A Regret Theory-Based Decision-Making Method with A Reference Set under the Hesitant Fuzzy Environment

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***Abstract*—People often face the problem of selecting a set of preferred alternatives from a large scale of alternatives, and due to the limited personal cognition, the judgments of decision-makers may distort a rational choice. This study puts forward a two-stage model to solve the decision-making problems with massive alternatives based on the regret theory. In the first stage, a method is proposed to narrow the set of alternatives so as to screen the relative better alternatives from a large number of options. In the second stage, by taking into accounts decision-makers' psychological behavior, a regret-theory-based method is applied to choose the optimal alternative from the narrowed candidate set. In addition, as it is a challenge for decision-makers to make precise evaluations in a highly uncertain decision-making environment, the evaluations in this study are represented by hesitant fuzzy sets to deal with ambiguous and uncertain information. An illustrative example is given to verify the applicability and effectiveness of the proposed model.**

***Keywords*—Multiple criteria decision making; regret theory; utility function; reference set; neighborhood relation**

I. INTRODUCTION

Due to the bounded rationality of decision-makers (DMs) resulted from the limited information and personal cognitive, the judgments of DMs may distort a rational choice, especially in a highly uncertain decision-making environment [1, 2]. That is why various contradictions of the expected utility theory (EUT) have been identified, such as the Allias paradox and Ellsberg paradox. To model the decision-making process involving human psychological behaviors, some classical behavior decision theories have been proposed, such as the prospect theory [3] and regret theory [4, 5].

The prospect theory [3] is used to make a decision based on the potential values of losses and gains rather than the final outcomes. In the prospect theory, different functions in terms of gains and losses with respect to one or more reference points are necessary to evaluate alternatives. The functions contain four parameters, which are difficult to determine since all of them rely on the psychological behavior of DMs. Additionally, the regret theory [4, 5] was developed to help people make a decision by taking into accounts the effect of anticipated regret. If one result is better, the DM will have a sense of regret; otherwise they will feel rejoice. Compared with the prospect theory, the reference points are not needed in the regret theory; furthermore, the calculation functions contain only two parameters, namely, the risk aversion coefficient and the regret aversion coefficient [6]. For these reasons, the regret theory has achieved extensive applications, such as the best seller publication [7], new energy resources investment risk evaluation [8], project investment [9], air-fighter selection [2], route choices and traffic equilibria [10], and outsourcing contractor and provider matching [11]. However, in these studies, the utility function in the regret theory failed to reflects the real utility of DM's evaluation values. In this study, we embed a new utility function in the regret theory-based decision process to characterize DMs' psychological behavior accurately.

When a decision-making problem involves a large scale of candidates, there is a big challenge for the DM: since the regret theory is essentially an outranking method based on pairwise comparisons, the large number of alternatives will increase the complexity of calculation; in addition, DM's evaluations may be disturbed so that the provided information is not accurate enough. To solve this problem, in this paper, we propose an approach to narrow the set of alternatives so as to screen the relative better alternatives from a large scale of options, which can help DMs reduce the workload and improve the evaluation efficiency.

In the environment with large amount of information, it is a challenging issue that DMs cannot make evaluations precisely. Since the hesitant fuzzy set [12] is a powerful tool to deal with ambiguous and uncertain information, this study is under the hesitant fuzzy environment.

The contributions of this study are highlighted as follows:

1) We propose an approach to narrow the set of massive alternatives, which can screen the alternatives with relative better performance from a large scale of options and thus will be helpful to improve the evaluation efficiency of the decision-making process.

2) Based on the narrowed candidate set, we introduce a regret-theory-based method to obtain the optimal alternative set under the hesitant fuzzy environment. The utility function in our regret theory-based decision-making process can characterize DMs' psychological behavior accurately.

II. PRELIMINARIES

This section introduces the preliminaries used in the rest of this study, including the regret theory and the hesitant fuzzy judgment information.

A. Regret Theory

Below we take the scenario of two alternatives as an example to give a specific description of the main ideas of the regret theory. Let a_1 and a_2 be the evaluation information of a DM on the alternatives A_1 and A_2, respectively. Then, the DM's comprehensive perceived utility for alternative A_1 is expressed as $Cu(a_1, a_2) = g(a_1) + R(g(a_1) - g(a_2))$, where $g(.)$ is a utility function, and $R(\Delta g)$ is the regret/rejoice function.

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When $R(\Delta g)<0$, it means the regret value is generated for the better result a_2 is not obtained; when $R(\Delta g)>0$, it refers the rejoice value is sensed since a_1 is currently the better result; while $R(\Delta g)=0$ implies that the two alternatives perform equally well and neither regret nor rejoice is felt.

To simulate the utility of the DM's evaluation information, the power function $g(a)=a^{\theta}$ with $g(.)'>0$ and $g(.)''<0$ is used in the previous study [2, 13], where the risk aversion coefficient of the DM is denoted by θ and satisfies $0<\theta<1$. However, in an uncertain evaluation environment, this may not be proper. We will discuss it in detail in Section III.

The regret/rejoice function $R(\Delta g)$ is with $R(\Delta g)'>0$ and $R(\Delta g)''<0$ [4, 14]. $R(\Delta g)'>0$ indicates that the utility is strictly increasing. Regret aversion, which generalizes the distinctive predictions of the regret theory and also caters for the law of diminishing marginal utility, implies that $R(\Delta g)$ is concave, which is reflected by $R(\Delta g)''<0$. Based on [10], the regret/rejoice function is constructed as $R(\Delta g)=1-e^{-\gamma*\Delta g}$, where γ is called the regret aversion coefficient and $\gamma\in[0,1]$. A larger γ corresponds to a larger regret aversion value. The function of $R(\Delta g)$ is shown in Fig. 1.

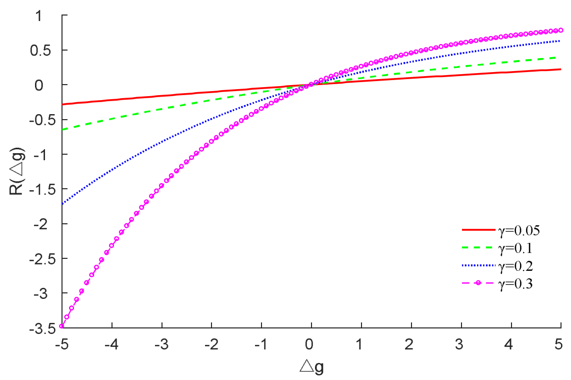

Fig. 1. The regret/rejoice function

B. *Hesitant Judgment Information*

Hesitant fuzzy sets (HFS) [12] is an effective tool to capture individual hesitancy. Let X be a fixed set, an HFS on X is represented as $H=\{<x,h_H(x)>|x\in X\}$, where $h_H(x)$ is a set of values in $[0,1]$, denoting the possible membership degrees of the element $x\in X$ to the set H. $h_H(x)$ is called a hesitant fuzzy element (HFE) [15].

For two HFEs $h_j=\{\gamma_j^l\,|\,l=1,2,\cdots,l_{h_j}\}(j=1,2)$, it is possible that the cardinality (l_{h_j}) of HFEs may be different. By the pessimistic principle [15], the smallest element of the shorter HFE is added to make two HFEs equivalent. Then, the hesitant normalized Hamming distance of two HFLEs h_1 and h_2 is calculated as [16]:

$$d(h_1,h_2)=\frac{1}{l_h}\sum_{t=1}^{l_h}|h_1^{\vartheta(t)}\text{-}h_2^{\vartheta(t)}|$$

where $h_j^{\vartheta(t)}(j=1,2)$ is the $\vartheta(t)$th largest element in $h_j(j=1,2)$.

To compare two HFEs, Liao *et al.* [17] introduced the following comparative technique: if $s(h_1)>s(h_2)$, then $h_1>h_2$; if $s(h_1)=s(h_2)$ and $\sigma(h_1)>\sigma(h_2)$, then $h_1>h_2$; if $s(h_1)=s(h_2)$ and $\sigma(h_1)=\sigma(h_2)$, then $h_1=h_2$, where $s(h)$ and $\sigma(h)$ are the score and variance of the HFE with $s(h)=\sum_{\gamma\in h}\gamma/l_h$ and $\sigma(h)=[\sum_{\gamma\in h}(\gamma\text{-}s(h))^2]^{1/2}/l_h$.

Let $h_i(i=1,2,\cdots,n)$ be a collection of HFEs, a hesitant fuzzy ordered weighted averaging (HFOWA) operator is [15]:

$$HFOWG(h_1,h_2,\cdots,h_n)$$

$$=\overset{n}{\underset{j=1}{\oplus}}(h_j^{w_j})=\bigcup_{\gamma_1^{\sigma(t)}\in h_1,\gamma_2^{\sigma(t)}\in h_2,\cdots,\gamma_n^{\sigma(t)}\in h_n}\left\{\prod_{j=1}^{n}(h_j^{\sigma(t)})^{w_j}\right\}$$

where $w=(w_1,w_2,\cdots,w_n)^T$ is the weight vector of h_j with $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$.

III. TWO-STAGE MULTIPLE ATTRIBUTE DECISION MAKING BASED ON THE REGRET THEORY

In this section, we introduce a two-stage model to solve the decision-making problems with massive alternatives. In the first stage, an approach is proposed to narrow the set of alternatives so as to screen the relative better alternatives from a large number of options. In the second stage, a regret-theory-based decision-making method is applied to choose the optimal alternatives from the candidate alternative set that only includes the relative better alternatives. A summarization graph of the proposed model is presented in Fig. 2.

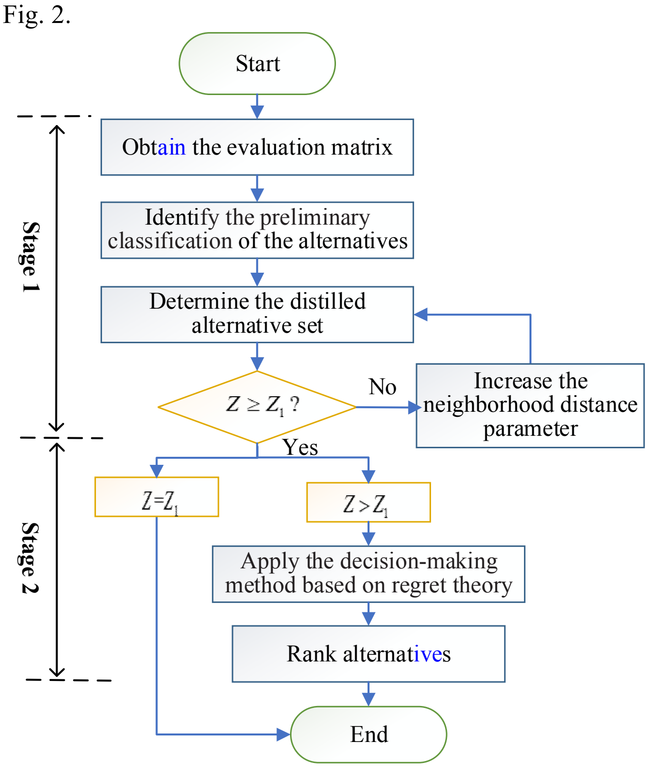

Fig. 2. Framework of the proposed model

A. A Reference Set Method to Narrow the Set of Alternatives

There is a big challenge for DMs when decision-making problems involves a large number of candidates. For example, a fierce competition for bids may cause some third-party reverse logistics providers fail to be selected in one situation, or worse ones to be selected when little competition exists [18]. Thus, it is of great importance to distill a larger set of candidates into a smaller preferred set.

To classify or distill the objects, the rough set theory is a classical method. The rough-set-based classification model relies on the equivalence relations and equivalence classes regarding the discrete data sets. However, this classification technology is so strict that the classification may be invalid. In addition, as many real-world attributes are assigned with continuous numerical values, the discretization method may result in information loss. Thirdly, the classification brought by the classical rough set theory is invalid without a decision attribute. Three-way decision theory, as another utilization of the rough set theory, classifies objects into three decisions outcomes, namely, the decision of acceptance, uncertainty (or deferment) and rejection [19, 20] on the basis of a given conditional probability of the classification and a loss function. However, both the conditional probability and loss function are not easy to determine. Liu, *et al.* [20] applied the concept of three-way decisions to group decision making, but the parameter they used to divide the objects were subjectively determined. If the parameter is assigned inappropriately, unreasonable three decision regions will be produced. Inspired by Bai *et al.* [21], who utilized a segmentation approach to help practitioners implement appropriate green supply development practice based on suppliers' categories, we use an objective way to narrow the set of alternatives.

Suppose that there are m alternatives with n attributes, denoted as $A_i(i=1,2,\cdots,m)$, $C_j(j=1,2,\cdots,n)$, respectively. The problem discussed in this study is that we need to select z proper alternatives from m alternatives. The evaluations of the alternatives are depicted in HFEs as $H(A_i)=\{h_{ij}(A_i)\}$, for $i=1,2,\cdots,m$, $j=1,2,\cdots,n$. The distillation of the alternatives can be realized in the following steps:

Step 1. Normalize the evaluation values by the pessimistic principle [15] and all evaluation information depicted by HFEs are transformed into the same length.

Step 2. Determine the ideal reference sequence S^+, medium reference sequence $\overline{S}$ and negative-ideal reference sequence S^- by the following equations:

$$\begin{aligned} S^+ &= \{h_{i1}^+(A_i),h_{i2}^+(A_i),\cdots,h_{in}^+(A_i)\} \\ &= \{\max_i h_{i1}^+(A_i),\max_i h_{i2}^+(A_i),\cdots,\max_i h_{in}^+(A_i)\} \end{aligned} \tag{1}$$

$$\begin{aligned} \overline{S} &= \{\overline{h}_{i1}(A_i),\overline{h}_{i2}(A_i),\cdots,\overline{h}_{in}(A_i)\} \\ &= \{\operatorname{average}_i \overline{h}_{i1}(A_i),\operatorname{average}_i \overline{h}_{i2}(A_i),\cdots,\operatorname{average}_i \overline{h}_{in}(A_i)\} \end{aligned} \tag{2}$$

$$\begin{aligned} S^- &= \{h_{i1}^-(A_i),h_{i2}^-(A_i),\cdots,h_{in}^-(A_i)\} \\ &= \{\min_i h_{i1}^-(A_i),\min_i h_{i2}^-(A_i),\cdots,\min_i h_{in}^-(A_i)\} \end{aligned} \tag{3}$$

Step 3. Identify the preliminary classification of the alternatives. Here, the idea for preliminarily dividing the alternatives is according to the membership degree of the distance D_i^v between each alternative and each reference alternative S^v under all criteria. The distances are calculated as:

$$D_i^v(H(A_i),S^v)=\sum_{j=1}^{n} d(h_{ij}(A_i),h_{kj}^+(A_k)) \tag{4}$$

where $S^v \in \{S^+,\overline{S},S^-\}$ and $D_i^v \in \{D_i^+,\overline{D}_i,D_i^-\}$. D_i^+, $\overline{D}_i$ and D_i^- can be used as a reference for three classes, namely, the class level "1", "2" and "3", respectively. More specifically, each alternative is associated with a membership value for each reference class D_i^v by

$$\varphi(A_i,D_i^v)=\left[\sum_{j=1}^{3}\left(\frac{D_i^v(H(A_i),S^\mu)}{D_i^j(H(A_i),S^j)}\right)\right]^{-1} \tag{5}$$

where $\varphi(A_i,D_i^v)$ is in the range $[0,1]$. Then, we can assign the alternative roughly into the class which has the highest membership degree.

Step 4. Calculate the neighborhood relational matrices for the candidates under all attributes. The neighborhood relations are used to group the sets of objects based on the similarity or indistinguishability using a neighborhood distance parameter. Here, we use the Hamming distance to obtain the matrices, which is denoted as follows:

$$\Delta\{A_i,A_k\}=\sum_{j=1}^{n} d(h_{ij}(A_i),h_{kj}(A_k)) \tag{6}$$

Step 5. Determine the neighborhood relation granule for each alternative by

$$NR_{A_i,A_k}=\begin{cases}1, \Delta\{A_i,A_k\}\le\delta \\ 0, \text{otherwise}\end{cases} \tag{7}$$

where $\delta\in[0,1]$ is the neighborhood distance parameter, which can be determined by the number of alternatives that need to be selected. The larger the value is, the more relation granule will be obtained.

Step 6. Find the final number of alternatives classified as the class level "1" according to the computed neighborhood relation granule by

$$z_1=\max_{A_{i*}}\left(\sum_{k=1}^{m} NR_{A_{i*},A_k}\right) \tag{8}$$

where A_{i*} is the alternatives that have been preliminarily classified in the class level "1" in Step 3. Finally, we get the reduced alternative set as:

$$\{A_k \mid \max_{A_{i*}}\left(\sum_{k=1}^{m} NR_{A_{i*},A_k}\right)\cup NR_{A_{i*},A_k}=1,\ k<m\} \tag{9}$$

Through the above steps, we can narrow a larger alternative set into a smaller preferred set, which can reduce the workload of DMs and improve the evaluation efficiency since bad alternatives are identified and rejected directly and DMs can focus on the non-inferior solutions. The number of distilled candidates can be controlled by the parameter δ.

Next, we discuss the relation between z and z_1.

1) $z_1=z$. If the number z of alternatives required to be selected is exactly equal to the number z_1 in the distilled set, then, z_1 alternatives are just the optimal choice.

2) $z_1>z$. If z is more than z_1, then we can apply the decision-making method present in the next section based on the regret theory to choose the optimal ones from the distilled alternatives.

B. Hesitant Fuzzy Decision-making Process Based on the Regret Theory

1) A new utility function based on human psychophysics

As we mentioned in Section II, usually, the power function $g(a)=a^{\theta}$ with $g(.)'>0$ and $g(.)''<0$ is used to simulate the utility of a DM's evaluation [2, 13]. However, this may be unreasonable when the psychology of the DM is taken into consideration. In fact, people are always careful in evaluating an object with an excellent performance since it may have a greater influence on the final decision result. Lootsma [22] explained in detail the psychophysical characteristic that people are more sensitive and careful to evaluate the objects which are closer to the target value than the objects far from the target one.

Previous studies that took into account the psychological behavior in the decision-making process did not simulate the real utilities of DMs' evaluation values or the function was supposed to be concave. Suppose the evaluation values $\{a_1,a_2,a_3,a_4,a_5,a_6\}$ of the alternatives $\{A_1,A_2,A_3,A_4,A_5,A_6\}$ given by a DM under an attribute are $\{0.8,0.4,0.3,0.4,0.7,0.6\}$. In this case, we can find that $a_5-a_6=a_2-a_3$=0.1. Since a_5 and a_6 are much closer to the target value than a_2 and a_3, the utility value of a_5-a_6 should be bigger than that of a_2-a_3. Assume that θ=0.88, then by the utility function $g(a)=a^{\theta}$, the utilities of the evaluations are obtained as $\{0.82,0.45,0.35,0.45,0.73,0.64\}$. In this way, we can find that the utility difference of h_5 and h_6 is 0.09, while that of h_2 and h_3 is 0.1, which is contrary to the law given by Lootsma [22].

Thus, in this paper, we give another utility function to characterize DMs' psychological behavior when they are invited to give the evaluation information, expressed as:

$$g(a)=\frac{e^{\xi * a}-1}{\xi} \tag{10}$$

where ξ is in the range $[0,1]$. The larger the value of ξ is, the more cautious the DM is with the target value. When $\xi\to 0$, $g(a)\to a$.

The function's first derivative, $g'(a)=e^{\xi a}$, is greater than zero ($g'(a)>0$, as $e^{\xi a}>0$), and its second derivative, $g''(a)=\xi e^{\xi a}$, is also greater than zero ($g''(a)>0$, as $\xi>0, e^{\xi a}>0$). This functional form means that the utility of DM's evaluations are monotonically increasing with the increase of the membership degrees of an element, and the utilities increase more and more rapidly with the evaluation value getting closer to the target value, which is in accordance with DM's psychophysical characteristic. For clarity, we plot the function with different parameter vlaues in Fig. 3.

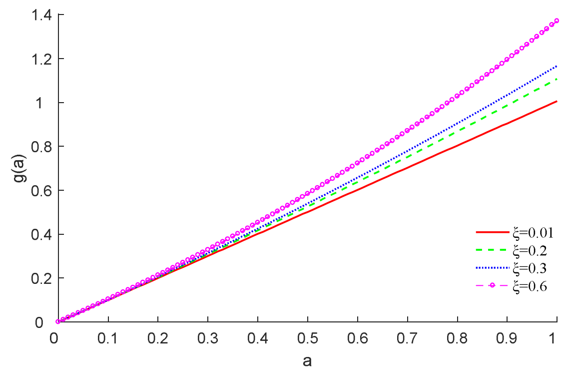

Fig. 3. The utility function of evaluation information

2) A hesitant fuzzy decision-making process based on the regret theory

Suppose that the weight vector of n criteria is $w=(w_1,w_2,\cdots,w_n)^T$, $\sum_{j=1}^{n} w_j=1$. z proper alternatives are selected from m alternatives. A hesitant fuzzy decision-making process based on the regret theory is given as follows:

Step 1. Calculate the utility values of the evaluation information. Based on Eq. (10), we can get the utility values of $h_{ij}(A_i)$ as:

$$g(h_{ij}(A_i))=\bigcup_{h_{ij}^{\vartheta(t)}\in h_{ij}(A_i)} \left(e^{\xi * h_{ij}^{\vartheta(t)}}-1\right)\Big/\xi \tag{11}$$

where $h_{ij}^{\vartheta(t)}$ is the $\vartheta(t)$th largest element in $h_{ij}(A_i)$.

Step 2. Compute the distances of the evaluation utilities between alternative A_i and alternative A_k under criterion C_j by

$$d_{ik,j}=\sum_{\substack{g_{ij}^{\vartheta(t)}\in g(h_{ij}(A_i))\\ g_{kj}^{\vartheta(t)}\in g(h_{kj}(A_i))}} \frac{1}{l_h}\sum_{t=1}^{l_h}|g_{ij}^{\vartheta(t)}\text{-}g_{kj}^{\vartheta(t)}| \tag{12}$$

where $g_{ij}^{\vartheta(t)}$ and $g_{kj}^{\vartheta(t)}$ are the $\vartheta(t)$th largest element in $g(h_{ij}(A_i))$ and $g(h_{kj}(A_i))$, respectively.

Step 3. Calculate the regret/rejoice value of alternative A_i to alternative A_k under criterion C_j. The regret value is calculated as:

$$R_{ik,j}(\Delta g)=\begin{cases}-(1-e^{-\gamma*\Delta g}), & g(h_{ij}(A_i))<g(h_{kj}(A_i))\\ 0, & \text{otherwise}\end{cases} \tag{13}$$

The rejoice value is calculated as:

$$\tilde{R}_{ik,j}(\Delta g)=\begin{cases}1-e^{-\gamma*\Delta g}, & g(h_{ij}(A_i))>g(h_{kj}(A_i))\\ 0, & \text{otherwise}\end{cases} \tag{14}$$

Step 4. Compute the regret or rejoice value of alternative A_i under criteria C_j by

$$\hat{R}_{ij}=\sum_{k=1}^{m}(R_{ik,j}(\Delta g)+\tilde{R}_{ik,j}(\Delta g)) \tag{15}$$

Step 5. Compute DMs' optimal comprehensive perceived utility value Cu_i for alternative A_i as:

$$Cu_i=g_i(h_i(A_i))+\sum_{j=1}^{n} w_j\hat{R}_{ij}\ ,\quad i=1,2,\cdots,z \tag{16}$$

where $g_i(h_i(A_i))=\prod_{j=1}^{n}(h_{ij}^{\sigma(t)})^{w_j}$ is the obtained utility of A_i. As $\Delta g \in [0,1]$, the regret/rejoice value is small. In order not to offset the value, we adopt the HFOWA operator since the aggregation results by the HFOWA operator is smaller than that of the hesitant fuzzy weighted averaging (HFWA) operator [15].

Step 6. Generate the ranking order of alternatives in an increasing order according to the comprehensive perceived utility values Cu_i, for $i=1,2,\cdots,z$.

IV. An Illustrative Example

In this section, an application of our proposed model is shown by selecting 3 infrastructure projects to tender from the government tender procurement information platform.

The tendering decision for infrastructure projects is important for the development of enterprises. If a contractor tenders involuntarily in the face of multiple projects in the market, it will lead to the chaotic management of the company and waste resources, and even endanger the company's survival. By focusing on infrastructure projects with high economic benefits, good development prospects and catering one company's advantages, the competitiveness of a company can increasingly improve. Thus, how to fully measure the company's capital, technical strength, and professional scope, and then choose a feasible and advantageous project to tender in a scientific and reasonable way is a significant decision-making problem faced by construction companies.

Zhongtian Construction Co., Ltd., whose main business is house construction, transportation roads and bridges, is a large construction enterprise in China. Suppose that there are 10 infrastructure projects tender notices $A_i(i=1,2,\cdots,10)$ published on the government tender procurement information platform. After the company's tender manager carefully checked the current personnel and funding status of the company, they decided to select five projects to tender. Six experts with rich tendering and bidding experience in the company were invited to form a group of evaluation team. For the purpose of evaluation, five attributes were defined, including: project profitability (C_1), promotion of competitive growth (C_2), project relevance (C_3), efficiency of government officials (C_4), and competitors (C_5).

A. Application of the Proposed Method

In accordance with Section □, the evaluation model is demonstrated through two stages.

Stage one is to narrow the set of alternatives. The hesitant fuzzy judgment information given by DMs is shown in Table I. First, we normalize the evaluation values. Then, by Eqs. (1)-(3), we find the ideal reference sequence S^+, medium reference sequence $\bar{S}$ and negative-ideal reference sequence S^-, and the preliminary classification of the alternatives can be identified by Eqs. (4)-(5). The corresponding results are shown in Table II. From the preliminary results, we can see that alternatives A_4, A_7, A_9 belong to the class level "1". Next, by setting the neighborhood distance parameter δ =0.2, we obtain the neighborhood relation granules for each alternative through Eqs. (6)-(7). The results are shown in Table III. Finally, with Eqs. (8)-(9), the alternative set are narrowed as $\{A_4, A_6, A_7, A_8, A_9\}$ (displayed in Table III) based on the obtained granules, which contains only the relative better alternatives.

Table I. Hesitant fuzzy judgments given by DMs

	C_1	C_2	C_3	C_4	C_5
A_1	{0.7,0.8}	{0.4,0.5}	{0.3,0.4}	{0.6}	{0.7,0.8}
A_2	{0.4,0.5}	{0.5,0.6}	{0.6,0.7}	{0.2,0.3}	{0.7,0.8}
A_3	{0.4,0.5}	{0.4,0.5}	{0.2,0.3}	{0.6,0.7}	{0.5,0.6}
A_4	{0.7,0.8}	{0.7}	{0.7,0.8}	{0.9}	{0.8,0.9}
A_5	{0.2,0.3}	{0.3,0.4}	{0.4,0.5}	{0.3}	{0.5}
A_6	{0.4,0.5}	{0.5,0.6}	{0.7,0.8}	{0.8}	{0.9}
A_7	{0.6,0.7}	{0.7,0.8}	{0.6,0.7}	{0.8,0.9}	{0.6,0.7}
A_8	{0.6,0.7}	{0.7,0.8}	{0.5,0.6}	{0.6,0.7}	{0.8,0.9}
A_9	{0.6,0.7}	{0.2,0.3}	{0.9}	{0.8,0.9}	{0.8}
A_{10}	{0.3,0.4}	{0.1,0.2}	{0.4}	{0.5}	{0.5,0.6}

Table II. Neighborhood relation granule for each alternative

	D_i^+	$\bar{D}_i$	D_i^-	$\varphi(A_i, D_i^+)$	$\varphi(A_i, \bar{D}_i)$	$\varphi(A_i, D_i^-)$	Class
A_1	1.30	1.50	0.55	23.52%	20.38%	56.10%	2
A_2	1.55	1.25	0.65	21.54%	26.71%	51.76%	2
A_3	1.85	0.95	0.65	17.20%	33.49%	49.32%	2
A_4	0.25	2.55	1.00	74.11%	7.27%	18.62%	1
A_5	2.35	0.45	1.11	11.98%	62.55%	25.47%	3
A_6	0.75	2.05	0.68	40.37%	14.77%	44.86%	2
A_7	0.65	2.15	0.73	45.48%	13.75%	40.77%	1
A_8	0.75	2.05	0.55	36.47%	13.34%	50.19%	2
A_9	0.75	2.05	0.99	47.01%	17.20%	35.79%	1
A_{10}	2.25	0.55	1.01	13.64%	55.81%	30.54%	3

Table III. Neighborhood relation granules and preliminary classification of the alternatives

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
A_1	1	0	1	0	0	0	0	1	0	0
A_2	0	1	0	0	1	1	0	0	0	0
A_3	1	0	1	0	1	0	0	0	0	1
A_4	0	0	0	1	0	1	1	1	1	0
A_5	0	1	1	0	1	0	0	0	0	1
A_6	0	1	0	1	0	1	1	1	1	0
A_7	0	0	0	1	0	1	1	1	1	0
A_8	1	0	0	1	0	1	1	1	0	0
A_9	0	0	0	1	0	1	1	0	1	0
A_{10}	0	0	1	0	1	0	0	0	0	1

Since $5 = z_1 > z = 3$, we need to go into the second stage to choose the optimal ones from the distilled alternative set based on the regret theory. First, we calculate the utility value of the evaluation information based on Eq. (11). The results are shown in Table IV.

Table IV. Utility value of the evaluation information

	C_1	C_2	C_3	C_4	C_5
A_4	{0.75, 0.87}	{0.75, 0.75}	{0.75, 0.87}	{0.99, 0.99}	{0.87, 0.99}
A_6	{0.42, 0.53}	{0.53, 0.64}	{0.75, 0.87}	{0.87, 0.87}	{0.99, 0.99}
A_7	{0.64, 0.75}	{0.75, 0.87}	{0.64, 0.75}	{0.87, 0.99}	{0.64, 0.75}
A_8	{0.64, 0.75}	{0.75, 0.87}	{0.53, 0.64}	{0.64, 0.75}	{0.87, 0.99}
A_9	{0.64, 0.75}	{0.20, 0.31}	{0.99, 0.99}	{0.87, 0.99}	{0.87, 0.87}

Then, through Eqs. (12), we can obtain the distances of the evaluation utilities between alternatives under each attribute, and on the basis of the distances, we can obtain the regret/rejoice values of pairwise alternatives under each criterion by Eqs. (13)-(14). Here, we set ξ=0.2, γ=0.4. Finally, the DMs' comprehensive perceived utility values Cu_i are obtained through Eqs. (15)-(16), as listed in Table V, and the ultimate set of preferred alternatives is $\{A_4, A_7, A_8\}$.

Table V. Comprehensive perceived utility values of DMs

	$\sum_{j=1}^{n} w_j \hat{R}_{ij}$	$g_i(h_i(A_i))$	Cu_i	Ranking
A_4	0.37	0.85	1.22	1
A_6	-0.01	0.72	0.71	4
A_7	0.06	0.76	0.81	2
A_8	-0.02	0.73	0.72	3
A_9	0.01	0.67	0.68	5

B. *Sensitive and Comparative Analyses*

In this section, the sensitive and comparative analyses are conducted to verify the validity and feasibility of the proposed method.

Firstly, as the number of the narrowed alternatives is controlled by the parameter δ, we conduct a sensitive analysis to verify the effectiveness of the classification approach by changing the values of δ in the range $[0.1, 0.3]$ with the increment 0.05. From the results yielded in Table VI, we can see that A_4, A_6, A_7 are always in the narrowed set when $Z \geq 3$, which indicates the effectiveness of our proposed screening method.

Table VI. The narrowed alternatives with different values of δ

δ	0.1	0.15	0.2	0.25	0.3
Alternative set	$\{A_4\}$ $\{A_7\}$ $\{A_9\}$	$\{A_4, A_6, A_7, A_8\}$	$\{A_4, A_6, A_7, A_8, A_9\}$	$\{A_1, A_2, A_3, A_4, A_6, A_7, A_8, A_9\}$	$\{A_1, A_2, A_3, A_4, A_6, A_7, A_8, A_9\}$

Secondly, we take the results of the first stage in comparison with the TOPSIS (technique for order preference by similarity to ideal solution) method since the technique that the positive-ideal and negative-ideal reference sequence used in the first stage to narrow the alternative set is similar to that in the TOPSIS method. The results are shown in Table VII.

Table VII. Scores obtained by the TOPSIS method

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Scores	0.54	0.45	0.34	0.91	0.16	0.73	0.77	0.73	0.73	0.20

From the yielded results in Table □, we obtain that $A_4 \succ A_7 \succ A_6 \approx A_8 \approx A_9 \succ A_1 \succ A_2 \succ A_3 \succ A_{10} \succ A_5$. We find that candidates A_6, A_8 and A_9 are without difference in the TOPSIS method, and a final suggestion fails to be given by the method since we aim to select three desired alternatives. In addition, the TOPSIS method cannot divide the alternative set flexibly. However, in Table VI, we can see that the alternative set can be narrowed flexibly by adjusting the values of the parameter δ. Hence, the reference set method proposed in this study to narrow alternatives is of advantages.

Thirdly, we compare our method with the prospect-theory-based ranking method. The results are illustrated in Fig. 4. From Fig. 4, we can find that the preferred alternative set will change with the changing of the reference point. That is to say, the method based on the prospect theory is not stable. Specifically, when the reference point value is less than 0.7, the alternative set $\{A_4, A_7, A_8\}$ is the recommended set; when the value is 0.8, the alternative set $\{A_4, A_7, A_9\}$ should be selected; while when the value is 0.9, the alternative set $\{A_4, A_6, A_7\}$ is recommended. Meanwhile, the utility differences among the candidates are not obvious. Further, we verify the stability of our proposed regret-theory-based method with the changes of parameter values. The results yielded are shown in Fig. 5, through which we can find that the alternative set $\{A_4, A_7, A_8\}$ is always the recommended set. Compared with the prospect theory-based ranking method, the differences of the obtained utilities among the alternatives are more prominent. Therefore, our propose model is feasible and effective.

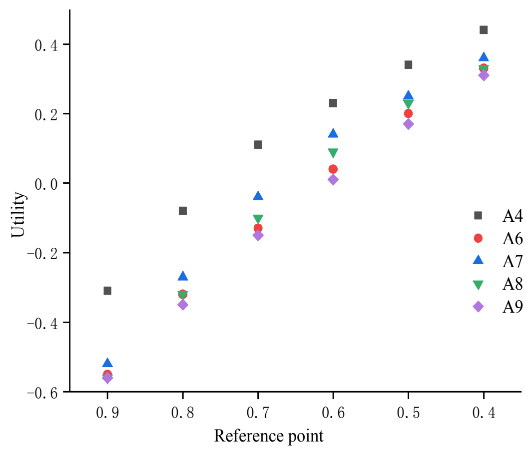

Fig. 4. The results of the prospect theory-based method

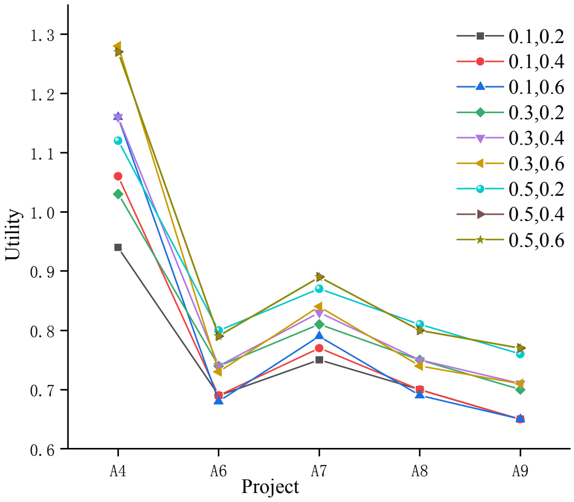

Fig. 5. The results of the regret theory-based method

V. CONCLUSUIONS

In this study, we proposed a two-stage model to solve the decision-making problems with massive alternatives. In the first stage, a method was proposed to screen the relative better alternatives from a large scale of options. In the second stage, a regret-theory-based decision-making method was applied to choose the optimal alternatives from the narrowed set that only covers the relative better alternatives. The proposed model could characterize DMs' psychological behavior accurately. Meanwhile, it is helpful to improve the evaluation efficiency of the decision-making process. The practicality of our proposed model was illustrated by an example of selecting three infrastructure projects to tender and the corresponding sensitive and comparative analyses further verified the validity and feasibility of the proposed decision-making method.

In the future, we will discuss both qualitative and quantitative criteria. Meanwhile, we will increase the scale of the alternative set. In this way, we hope to further demonstrate the advantages and effectiveness of our proposed model.

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