A Mixed Gaussian Membership Function Fuzzy CMAC for a Three-Link Robot

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Abstract—This research produces a mixed Gaussian membership function (GMF) fuzzy cerebellar model articulation controller (CMAC) for a three-link robot. A mixed GMF is created using the current and the previous GMFs on each layer of CMAC to detect errors efficiently, so a mixed GMF fuzzy CMAC (MGMFFC) is able to train parameters efficiently and constructs the MGMFFC structure automatically. A Lyapunov cost function and the gradient descent technique are utilized to get the adaptive with guaranteed system’s stable. Simulation studies for a three-link robot show that the MGMFFC attains favorable tracking performance.

Index Terms—Gaussian membership function, fuzzy inference system, CMAC, three-link robot.

I. INTRODUCTION

Nowadays, intelligent controllers have been studied for many fields, such as medical diagnosis system, machine learning system, noise filtering system, and control system, etc. [1-4]. The intelligent control structures using fuzzy inference systems, various neural networks, traditional or modified CMACs have been used for uncertain nonlinear systems for improving the learning ability [5-8].

The control for a practical system is usually affected by uncertainty, nonlinearity and computational load problems. Therefore, some new techniques have been designed to detect error efficiently over the years. Recently, a parameterized type-2 fuzzy membership function was designed for practical applications [9]. Memari et al. proposed an intuitionistic fuzzy technique for choosing the correct sustainable supplier that involves many sub-criteria for a manufacturer [10]. A type-2 wavelet membership function was embedded in a CMAC for uncertain nonlinear systems [11]. Mohammadzadeh et al. introduced a type-3 fuzzy inference set for modeling the uncertain systems [12].

This research presents a mixed Gaussian membership function fuzzy CMAC (MGMFFC) for a three-link robot. The proposed controller uses a mixed GMF integrated into a fuzzy CMAC to detect the error values efficiently. A Lyapunov function and the gradient descent algorithm are employed to provide the online learning laws and to prove the system’s stability. Simulation results using the proposed MGMFFC for a three-link robot will be given to illustrate the effectiveness of the proposed control algorithm.

II. PROBLEM FORMULATION

An n-th-order MIMO nonlinear system is given as:

\[
\begin{align*}
\dot{q}(t) &= G(q(t)) + H(q(t))u(t) + I(q(t)) \\
y_{out}(t) &= q(t)
\end{align*}
\]

where \(q(t) \in \mathbb{R}^n\) is a state vector, \(y_{out}(t) \triangleq [y_{out1}(t), y_{out2}(t), \ldots, y_{outm}(t)]^T \in \mathbb{R}^m\) is a vector for the system outputs, and \(u(t) = [u_{c1}(t), u_{c2}(t), \ldots, u_{cm}(t)]^T \in \mathbb{R}^m\) is the vector for the control inputs. Define \(\psi(t) = [q^T(t), q^2(t), \ldots, q^{n-1}(t)]^T \in \mathbb{R}^{n^m}\) as the vector for system states that is measurable. \(G(q(t)) \in \mathbb{R}^n\) and \(H(q(t)) = \text{diag}(h_1, h_2, \ldots, h_m) \in \mathbb{R}^{n \times m}\) are bounded nonlinear functions, \(H^T(q(t))\) is invertible and \(I(q(t)) \in \mathbb{R}^n\) is the bounded unknown disturbance.

The desired trajectory signal \(y_{REF}(t) \triangleq [y_{REF1}(t), y_{REF2}(t), \ldots, y_{REFm}(t)]^T \in \mathbb{R}^m\). The tracking error is then defined as:

\[
e(t) \triangleq y_{REF}(t) - y_{out}(t)
\]

Define the vector of tracking error as:

\[
e_5(t) \triangleq [e^T(t), e^2(t), \ldots, e^{(n-1)}(t)]^T \in \mathbb{R}^{n^m}
\]

An sliding function is defined as:

\[
s(e_5(t), t) \triangleq e^{n-1}(t) + \Omega e^{n-2}(t) + \cdots + \Omega e^{0}(t)
\]

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where \( \Omega_j \in \mathbb{R}^{m \times m} \), for \( j = 1, 2, \ldots, L \) are the matrices with positive constant and \( \Omega = [\Omega_1, \Omega_2, \ldots, \Omega_L] ^T \in \mathbb{R}^{L \times m} \).

If \( G(\psi(t)), H(\psi(t)) \) and \( I(\psi(t)) \) are known exactly, then an ideal controller can be given as:

\[
u^*_e(t) = H^{-1}(\psi(t)) [y_{ref}^{(t)} - G(\psi(t)) - I(\psi(t)) + \Omega^T e_s(t)].
\]

Substituting (5) into (1), gives:

\[
s(e_s(t), t) = e^{(t)} + \Omega^T e_s(t) = 0.
\]

However, \( \nu^*_e(t) \) is unattainable due to \( I(\psi(t)) \) is always unknown in practical applications. Therefore, a MGMFFC is proposed to imitate the ideal controller. The proposed MGMFFC is shown in Fig. 1, which consists of a MGMFFC, \( u_{MGMFFC} \), and a robust controller, \( u_{RB} \), as follows:

\[
u_e(t) = u_{MGMFFC}(t) + u_{RB}(t)
\]

where \( u_{MGMFFC}(t) \) is the main controller and the robust controller, \( u_{RB}(t) \), compensates the approximation error for \( u_{id}(t) \) and \( u_{MGMFFC}(t) \) to give robust control performance.

This research uses a mixed GMF as shown in Fig. 3 [13]. There are two GMFs on a layer (i.e. previous and current GMFs), and they are activated simultaneously. It implies that the GMF of the previous state will remain active while the next state is activated.

![A mixed GMF](Fig. 3. Architecture of the proposed MGMFFC)

This MGMFFC contains five structures:

1) **Input**: \( I = [I_1 \ I_2 \ \ldots \ I_n]^T \).

2) **Association memory**: Two GMFs, \( \varphi_{ik}^{\text{current}} \) and \( \varphi_{ik}^{\text{previous}} \), are handled as a mixed GMF. The value of \( \varphi_{ik}^{\text{previous}} \) and \( \varphi_{ik}^{\text{current}} \) are defined as:

\[
\begin{align*}
\varphi_{ik}^{\text{current}} &= \exp \left[ -\frac{(I_i - m_{ik}^{\text{current}})^2}{2v_{ik}^{\text{current}}} \right], \\
\varphi_{ik}^{\text{previous}} &= \exp \left[ -\frac{(I_i - m_{ik}^{\text{previous}})^2}{2v_{ik}^{\text{previous}}} \right],
\end{align*}
\]

where \( m_{ik}^{\text{previous}} \) and \( m_{ik}^{\text{current}} \) are the previous and current means and \( \sigma_{ik}^{\text{previous}} \) and \( \sigma_{ik}^{\text{current}} \) are the previous and current variances for \( k \) th layer and \( i \) th input. \( i = 1, 2, \ldots, n \) and \( k = 1, 2, \ldots, n_k \).

The previous GMF, \( \varphi_{ik}^{\text{previous}} \), is saved in memory to forecast the subsequent state.

3) **Receptive-field**: A 2D example for calculating the synthesis hypercubes is presented in Fig. 4. The synthesis hypercube is the sum of two values of the current state (6, 5) and the previous state (4, 7). The synthesis hypercube is \( b_k^{\text{synthesis}} \), which is defined as:

\[
b_k^{\text{synthesis}} = \Lambda_k b_k^{\text{previous}} + \beta_k \prod_{i=1}^{n} \varphi_{ik}^{\text{current}}
\]

where \( \beta_i \) and \( \Lambda_k \) are respectively adaptive gains and \( b_k^{\text{previous}} \) is the previous hypercube value.
4) Weight memory: The weight is defined as:
\[ w_j = \begin{bmatrix} w_{j_1} & \cdots & w_{j_i} & \cdots & w_{j_{nj}} \end{bmatrix}^T \]  
where \( w_{j} \) is the output weight.

\[ W = \begin{bmatrix} w_1 & \cdots & w_j & \cdots & w_m \end{bmatrix} \]  

5) Output: The \( j \)-th output of MGMFFC is calculated as
\[ u_j = \sum_{k=1}^{n} w_{j_k} b_k^{\text{synthesis}} = \sum_{k=1}^{n} w_{j_k} \left[ \Lambda_k b_k^{\text{previous}} + \beta_k \prod_{i=1}^{n} \phi_i^{\text{current}} \right]. \]  
The MGMFFC output is rewritten via a vector form:
\[ u = Wb = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} & w_{21} & w_{22} & \cdots & w_{2n} & \vdots & \vdots & \vdots & w_{nj_1} & \cdots & w_{nj_{nj}} \end{bmatrix} \]  

B. Adaptive Learning Laws for MGMFFC

Taking the derivative of \( s(e_S(t), t) \) and using (1), yields
\[ \dot{s}(e_S(t), t) = e^o(t) + \Omega e_S(t) \]
\[ = -G(p(t))H(p(t))u(t) + y_{ef}(t) - I(p(t)) + \Omega e_S(t) \]  
Insert (7) into (15) and multiply both sides by \( s^T(e_S(t), t) \),
gives:
\[ s^T(e_S(t), t) \dot{s}(e_S(t), t) = -s^T(e_S(t), t)G(p(t)) - \]
\[ s^T(e_S(t), t)H(p(t))\left[u_{\text{MGMFFC}}(t) + u_{rb}(t) \right] + \]
\[ s^T(e_S(t), t)\left[y_{ref}^{(o)}(t) - I(p(t)) + \Omega e_S(t) \right] \]  
where
\[ u_{\text{MGMFFC}}(t) = H^{-1}(p(t)) \left[y_{ef}^{(o)}(t) - G(p(t)) - I(p(t)) + \Omega e_S(t) \right] \]  
Define a Lyapunov cost function as:
\[ V(s(e_S(t), t)) = \frac{1}{2} s^T(e_S(t), t) s(e_S(t), t) \]
\[ \Rightarrow \dot{V}(s(e_S(t), t)) = s^T(e_S(t), t) \dot{s}(e_S(t), t). \]  
The aim is to minimize \( s^T(e_S(t), t) \dot{s}(e_S(t), t) \) for reaching fast convergence of \( s \). The gradient descent technique is then used so the parameters are updated through the online adaptive laws as follows:
The approximation error $e$ is the minimum reconstructed error between $u^*_n(t)$ and $u_{MGMFFC}(t)$, and it is assumed to be bounded by $0 \leq |e| \leq \chi$.

If the adaptive law of the error bound is selected as
\[
\dot{\chi} = -\chi = -\gamma |e|, \tag{28}
\]
then (27) becomes:
\[
\dot{V}(s(e_s(t), t)) = s^T \dot{e} - |e|s - (\chi - \dot{\chi})\dot{e} \leq (s^T \dot{e}) - |e|s - (\chi - \dot{\chi})|e|
= -(\chi - |e|)|e| \leq 0. \tag{29}
\]
Since $\dot{V}(s(e_s(t), t), \tilde{\chi})$ is negative semi-definite that is $\dot{V}(s(e_s(t), t), \tilde{\chi}) \leq \dot{V}(s(0), \tilde{\chi}(0))$, it implies that $s$ and $\tilde{\chi}$ are bounded. Let function $\xi \equiv (\chi - |e|) s$ be bounded, so
\[
\lim_{t \to \infty} \xi(t) = 0.
\]
Therefore, the MGMFFC is asymptotically stable.

All system parameters for the three-link robot are listed in Table 1.

\[\begin{array}{|c|c|c|c|c|}
\hline
\text{Parameter} & \text{Value} \\
\hline
a_i & 0.5 m \\
m_i & 1.2 kg \\
i_i & 43.33 \times 10^{-3} \text{ kgm}^2 \\
\hline
\end{array}\]

Definitions of variables in (32) are described in Table 2 as
Equation (32) is rewritten as:

$$\dot{\psi}(t) = G(\psi(t)) + H(\psi(t)) u(t) + D(\psi(t)),$$  \hspace{1cm} (33)

where

\[
\begin{align*}
G(\psi(t)) &= \mathbf{J} [\dot{q}_1(t), \dot{q}_2(t), \dot{q}_3(t)]^T = \begin{bmatrix} x_1(t), x_2(t), x_3(t) \end{bmatrix}^T, \\
H(\psi(t)) &= -\mathbf{J}^{-1}(\mathbf{q}) \begin{bmatrix} \Gamma(q, \dot{q}) \dot{q} + g(q) \end{bmatrix}, \\
D(\psi(t)) &= -\mathbf{J}^{-1}(\mathbf{q}) \begin{bmatrix} \alpha_1(t), \alpha_2(t), \alpha_3(t) \end{bmatrix}^T.
\end{align*}
\]

The initial means of GMFs for the previous state and the current state set as:

$$\begin{align*}
\mu_{i0} &= \cos(q_{i0} + q_{i1}), \\
\mu_{i1} &= \sin(q_{i0} + q_{i1}).
\end{align*}$$

The short hand notations, in which $\tilde{\mathbf{J}}_i$ is the moment of inertia ($k_2 \mathbf{m}_i^2$) and $d_i$ is defined in the first row of Table 1

The learning rates for MGMFFC are $\gamma_I = 1, \gamma_P = 1, \gamma_U = \gamma_A = 0.05, \eta_I = \eta_P = \eta_U = \eta_A = \eta_I = \eta_P = \eta_U = \eta_A = 0.01, \delta = 0.1, \gamma_D = 0.1, \gamma_B = 1,$ and $\gamma_A = 0.1$. The initial value of $\mathbf{A}_0$ and $\mathbf{B}_0$ are set randomly between -1 and 1. In order to show the effectiveness of the MGMFFC, the TFLFCM (TOPSIS Function-link CMAC) [15] is also used for the three-link robot to compare their performance. The angular trajectories and joint velocities are respectively plotted in Figs. 6 and 8 (a)-(c). Enlarge of angular trajectories and joint velocities are shown in Figs. 7 (a)-(c) and Figs. 9 (a)-(c), respectively. The control efforts and their enlargements are displayed in Figs. 10 and 11 (a)-(c). The tracking errors for three links converge quickly to zero (see Figs. 12 and 13 (a)-(c)). The tracking angular positions of three links follow the reference angular positions well (see Figs. 6 and 7 (a)-(c)). The response of the proposed MGMFFC is fast (see Fig. 10 and 11 (a)-(c)). The proposed MGMFFC achieves a favorable tracking response when two adaptive prediction gains $\mathbf{A}$ and $\mathbf{B}$ are adjusted online as shown in Fig. 14. The total RMSE (root mean square error) for the TFLCM and the proposed MGMFFC are measured in Table 3. In summary, Table 3 and the simulation results confirm that the proposed MGMFFC achieves better tracking performance with quicker convergence and smaller tracking error than for TFLCM.
The tracking errors for the three-link robot proposed MGMFFC is the main controller and its robust ability model.

Fig. 9. Enlarge of joint velocities for the three-link robot

Fig. 10. The control efforts for the three-link robot

Fig. 11. Enlarge of control efforts for the three-link robot

Fig. 12. The tracking errors for the three-link robot

Fig. 13. Enlarge of tracking errors for the three-link robot

Fig. 14. Two adaptive gains $\alpha$ and $\beta$ of the MGMFFC

V. CONCLUSION

This research shows a mixed Gaussian membership function-based fuzzy CMAC for a three-link robot. The proposed MGMFFC is the main controller and its robust ability for handling system uncertainty has been demonstrated. A $\text{sgn}(\cdot)$ robust compensation controller is also applied to cover approximation errors. The online learning laws for all system parameters are derived from the gradient descent algorithm. Future studied can apply the proposed method for a practical model.

\begin{table}[h]
\centering
\caption{Total RMSE for Three-Link Robot}
\begin{tabular}{|c|c|c|c|}
\hline
 & RMSE1 & RMSE2 & RMSE3 \\
\hline
TFLFCM & 0.11 & 0.06 & 0.04 \\
MGMFFC & 0.09 & 0.04 & 0.01 \\
\hline
\end{tabular}
\end{table}

REFERENCES


