < 0$. Now, by extracting the fuzzy premise variables, it yields

$$\Psi_{ll} < 0, 1 \leq l \leq r, \tag{51}$$

$$\Psi_{lp} + \Psi_{pl} < 0, 1 \leq l < p \leq r \tag{52}$$

where $\Psi_{lp} = \left[\begin{array}{cc} \text{Sym}\left(\bar{A}_{lp}^T\bar{P}\right) + \eta\bar{E}^T\bar{P} & \bar{P}^T\bar{D} \\ \star & -\frac{\eta}{\delta}\mathbf{I} \end{array}\right]$, $\bar{A}_{lp} = \left[\begin{array}{ccc} 0 & A_l & B_l \\ \mathbf{I} & -\mathbf{I} & 0 \\ GB_lK_p & 0 & -GB_l \end{array}\right]$.

To cast the conditions (51) and (52) into LMIs, we define

$$\bar{P}^{-1} = \bar{X}, \bar{P} = \left[\begin{array}{ccc} P & 0 & 0 \\ P_{(21)} & P_{(22)} & 0 \\ P_{(23)} & P_{(24)} & P_{(25)} \end{array}\right],$$

$$\bar{X} = \left[\begin{array}{ccc} X & 0 & 0 \\ X_{(21)} & X_{(22)} & 0 \\ X_{(23)} & X_{(24)} & X_{(25)} \end{array}\right], \tag{53}$$

where $P = \left[\begin{array}{cc} P_1 & 0 \\ P_2 & P_3 \end{array}\right]$, $X = \left[\begin{array}{cc} X_1 & 0 \\ X_2 & X_3 \end{array}\right]$, $0 < P_1 = P_1^T \in \Re^{n_x \times n_x}$, $0 < X_1 = X_1^T \in \Re^{n_x \times n_x}$, $\{P_2, P_3, X_2, X_3\} \in \Re^{n_x \times n_x}$, $\left\{P_{(21)}, P_{(22)}, X_{(21)}, X_{(22)}\right\} \in \Re^{n_x \times n_x}$, $\left\{P_{(23)}, P_{(24)}, P_{(25)}, X_{(23)}, X_{(24)}, X_{(25)}\right\} \in \Re^{n_u \times n_x}$.

Now, define $\Gamma = \text{diag}\left\{\bar{X}, \mathbf{I}\right\}$, and use a congruent transformation to (51) and (52) by $\Gamma$. The inequalities in (43) and (44) can be directly obtained.

In addition, $J_2(t) < 0$ implies that

$$\dot{V}_3(t) + \eta V_3(t) < \frac{\eta}{\delta}\omega^T(t)\omega(t). \tag{54}$$

Pre- and post-multiplying both sides of the inequality (54) by $e^{\eta t}$ and integrating the successive inequality from $T^*$ to $T$. It is easy to see that

$$V_3(T) < e^{\eta(T^* - T)}V_3(T^*) + 1 - e^{\eta(T^* - T)}. \tag{55}$$

In addition, it can be seen from (47) that

$$V_3(t) = x_{1,2}^T(t)P_1x_{1,2}(t), \tag{56}$$

which implies that

$$x_{1,2}^T(t)P_1x_{1,2}(t) < e^{\eta(T^* - T)}\left(V_3(T^*) - 1\right) + 1, t \in [T^*, T]. \tag{57}$$

It follows from (37) that

$$\begin{aligned} x_{1,2}^T(t)P_1x_{1,2}(t) &< e^{-\eta T}x_{1,2}^T(0)P_1x_{1,2}(0) \\ &\quad + e^{\eta(T^* - T)} - 2e^{-\eta T} + 1, t \in [T^*, T]. \end{aligned} \tag{58}$$

Similar to the work of [23], the smallest bound for the reachable set in (11) can be obtained by maximizing $\xi$ subject to $\xi\mathbf{I} < \frac{P_1}{e^{-\eta T}x_{1,2}^T(0)P_1x_{1,2}(0) + e^{\eta(T^* - T)} - 2e^{-\eta T} + 1}$. By using Schur complement, and use a congruent transformation by $\Gamma$ =diag$\{\mathbf{I}, X_1\}$, one can easily solve the optimization problem as shown in Theorem 3.3. This completes the proof.

It is easy to see from (4) and Theorem 3.3 that $x_1(t) < \sqrt{\bar{\psi}}$, and recalling the fast dynamic subsystem in (5) as below:

$$\sum_{l=1}^{r}\mu_l\left(1 - n_p\gamma I_{rs}\mathcal{F}_4^l\right)\phi_{pv} - \sum_{l=1}^{r}\mu_l n_p\gamma I_{rs}\mathcal{F}_3^l\varepsilon_{pv} = 0. \tag{59}$$

We can calculate the tracking error of maximum power point $\varepsilon_{pv}$ as below:

$$\varepsilon_{pv} < \left|\frac{\sum\limits_{l=1}^{r}\mu_l\left(1 - n_p\gamma I_{rs}\mathcal{F}_4^l\right)}{\sum\limits_{l=1}^{r}\mu_l n_p\gamma I_{rs}\mathcal{F}_3^l}\right|\sqrt{\bar{\varphi}}. \tag{60}$$

### D. Design of controller gain $K_l$

In the following, we will design the fuzzy controller gain $K_l$ in the sliding surface function (13), which guarantees that the conditions in Theorems 3.2 and 3.3 are feasible synchronously. The corresponding result can be summarized as follows:

**Theorem 3.4.** Consider the fuzzy PV system in (8) and the sliding surface function in (10). For the specified finite time $T$, the resulting FSMC system is bouned, if there exist matrices $0 < X_1 = X_1^T \in \Re^{n_2 \times n_2}, X_2 \in \Re^{n_1 \times n_2}, \left\{X_{(21)}, X_{(22)}\right\} \in \Re^{n_3 \times n_3}$, $\left\{X_{(23)}, X_{(24)}\right\} \in \Re^{n_1 \times n_3}$, the scalars $\left\{X_3, X_{(25)}\right\}$, and the control gain $\bar{K}_l$, and the positive scalars $\{\delta, \tilde{\rho}, \eta\}$, such that the following LMIs hold:

$$\bar{\Phi}_{ll} < 0, 1 \leq l \leq r \tag{61}$$

$$\bar{\Phi}_{lp} + \bar{\Phi}_{pl} < 0, 1 \leq l < p \leq r \tag{62}$$

$$\bar{\Psi}_{ll} < 0, 1 \leq l \leq r, \tag{63}$$

$$\bar{\Psi}_{lp} + \bar{\Psi}_{pl} < 0, 1 \leq l < p \leq r \tag{64}$$

where

$$\bar{\Phi}_{lp} = \left[\begin{array}{ccc} \bar{\Phi}_{lp(1)} & D & B_l\left[GB_p\right]^{-1} \\ \star & -\frac{\eta}{\delta}\mathbf{I} & 0 \\ \star & \star & -\frac{\eta}{\tilde{\rho}}\mathbf{I} \end{array}\right],$$

$$\bar{\Phi}_{lp(1)} = \text{Sym}\left\{X^TA_l^T + \bar{K}_p^TB_l^T\right\} + \eta X^TE^T. \tag{65}$$

and

$$\bar{\Psi}_{lp} = \begin{bmatrix} \mathrm{Sym}\left(\bar{\Psi}_{lp(1)}\right) + \eta \bar{E}\bar{X} & \bar{D} \\ \star & -\frac{\eta}{\delta}\mathbf{I} \end{bmatrix},$$

$$\bar{X} = \begin{bmatrix} X & 0 & 0 \\ X_{(21)} & X_{(22)} & 0 \\ X_{(23)} & X_{(24)} & X_{(25)} \end{bmatrix}, X = \begin{bmatrix} X_1 & 0 \\ X_2 & X_3 \end{bmatrix},$$

$$\bar{\Psi}_{lp(1)} = \begin{bmatrix} \bar{\Psi}_{lp(11)} & \bar{\Psi}_{lp(12)} & B_l X_{(25)} \\ X - X_{(21)} & -X_{(21)} & 0 \\ \Phi_{lp(31)} & -GB_l X_{(24)} & -GB_l X_{(25)} \end{bmatrix},$$

$$\bar{\Psi}_{lp(11)} = A_l X_{(21)} + B_l X_{(23)}, \bar{\Psi}_{lp(12)} = A_l X_{(22)} + B_l X_{(24)},$$
$$\bar{\Psi}_{lp(31)} = GB_l \bar{K}_p - GB_l X_{(23)}. \quad (66)$$

Furthermore, the fuzzy controller gain is calculated as below:

$$K_p = \bar{K}_p X^{-1}. \quad (67)$$

### *E. Design procedure for MPPT algorithm*

The detailed calculating steps of the proposed MPPT algorithm for the nonlinear PV system is summarized as below:

*i*) Use the descriptor system approach to represent the MPPT control problem of the PV system, as shown in (4);

*ii*) Use the T-S fuzzy model method to describe the nonlinear descriptor system as shown in (8);

*iii*) Choose a suitable matrix $G$, and solve Theorem 3.4 to obtain the fuzzy controller gain $K_l$;

*iv*) Given the finite time $T$, and the initial state $x(0)$. Construct the sliding mode controller as shown in Theorem 3.1;

*v*) Use Theorems 3.2 and 3.3 to minimize $\bar{\varphi}$, and calculate the bounding for the MPPT error $\varepsilon_{pv}$.

## IV. Simulation Study

In order to testify the effectiveness of the proposed MPPT control method, we consider a solar PV system, and its dynamic model can be described as shown in (1). The parameters are chosen as below: $L = 150\mu\mathrm{H}, C_0 = 1000\mu\mathrm{F}, n_p = 36, \gamma = 0.03863, I_{rs} = 4\mathrm{A}, T = 300\mathrm{K}$. Now, the proposed MPPT algorithm can be implemented as below:

*i*) Use the descriptor system approach to represent the MPPT control problem of the PV system as shown in (4).

*ii*) For simplicity, we only choose $z_1 = \frac{v_{pv}}{\phi_{pv}}$, $z_2 = e^{\gamma v_{pv}^*}$, $z_3 = \frac{v_{pv}}{\phi_{pv}} e^{\gamma v_{pv}^*}$ as fuzzy premise variables, and linearize the above mentioned nonlinear system around the operation points $z_1 = (5, 3.25)$, $z_2 = (0.3636, 0.5)$, and $z_3 = (1.5295, 1.5897)$. Further assume that $\phi_0 = 4\mathrm{A}, v_{dc} = 11\mathrm{V}$, $\phi_{pv} = 2\mathrm{A}$. Then, the nonlinear descriptor system is represented by the following T-S fuzzy model:

$$E\dot{x}(t) = A(\mu)x(t) + Bu(t) + D\omega(t),$$

where the system's parameters are omitted because of space limitations.

*iii*) Choose a suitable matrix $G = \begin{bmatrix} 0.1 & -0.1 & 0 \end{bmatrix}$, and solve Theorem 3.4 to obtain the controller gains as below:

$$\begin{aligned} K_1 &= \begin{bmatrix} -0.8698 & -1.0478 & -0.0069 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -0.6913 & -0.8082 & -0.0120 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -0.8779 & -1.0490 & -0.0082 \end{bmatrix}, \\ K_4 &= \begin{bmatrix} -0.6969 & -0.8089 & -0.0129 \end{bmatrix}, \\ K_5 &= \begin{bmatrix} -0.9220 & -1.0538 & -0.0153 \end{bmatrix}, \\ K_6 &= \begin{bmatrix} -0.7318 & -0.8157 & -0.0187 \end{bmatrix}, \\ K_7 &= \begin{bmatrix} -0.9298 & -1.0581 & -0.0168 \end{bmatrix}, \\ K_8 &= \begin{bmatrix} -0.7382 & -0.8164 & -0.0197 \end{bmatrix}. \end{aligned}$$

*iv*) Given the finite time $T = 1\mathrm{s}$, and the initial state $x(0) = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^T$. We construct the sliding mode controller as follows:

$$u(t) = u_b(t) + u_c(t),$$

with

$$u_b(t) = \sum_{l=1}^{r} \mu_l K_l x(t), u_c(t) = -93.3333\rho(t)\,\mathrm{sgn}\left(s(t)\right),$$

where $\rho(t) = \varrho + \|\omega(t)\|, \varrho \geq 0.4, \omega(t) = 0.5\sin t$.

*v*) Use Theorems 3.2 to minimize $\bar{\varphi}$, we can obtain $\bar{\varphi} = 42.2729$. When considering Theorems 3.2, we can also obtain $\bar{\psi} = 47.0847$. Calculate the bounding for the MPPT error $\varepsilon_{pv}$. It yields $\varepsilon_{pv} < 206.3961$ in the finite-time interval $[0, T^*]$ and $\varepsilon_{pv} < 229.8891$ in the finite-time interval $[T^*, T]$, respectively.

Note that the open-loop system is unstable. With the above solution, the responses of the sliding surface function are shown in Fig.3. It has been shown that the state trajectories can be driven into the sliding surface in the finite-time $T = 1\mathrm{s}$. Accordingly, Figs 4 and 5 show that the proposed FSMC can force the tracking error around the zero.

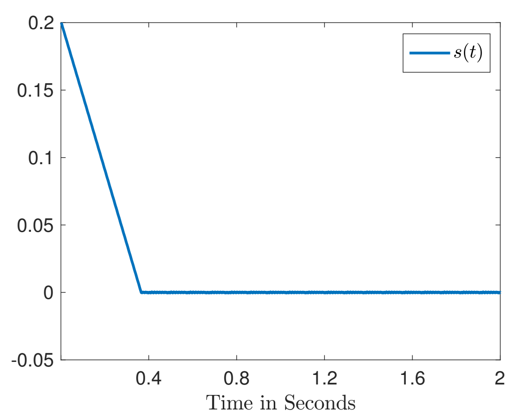


Fig. 3. Responses for sliding surface function

## V. Conclusions

This paper proposes a novel MPPT control strategy for the nonlinear PV systems. The MPPT control problem of PV systems is reformulated into the framework of fuzzy descriptor systems. An FSMC law is constructed to drive

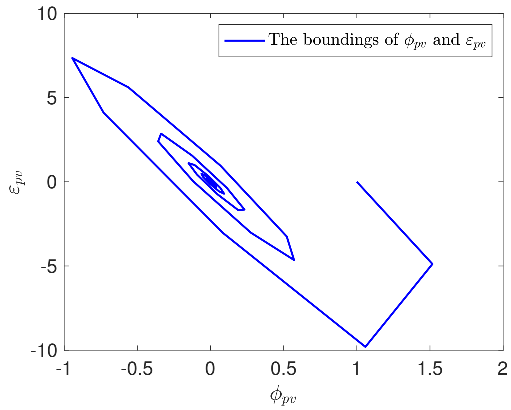


Fig. 4. Responses for $\phi_{pv}$ and $\varepsilon_{pv}$

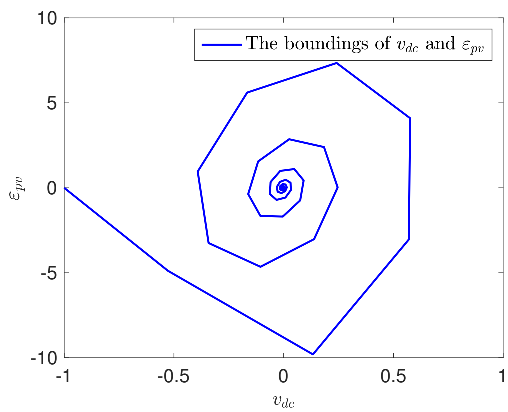


Fig. 5. Responses for $v_{dc}$ and $\varepsilon_{pv}$

the state trajectories onto the specified sliding surface within the finite-time $T^*$ with $T^* < T$. And then, the sufficient conditions are derived to ensure the reachable set boundings of the closed-loop PV systems in the finite-time intervals $[0, T^*]$ and $[T^*, T]$, respectively. Through a numerical simulation, it has been shown that fast and accurate MPPT performance can be achieved.

## REFERENCES


[1] X. Li, Y. Li, and J. Seem. "Maximum power point tracking for photovoltaic system using adaptive extremum seeking control," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 6, pp. 2315-2322, 2013.

[2] Z. Zhong. "Tracking synchronization for DC microgrid with multiple-photovoltaic arrays: An even-based fuzzy control scheme," *IEEE Access*, vol. 6, pp. 24996-25006, 2018.

[3] R. Wai and W. Wang. "Grid-connected photovoltaic generation system," *IEEE Transactions on Circuits & Systems*, vol. 55, no. 3, pp. 953-964, 2008.

[4] F. Attivissimo, A. Nisio, M. Savino, and M. Spadavecchia. "Uncertainty analysis in photovoltaic cell parameter estimation," *IEEE Transactions on Instrumentation & Measurement*, vol. 61, no. 5, pp. 1334-1342, 2012.

[5] C. Hua, J. Lin, and C. Shen. "Implementation of a DSP-controlled photovoltaic system with peak power tracking," *IEEE Transactions on Industrial Electronics*, vol. 45, no. 1, pp. 99-107, 1998.

[6] Z. Zhong. "*Modeling, Control, Estimation, and Optimization for Microgrids: A Fuzzy-Model-Based Method*," , Boca Raton, FL, USA: CRC Press, 2019.

[7] J. Ahmed and Z. Salam. "A maximum power point tracking (MPPT) for PV system using Cuckoo search with partial shading capability," *Applied Energy*, vol. 119, pp. 118-130, 2014.

[8] Y. Liu, C. Liu, J. Huang, and J. Chen. "Neural-networkbased maximum power point tracking methods for photovoltaic systems operating under fast changing environments," *Solar Energy*, vol. 89, pp. 42-53, 2013.

[9] P. Manganiello, M. Ricco, G. Petrone, E. Monmasson, and G. Spagnuolo. "Optimization of perturbative PV MPPT methods through online system identification," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 12, pp. 6812-6821, 2014.

[10] S. Beid, and S. Doubabi. "DSP-based implementation of fuzzy output tracking control for a boost converter," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 1, pp. 196-209, 2014. 61(1): 196-209, 2014.

[11] Z. Zhong, Y. Zhu, C. Lin, and T. Huang. "A fuzzy control framework for interconnected nonlinear power networks under TDS attack: Estimation and compensation," *Journal of the Franklin Institute*, doi.org/10.1016/j.jfranklin.2018.12.012.

[12] C. Chiu and Y. Ouyang. "Robust maximum power tracking control of uncertain photovoltaic systems: A unified T-S fuzzy model-based approach," *IEEE Transactions on Control Systems and Technology*, vol.19, no.6, pp. 1516-1526, 2011.

[13] Y. Xia and Y. Jia. "Robust sliding-mode control for uncertain time-delay systems: An LMI approach," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 1086-1091, 2003.

[14] C. Chiu, Y. Ouyang, and C. Ku. "Terminal sliding mode control for maximum power point tracking of photovoltaic power generation systems," *Solar Energy*, vol. 86, no. 10, pp. 2986-2995, 2012.

[15] J. Song, Y. Niu, and Y. Zou. "Finite-time stabilization via sliding mode control," *IEEE Transactions on Automatic Control*, vol. 62, no. 3, pp. 1478-1483, 2017.

[16] S. Kim. "Output voltage-tracking controller with performance recovery property for DC/DC boost converters," *IEEE Transactions on Control Systems Technology*, doi: 10.1109/TCST.2018.2806366

[17] M. Chadli, H. Karimi, and P. Shi. "On stability and stabilization of singular uncertain Takagi-Sugeno fuzzy systems," *Journal of the Franklin Institute*, vol. 351, no. 3, pp. 1453-1463, 2014.

[18] Z. Zhong, Y. Zhu, and H. Lam. "Asynchronous piecewise output-feedback control for large-scale fuzzy systems via distributed event-triggering schemes," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 3, pp. 1688-170, 2018.

[19] L. Wu, X. Su, and P. Shi. "Sliding mode control with bounded $\mathcal{L}_2$ gain performance of Markovian jump singular time-delay systems," *Automatica*, vol.48, pp. 1929-1933, 2012.

[20] W. Haddad and A. L'Afflitto. "Finite-time stabilization and optimal feedback control," *IEEE Transactions on Automatic Control*, vol. 61, no. 4, pp. 1069-1074, 2016.

[21] A. Bartoszewicz and J. Zuk. "Sliding mode control-basic concepts and current trends," *IEEE International Symposium on Industrial Electronics*, pp. 3772-3777, 2010.

[22] Z. Zhong, Y. Zhu, and C. Ahn, "Reachable set estimation for Takagi-Sugeno fuzzy systems against unknown output delays with application to tracking control of AUVs," *ISA Transactions*, vol. 78, pp. 31-38, 2018.

[23] Z. Zhong, R. Wai, Z. Shao, and M. Xu, "Reachable set estimation and decentralized controller design for large-scale nonlinear systems with time-varying delay and input constraint," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 6, pp. 1629-1643, 2017.