Perceptual Computing with Comparative Linguistic Expressions

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***Abstract*—A perceptual computer generates appropriate word recommendations, rankings and/or classifications for any given application context. During the preparation of the codebook, perceptions of individuals are obtained in the form of end-points of intervals, followed by the selection of words from the codebook at a later stage. Keeping in mind the fact that humans are hesitant while providing responses, the perceptual computing paradigm does not handle such hesitancy at any stage. To conquer this deficiency within the framework of perceptual computing, we propose the idea of perceptual computing with comparative linguistic expressions. Through this proposal, we engage the hesitancy faced by people at different stages of the perceptual computing framework. We also introduce two different types of hesitancies that can occur, depending on the types of elements.**

Keywords—Comparative linguistic expressions, hesitancy, perceptual computing

I. Introduction

With the growth in tendency of catering to a large group of people with highly personalized results, there has been a rapid increase in the development of models that process the choices obtained from such individuals. Such processes are aided by options that are presented to the target individual, who then responds with appropriate selections based on his/her preferences. Similarly, one must also consider the rise in computer aided models that are made available which accept the responses of individuals and produce results relevant to the end user.

Currently, also on the rise is the development of various other computer aided models that aim to simulate the behavioural aspects of a human being. One such aspect is the ability of human beings to react under the impreciseness and vagueness that words have to offer. A dominant portion of the communication that occurs between humans involves linguistic features, which includes words. Since words mostly represent qualitative aspects, their meanings vary amongst individuals. This is due to the dissimilar and diverse *perceptions* of individuals with respect to a context.

To tackle this inherent impreciseness and vagueness within linguistic features, Zadeh in 1975 [1] proposed the idea of linguistic variables within the domain of computing with words (CWW) [2]. According to this proposal, linguistic variables are used to model linguistic terms within systems. These linguistic variables find their basis in fuzzy sets (FS) [3] which handle the linguistic uncertainty in words. Each linguistic variable is a quintuple conveying the name of the associated linguistic term, its syntactic and semantic definition (associated membership function (MF)), the linguistic term set (LTS) fixed a priori to which the linguistic term belongs, along with the universe of discourse.

Many existing approaches have constructed their models on CWW, such as the models based on extension principle, symbolic method, 2-tuple and the likes [4-6]. Each of these models has been widely applied to many practical domains such as supply chain, risk evaluation, information retrieval, engineering systems [7], etc. Each of the models discussed above draws linguistic terms from an LTS which is fixed a priori, often by experts.

However, this is not one of the most practical assumptions to be made when it comes to real-life applications. None of the above model discusses about the formation of the LTS. Also, since the proposal of type-2 FSs (T2 FSs) [8], they are considered to be better in modelling uncertainty than their previous counterparts, namely type-1 (T1) FSs. This is because T2 FSs assign another degree of membership to the primary membership. This holds true within the domain of CWW [9] because a linguistic term usually encapsulates two types of uncertainties namely the *intra-uncertainty* and the *inter-uncertainty* which a T1 FS fails to model due to its less degrees of freedom [10]. The intra-uncertainty is faced by a person while providing an appropriate definition for the linguistic term in concern, whereas, the inter-uncertainty arises due to a disagreement amongst various individuals regarding the definition of a linguistic term. Mendel has hence, recently stated that it is scientifically correct to model a linguistic term using T2 FSs or higher representational models [11].

T2 FSs were employed in decision making problems in [12]. Also, recently IT2 FSs were used to model timing constraints in an Industry 4.0 ecosystem in [13]. Apart from this, IT2 FSs have also been utilized in improving the deep learning abilities within restricted Boltzmann machines [14] and deep belief networks [15]. In [16], uncertainty modelling in gene expression datasets was done using IT2 FSs.

A CWW framework called the *perceptual computer* (Per-C) [10] takes into consideration the aforementioned drawbacks of the previously existing CWW models. A Per-C when applied to an application collects end-point data for intervals that define the linguistic terms from various individuals. These data are then processed to give an interval T2 (IT2) FS based representation for the linguistic terms, which results in a *codebook*. Later in the context of the considered application, the decision maker selects words from the codebook for different parameters, which are aggregated together to obtain a final recommendation. To this day, the framework of Per-C has proven to possess good applicability within various domains such as health monitoring of heart failure patients [17], hierarchical decision making [18], investment judgment advising [10] and, power optimization [19-20].

Even though Per-C conquers the drawbacks of the existing CWW approaches, its framework still suffers from a drawback. Many times, humans feel *hesitant* while choosing or selecting the best option according to their preference. This reflects within the phrases that humans use on a regular basis, such as 'my scores are *between good and very good*' or 'the performance of the candidate is *more than average*'. Such phrases are called comparative linguistic expressions (CLE) [21]. They are more complex than single linguistic terms to be handled with traditional linguistic models. Such expressions were first handled by the hesitant fuzzy linguistic term sets (HFLTS) based linguistic model [22]. The idea of HFLTS was studied further in [23] where its authors augmented the framework for large-scale group decision making. In [24], a consensus model for multiple attribute group decision making was developed for multi-granular HFLTSs. Due to the ability of T2 FSs to handle linguistic uncertainties efficiently, T2 FS based HFLTSs were recently proposed [25]. Within this framework, linguistic terms were represented using T2 FSs, which were used for multiple criteria group decision making.

Given the practicality of CLEs and the advantages of Per-C over the other existing linguistic models, it is motivating to augment the existing framework of Per-C to handle hesitancy along with the linguistic uncertainties associated with linguistic terms described above. Hence, in this paper we propose to introduce the idea of perceptual computing with CLEs, wherein the hesitancy of individuals are handled appropriately whenever such a situation arises throughout the Per-C framework. Moving along these lines, we define two different forms of hesitancies, namely the *definitive* and *non-definitive* hesitancy. These hesitancies provide better robustness to the Per-C framework as they enable the Per-C to handle even more uncertainties associated with linguistic features. This enriches the quality of linguistic information elicited within the Per-C framework.

In summary, our contributions presented through this paper are given below:

- *Non-definitive and definitive hesitancy*: Concept of two types of hesitancies are introduced which are associated with linguistic and numeric data respectively.
- *Definitions of new hesitancies*: Proper grammar rules to generate CLEs corresponding to the new hesitancies are introduced.
- *New hesitant term*: A new hesitant term that expresses a certain type of hesitancy amongst individuals is introduced.
- *Handle CLEs in augmented Per-C framework*: We handle linguistic phrases as inputs when the individual interacting with the Per-C hesitates while providing her/his input.

The subsequent sections of this article are organized as follows: Section II describes the prefatory knowledge required to understand our proposed framework properly, Section III presents our proposed framework while introducing the new types of hesitancies. We conclude the paper with some conclusions and discussions on future work in Section V.

II. Prefatory Knowledge

A. T1 and T2 FSs [26]

A T1 FS is defined on a set of elements, where each element belongs to this set with some degree of membership.

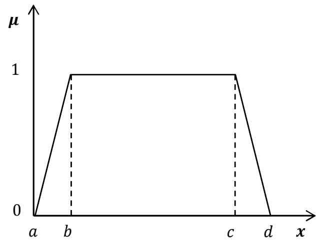

Fig. 1. A T1 FS

Let X be a set of elements, the T1 FS A defined on X, assigns graded memberships to every $x \in X$, based on a MF denoted by $\mu_A(x) \in [0,1]$. Formally,

$$A = \{(x, \mu_A(x)) | \forall x \in X, \mu_A(\cdot) \in [0,1]\} \quad (1)$$

A trapezoidal T1 FS is shown in Fig. 1.

On the other hand, T2 FSs assign a secondary degree of membership ($\mu_{\tilde{A}}$) in addition to a primary degree of membership (J_x) on the elements of the set in consideration. Consider the same set X, for which the T2 FS is denoted by $\tilde{A}$, whose MF is characterized by $\mu_{\tilde{A}}(x,u)$, where $u \in [0,1]$, for every $x \in X$. Formally,

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) | \forall x \in X, u \in J_x \subseteq [0,1]\}, \quad (2)$$

where $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$.

When, the secondary MF is set to 1 for all elements, the T2 FS becomes an IT2 FS. Therefore,

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u) = 1) | \forall x \in X, u \in J_x \subseteq [0,1]\} \quad (3)$$

Fig. 2 demonstrates an IT2 FS, $\tilde{A}$. Notice that the shaded region, called the FOU conveys the uncertainty in the IT2 FS, and uniquely determines the same. The FOU comprises of an upper MF (UMF) and a lower MF (LMF). All the three are formally defined as follows:

$$FOU(\tilde{A}) = \left\{(x,u): x \in X, u \in \left[\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)\right]\right\} \quad (4)$$

$$\underline{\mu}_{\tilde{A}}(x) = \inf\{u | u \in [0,1], \mu_{\tilde{A}}(x,u) > 0\}, \forall x \in X \quad (5)$$

$$\overline{\mu}_{\tilde{A}}(x) = \sup\{u | u \in [0,1], \mu_{\tilde{A}}(x,u) > 0\}, \forall x \in X \quad (6)$$

Eq. (5) denotes the LMF whereas Eq. (6) denotes the UMF for an IT2 FS $\tilde{A}$.

B. Comparative Linguistic Expressions

CLEs are expressions that convey richer linguistic information than single terms. Humans use such CLEs to express the hesitancy that they face while selecting appropriate linguistic terms.

Just like generation of any expression in a language is based on a grammar, the CLEs are also generated with the help of a context-free grammar, with specific production rules. Note that we follow production rules defined in the extended Backus Naur form. They are defined below:

Definition 1 [21]: Let $S = \{s_0, s_1, \ldots, s_n\}$ be an LTS where $s_k, k \longrightarrow 1\ to\ n$ is a linguistic term. Let G_H be a context-free grammar which is a quadruplet $G_H = (V_N, V_T, I, P)$. The

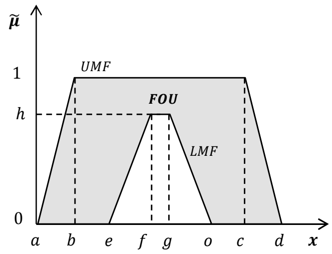

Fig. 2. An IT2 FS

elements of the quadruplet along with the production rules, P of G_H are defined as follows:

$V_N = \{\langle primary\ term\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunction\rangle\}$

$V_T = \{\text{less than, more than, between, and}, s_0, s_1, \ldots, s_n\}$

$I \in V_N$

$P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle$
$\langle composite\ term\rangle ::= \langle unary\ relation\rangle \langle primary\ term\rangle |$
$\langle binary\ relation\rangle \langle primary\ term\rangle \langle conjunction\rangle \langle primary\ term\rangle$
$\langle primary\ term\rangle ::= s_0 | s_1 | \ldots | s_n$
$\langle unary\ relation\rangle ::= \text{less than}|\text{more than}$
$\langle binary\ relation\rangle ::= \text{between}$
$\langle conjunction\rangle ::= \text{and}\}$

C. Perceptual Computer

A Per-C consists of three components: the encoder, the CWW engine and decoder. Words, which are perceptions of individuals activate the Per-C. Below, we give a pointwise working of the Per-C. A bird's eye view of the same is depicted through a pictorial representation in Fig. 3.

1. After an appropriate application is selected, a vocabulary of words is constructed to be used.
2. Interval end points are collected from various individuals to obtain an understanding of varying perceptions.
3. Encoder: The obtained end-points are processed to obtain T2 FS based representation for the words in vocabulary. The Interval Approach (IA) [27] is the most commonly used approach and is described below.
    a. Data preprocessing: includes bad data processing, outlier processing, tolerance limit processing, reasonable interval processing.
    b. Computation of data statistics for each interval
    c. Choosing a T1 FS model
    d. Establishing FS uncertainty measures
    e. Computing uncertainty measures for T1 FSs
    f. Computing general formulas for parameters of T1 FS models
    g. Establishing nature of FOU
    h. Computing embedded T1 FSs
    i. Deletion of inadmissible T1 FSs
    j. Computation of IT2 FS and corresponding mathematical model using union operation.

    This step results in a codebook of IT2 FSs of words in the vocabulary.
4. CWW Engine: One of the most commonly used CWW engine is the linguistic weighted average (LWA) [10], which aggregates the IT2 FS representations of words chosen from the codebook and results in an IT2 FS based representation of the aggregation just performed.
5. Decoder: Map the IT2 FS obtained from the CWW engine to a suitable recommendation either through similarity analysis, ranking or classification.

To obtain in-depth knowledge about each of the steps presented above, please refer to [10].

III. Perceptual Computing with Comparative Linguistic Expressions

In this section, we discuss in length about our proposed framework of perceptual computing with CLEs.

A. Definitve and Non-definitive Hesitancies

Before delving into a detailed discussion about the proposed idea, we first define two different types of hesitancies that a human faces.

Hesitancy comes into the picture only when an individual is provided with an array of options to choose from, i.e. hesitancy is not an 'absolute', but a 'relative' concept. Having said that, it can be observed that there can be two types of elements, one with crisp definitions and one with uncertainty. To elaborate, elements falling within the first category have universal definitions and are same for every individual, whereas elements belonging to the second category may have different definitions for different individuals. For e.g. numbers such as 45, 2.3, 8 etc. quantify the exact same amount for every individual, however adjectives such as $high$, $great$, $average$ etc., depend on perceptions of people, as already discussed in Section I. Referring to this discussion above, we give the following definitions.

Definition 2: Let $M = \{m_0, m_1, \ldots, m_p\}$ be a set of elements that have crisp definitions. The hesitancy associated with the selection of such elements is called the *definitive hesitancy*.

Definition 3: Let $L = \{l_0, l_1, \ldots, l_q\}$ be a set of elements that have different meanings for different individuals. The hesitancy associated with such elements is called *non-definitive hesitancy*.

As already discussed, CLEs are utilized within linguistic models to express the hesitancy faced by individuals. Therefore, we must define the generation of CLEs for the different hesitancies proposed above. The associated grammar and production rules are given below. In the definitions below, we also add a new type of hesitant term around.

Definition 4: Let $M = \{m_0, m_1, \ldots, m_p\}$ be a set of elements that have crisp definitions. Let G_{H_D} be a context-free grammar which is a quadruplet $G_{H_D} = (V_N, V_T, I, P)$. The elements of the quadruplet along with the production rules, P of G_{H_D} are defined as follows:

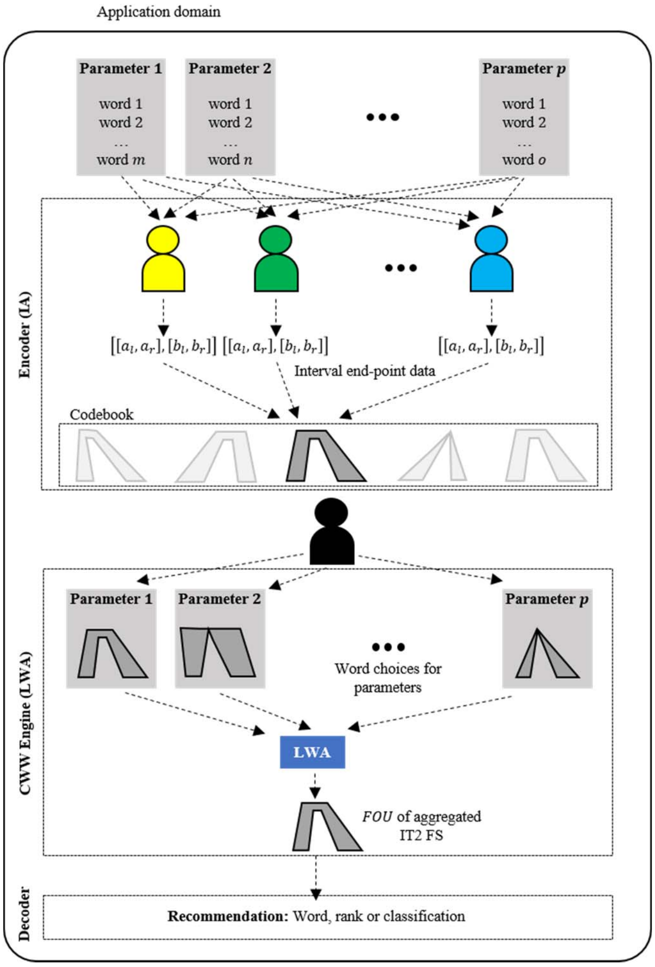

Fig. 3. Bird's eye view of the working of a Per-C.

$$V_N = \{\langle \boldsymbol{crisp\ term}\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunction\rangle\}$$

$$V_T = \left\{ \begin{matrix} \text{less than, more than, } \textbf{around}\text{, between, and,} \\ \boldsymbol{m_0, m_1, \dots, m_p} \end{matrix} \right\}$$

$$I \in V_N$$

$$\begin{aligned}
P = \{&I ::= \langle \boldsymbol{crisp\ term}\rangle | \langle composite\ term\rangle \\
&\langle composite\ term\rangle ::= \langle unary\ relation\rangle\langle \boldsymbol{crisp\ term}\rangle | \\
&\langle binary\ relation\rangle\langle \boldsymbol{crisp\ term}\rangle\langle conjunction\rangle \\
&\langle \boldsymbol{crisp\ term}\rangle | \\
&\langle \boldsymbol{crisp\ term}\rangle ::= \boldsymbol{m_0} | \boldsymbol{m_1} | \dots | \boldsymbol{m_p} \\
&\langle unary\ relation\rangle ::= \text{less than} | \text{more than} | \textbf{around} \\
&\langle binary\ relation\rangle ::= \text{between} \\
&\langle conjunction\rangle ::= \text{and}\}
\end{aligned}$$

Example 1: Some CLEs generated by G_{H_D} are given below:

$$\begin{aligned}
cle_1 &= \text{less than } 57 \\
cle_2 &= \text{between } 2 \text{ and } 7 \\
cle_3 &= \text{more than } 19 \\
cle_4 &= \text{around } 5
\end{aligned}$$

We use Definition 1 for the generation of CLEs corresponding to non-definitive hesitancies.

B. Fuzzy representations of CLEs corresponding to definitive and non-definitive hesitancies

Since linguistic models are based on FS representations, obtaining the same for the hesitancies just introduced is crucial. We do so in the current subsection.

a. Definitive hesitancy

Looking at the CLEs generated in Example 1, it is fairly clear that each of them has different importance. Hence, each CLE is treated differently according to its importance.

Let for the following discussion, M[1] $= \{m_0, m_1, \dots, m_p\}$ be a set of elements that have crisp definitions. Let the Universe of Discourse for the sake of discussion be $[u_l, u_r]$.

1. less than $m_i, i \in [0, p]$

The importance of elements less than m_i increases as one moves leftwards on the Universe, from m_i. Accordingly, the corresponding MF of the related T1 FS, $\mu(x)$ is given as:

$$\mu(x) = \begin{cases} \dfrac{m_i - x}{m_i}, & u_l \le x \le m_i \\ 0, & otherwise \end{cases}$$

The corresponding FS representation is given in Fig. 4(a).

2. more than $m_i, i \in [0, p]$

The importance of elements more than m_i increases as one moves towards the right on the Universe, from m_i. Accordingly, the corresponding MF of the related T1 FS, $\mu(x)$ is given as:

$$\mu(x) = \begin{cases} \dfrac{x - m_i}{u_r - m_i}, & m_i \le x \le u_r \\ 0, & otherwise \end{cases}$$

The corresponding FS representation is given in Fig. 4(b).

3. between m_i and $m_j, i, j \in [0, p], i < j$

The importance of elements between m_i and m_j is the highest. It decreases when moving right towards u_r from m_j. It also decreases when moving left towards u_l from m_i. Since for larger Universes, not all elements in $[u_l, u_r]$ might be of importance, we define the *bounded importance factor*, $\delta \ge 0$. This factor controls the spread of the MF. Accordingly, the corresponding MF of the related T1 FS, $\mu(x)$ is given as:

$$\mu(x) = \begin{cases} 0, & (x < (m_i - \delta)) \ or \ \left(x > (m_j + \delta)\right) \\ \dfrac{x - m_i + \delta}{\delta}, & (m_i - \delta) \le x \le m_i \\ 1, & m_i \le x \le m_j \\ \dfrac{m_j + \delta - x}{\delta}, & m_j \le x \le (m_j + \delta) \end{cases}$$

The corresponding FS representation is given in Fig. 4(c).

4. around $m_i, i \in [0, p]$

The importance of elements towards the left and right of m_i decrease while moving away from m_i, the highest being at m_i. The spread of the corresponding MF is again bounded using

[1] Here, we assume that the elements are real numbers.

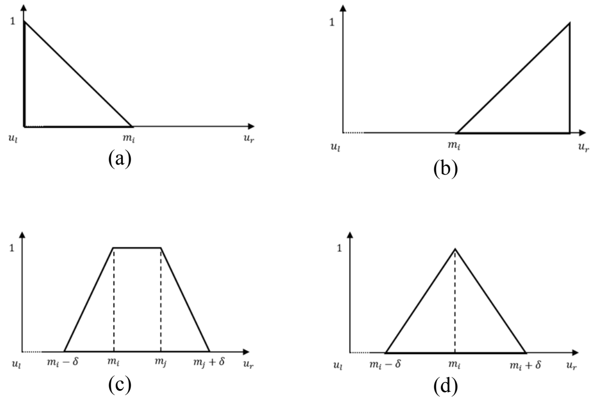

Fig. 4. Fuzzy representations of CLEs corresponding to definitive hesitancy (a) less than $m_i, i \in [0,p]$, (b) more than $m_i, i \in [0,p]$, (c) between m_i and $m_j, i,j \in [0,p], i < j$, (d) around $m_i, i \in [0,p]$

$\delta \geq 0$. Accordingly, the corresponding MF of the related T1 FS, $\mu(x)$ is given as:

$$\mu(x) = \begin{cases} 0, & x \leq (m_i - \delta) \\ \dfrac{x - m_i + \delta}{\delta}, & (m_i - \delta) \leq x \leq m_i \\ \dfrac{m_i + \delta - x}{\delta}, & m_i \leq x \leq (m_i + \delta) \\ 0, & x \geq (m_i + \delta) \end{cases}$$

The corresponding FS representation is given in Fig. 4(d).

b. Non-definitive hesitancy

In this paper, we consider the non-definitive hesitancy to be the one which is faced when an individual is selecting linguistic terms. Since linguistic terms are best represented using IT2 FSs, all the IT2 FSs of the corresponding terms are aggregated to obtain one IT2 FS.

Any aggregation method can be used to combine the linguistic terms in question. However, we suggest the usage of a weighted average such as the LWA that would consider the importance of the terms as weights, computed differently for different CLEs. A simple method of generating the 'importance weights' of the terms resulting from the CLE would be defining them as a function of the indices of the terms resulting from the CLE.

First let $L_k = \{l_0^k, l_1^k, \ldots, l_q^k\}$ be a set of terms that corresponds to the possible values for the k^{th} parameter, such that for any l_i^k and l_j^k, $i,j \in [0,q], i < j$, l_i^k has a lower semantic rank[2] than l_j^k. Since each element in the set L_k is a word, each of them is encoded into an IT2 FS. Also, let $T = \{l_i^k, l_r^k, \ldots, l_j^k\}, i,r,j \in [0,q]$ be a set of terms resulting from a CLE defined on the set of terms L_k, also following the semantic ranking. Let $W = \{\omega_0, \omega_1, \ldots, \omega_t\}$ be the weight vector, with a one-to-one mapping between every element of T and W. Aggregation using LWA is computed as:

$$C = \frac{\sum_{s=0}^{t} l_i^k \omega_s}{\sum_{s=0}^{t} \omega_s}$$

[2] Semantic rank refers to the ranking of words in natural language. For e.g. the word 'short' would always rank lower than the word 'tall' in context of the height of a person.

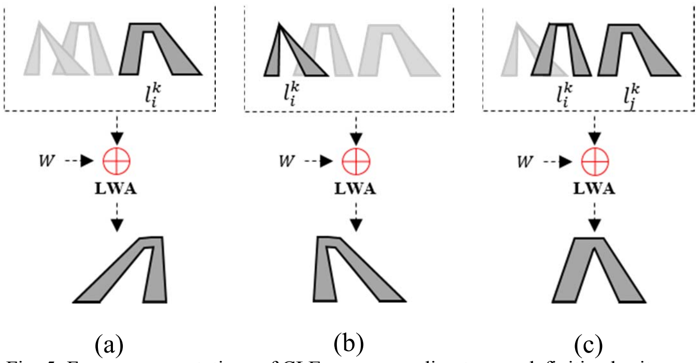

Fig. 5. Fuzzy representations of CLEs corresponding to non-definitive hesitancy (a) less than $l_i^k, i \in [0,q]$, (b) more than $l_i^k, i \in [0,q]$, (c) between l_i^k and $l_j^k, i,j \in [0,q], i < j$

We now consider specific cases to obtain the fuzzy representations for CLEs corresponding to non-definitive hesitancy.

1. less than $l_i^k, i \in [0,q]$

   The importance of terms less than l_i^k increases as one goes from i to 0. Therefore, a suitably chosen W would result in an IT2 FS as shown in Fig. 5(a), after aggregation.

2. more than $l_i^k, i \in [0,q]$

   The importance of terms more than l_i^k increases as one goes from i to q. Therefore, a suitably chosen W would result in an IT2 FS as shown in Fig. 5(b), after aggregation.

3. between l_i^k and $l_j^k, i,j \in [0,q], i < j$

   The importance of terms between l_i^k and l_j^k is the highest. It increases as one goes from i to q and decreases when one goes from i to 0. Therefore, a suitably chosen W would result in an IT2 FS as shown in Fig. 5(c), after aggregation.

C. *Hesitancy within the Per-C framework*

As discussed earlier, hesitancy occurs in the presence of choices from which selections must be made. We now go through every component of a Per-C and analyze the role of hesitancy within every component.

a. Encoder

Looking closely at the IA in the encoder component of a Per-C, we observe that end-points of intervals defining the words in the vocabulary are obtained from a group of individuals to capture their perceptions. Keeping in mind that the universe of discourse provided to these individuals is [0,10], the individuals are presented with some options to be assessed.

Consequently, an individual might face hesitancy in choosing the end points for the interval data. For e.g., the question posed to the group is as follows:

"What according to you should be the end points of the interval $[a,b] \in [0,10]$ *that you would associate with the word old with respect to the age of a person?"*

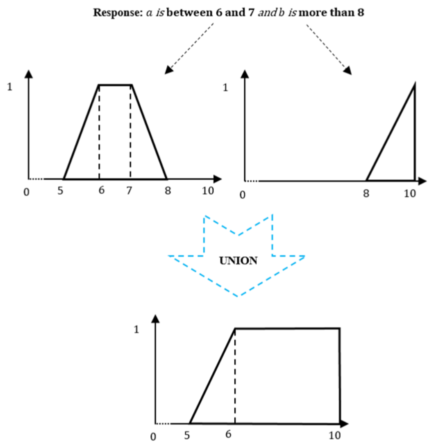

Fig. 6. Computation of union of fuzzy representations of CLEs corresponding to definite hesitancy

For the sake of initial research, we consider the existing definition of IA wherein a person must respond with a left-end point and a right end-point associated each a and b. However, a more realistic response to the question above is:

"*a is* **between 6 and 7** *and b is* **more than 8**."

It is clearly visible that the above response consists two FSs, one each for the end point of the interval, as opposed to consisting of two crisp numbers.

Therefore, to obtain a standard representation, which is a T1 FS for such responses, we compute the union of the two T1 FSs defining the end points of the interval. It is obvious that the membership degree of all other elements between these two T1 FSs is 1. As an example, the T1 FS representation for the response given above is shown in Fig. 6.

Due to the nature of linguistic features, even these responses might vary within individuals, therefore, giving rise to d T1 FSs for d individuals. A union computed for these T1 FSs results in an IT2 FS, which is the encoded version of the considered word in the vocabulary.

Notice how the FOU for the IT2 FS of the encoded word is obtained with having to go through lesser number of steps, when compared with the existing definition of IA based encoding.

b. CWW Engine

Prior to aggregation, the decision maker interacting with the Per-C is required to input words according to his/her preference for the concerned parameters. Since the decision maker is presented with multiple options to choose from, hesitancy is bound to occur at this stage. A sample response from a decision maker might be of the following form:

"*Age of this person is* **more than *average***."

Notice that the hesitancy occurring here is the non-definitive hesitancy as the elements involved do not have a crisp definition.

After choosing the appropriate terms and resulting with the IT2 FS representation for the same based on the discussion in Section III-B, they are combined together depending on the CWW engine chosen. This step of combining the terms results in one IT2 FS which the decoder utilizes to generate the final recommendation at the next stage. Consequently, the actual definition of the CWW engine of a Per-C remains intact implying that hesitancy does not affect the choice and working of the CWW engine.

c. Decoder

The decoder maps the IT2 FS obtained from the CWW engine to a final appropriate recommendation. Since this step only requires the Per-C to map the recommendation, no hesitancy can occur within this component. Therefore, the original definition of the decoder within a Per-C remains unchanged.

Having defined the role of hesitancy within various components of a Per-C, one can see how the framework of Per-C is augmented.

To summarize the proposed idea of perceptual computing with CLEs, a step-wise flow chart is given in Fig. 7.

IV. Conclusion

In this paper, the idea of perceptual computing with CLEs was introduced. To formalize the proposal, two different types of hesitancies were presented, namely: the definitive and non-definitive hesitancy. These hesitancies differ in the elements within which the hesitancy occurs. Post this, fuzzy representations of CLEs obtained from these hesitancies were introduced, that were used in the linguistic framework to generate responses. These ideas were then used to augment the existing framework of Per-C.

The obvious advantages of this proposed idea are now discussed. The new types of hesitancies, i.e. definitive and non-definitive, defined in this paper open more avenues for research within the other existing linguistic models. They also enrich the linguistic information conveyed by the CLEs. Also, the idea of CLEs is used to augment the existing framework of Per-C, making it more robust to linguistic uncertainties. Moreover, the fuzzy representations of the CLEs corresponding to the hesitancies possess a modular structure which fit well within the Per-C framework, without hampering the working of all its components. At a deeper level, more uncertainties are handled in the modified definition of the IA for encoding data, as the intervals are now modeled as T1 FSs. One can also notice that, obtaining an FOU for the IT2 FS representation of a word requires lesser steps as compared to the original definition of the encoding process based on the IA approach.

With the multitude of advantages that our proposed idea offers, we list some future works. In the future, we shall consider the different encoders available for use within the Per-C. We shall also consider generating recommendations while applying multiple existing CWW engines, and decoders. Additionally, some very good practical domains can be considered to generate useful recommendations using Per-C with CLEs.

References

[1] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning–i, ii, iii," *Information sciences*, vol. 8, 9, no. 3, pp. 199–249, 301–357, 43–80, 1975.

[2] L. A. Zadeh, "Fuzzy logic = computing with words," *IEEE Transactions on Fuzzy Systems*, vol. 4, no. 2, pp. 103–111, May 1996.

[3] L. A. Zadeh, "Fuzzy sets," *Information Control*, vol. 8, pp. 338-353, 1965.

[4] G. J. Klir and B. Yuan, *Fuzzy sets and fuzzy logic : theory and applications*. India: Pearson, Imp, 2015.

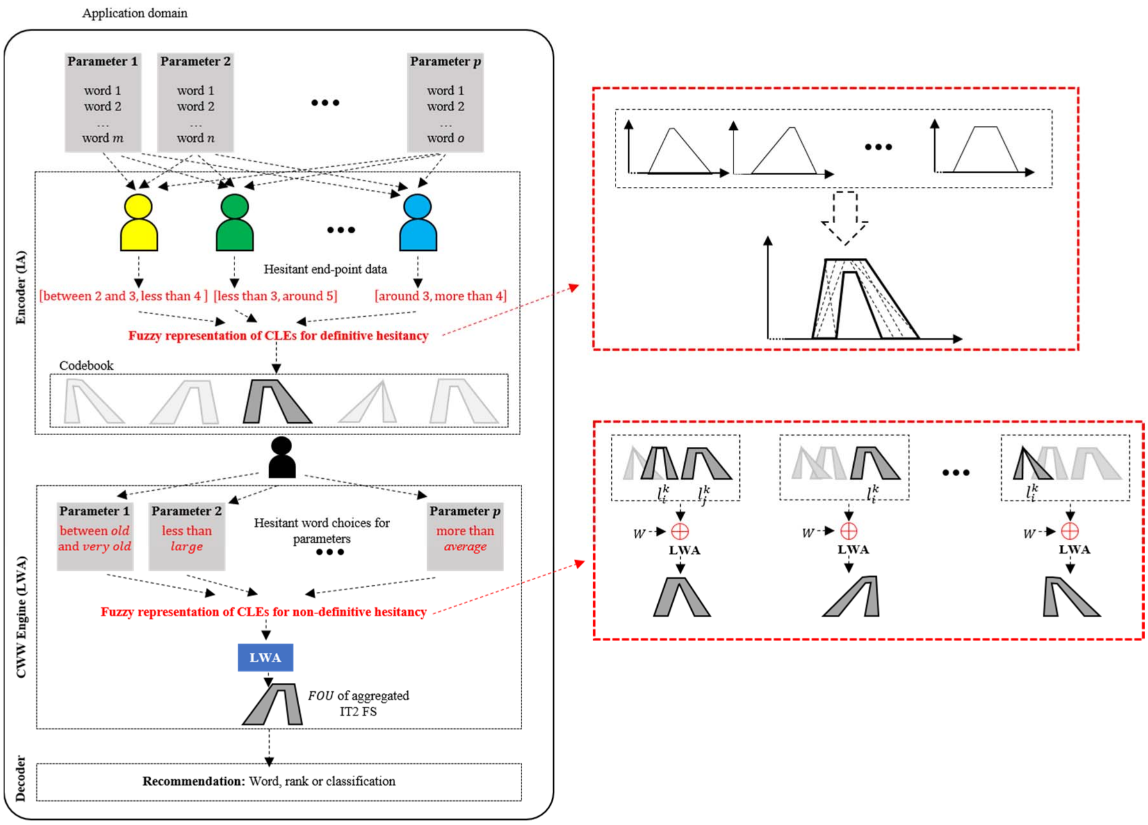

Fig. 7. Modified framework of Per-C with augmentation done by CLEs corresponding to different hesitancies in components. The red markings denote the addition of hesitancies within the Per-C based on our proposal.

[5] R. R. Yager, "A New Methodology for Ordinal Multiobjective Decisions Based on Fuzzy Sets," *Decision Sciences*, vol. 12, no. 4, pp. 589–600, 1981.

[6] L. Martinez and F. Herrera, "A 2-tuple fuzzy linguistic representation model for computing with words," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 6, pp. 746–752, 2000.

[7] L. Martı´nez and F. Herrera, "An overview on the 2-tuple linguistic model for computing with words in decision making: Extensions, applications and challenges," *Information Sciences*, vol. 207, pp. 1–18, Nov. 2012.

[8] A. K. Shukla, S. K. Banshal, T. Seth, A. Basu, R. John, and P. K. Muhuri, "A Bibliometric Overview of the Field of Type-2 Fuzzy Sets and Systems [Discussion Forum]," *IEEE Computational Intelligence Magazine*, vol. 15, no. 1, pp. 89–98, Feb. 2020.

[9] J. M. Mendel, "Computing with words and its relationships with fuzzistics," *Information Sciences*, vol. 177, no. 4, pp. 988–1006, Feb. 2007.

[10] J. M. Mendel and Dongrui Wu, *Perceptual Computing : Aiding People in Making Subjective Judgments*. New Jersey: John Wiley & Sons, 2010.

[11] D. Wu and J. M. Mendel, "Recommendations on designing practical interval type-2 fuzzy systems*," Engineering Applications of Artificial Intelligence*, vol. 85, pp. 182–193, Oct. 2019.

[12] A. K. Shukla and P. K. Muhuri, "General Type-2 fuzzy decision making and its application to travel time selection," *Journal of Intelligent & Fuzzy Systems*, vol. 36, no. 6, pp. 5227–5244, Jun. 2019.

[13] A. K. Shukla, R. Nath, P. K. Muhuri, and Q. M. D. Lohani, "Energy efficient multi-objective scheduling of tasks with interval type-2 fuzzy timing constraints in an Industry 4.0 ecosystem," *Engineering Applications of Artificial Intelligence*, vol. 87, p. 103257, Jan. 2020.

[14] M. Janmaijaya, A. K. Shukla, T. Seth, and P. K. Muhuri, "Interval Type-2 Fuzzy Restricted Boltzmann Machine for the Enhancement of Deep Learning," *2019 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, New Orleans, USA, 2019.

[15] A. K. Shukla, T. Seth, and P. K. Muhuri, "Interval type-2 fuzzy sets for enhanced learning in deep belief networks," *2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEE)*, Naples, Italy, 2017.

[16] A. K. Shukla and P. K. Muhuri, "Big-data clustering with interval type-2 fuzzy uncertainty modeling in gene expression datasets," Engineering Applications of Artificial Intelligence, vol. 77, pp. 268–282, Jan. 2019.

[17] P. K. Gupta and P. K. Muhuri, "A novel approach based on computing with words for monitoring the heart failure patients," *Applied Soft Computing*, vol. 72, pp. 457–473, Nov. 2018.

[18] D. Wu and J. M. Mendel, "Computing With Words for Hierarchical Decision Making Applied to Evaluating a Weapon System*," IEEE Transactions on Fuzzy Systems*, vol. 18, no. 3, pp. 441–460, Jun. 2010.

[19] P. K. Muhuri, P. K. Gupta, and J. M. Mendel, "User-Satisfaction-Aware Power Management in Mobile Devices Based on Perceptual Computing," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 4, pp. 2311–2323, Aug. 2018.

[20] P. K. Muhuri, P. K. Gupta, and J. M. Mendel, "Person Footprint of Uncertainty Based CWW Model for Power Optimization in Handheld Devices," *IEEE Transactions on Fuzzy Systems (Early Access)*, 2019.

[21] R. M. Rodr´ıguez, L. Mart´ınez, and F. Herrera, "Hesitant fuzzy linguistic term sets for decision making*," IEEE Transactions on Fuzzy Systems*, vol. 20, no. 1, pp. 1109–119, 2012.

[22] R. M. Rodr´ıguez, L. Mart´ınez, and F. Herrera, "A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets," *Information Sciences*, vol. 241, no. 1, pp. 28–42, 2013.

[23] Z. Zhang, W. Yu, L. Martinez, and Y. Gao, "Managing Multigranular Unbalanced Hesitant Fuzzy Linguistic Information in Multiattribute Large-Scale Group Decision Making: A Linguistic Distribution-Based Approach," *IEEE Transactions on Fuzzy Systems (Early Access)*, pp. 1–1, 2019.

[24] W. Yu, Z. Zhang, and Q. Zhong, "Consensus reaching for MAGDM with multi-granular hesitant fuzzy linguistic term sets: a minimum adjustment-based approach," *Annals of Operations Research*, Oct. 2019.

[25] T. Seth and P. K. Muhuri, "Type-2 Fuzzy Set based Hesitant Fuzzy Linguistic Term Sets for Linguistic Decision Making," *arXiv preprint* arXiv:2002.11714, pp. 1–19, 2020.

[26] J. M. Mendel, *UNCERTAIN RULE-BASED FUZZY SYSTEMS : introduction and new directions*, 2nd edition. Springer, 2019.

[27] F. Liu and J. M. Mendel, "Encoding Words Into Interval Type-2 Fuzzy Sets Using an Interval Approach," *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 6, pp. 1503–1521, Dec. 2008.