Multicriteria Decision Making: Scale, Polarity, Symmetry, Interpretability

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***Abstract*—In this study, we discuss the problem of interpretability of scale versus polarity in multicriteria decision-making problem. Decision making requires aggregation of premises of different characters and types. The influence of premises on a decision to be made may have a different characteristics as well. Some premises may have a positive character, i.e. they vote/agitate towards making a decision, others may fight against a decision. On the other hand, premises can be tempered by priorities, which may affect their character. Therefore, there is a need to discuss different configurations of premises and their priorities. This is the first aspect of our discussion on multicriteria decision making. The second one under discussion is the interpretability of all aspects mentioned so far. In this case, we discuss the representation problems of both premises and priorities. They are usually exhibited as numbers taken from some scale as, for instance, the unipolar unit interval $[0, 1]$ or the bipolar unit interval $[-1, 1]$. On the other hand, there is a question raised about a character of premises/priorities, that is, whether they vote pro or contra a decision to be taken and what relations between scales and polarities are.**

***Index Terms*—muticriteria decision making, scale, polarity, interpretability**

I. Introduction

In this study, we discuss the problem of interpretability in the context of multicriteria decision making with data exhibited in different scales and polarities. Multicriteria decision making requires aggregation of data of different properties and types. The influence of data on a decision to be made may have a different impact as well. Some data may have a positive character, i.e. they vote towards making a decision, others may act against a decision. On the other hand, data affecting a decision may be tempered by factors, which may alter their character.

We focus attention on a specific aspect of scale, polarity, and interpretability related to human cognitive processes. However, mechanisms discussed here could be easily adapted to other domains and be generalized in an abstract way. It is worth to underline that this study has a practical background. Readers interested in much more general and theoretical views on undertaken ideas are referred to [7].

From now on we refer to ideas exhibited in [13] and also adapt the terminology employed there. Specifically, we use terms premises and priorities for data affecting a decision and for factors tempering data:

- premises describe attitudes toward certain features or possibilities associated with the object of the decision. From our perspective, premises are motivational stimuli that elicit, control, and sustain certain behaviors. They are general factors relevant to the current motivational state. Premises can be somehow called initial or a priori motivation,
- priorities is the second term used in the discussed framework. This term is applied in the context of the second set of motivational stimuli evaluations. Priorities concern qualitatively the same arguments, but evaluated later, in the context of one particular decision. Priorities allow us to take into account reassessed attitudes toward a particular decision. Priorities provide a perspective of how one particular choice satisfies the stated conditions. In this context, they may be perceived as a posteriori

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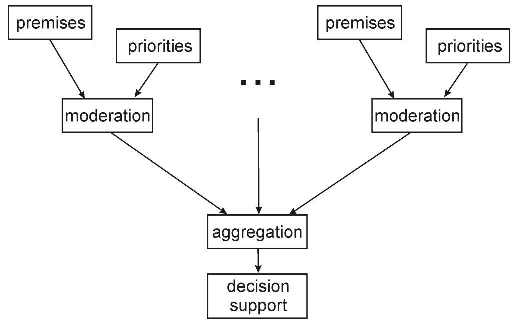

Fig. 1. Step-by-step illustration of the decision-making model.

motivation, arising when the decision maker reasons about a particular item. Of course, a set of priorities evaluations might be different than the premises.

Priorities moderate premises. We can capture causality because the decision takes into account not only final attitudes towards one particular decision (priorities), but also general attitudes (gathered in the premises vector). Processing with chosen aggregators is performed on such input data.

A concept of criteria representation based on a twofold evaluation (as in our premises/priorities approach) is present in the literature. Researchers from the field of social and managerial sciences introduced and applied such an approach in the form of the Kepner-Tregoe method, cf. [15], [18].

Figure 1 illustrates step-by-step a decision-making process based on premises and priorities. To sum up, the proposed approach is based on a two-step method of criteria evaluation. First, the basic criteria named premises and priorities are combined. Then, evaluated pairs of premises and priorities are aggregated to produce decision support.

Let us consider vectors of premises $\mathbf{p} = [p_1, p_2, \ldots, p_c]^T$ and priorities $\mathbf{r} = [r_1, r_2, \ldots, r_c]^T$, where $p_i \in I$ and $r_i \in J$. We can depict the process of decision support production with the following general formula:

$$d = \mathbf{p} \star \mathbf{r} \tag{1}$$

where $\star$ denotes an abstract aggregating operation performed on premises and priorities like, for instance, squashed scalar product.

There is a need to discuss different configurations of premises and their priorities. This is the first aspect of our discussion on multicriteria decision making. The second facet under discussion is the interpretability of all aspects mentioned so far. In this case, we would like to discuss the representation problems of both premises and priorities. They are usually exhibited as numbers taken from some scales as, for instance, the unipolar unit interval $[0, 1]$ or the bipolar one $[-1, 1]$. On the other hand, there is a question raised about a character of premises/priorities, that is, whether they vote pro or contra a decision to be taken. The third and the most important aspect of multicriteria decision making is in aggregation properties of premises and priorities. Properties of aggregation overhead types of scales and polarities. This means that in our opinion types of them firmly depend on the character and properties of aggregation mechanisms. In other words, polarities of scales cannot be uncovered unless operations employed to process data are not established.

Preliminary notes on scales and polarities are outlined in Section II. Sections III and IV are devoted to detailed discussion on unipolarity and bipolarity, respectively, and to related aspects of scaling and symmetry. An experiment on real world data is exhibited in Section V. The experiment can be repeated simply based on concepts introduced in previous Sections applied to referred data. Finally, comclusions are outlined in the last Section.

II. Scales and Polarities

A. Scales

Scales can be exhibited quantitatively as the real line or its fragments - intervals. Intervals are the conceptually simplest manifestation of a wide spectrum of different (algebraic) structures, like for instance an ordered set $(L, <)$. In this study, we limit the meaning of scales only to intervals of the real line. From the simplest perspective, i.e. without considering other aspects of scaling, we can distinguish a few types of such scales: open, closed and half-open intervals which can be finite or infinite ones. Any two intervals of the same type are qualitatively indistinguishable in the sense that there is a rescaling of one of them into another. A difference between them is just quantitative: different values of scales express the same measure of a phenomenon occurrence. Let us briefly comment on this observation.

Any two finite intervals of the same type, i.e. both open, both closed, both left-open or both right-open, are qualitatively indistinguishable in the sense that we have a bijection between them. For instance, we may take the simplest bijection: a linear mapping, $b : [a, b] \to [c, d]$, $b(x) = c+(x-a)/(b-a)*(d-c)$, where $-\infty < a < b < +\infty$, $-\infty < c < d < +\infty$. In this sense, the unit intervals $[0, 1]$ and $[-1, 1]$ are (paradoxically) qualitatively indistinguishable, which contradicts our intuition. Later on, we will put more light on this aspect.

For the real line $(-\infty, +\infty)$ and any finite open interval (a, b) we can find a bijection between them, though not linear, which also makes them quantitatively indistinguishable. As an example of a bijection between $(-\infty, +\infty)$ and $(-1, 1)$ we may take $\tanh : (-\infty, +\infty) \to (-1, 1)$. Any finite open interval (a, b) can be mapped to the real line $(-\infty, +\infty)$ by the composition of $\tanh \circ b$, where b is the linear bijection from (a, b) to $(-1, 1)$.

Alike, a bijection between the closed real line $[-\infty, +\infty]$ and a finite interval $[a, b]$ can be easily set up. Pairs of other types of intervals are related likewise. Details of correspondence between closed and open intervals can be restricted to intervals without endpoints and then extended to endpoints.

As an example of qualitatively indistinguishable scales, we can consider Kelwin $[0, +\infty)$, Celsius $[-273.15, +\infty)$ and Fahrenheit $[-459.67, +\infty)$ temperature scales. The beginning of scales and another reference point allow to raise linear mappings between these scales, for instance, $273.15°$K, $0°$C and $32°$F may be used as a second reference point.

B. Polarities

As mentioned before, we will consider two types of polarity: unipolarity and bipolarity. Intuitively polarities are interrelated with positive and negative aspects of information. At first glance, we may distinguish two types of scales associated with two types of polarity. Polarities can be exhibited in simple quantitative ways related to the real line and its fragments: the whole real line $(-\infty, +\infty)$ corresponds to a bipolar scale while the nonnegative semi-line $[0, +\infty)$ - to a unipolar scale. Defining closed counterparts $[-\infty, +\infty]$ and $[0, +\infty]$ is direct and seems to not need extra comments. Seemingly, these scales are rarely employed than most popular unit intervals: the bipolar unit intervals, closed or open, $[-1, 1]$ or $(-1, 1)$, and the unipolar unit intervals $(0, 1)$ or $[0, 1]$. In light of II-A other open and closed intervals are qualitatively equivalent to mentioned here.

However, the interrelation of isolated scales and polarities is supported by intuition only, which may not be correct. For instance, Celsius and Fahrenheit scales mentioned above look like bipolar ones, though this statement is arguable. So then let us look at the polarity of scales in a wider context, in the context of operations performed on scales. Let us also consider more convincing examples of scales and polarities. For the sake of simplicity, the discussion will be limited to fuzzy sets, though some generalization seems to be direct.

III. Unipolarity

Fuzzy set theory is often used to partition a universe into two subsets if partition criteria are not crisp. This statement directly corresponds to decision-making process: a space of premises and priorities should be split into two partitions corresponding to positive and negative decisions.

Polarity is a feature of operations performed on a scale rather than the scale itself. The prominent example, as assumed above, is the fuzzy set theory. Fuzzy sets defined in a universe X are represented by membership functions. A fuzzy set F is expressed as the membership function $\mu_F : X \to [0, 1]$, where the unipolar unit interval is a scale of possible values. What is important is that both the scale and the operations performed are considered together. The operations correspond to membership functions $\max$, $\min$ and $1-$ what formally defines a system $\mathcal{F} = ([0, 1]^X, \max, \min, 1-)$, where $[0, 1]$ is the scale, $\max$, $\min$ and $1-$ are mappings with the scale is the co-domain. It is worth drawing attention that the character of both union and intersection is independent on values of arguments, i.e. $\max$/$\min$ always take the value of the greater/smaller argument respectively. Likewise, $1-$ always completes to one.

A. Interpretation

Partitioning the universe into two complementary sets suggests a comparable significance of both sets unless additional principle is given. In such partitioning elements of the universe can be classified as *true* and *false*, *like* and *dislike*, *good* and *bad*, etc. without any emotional evaluation of these terms. We will simply talk about *positive* and *negative* information, again - without emotional evaluation of both terms. Using classical aggregators we can choose between *all good criteria* or *one good criterion*. The first one, where the elements classified as *good* one must have all criteria *good*, is implemented by conjunction. The second one, where the element classified as *good* can have only one *good* criterion, is implemented by disjunction. Aggregation connectives, conjunction, and disjunction, raise clear asymmetry under complement. If elements of one set are classified as having all criteria *good*, elements of the complementary set must have at least one criterion *bad* instead of expected the same condition of all criteria *bad*. Keeping the same condition (either *all criteria* or *at least one criterion*) in the definition of both sets raises troubles concerning the law of excluded middle. Following this way of thinking we need other connectives that will balance aggregation of decisions based on a singular criterion. The above discussion leads to the conclusion that classical fuzzy set theory is asymmetrical concerning processing opposite values of given attributes.

B. Triangular Norms

1) t-norms and t-conorms: Triangular norms were introduced in [20] and then studied in [16], [22]. Triangular norms: t-norm t and t-conorm s, are mappings $t : [0, 1]\times[0, 1] \to [0, 1]$ and $s : [0, 1] \times [0, 1] \to [0, 1]$ satisfying the following axioms:

1.	$t(a, t(b, c)) = t(t(a, b), c)$ $s(a, s(b, c)) = s(s(a, b), c)$	associativity
2.	$t(a, b) = t(b, a)$ $s(a, b) = s(b, a)$	commutativity
3.	$t(a, b) \le t(c, d)$ if $a \le c$ & $b \le d$ $s(a, b) \le s(c, d)$ if $a \le c$ & $b \le d$	monotonicity
4.	$t(1, a) = a$ for $a \in [0, 1]$ $s(0, a) = a$ for $a \in [0, 1]$	boundary conditions

$\min$ and $\max$ mappings are special examples t-norm and t-conorm respectively. Therefore, t-norms and t-conorms can be seen as generalization of operations on fuzzy sets.

t-norms and t-conorms are dual operations in the sense that any given dual t-norm t and t-conorm s satisfy the De Morgan's laws:

$$s(a, b) = 1-t(1-a, 1-b) \text{ and } t(a, b) = 1-s(1-a, 1-b) \quad (2)$$

In this study, the following well-known pairs of dual t-norms and t-conorms are used:

- min/max, $\min(x, y)/\max(x, y)$,
- product/probabilistic sum, $xy/x + y - xy$.

2) Additive Generators: t-norms can be generated by additive or multiplicative generators, cf. [16], [22]. In this study, additive generators are used to define t-norms and t-conorms. Let $f : [0, 1] \to [0, d]$ be a non-decreasing mapping with $[0, d]$ being a closed subinterval of the extended real semiline $[0, +\infty]$. Then the pseudo-inverse of the mapping f is defined by the formula $f^{-1} : [0, +\infty] \to [0, 1]$ such that $f^{-1}(y) = \sup\{x \in [0, 1] : f(x) < y\}$. Likewise, if the mapping f is non-increasing, then the pseudo-inverse is defined by the formula $f^{-1} : [0, +\infty] \to [0, 1]$ such that $f^{-1}(y) = \sup\{x \in [0, 1] : f(x) > y\}$. We restrict our discussion to strictly monotonic and continuous bijections with $f(0) = 0$ for an increasing mapping and $f(1) = 0$ for a decreasing mapping f. Therefore we get $(f^{-1})^{-1} = f$ in the interval $[0, 1]$ and $f^{-1}(y) = 1$ for $y > d$ for an increasing mapping f and $f^{-1}(y) = 0$ for $y > d$ for a decreasing mapping f. A mapping $q : [0, 1]^2 \to [0, 1]$ such that $q(x, y) = f^{-1}(f(x) + f(y))$ is a t-norm for the decreasing f and t-conorm for the increasing f. Moreover, norms that are monotonic and continuous are called *strict* norms assuming strict monotonicity. A detailed discussion on additive generators of triangular norms and conorms is presented in [16].

The following mappings are employed to illustrate additive generation of t-norms and t-conorms: (i) arctanh and its inverse and (ii) arcsin and its pseudoinverse:

(i) $f : [0, 1] \to [0, +\infty]$, $f^{-1} : [0, +\infty] \to [0, 1]$ such that $f(x) = \text{arctanh}(x)$, $f^{-1}(x) = \tanh(x)$

(ii) $f : [0, 1] \to [0, +\infty]$, $f^{-1} : [0, +\infty] \to [0, 1]$ such that $f(x) = \arcsin(x)$, $f^{-1}(x) = \sin(\min(x, \pi/2))$.

3) Limits for t-norms and t-conorms: It is worth to underline that t-norms and t-conorms are bounded by t-norm min and t-conorm max, i.e. for any t-norm t, any t-conorm s and any $x, y \in [0, 1]$ the following inequalities hold:

$$t(x, y) \leq \min(x, y) \leq \max(x, y) \leq s(x, y) \tag{3}$$

IV. Bipolarity

A. Symmetrization of the scale

A classical fuzzy set A in the universe X can be defined in terms of its membership function $\mu : X \to [0, 1]$, where the value 0 means the exclusion of the element from the set while values greater than 0 express the grade of inclusion of the element into the set. However, membership function does not define a grade of exclusion, the grade of negative information. Therefore, the fuzzy sets theory distinguishes grades of inclusion and reserves only one value - 0 - for exclusion what raises the asymmetry of this interpretation.

We can split values of a given criterion in the spirit of *good* and *bad* allocating the values of the interval $[0, 0.5)$ as pieces of *negative* information relevant to *bad* values and the values of the interval $(0.5.1]$ as pieces of *positive information* relevant to *good* values. The value 0.5, the center of the unit interval $[0, 1]$, is a numerical representation of the state of no negative/positive information. Being compatible with the common meaning of membership function lets assume that the greater the value of positive information, the stronger the *good* value of the criterion. By symmetry, the smaller the value of negative information, the stronger the *bad* value of the criterion, cf. Figure 2. An extension of this discussion leads to necessity/possibility measures, cf. for instance [6].

This interpretation is well-matched with the common sense of the ordering of the negative/positive values. The ordering could be seen as *monotonicity* of negative/positive information mapping: it starts from the left end of the unit interval representing strong negative information then goes toward the middle of the unit interval diminishing the strength of negative information, then crosses the middle point of the unit interval and then goes towards the right end of the unit interval increasing strength of positive information.

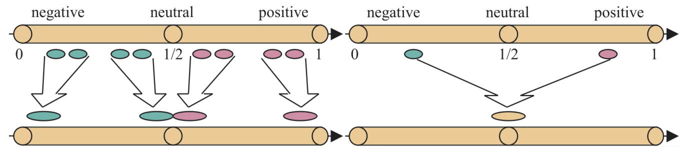

Fig. 2. Symmetry expectation in a process of information aggregation

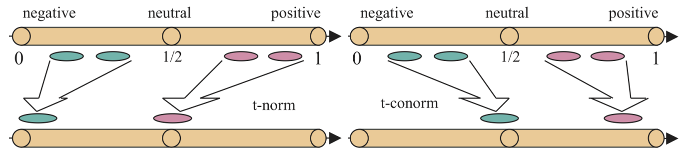

Fig. 3. Asymmetry of classical triangular norms

This interpretation is also well-matched with the common sense of symmetry of the negative/positive values with the symmetry center at the value 0.5. The linear transformation $b(x) = 2x - 1$ of the unit interval $[0, 1]$ into the symmetrical interval $[-1, 1]$ points out the symmetry. In this transformation negative information is mapped to the interval $[-1, 0)$, positive information - to the interval $(0, 1]$ and the state of no information - to the value 0.

B. Asymmetry of connectives

The fuzzy connectives min and max stay asymmetrical even if adapted to the symmetrical bipolar scale of the interval $[-1, 1]$, cf. II-A. They get their values from the maximal argument (maximum) and the minimal argument (minimum). Fuzzy connectives min and max were generalized to triangular norms: maximum is an example of t-conorms, the minimum is an example of t-norm, cf. [22]. Strong t-norms and t-conorms, the special cases of triangular norms, have an interesting property: if both arguments are greater than 0 and smaller than 1, the result of strong t-conorm exceeds the greater argument while the result of strong t-norm is less than smaller argument, cf. [16]. This property might be interpreted as asymmetry: union tends to positive information while intersection tends to negative information despite the values of their arguments. In other words, symmetrical interpretation of the unipolar scale makes that strong t-norm increases the certainty of negative information and decreases the certainty of positive information. And vice versa, strong t-conorm decreases the certainty of negative information and increases the certainty of positive information. This observation emphasizes again the asymmetry of fuzzy connectives, cf. Figure 3.

The problem of asymmetry of fuzzy connectives was discussed in number of papers, e.g. [5], [11], [12], [23], [25], [29]. In these papers, discussion on the asymmetry of fuzzy sets and uncertain information processing was undertaken for different reasons, though common conclusions led to the importance of the symmetry problem in fuzziness and uncertainty.

C. Negativeness versus Symmetry of Scale and Connectives

The mapping of negative and positive information in the scale of unit interval $[0, 1]$ as well as in the symmetrical interval $[-1, 1]$ bring incompatibility with connectives, so the question is raised if negative information can be considered as a subject of uncertainty. The question seems justified since negative information is hardly interpretable in classical set theory and classical fuzzy sets theory. However, negative information plays an important role in different fields. From psychological studies, it is known that human beings convey symmetry in their behavior, cf. [17]. One can be faced with positive (gain, satisfaction, etc.) or negative (loss, dissatisfaction, etc.) quantities, but also with a kind of disinterest (does not matter, not interested in, etc.). For instance, one either likes to listen to the music while reading an interesting novel or does not like to listen to the music then or even music is only a background not affecting him at all. These quantities could be interpreted in the context of positive/negative/neutral information. On the other hand, in economics a psychology-related attempt to decision making process with uncertain premises overheads traditional models of customer behavior. The pseudocertainty effect is a concept from prospect theory. It refers to people's tendency to make risk-averse choices if the expected outcome is positive, but risk-seeking choices to avoid negative outcomes. Their choices can be affected by simply re-framing the descriptions of the outcomes without changing the actual utility, cf. [14]. Aggregation of positive and negative premises leads to the implementation of a crisp decision. Modeling of such an attempt requires the processing of positive/neutral/negative information.

An interesting contribution to positive/negative information maintenance could be found in the theory of intuitionistic fuzzy sets [1] and the very similar theory of vague sets [8]. Another approach to positive/negative information is discussed in twofold fuzzy sets, cf. [6]. In these theories, uncertain information is represented as a pair of positive/negative components numerically described by membership values from the unit interval $[0, 1]$. Both components are tied with a degree of indeterminacy which stays that the sum of membership values of both components cannot exceed the value 1. However, no tool to combine both components is provided in these theories. Since information aggregation leading to non-ambiguous result is a clue issue in decision-making process, these theories must be supported by information aggregators in such a process.

The very early medical expert system MYCIN, cf. [2], combine positive and negative information by somewhat ad hoc invented aggregation operator. It was shown that the MYCIN aggregation operator is a particular case in a formal study on the aggregation of truth and falsity values, cf. [4] for further discussion on the aggregation of positive and negative information.

Having many premises, usually uncertain ones, we need to produce unique, non-ambiguous information that yields the decision. Therefore aggregation of information is crucial in decision-making process. The topic of information aggregating has been studied in number of papers, cf. interesting considerations in e.g. [3], [19], [24], [26], [29].

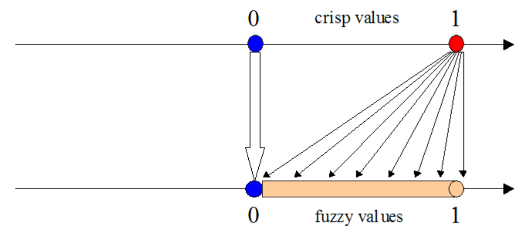

Fig. 4. Extension of crisp sets to fuzzy sets

D. *Symmetrizing fuzziness*

Fuzzy connectives stay asymmetrical with a symmetrized scale. The incompatibility of symmetrical interpretation of the scale and asymmetrical behavior of fuzzy connectives suggests incorrectness of scale symmetrization. This discussion leads to the hypothesis that Zadeh's extension of crisp sets to fuzzy sets, cf. [28], relied on dispersion of positive information of the crisp point $\{1\}$ into the interval $(0, 1]$. However, negative information of the point $\{0\}$ was still bunched in this point, cf. Figure 4. This hypothesis can be supported by ideas of balanced triangular norms, cf. [10], and uninorms and nullnorms, cf. [16], [27]. These ideas raise qualitatively different approaches to the extension of fuzzy connectives.

E. *Balanced symmetrization of the scale*

Both balanced t-norms and t-conorms, as well as uninorms and nullnorms, are created dispersion of information applied to the point $\{0\}$, cf. Figure 5. This operation extends classical fuzzy sets to balanced fuzzy sets, c.f. [9], [10]. The extension is being done by dispersion of crisp negative information bunched in the point $\{0\}$ into the interval $[-1, 0)$ without affecting fuzzy sets based on the unit interval $(0, 1]$. Thus, classical fuzzy sets will be immersed in a new space of balanced fuzzy sets. Since both kinds of information - positive and negative - are assumed to be equally important, it would be reasonable to expect that such an extension will provide a kind of symmetry of positive/negative information.

Concluding, the following symmetry principle can be formulated: *the extension of fuzzy sets to balanced fuzzy sets relies on spreading negative information (information about exclusion) that fits the crisp point $\{0\}$ of fuzzy set into the interval $[-1, 0)$*. The extension will retain properties of classical operators for positive information. It will provide the symmetry of positive/negative information with the center of symmetry placed in point 0, c.f. Figure 5. It is worth to underline that this operation is entirely different than the simple re-scaling of the unit interval $[0, 1]$. The linear mapping $f(x) = 2x - 1$ was replaced by the transformation that is not a function: it allocates the whole interval $[-1, 0)$ as a "value" in the point 0. Consequently, point 0 becomes "empty" what can be seen, intuitively and eventually, as its neutrality, i.e. it brings no positive or negative information.

F. *The symmetry of balanced connectives*

Triangular norms referred to in Section III-B generalize the concept of set operations union and intersection, cf. also [22]. Triangular norms, t-norms, and t-conorms, together

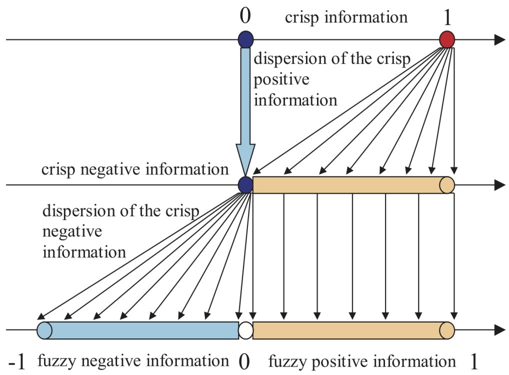

Fig. 5. Extension of fuzzy sets to balanced fuzzy sets

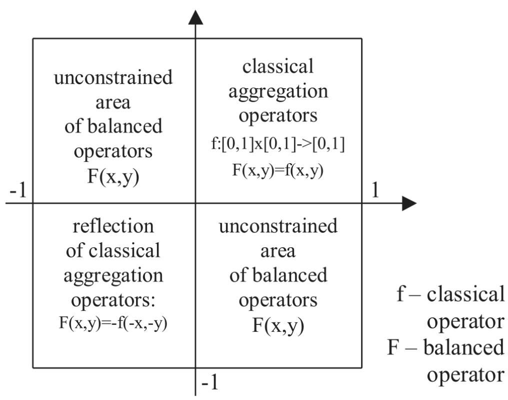

Fig. 6. Balanced extension of fuzzy operators

with negation, the basic fuzzy connectives are the subject of the discussion of connectives symmetrization.

The idea of a balanced extension of classical fuzzy connectives must be compatible with the concept of a balanced extension of the unipolar scale and with the symmetry principle formulated in Section IV-E. This requirements and symmetry of the balanced fuzzy scale of the interval $[-1, 1]$ determines the domain of symmetrized balanced connectives to the square $[-1, 1] \times [-1, 1]$. Preservation of fuzzy sets properties requires the protection of properties of classical fuzzy connectives on the unit square $[0, 1] \times [0, 1]$. Conversely, the expected symmetry of positive and negative information puts strict restrictions on the balanced extension of fuzzy connectives on the square $[-1, 0] \times [-1, 0]$. On the other hand, the same factors determine the co-domain of symmetrical fuzzy connectives to the interval $[-1, 1]$. This idea of the balanced extension of classical fuzzy connectives is outlined in Figure 6. It is clear, that balanced connectives on the square $[-1, 0] \times [-1, 0]$ are a simple reflection of respective classical connectives while balanced t-conorm is not explicitly constrained in the remaining quarters of the domain. However, monotonicity forces balanced t-conorms to be equal to 0 in these two quarters.

This discussion leads to the definition of balanced negation, balanced t-norms and balanced t-conorms: $N : [-1, 1] \to [-1, 1]$, $N(x) = -x$, $T : [-1, 1] \times [-1, 1] \to [-1, 1]$ and $S : [-1, 1] \times [-1, 1] \to [-1, 1]$ assuming that they satisfy the following axioms in the whole domain $[-1, 1] \times [-1, 1]$ unless defined explicitly:

1., 2., 3.: associativity, commutativity, and monotonicity

4. $T(1, a) = a \quad for \quad a \in [0, 1]$ boundary
   $S(0, a) = a \quad for \quad a \in [0, 1]$ conditions
5. $T(x, y) = N(T(N(x), N(y)))$
   $S(x, y) = N(S(N(x), N(y)))$ symmetry

The direct conclusions are:

(i) balanced t-norm and balanced t-conorm restricted to the unit square $[0, 1] \times [0, 1]$ are equivalent to classical t-norm and classical t-conorm, respectively,

(ii) balanced t-norm and balanced t-conorm restricted to the square $[-1, 0] \times [-1, 0]$ are isomorphic with the classical t-conorm and classical t-norm, respectively,

(iii) balanced t-conorm vanishes on the squares $[-1, 0] \times [0, 1]$ and $[0, 1] \times [-1, 0]$

V. Processing Real-World Data

As an illustration of the employment of the introduced unipolar and bipolar models, we follow discussion carried out in [13]. The experimental evaluation of models under interest refers to the Analytic Hierarchy Process (AHP) as a tool for multi-criteria decision-making, cf. [21]. In the framework of AHP, Saaty evaluated four US cities (Bethesda, Boston, Pittsburgh, and Santa Fe) based on five criteria (Culture, Family, Housing, Job, and Transport) and priorities for these criteria. The data used for processing with the unipolar model were taken directly from [21]. The values of priorities and premises (evaluation of each criterion for each city) are outlined in the left half of Table I. Those data have positive lineament, which can be roughly described with the phrase: "the more intensive the feature, the more desirable the object".

However, it is apparent that when a human being evaluates an object in a natural scenario, he does not restrict himself to positive features only. She/he considers negative aspects too. The main drawback of the model introduced in [21] is that it does not take into account negative features. About the presented real-life example, we may propose Extreme weather, Pollution, Noise, Corruption, and Crime as important negative features that may be considered when we compare cities. The values of the premises and priorities are outlined in the right half of Table I. The values of negative features are cited from [13] since neither [21] discusses them, nor is this example available elsewhere in the literature. The premises and priorities for negative features are expressed using negative values from the $[-1, 0]$ interval. The meaning of a negative value is understood as follows: the more negative the value (the closer to -1) the more unwanted this feature is.

In this section, we make use of the data explained above processed with different instantiations of the formula (1). We consider three specific models with an operation $\star$ fit with different triangular norms and balanced triangular norms. Namely, the pairs of following t-conorms and t-norms and their balanced counterparts are used: (i) maximum / minimum and probabilistic sum / product arc, (ii) norms generated with asin

TABLE I
EVALUATION OF CITIES FROM [21] AUGMENTED WITH NEGATIVE FEATURES: PRIORITIES, PREMISES

Priorities:	0.152	0.433	0.072	0.305	0.038	-0.152	-0.433	-0.072	-0.305	-0.038
Premises:	Cultur.	Family	Housing	Jobs	Transp.	Pollut.	Crime	Ex.Weat.	Noise	Corrupt.
Bethesda	0.474	0.330	0.459	0.500	0.467	-0.250	-0.125	-0.125	-0.125	-0.125
Boston	1.000	0.155	0.082	1.000	0.295	-0.125	-0.875	-0.750	-0.500	-0.500
Pittsburgh	0.424	1.000	1.000	0.155	1.000	-0.200	-0.875	-1.000	-0.625	-0.750
Santa Fe	1.000	0.089	0.209	0.135	0.115	-0.375	-0.625	-0.675	-0.700	-0.750

and atanh additive generators. Outcomes from these operations are compared with Saaty's sum / product employed in [21].

A. Unipolar Models with Positive Information

In this discussion, t-norms and t-conorms exemplify the formula (1). Namely, having vectors of premises $\mathbf{p} = [p_1, p_2, \ldots, p_c]^T$ and priorities $\mathbf{r} = [r_1, r_2, \ldots, r_c]^T$, $i = 1, 2, \ldots n$ and $p_i, r_i \in [0, 1]$, we can compute an outcome for given t-conorm s and t-norm t:

$$d = s\Big(t(p_1, r_1), t(p_2, r_2), \ldots, t(r_{n-1}, p_{-1}), t(r_n, p_n)\Big) \quad (4)$$

where, due to associativity of the t-conorm s:
$s\Big(x_1, x_2, ..., x_{n-1}, x_n\Big) = s\Big(x_1, s\big(x_2, s(..., s(x_{n-1}, x_n)...)\big)\Big)$
The outcome of Saaty's sum/product is computed with the formula $d = \sum_{i=1}^{n} p_i \cdot r_i$. Of course, the Saaty's outcome falls into the non negative real semiline, $0 \leq d$, while t-conorm/t-norm's result is in the unit interval $[0, 1]$.

B. Unipolar Models with Positive and Negative Information

Processing positive and negative information in the unipolar model requires assumptions concerning the numerical representation of such data. In this study, we employ a standard assumption: fiercely positive data is represented with values close to one, while strongly negative data - with values close to zero. Based on this assumption, data from the right half of Table I are linearly mapped to the unit interval, cf. Section II-A. Then, the processing is performed as described in the previous Section V-A. Of course, the outcome falls into the unit interval $[0, 1]$ for triangular norms used or can be (theoretically) any non-negative real number for Saaty's sum/product.

C. Positive and Negative Information and Bipolar Model

The bipolar model is alike the unipolar model outlined in Section V-A. Unlike in the unipolar model, balanced t-conorms and balanced t-norms are employed in formula (4) in place of t-conorms and t-norms. Of course, in this case, computed outcome falls into the bipolar unit interval $[-1, 1]$ instead of the unipolar one $[0, 1]$. With regard to Saaty's computation, the product of opposite factors is taken as a negative number while we do not have a case with factors of opposite signs. Anyway, in a case of factors of opposite signs would be reasonable to assume that Saaty's product vanishes.

D. Results

In [21], the final evaluation of each city was aggregated using the sum of multiplied premises and corresponding priorities. In this paper, additionally, the data are subjected to aggregation with triangular norms. Hence, we can compare the literature-based method with our approach.

Then, the same methods of aggregation were employed to process positive and negative data with the unipolar model. In particular, we used the sum of products (following Saaty's line of thought) and, in addition, we used the aforementioned four pairs of dual t-conorms and t-norms.

Finally, to process positive and negative data within the bipolar model, we applied aggregation methods derived from those methods that were used in unipolar models, i.e. the sum of products and balanced extension of the mentioned pairs of dual triangular norms.

The outcome of the cities evaluation is presented in Figure 7. We see that, in general, results are qualitatively consistent. Only two estimations differ: positive data in the unipolar model aggregated with triangular norms generated by the sin additive generator and in positive and negative data in the bipolar model aggregated with max/min triangular norms.

Processing original Saaty's positive data ranks Pittsburgh as the best city, while Santa Fe is ranked as the worst. Notice, though, that the differences in evaluations are not big. The biggest numerical difference is in Bethesda's evaluation with sin/arcsin generated triangular norms. Characteristics of sin/arcsin functions are similar to identity mapping for small arguments and this causes the t-norm to be strongly decreasing for such arguments. This makes the final result smaller than in other cases.

We can draw similar conclusions when positive and negative data are processed with the unipolar model for the first two aggregators: the sum of products and max/min triangular norms. However, the other three triangular norms do not behave in this way, for them, the evaluation reaches the highest value for all cities. This is not surprising, because these t-conorms used as aggregators accumulate positive arguments, despite their values.

Processing the same positive and negative data as before, but with the bipolar model, produces interesting results. All methods evaluate Santa Fe as the worst city. Bethesda is the leader in Saaty's sum of products, while Pittsburgh is the best according to four other evaluation methods. Bethesda overtakes Boston. Notice that the differences in evaluations

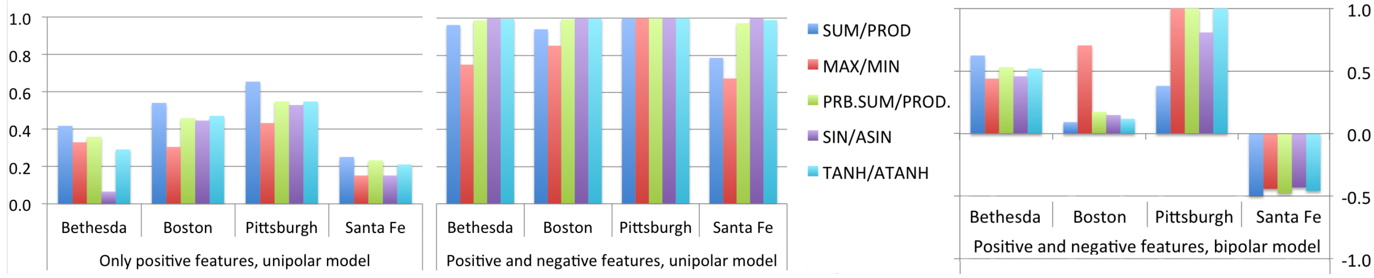

Fig. 7. Cities evaluation in different models.

are more significant than in unipolar models. The substantial numerical difference in max/min evaluation for Boston results from the insensitivity of these triangular norms, cf. Section III-B.

VI. Conclusions

In this paper, we study multi-criteria decision-making models in the context of different scales and polarities employed in model construction. The concepts introduced in this study rely on the two-step procedure of premises and priorities aggregation in different quantitative environments. The discussion concerns different types of scales used in data processing and of the polarity of employed operations. These aspects are reflected in the human intuitive expectation of symmetry and asymmetry of information processing. We can capture nontrivial aspects of decision-making including imprecise knowledge and polarities of information. The key concepts introduced in this study are reflected in human expectation and interpretation of information processing and decision making. Besides, we present a discussion of different types of operators that could be used in decision making. This discussion is oriented on operators' suitability to represent human expectation and interpretation. The developed methodology has been applied to an experiment on real data.

In future research, one could extend the study by looking into other imperfect information representation models, especially including intuitionistic fuzzy sets, interval-based fuzzy sets and interval based balanced fuzzy sets.

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