Cooperative Adaptive Fuzzy Control of Uncertain Affine Nonlinear Multi-agent Systems Based on Artificial Potential Functions

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***Abstract*— Cooperative control of multi-agent systems deals with various challenges such as uncertain dynamics, external disturbances, and limitations on interactions with other agents or the leader. Here, a cooperative adaptive fuzzy controller is introduced for a class of higher-order affine nonlinear multi-agent systems. The agents can only interact with their neighbors, and only a portion of them have access to the leader states. The unknown dynamics of the agents are approximated by fuzzy systems. In addition, the artificial potential functions (APFs) are employed to model the interaction among the agents. The APFs could provide a more transparent and straightforward design in comparison with usual graph-based methods. By modifying the error function, the proposed controller extends our previous work by giving only a portion of the agents access to the leader states. Lyapunov stability theory is used to design appropriate update laws for the weights of the fuzzy system and verify the overall stability of the system. Simulations results indicate a lower steady-state tracking error and lesser control energy consumption compared with a competing neural network-based approach.**

Keywords—Multi-agent systems, cooperative adaptive control, fuzzy logic, artificial potential functions, Lyapunov stability theory.

I. INTRODUCTION

Multi-agent control has gained significant attention in recent years because of its wide-ranging applications such as the control of unmanned vehicles, synchronization of satellites, and multiple-robotic systems [1]-[2]. The uncertainties and complexities of the real-world multi-agent problems such as their uncertain dynamics, external disturbances, and interaction limitations necessitate the design of efficient control structures that could guarantee stability while dealing with such challenges in the agents and their environment.

Fuzzy systems are intelligent paradigms inspired by the human mind. They have properties such as universal approximation, uncertainty handling, and incorporating human knowledge into the controller design. They are thus good candidates to deal with the uncertainties of the multi-agent domain. Adaptive fuzzy controllers have been employed in various multi-agent problems such as formation control [3]-[4], cooperative and consensus control [5]-[8], flocking control [9]-[10], and containment control [11]. These fuzzy systems are used mainly for the approximation of unknown dynamics of these multi-agent systems (MAS).

In most of the control structures for MAS, the interaction among the agents is modeled by graph-based methods (e.g. [5]-[8], [12]-[13]). The artificial potential functions (APFs) are another way of modeling the interaction among the agents. They should be strictly convex, continuously differentiable, and positive definite [14]-[15]. For instance, the shape control of swarm robotics in [16] and the coordinated problem of wheeled vehicles in [17] are investigated based on APFs. Fuzzy controllers for MAS based on potential functions are also investigated. For example, in [3]-[4] adaptive fuzzy formation control for robotic systems is investigated, where the interactions among the agents are modeled using APFs. Also, in [9]-[10] and [18], APFs are modeled by fuzzy logic systems in flocking control problems.

In comparison with the graph-based methods, APFs are energy-like functions that could offer a more straightforward design for extending the adaptive controllers of the single-agents to the multi-agent domain. Previously, we designed adaptive fuzzy controllers for cooperative control ([19]) and formation control ([20]) of nonlinear MAS. APFs were used to model the interaction among the agents. The agents could only interact with their neighboring agents. However, all of the follower agents required access to the leader state. This limits the applicability of the methods in [19]-[20] to the real-world problems. Therefore, here it is assumed that only a portion of the agents may access the leader state, and hence, the error function and the controller design are appropriately modified. Similar to [19]-[20], the interaction among the agents is presented using APFs.

Therefore, we introduce a cooperative adaptive fuzzy controller for a class of higher-order uncertain nonlinear MAS. The agents interact only with their neighbors, and only a portion of them has access to the state of the leader. The interaction among the agents is modeled by APFs. The unknown dynamics of the agents are approximated using fuzzy systems. The fuzzy system input is chosen to be a function of error, which is one dimensional instead of the states of the higher-order MAS, to decrease the number of fuzzy rules and the computational

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burden. Lyapunov stability theory is employed to design appropriate update laws fuzzy system parameters and verify the overall stability of the MAS.

The rest of the paper is organized as follows. Section II presents problem formulation and preliminaries on the fuzzy logic systems. In Section III, the proposed cooperative control structure is explained. Simulation results are investigated in Section IV; and finally, the conclusions are provided in Section V.

II. PRELIMINARIES

A. Problem formulation

Consider a group of n^{th}-order nonlinear heterogeneous MAS as follows,

$$x_i^{(n)} = f_i(\underline{x_i}) + g_i(\underline{x_i})u_i + d_i, \quad (1)$$

where $\underline{x_i} = [x_{1i}, x_{2i}, \dots, x_{ni}]^T = [x_i, \dot{x}_i, \dots, x_i^{(n-1)}]^T \in \mathbb{R}^n$ indicates the vector of the states for i^{th} agent ($i = 1, \dots, N$), u_i shows the control input, d_i is the external disturbance with $|d_i| \le \varepsilon_d$ and unknown ε_d, $f_i(\underline{x_i})$ is an unknown smooth nonlinear function that satisfies $|f_i(\underline{x_i})| \le f_1 < \infty$ with unknown upper bound $f_1 > 0$, and $g_i(\underline{x_i}) > 0$ is a known smooth nonlinear function ($g_i(\underline{x_i}) \neq 0$ ensures the MAS to be controllable). The desired state of the leader, x_d, and its derivatives up to order n should be bounded. The goal is to design a controller such that the follower agents track the desired state of the leader, and all of the signals remain bounded.

The communication topology between the agents is undirected fixed as $G = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \{1,2,\dots,N\}$ as the set of nodes and $\mathcal{E} \subseteq \{(i,j): i,j \in \mathcal{V}, j \neq i\}$ as the set of edges. The set of neighbors of i^{th} agent is $\mathcal{N}_i = \{j \in \mathcal{V}: (i,j) \in \mathcal{E}\}$, and the adjacency matrix A is $A = [a_{ij}] \in \mathbb{R}^{N*N}$ with $a_{ij} = 1$ if $(i,j) \in \mathcal{E}$ and $a_{ij} = 0$ if $(i,j) \notin \mathcal{E}$. Also, the diagonal in-degree matrix D is defined as $D = \text{diag}(d_1, \dots, d_N)$, where $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$. The Laplacian matrix L is denoted by $L = D - A$. The access to the leader states by the i^{th} agent is shown by c_i ($i = 1, \dots, N$), where $c_i = 1$ if the i^{th} agent has access to the leader states, else $c_i = 0$. At least one of the followers must access the state of the leader. According to [21], if the undirected graph G is connected, $c_i + d_i > 0, i = 1,2,\dots,N$.

The following APF indicates the interaction between agent i and its neighboring agent $j \in \mathcal{N}_i$.

$$H_{ij} = \frac{1}{2}\left\|(x_i - x_j)\right\|^2, j \in \mathcal{N}_i, \quad (2)$$

where $\|x\| = \sqrt{x^T x}$. The overall APF for each agent shows its interaction with all of its neighbors and is defined as,

$$H_i = \sum_{j \in \mathcal{N}_i} a_{ij} H_{ij}. \quad (3)$$

The tracking error for each agent is defined as below,

$$e_i = c_i(x_i - x_d) + \nabla_{x_i} H_i. \quad (4)$$

As (4) shows, the tracking error comprises of two terms: the error with respect to the state of the leader ($x_i - x_d$), and the gradient of the APF ($\nabla_{x_i} H_i$). If $c_i = 1$, the i^{th} agent has access to the leader states.

Computing $\nabla_{x_i} H_i$, the tracking error for the i^{th} agent is attained as,

$$e_i = c_i(x_i - x_d) + \sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j). \quad (5)$$

The n^{th} order derivative of e_i is then calculated as,

$$e_i^{(n)} = h_i x_i^{(n)} - c_i x_d^{(n)} - \sum_{j \in \mathcal{N}_i} a_{ij} x_j^{(n)}, \quad (6)$$

where $h_i = c_i + d_i = c_i + \sum_{j \in \mathcal{N}_i} a_{ij}$.

The error function is defined as,

$$s_i = e_i^{(n-1)} + \Lambda_{n-1} e_i^{(n-2)} + \dots + \Lambda_1 e_i, \quad (7)$$

where $\Lambda_m > 0$ ($m = 1, \dots, n-1$) is a constant parameter.

Taking the derivative of (7) and using (1), $\dot{s}_i$ is calculated as,

$$\dot{s}_i = h_i\left(f_i(\underline{x_i}) + g_i(\underline{x_i})u_i + d_i\right) - \sum_{j \in \mathcal{N}_i} a_{ij}\left(f_j(\underline{x_j}) + g_j(\underline{x_j})u_j + d_j\right) + q_i, \quad (8)$$

where $q_i = \Lambda_{n-1} e_i^{(n-1)} + \dots + \Lambda_1 \dot{e}_i - c_i x_d^{(n)}$.

B. Fuzzy logic systems

Fuzzy logic systems are comprised of four subsystems: fuzzifier, inference engine, rule base, and the defuzzifier. The input data enters the fuzzifier block for producing the membership functions (MFs) of the input space. The inference engine combines the input MFs according to the fuzzy rules in the rule base and creates the output MF. Finally, a crisp output is generated from the output MF by the defuzzifier.

In this paper, the unknown dynamics $f_i(\underline{x_i})$ in (1) is approximated using the fuzzy system. The fuzzy system input is chosen to be the one-dimensional s_i in (7) instead of states of the agents to decrease the number of fuzzy rules.

The Mamdani fuzzy rule can be described as,

Rule l: If s_i is A^l, then $\hat{f}_i(\underline{x})$ is w_i^l,

where s_i is the input to the l^{th} ($l = 1, \dots, M$) fuzzy rule, M is the total number of rules, A^l is the MF of the antecedent part of the l^{th} rule, $\hat{f}_i(\underline{x})$ is the approximated value of $f_i(\underline{x_i})$ as the output of the fuzzy system, and w_i^l is the MF of the consequent of the l^{th} rule.

Using singleton fuzzifier, Mamdani product inference engine, and weighted average defuzzifier, the overall output of the fuzzy system is computed as follows,

$$\hat{f}_i(\underline{x_i}) = \frac{\sum_{l=1}^{M} \overline{w}_i^l\left(\mu_{A^l}(s_i)\right)}{\sum_{l=1}^{M} \mu_{A^l}(s_i)} = W_i^T \phi_i, \quad (9)$$

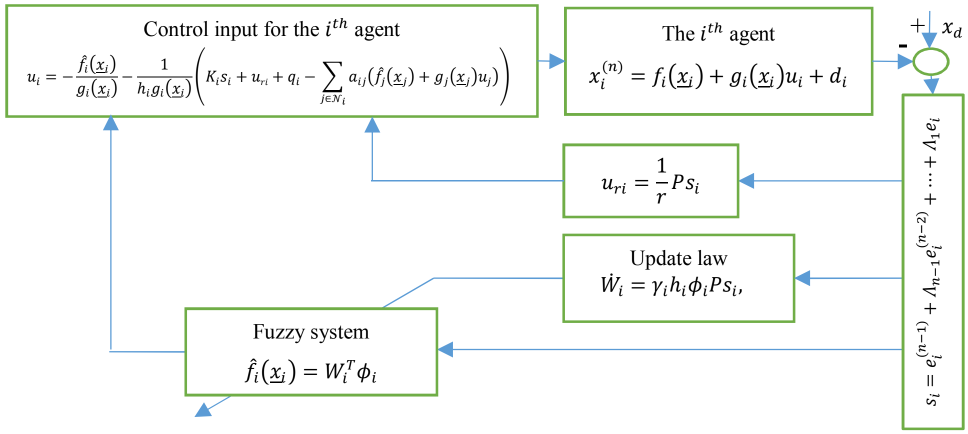

Fig. 1. The overall closed-loop scheme for the proposed controller.

in which $\overline{w}_i^l$ $(l = 1, \dots, M)$ indicates the center of fuzzy set in the consequent of the l^{th} rule, $\varphi_i^l = \mu_{A^l}(s_i)/\sum_{l=1}^{M}\mu_{A^l}(s_i)$ is the basis function, $W_i^T = [\overline{w}_i^1, \dots, \overline{w}_i^M]$ and $\phi_i^T = [\varphi_i^1, \varphi_i^2, \dots, \varphi_i^M]$ with $i = 1, \dots, N$.

III. THE PROPOSED COOPERATIVE ADAPTIVE CONTROLLER

The proposed controller is designed as follows,

$$u_i = -\frac{\hat{f}_i(\underline{x}_i)}{g_i(\underline{x}_i)} - \frac{1}{h_i g_i(\underline{x}_i)}\Big(K_i s_i + u_{ri} + q_i - \sum_{j\in\mathcal{N}_i} a_{ij}\big(\hat{f}_j(\underline{x}_j) + g_j(\underline{x}_j)u_j\big)\Big), \tag{10}$$

where u_{ri} is a robust compensator for decreasing the effect of uncertainties, $K_i > 0$, and $\hat{f}_i(\underline{x}_i)$ is the approximated value of $f_i(\underline{x}_i)$ using fuzzy logic system (9). Fig. 1 depicts the overall closed-loop scheme for the proposed controller.

It is assumed that the state vector $\underline{x}_i$, and the weights of the fuzzy system, W_i ($i = 1, \dots, N$), satisfy the following properties: $\Omega_x = \{\underline{x}_i | \|\underline{x}_i\| \le M_x\}$ and $\Omega_{w_i} = \{W_i | \|W_i\| \le M_w\}$, where M_x and M_w are positive constants and Ω_x and Ω_{w_i} are compact sets.

The minimum fuzzy approximation error is defined as,

$$\omega_{fi} = f_i(\underline{x}_i) - \hat{f}_i(\underline{x}_i | W_i^*), \tag{11}$$

where W_i^* is the optimal value of W_i defined by,

$$W_i^* = \arg\min_{W_i\in\Omega_{w_i}}\left[\sup\|\hat{f}_i(\underline{x}_i|W_i) - f_i(\underline{x}_i)\|\right]. \tag{12}$$

Substituting (10) in (8) attains,

$$\dot{s}_i = h_i\left(f_i(\underline{x}_i) - \hat{f}_i(\underline{x}_i|W_i)\right) - K_i s_i - u_{ri} + h_i d_i - \sum_{j\in\mathcal{N}_i} a_{ij}\left(f_j(\underline{x}_j) - \hat{f}_j(\underline{x}_j|W_j) + d_j\right), \tag{13}$$

Using (9) and (11), (13) is turned to,

$$\dot{s}_i = h_i \widetilde{W}_i^T \phi_i - K_i s_i - u_{ri} + \omega_i, \tag{14}$$

where $\omega_i = h_i\omega_{fi} + h_i d_i - \sum_{j\in\mathcal{N}_i} a_{ij}\left(\widetilde{W}_j^T\phi_j + d_j + \omega_{fj}\right)$ is the new perturbation, and $\widetilde{W}_i = W_i^* - W_i$. The assumptions on d_i, ω_{fi} and W_i make ω_i to be bounded.

The following update law is designed for W_i,

$$\dot{W}_i = \gamma_i h_i \phi_i P s_i, \tag{15}$$

where $\gamma_i > 0$ is the learning rate and $P \ge 0$ is attained solving the Riccati-like equation (16),

$$-PK_i - K_i^T P + Q + P\left(\frac{1}{\rho^2} - \frac{2}{r}\right)P = 0, \tag{16}$$

where $2\rho^2 \ge r$ and $Q \ge 0$.

Theorem 1: Consider the nonlinear MAS (1) with the control input (10), where the unknown function $f_i(\underline{x}_i)$ is approximated by the fuzzy system (9) with the update law (15). The robust control term is also defined as follows,

$$u_{ri} = \frac{1}{r}Ps_i. \tag{17}$$

Therefore, the H∞ performance criterion (18) is attained for a pre-defined attenuation value ρ, and the error converges to zero.

$$\sum_{i=1}^{N}\left[\int_0^T s_i^T Q s_i dt\right] \le \sum_{i=1}^{N}\left[s_i^T(0)Ps_i(0) + \frac{1}{\gamma_i}tr\left(\widetilde{W}_i^T(0)\widetilde{W}_i(0)\right)\right] + \rho^2\sum_{i=1}^{N}\left[\int_0^T \omega_i^T\omega_i\, dt\right], \tag{18}$$

Proof: The following Lyapunov function is considered,

$$V_L = \sum_{i=1}^{N}\left[\frac{1}{2}s_i^T P s_i + \frac{1}{2\gamma_i}\widetilde{W}_i^T\widetilde{W}_i\right], \tag{19}$$

Taking the derivate of (19) with respect to time and using (14) and (17) gives,

$$\dot{V}_L = \sum_{i=1}^{N}\left[\frac{1}{2}s_i^T\left(-K_i^T P - PK_i - \frac{2}{r}P^2\right)s_i + \frac{1}{2}(\omega_i^T P s_i + s_i^T P\omega_i) + \widetilde{W}_i^T\left(h_i\phi_i P s_i - \frac{1}{\gamma_i}\dot{W}_i\right)\right]. \tag{20}$$

Substituting the adaptive law (15) and using Riccati-like equation (16) one attains,

$$\dot{V}_L = \sum_{i=1}^{N}\left[-\frac{1}{2}s_i^T(Q+\frac{P}{\rho^2}P)s_i + \frac{1}{2}(\omega_i^T P s_i + s_i^T P\omega_i)\right]. \quad (21)$$

By simple manipulation, the following is attained,

$$\dot{V}_L = \sum_{i=1}^{N}\left[-\frac{1}{2}s_i^T Q s_i + \frac{1}{2}\rho^2\omega_i^T\omega_i\right]. \quad (22)$$

Integrating (22) from $t = 0$ to $t = T$, (18) is achieved. As ω_i is bounded, using Barbalat's lemma in [23], the convergence of error to zero is concluded and all of the system signals ensure to be bounded. ■

IV. SIMULATION RESULTS

Here, the proposed method is applied to the third-order nonlinear MAS of the numerical example in [22] and is compared with the neural network-based approach in [22].

The dynamics of the leader are as follows,

$$\dot{x}_{1d} = x_{2d},$$
$$\dot{x}_{2d} = x_{3d},$$
$$\dot{x}_{3d} = -x_{2d} - 2x_{3d} + 1 + 3\sin(2t) + 6\cos(2t) - \frac{1}{3}(x_{1d} + x_{2d} - 1)^2(x_{1d} + 4x_{2d} + 3x_{2d} - 1). \quad (23)$$

There are 5 follower agents with the following dynamics,

$$\dot{x}_{1i} = x_{2i},$$
$$\dot{x}_{2i} = x_{3i},$$
$$\dot{x}_{3i} = f_i(\underline{x}_i) + u_i + d_i. \quad (24)$$

where $f_1(\underline{x}_1) = x_{21}\sin(x_{11}) + \cos(x_{31}^2)$, $f_2(\underline{x}_2) = -x_{12}x_{22} + 0.01x_{12} - 0.01x_{12}^3$, $f_3(\underline{x}_3) = x_{23} + \sin(x_{33})$, $f_4(\underline{x}_4) = -3(x_{14} + x_{24} - 1)^2(x_{14} + x_{24} + x_{34} - 1) - x_{24} - x_{34} + 0.5\sin(2t) + \cos(2t)$, and $f_5(\underline{x}_5) = \cos(x_{15})$. The initial values of the state of agents are $x_{11}(0) = -1$, $x_{12}(0) = 2.7$, $x_{13}(0) = 2.7$, $x_{14}(0) = 2$, and $x_{15}(0) = -2$.

To make a fair comparison, the communication graph here is similar to [22], as shown in Fig. 2, with the difference in being undirected and the non-zero elements of A being equal to 1, i.e. $A(1,:) = [0\ 1\ 1\ 1\ 0]$, $A(2,:) = [1\ 0\ 1\ 0\ 0]$, $A(3,:) = [1\ 1\ 0\ 0\ 0]$, $A(4,:) = [1\ 0\ 0\ 0\ 0]$, $A(5,:) = [0\ 0\ 0\ 0\ 0]$, and $b_i = 1\ (j = 1,5)$.

The parameters of the competing approach in [22] are set at $\lambda_1 = 5$, $\lambda_2 = 4$, $c = 15$, $\kappa = 0.01$, $F_i = 1000$, and $P = 0.01$. There are 30 neurons in the neural network with centers that are evenly spaced in [-3,3].

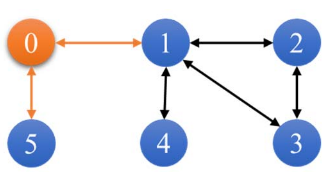

Fig. 2. The communication graph among the agents. Node "0" is the leader.

Similarly, the parameters of the proposed controller are set at $\Lambda_1 = 4$, $\Lambda_2 = 5$, $K_i = 15$, $\gamma_i = 1000$, $r = 0.08$, $P = 0.01$, and $\rho = 0.2$. The input space of the fuzzy system is partitioned to 30 MFs of Gaussian type with means spaced evenly in [-5,5] and standard deviation $\sigma = 1$. The number of rules is $M = 30$. The initial values of the weights ($W_i(0)$) are equal to zero. The total simulation time is 20 s with time step 0.01. The measurement criteria are Mean Squared Error (MSE), MSE at Steady-State (MSESS), the average control energy (J), and the maximum absolute value of the control input ($\text{MaxU} \triangleq \max_{i=1,\dots,5}|u_i|$). The MSE for each method averages the MSE of five agents, MSESS is the MSE for $t \geq 2\ s$, and $J = \frac{1}{5}\sum_{i=1}^{5} J_i$, where $J_i = \frac{1}{k_T}\sum_{k_t=1}^{k_T}|u_i(k_t)|$ and k_T is the total number of sample points. The results of MSE, MSESS, J, and MaxU are presented in Table I.

Simulations are performed in three cases: A) without disturbance, B) with external disturbance of sinusoidal type, and C) with Gaussian white noise.

A. Simulation results without disturbance

In this case, the measurement noise and the external disturbance are zero. Fig. 3 and Fig. 4 show the state trajectories of the agents for the proposed method and the competing approach, respectively. In both approaches, the agents could follow the leader trajectory satisfactory. The control inputs of all of the agents for both methods are depicted in Fig. 5. As could be seen, the proposed method uses a lower amount of control input at first steps (MaxU in Table I is 75% lower for the proposed method). The MSE in Table I is 0.04% lower for the competing approach. In contrast, the proposed method achieves considerably lower MSESS (by 50%), which confirms its better steady-state tracking. The proposed method also reaches a 12% lower J (Table I).

B. Simulation results with sinusoidal disturbance

In this case, the measurement noise is zero while an external disturbance $d_i = 3\sin(5\pi t)$ is applied to the agents. Fig. 6 and Fig. 7 show the trajectories of the states of the agents for the proposed method and the competing approach, respectively. These figures indicate that both methods could handle external disturbances. However, the third state (x_{3i}) shows slightly higher fluctuations for the proposed controller. As Table I shows, similar to Case A, the MSE for both methods is similar, and the proposed method attains 51% lower MSESS. Fig. 8 shows the control input for both methods. Similar to Case A, the control input at initial steps is considerably lower for the proposed method (75% lower MaxU in Table I). The data of J in Table I shows that both methods use similar average control energy (3% higher for the proposed method).

C. Simulation results with measurement noise

In this case, the external disturbance is zero, and the output is corrupted with measurement noise of white Gaussian type with SNR=35. For reliable results, the simulations are performed 20 times, and the values of MSE, MSESS, J, and MaxU are averaged and presented in Table I. As illustrated, both methods reach comparable MSE and MSESS (4% lower MSE for the proposed method). The average control energy (J) and

MaxU for the proposed method are lower (by 9% and 75%, respectively).

TABLE I. MSE, MSESS J, AND MAXU FOR THE PROPOSED METHOD AND THE COMPETING APPROACH. BOLD NUMBERS INDICATE BETTER RESULTS.

Case	Criteria	*MSE*	*MSESS*	*J*	*MaxU*
A. Without disturbance	Competing	**0.1236**	0.0016	4.663	901.073
	Proposed	0.1240	**7.99e-04**	**4.101**	**229.174**
B. Sinusoidal disturbance	Competing	**0.1237**	0.0016	**4.999**	900.995
	Proposed	0.1241	**7.90e-04**	5.138	**229.151**
C. Noise (averaged over 20 runs)	Competing	0.1910	0.0694	8.995	904.644
	Proposed	**0.1835**	**0.0691**	**8.212**	**229.386**

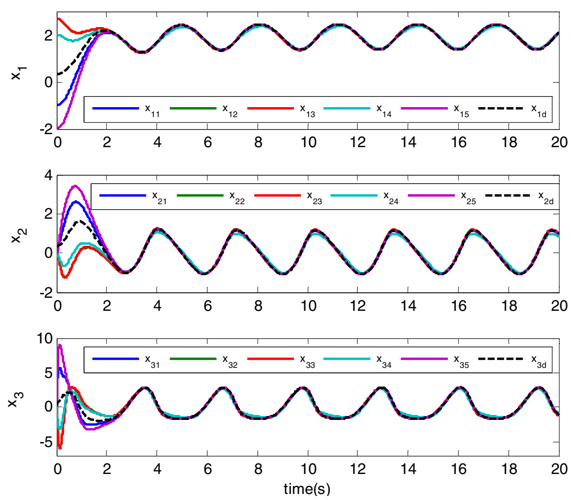

Fig. 3. Case A. The trajectories of the states of the MAS for the proposed method without disturbance.

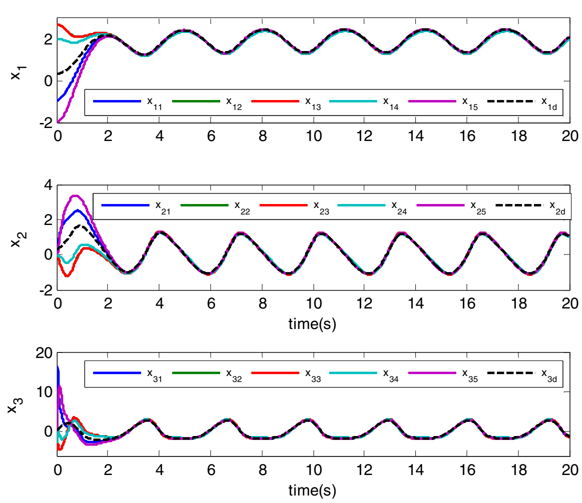

Fig. 4. Case A. The trajectories of the states of the MAS for the competing approach without disturbance.

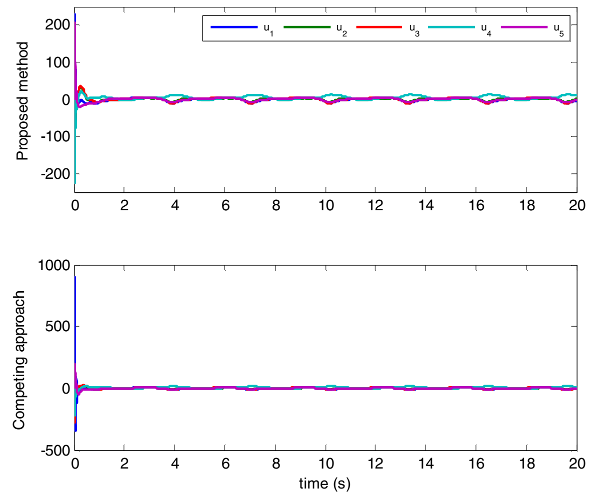

Fig. 5. Case A. the control input for the proposed method (above) and the competing approach (below) without disturbance.

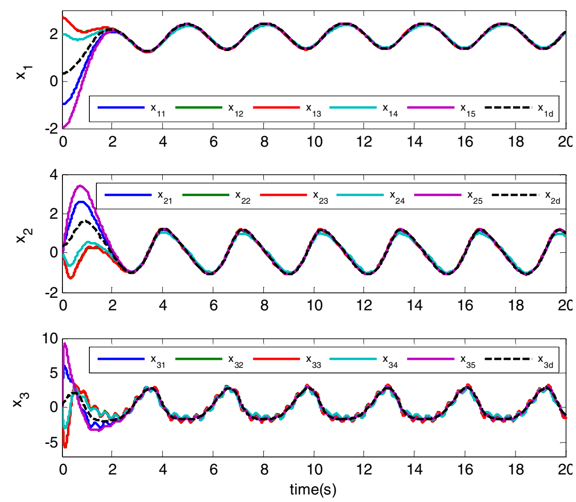

Fig. 6. Case B. The trajectories of the states of the MAS for the proposed method with sinusoidal disturbance.

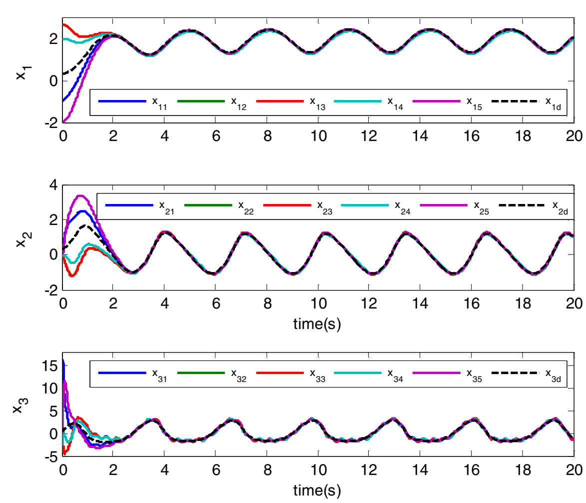

Fig. 7. Case B. The trajectories of the states of the MAS for the competing approach with sinusoidal disturbance.

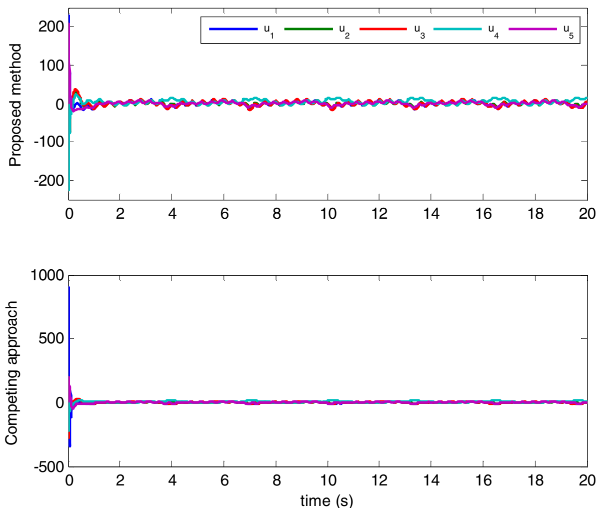

Fig. 8. Case B. The control input for the proposed method (above) and the competing approach (below) with sinusoidal disturbance

V. CONCLUSION

Here, a cooperative adaptive fuzzy controller is designed for a class of higher-order nonlinear MAS. The agents have partially unknown dynamics, and only a portion of them have access to the state of the leader. Their interaction is modeled by artificial potential functions. Fuzzy systems are used to approximate the unknown dynamics of each agent. The one-dimensional error function is used as the input to the fuzzy system to reduce the number of fuzzy rules. The derivation of the adaptive laws and the overall stability of the closed-loop system are based on the Lyapunov stability theory. Simulation results show a lower steady-state tracking error in comparison with a competing approach with and without disturbance. In the presence of measurement noise, both methods indicate comparable results. In all cases, the proposed method shows a considerably lower value for the maximum absolute value of the control input. In the future, we hope to investigate cooperative control structures based on APFs for MAS with more realistic challenges such as control input constraints or unmeasurable states of the agents.

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