Robust actuator and sensor fault estimation for Takagi-Sugeno fuzzy systems under ellipsoidal bounding

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***Abstract*—The paper undertakes the problem of designing a simultaneous sensor and actuator fault estimation scheme for Takagi-Sugeno fuzzy systems. Thus, the paper starts with the development of such a fault estimation scheme capable of estimating these faults simultaneously. Apart from estimating the faults, the proposed scheme provides the so-called uncertainty intervals, which overbound an unknown state and faults. These intervals can be applied for both reliable fault diagnosis as well as assessment of the estimation quality. Indeed, the smaller the intervals the better the estimation. To settle the above problem, an assumption is imposed, which yields that the external disturbances are overbounded by an ellipsoid. This permits employing the quadratic boundedness approach both both the estimator convergence analysis as well as determination of lower and upper bounds of uncertainty intervals. Finally, the performance of the proposed approach is examined by exploiting the laboratory three-tank system. In particular, the effectiveness of the proposed scheme is tested against a set of simultaneous actuator and sensor faults, respectively.**

***Index Terms*—fault diagnosis, fault detection, Takagi-Sugeno model, estimation**

I. Introduction

Industrial processes are usually expensive, because of that a malfunction may cause serious losses, whether the equipment is damaged. This issue pertains both sensors and actuators, which are inevitable in modern industrial systems. Moreover, their number will proliferate in the upcoming future due to Industry 4.0 and the related Internet of Things (IoT) tools. Owing to the above situation, fault diagnosis, and particularly, fault estimation have received a growing attention both from theoretical and practical viewpoint. There is another important reason behind making fault estimation, which concerns Fault-Tolerant Control (FTC) [1]–[4]. Indeed, it is the main component of the so-called integrated FTC [1], [5], [6]. Nevertheless, it is permanently playing an important role in the implementation of FDI (Fault Detection and Isolation) [7], [8], which provide a solution to maintain a safe and reliable operation of the system.

The paper concerns the design of a fault estimator for Takagi-Sugeno (T–S) fuzzy systems, which is motivated by enumerable successful implementations of this system modelling paradigm [9], [10]. In the literature, one can find numerous fault estimation approaches for T–S fuzzy systems [11], [12], which can be used either sensor [13], [14] or actuator [15], [16] faults. There are, of course, approaches, which can deal with both faults simultaneously, e.g., the data-driven approaches [17], [18]. On the other hand, Youssef et al. [19], proposed the the T–S proportional integral estimator design for actuator-sensor fault estimation under unmeasurable premise variables. Hadi et al. [20], proposed a robust development of T–S multiple-integral unknown input observer for actuator-sensor fault estimation. Another approach is a Sliding Mode Observer (SMO) based on T–S model [21], which was used to estimate the actuator fault and disturbances of a doubly fed induction generator. Furthermore, a SMO-based fault estimation method presented in [22] was used for detecting, isolating and reconstructing actuator and sensor faults in a dedicated benchmark devoted to wind turbine problem. Martinez-Gracia et al. [23] developed the so-called T–S unknown input interval estimator for a simultaneous state as well as actuator fault estimation under parametric uncertainties. Sun et al. [24], used robust actuator and sensor fault estimation for T–S systems with time-varying state delay. A delay-partitioning approach where presented in [25], it was used to determine the state as well as sensor-actuator faults under time-delay. Jun Yoneyama [26], developed a method for obtaining a filter that estimates the state of the T–S fuzzy bilinear system along with an unknown input coupled with external disturbances.

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Apart from an incontestable appeal of the above-listed approaches, none of them provide an information about an estimated fault uncertainty, which can be shaped by the so-called *uncertainty intervals* guarantying to overbound both an unknown real state and faults. Thus, the objective of this paper is to extend the actuator fault estimation approach capable of providing such uncertainty intervals [27], [28] to the case of simultaneous sensor-actuator estimation.

The approach proposed in this paper amounts to a novel estimator structure, which operates under the assumption that external disturbances are bounded within a known ellipsoid. This makes it possible to apply the Quadratic Boundedness (QB) framework [29], [30] for an underlying convergence analysis and the development of uncertainty intervals. The paper is organized as follows. The estimator development problem is given in the Section II, whilst Section III provides an approach for determining the upper/lower bound of the uncertainty intervals, respectively. Section IV presents a verification of the proposed strategy using the three-tank system [31] modeled within T–S framework [32]. Finally, the last section provides concluding remarks.

II. Estimator Design

Let us start by defining a nonlinear system

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}\left(\boldsymbol{x}_k, \boldsymbol{u}_k\right), \tag{1}$$

which can be efficiently modeled by a Takagi-Sugeno (T–S) system involving faults as well as uncertainties:

$$\begin{aligned}\boldsymbol{x}_{k+1} &= \boldsymbol{A}(\boldsymbol{v}_k)\boldsymbol{x}_k + \boldsymbol{B}(\boldsymbol{v}_k)\boldsymbol{u}_k + \boldsymbol{B}(\boldsymbol{v}_k)\boldsymbol{f}_{a,k} + \boldsymbol{W}_1\boldsymbol{w}_{1,k}\\ &= \sum_{i=1}^{M}\boldsymbol{h}_i\left(\boldsymbol{v}_k\right)\left[\boldsymbol{A}^i\boldsymbol{x}_k + \boldsymbol{B}^i\boldsymbol{u}_k + \boldsymbol{B}^i\boldsymbol{f}_{a,k}\right] + \boldsymbol{W}_1\boldsymbol{w}_{1,k},\end{aligned} \tag{2}$$

$$\boldsymbol{y}_k = \boldsymbol{C}\boldsymbol{x}_k + \boldsymbol{C}_f\boldsymbol{f}_{s,k} + \boldsymbol{W}_2\boldsymbol{w}_{2,k}, \tag{3}$$

with

$$\boldsymbol{h}_i\left(\boldsymbol{v}_k\right) \geq 0 \quad \forall i = 1,\ldots,M \qquad \sum_{i=1}^{M}\boldsymbol{h}_i\left(\boldsymbol{v}_k\right) = 1 \tag{4}$$

where $\boldsymbol{x}_k \in \mathbb{X} \subset \mathbb{R}^n$, $\boldsymbol{u}_k \in \mathbb{R}^r$ and $\boldsymbol{y}_k \in \mathbb{R}^m$ indicate the state, input and output, respectively. Moreover, $\boldsymbol{f}_{a,k} \in \mathbb{F}_a \subset \mathbb{R}^r$ and $\boldsymbol{f}_{s,k} \in \mathbb{F}_s \subset \mathbb{R}^{n_s}$ are the actuator-sensor faults, respectively. Matrix $\boldsymbol{C}_f$ stands for the sensor fault distribution one, where $\mathrm{rank}(\boldsymbol{C}_f) = n_s$ with $r + n_s \leq m$, means that there is no possibility to estimate more faults than the measured outputs. Additionally, $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ denote the distribution matrices of $\boldsymbol{w}_{1,k}$ and $\boldsymbol{w}_{2,k}$ signifying exogenous disturbance vectors, respectively. Furthermore, $\boldsymbol{h}_i(\cdot)$ stands for the rule fire strength [33] depending on the known vector of premise variables $\boldsymbol{v}_k = \left[\boldsymbol{v}_k^1, \boldsymbol{v}_k^2, \ldots, \boldsymbol{v}_k^p\right]^T$.

Let us first extract $\boldsymbol{C}_f\boldsymbol{f}_{s,k}$ from (3) to show that

$$\boldsymbol{C}_f\boldsymbol{f}_{s,k} = \boldsymbol{y}_k - \boldsymbol{C}\boldsymbol{x}_k - \boldsymbol{W}_2\boldsymbol{w}_{2,k}. \tag{5}$$

Additionally, the sensor fault can be defined by

$$\boldsymbol{f}_{s,k} = \boldsymbol{E}\boldsymbol{y}_k - \boldsymbol{E}\boldsymbol{C}\boldsymbol{x}_k - \boldsymbol{E}\boldsymbol{W}_2\boldsymbol{w}_{2,k}. \tag{6}$$

with

$$\boldsymbol{E} = \left(\boldsymbol{C}_f\right)^{\dagger}, \quad \text{where } \boldsymbol{E}\boldsymbol{C}_f = \boldsymbol{I}. \tag{7}$$

Accordingly, the sensor fault estimator is proposed to have the following

$$\begin{aligned}\hat{\boldsymbol{f}}_{s,k} = \boldsymbol{E}\boldsymbol{y}_k &- \boldsymbol{E}\boldsymbol{C}\boldsymbol{A}\left(\boldsymbol{v}_k\right)\hat{\boldsymbol{x}}_{k-1} - \boldsymbol{E}\boldsymbol{C}\boldsymbol{B}\left(\boldsymbol{v}_k\right)\boldsymbol{u}_{k-1}\\ &- \boldsymbol{E}\boldsymbol{C}\boldsymbol{B}\left(\boldsymbol{v}_k\right)\hat{\boldsymbol{f}}_{a,k-1}.\end{aligned} \tag{8}$$

Furthermore, the actuator fault and state estimator are described by:

$$\hat{\boldsymbol{f}}_{a,k+1} = \hat{\boldsymbol{f}}_{a,k} + \boldsymbol{F}\left(\boldsymbol{y}_k - \boldsymbol{C}\hat{\boldsymbol{x}}_k - \boldsymbol{C}_f\hat{\boldsymbol{f}}_{s,k}\right). \tag{9}$$

$$\begin{aligned}\hat{\boldsymbol{x}}_{k+1} = \boldsymbol{A}\left(\boldsymbol{v}_k\right)\hat{\boldsymbol{x}}_k &+ \boldsymbol{B}\left(\boldsymbol{v}_k\right)\boldsymbol{u}_k + \boldsymbol{B}\left(\boldsymbol{v}_k\right)\hat{\boldsymbol{f}}_{a,k}\\ &+ \boldsymbol{K}\left(\boldsymbol{y}_k - \boldsymbol{C}\hat{\boldsymbol{x}}_k - \boldsymbol{C}_f\hat{\boldsymbol{f}}_{s,k}\right),\end{aligned} \tag{10}$$

where $\boldsymbol{F}$ and $\boldsymbol{K}$ indicate the gain matrices of the estimator. Let us define the underlying state, actuator as well as sensor fault estimation errors for the subsequent convergence analysis:

$$\begin{aligned}\boldsymbol{e}_{k+1} = \boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1} &= \left(\boldsymbol{A}\left(\boldsymbol{v}_k\right) - \boldsymbol{K}\boldsymbol{C}\right)\boldsymbol{e}_k\\ &+ \boldsymbol{B}\left(\boldsymbol{v}_k\right)\boldsymbol{e}_{a,k} - \boldsymbol{K}\boldsymbol{C}_f\boldsymbol{e}_{s,k} + \boldsymbol{W}_1\boldsymbol{w}_{1,k}\\ &- \boldsymbol{K}\boldsymbol{W}_2\boldsymbol{w}_{2,k},\end{aligned} \tag{11}$$

$$\begin{aligned}\boldsymbol{e}_{a,k+1} = \boldsymbol{f}_{a,k+1} - \hat{\boldsymbol{f}}_{a,k+1} &= \boldsymbol{\varepsilon}_{a,k} + \boldsymbol{e}_{a,k} - \boldsymbol{F}\boldsymbol{C}\boldsymbol{e}_k\\ &- \boldsymbol{F}\boldsymbol{C}_f\boldsymbol{e}_{s,k} - \boldsymbol{F}\boldsymbol{W}_2\boldsymbol{w}_{2,k},\end{aligned} \tag{12}$$

$$\begin{aligned}\boldsymbol{e}_{s,k} = \boldsymbol{f}_{s,k} - \hat{\boldsymbol{f}}_{s,k} &= -\boldsymbol{E}\boldsymbol{C}\boldsymbol{A}\left(\boldsymbol{v}_k\right)\boldsymbol{e}_{k-1} - \boldsymbol{E}\boldsymbol{C}\boldsymbol{B}\left(\boldsymbol{v}_k\right)\boldsymbol{e}_{a,k-1}\\ &- \boldsymbol{E}\boldsymbol{C}\boldsymbol{W}_1\boldsymbol{w}_{1,k-1} - \boldsymbol{E}\boldsymbol{W}_2\boldsymbol{w}_{2,k}.\end{aligned} \tag{13}$$

Accordingly, the estimation errors can be obtained by substituting (13) into (11)–(12):

$$\begin{aligned}\boldsymbol{e}_{k+1} = \left(\boldsymbol{A}\left(\boldsymbol{v}_k\right) - \boldsymbol{K}\boldsymbol{C}\right)\boldsymbol{e}_k &+ \boldsymbol{B}\left(\boldsymbol{v}_k\right)\boldsymbol{e}_{a,k} + \boldsymbol{W}_1\boldsymbol{w}_{1,k}\\ &+ \boldsymbol{K}\boldsymbol{C}\boldsymbol{A}\left(\boldsymbol{v}_k\right)\boldsymbol{e}_{k-1} + \boldsymbol{K}\boldsymbol{C}\boldsymbol{B}\left(\boldsymbol{v}_k\right)\boldsymbol{e}_{a,k-1}\\ &+ \boldsymbol{K}\boldsymbol{C}\boldsymbol{W}_1\boldsymbol{w}_{1,k-1},\end{aligned} \tag{14}$$

$$\begin{aligned}\boldsymbol{e}_{a,k+1} = \boldsymbol{\varepsilon}_{a,k} + \boldsymbol{e}_{a,k} &- \boldsymbol{F}\boldsymbol{C}\boldsymbol{e}_k + \boldsymbol{F}\boldsymbol{C}\boldsymbol{A}\left(\boldsymbol{v}_k\right)\boldsymbol{e}_{k-1}\\ &+ \boldsymbol{F}\boldsymbol{C}\boldsymbol{B}\left(\boldsymbol{v}_k\right)\boldsymbol{e}_{a,k-1} + \boldsymbol{F}\boldsymbol{C}\boldsymbol{W}_1\boldsymbol{w}_{1,k-1}.\end{aligned} \tag{15}$$

It is easy to check that the sensor fault estimation error depends on the state as well as the actuator fault estimation error. They also are independent of the sensor fault estimation one.

Nevertheless, let us combine (14)–(15) to obtain the following super-vectors:

$$\bar{\boldsymbol{e}}_k = \begin{bmatrix}\boldsymbol{e}_k\\ \boldsymbol{e}_{a,k}\end{bmatrix}, \qquad \bar{\boldsymbol{w}}_k = \begin{bmatrix}\boldsymbol{w}_{1,k}\\ \boldsymbol{\varepsilon}_{a,k}\end{bmatrix}. \tag{16}$$

Consequently

$$\begin{aligned}\bar{\boldsymbol{e}}_{k+1} = &\begin{bmatrix}\boldsymbol{A}\left(\boldsymbol{v}_k\right) - \boldsymbol{K}\boldsymbol{C} & \boldsymbol{B}\left(\boldsymbol{v}_k\right)\\ -\boldsymbol{F}\boldsymbol{C} & \boldsymbol{I}\end{bmatrix}\bar{\boldsymbol{e}}_k\\ &+ \begin{bmatrix}\boldsymbol{K}\boldsymbol{C}\boldsymbol{A}\left(\boldsymbol{v}_k\right) & \boldsymbol{K}\boldsymbol{C}\boldsymbol{B}\left(\boldsymbol{v}_k\right)\\ \boldsymbol{F}\boldsymbol{C}\boldsymbol{A}\left(\boldsymbol{v}_k\right) & \boldsymbol{F}\boldsymbol{C}\boldsymbol{B}\left(\boldsymbol{v}_k\right)\end{bmatrix}\bar{\boldsymbol{e}}_{k-1}\\ &+ \begin{bmatrix}\boldsymbol{W}_1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\bar{\boldsymbol{w}}_k + \begin{bmatrix}\boldsymbol{K}\boldsymbol{C}\boldsymbol{W}_1 & \boldsymbol{0}\\ \boldsymbol{F}\boldsymbol{C}\boldsymbol{W}_1 & \boldsymbol{0}\end{bmatrix}\bar{\boldsymbol{w}}_{k-1}.\end{aligned} \tag{17}$$

Additionally, its simpler form is given as follows

$$\bar{e}_{k+1} = A_1(v_k)\bar{e}_k + A_2(v_k)\bar{e}_{k-1} + \bar{W}_1\bar{w}_k + \tilde{W}_1\bar{w}_{k-1}, \tag{18}$$

where:

$$A_1(v_k) = \bar{A}(v_k) - \bar{K}\bar{C},$$
$$A_2(v_k) = \bar{K}\bar{B}(v_k), \quad \tilde{W}_1 = \bar{K}\bar{W}_3,$$

with:

$$\bar{A}(v_k) = \begin{bmatrix} A(v_k) & B(v_k) \\ 0 & I \end{bmatrix}, \quad \bar{K} = \begin{bmatrix} K \\ F \end{bmatrix},$$
$$\bar{B}(v_k) = \begin{bmatrix} CA(v_k) & CB(v_k) \end{bmatrix},$$
$$\bar{W}_1 = \begin{bmatrix} W_1 & 0 \\ 0 & I \end{bmatrix}, \quad \bar{W}_3 = \begin{bmatrix} CW_1 & 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix},$$

Let us define the Lyapunov candidate function of the form

$$V_k = \bar{e}_k^T P\bar{e}_k + \bar{e}_{k-1}R\bar{e}_{k-1}, \tag{19}$$

with $P \succ 0$ and $R \succ 0$. Furthermore, let us start with reminding the following definitions [34], [35]:

Definition 1: The system signified by (2)–(3) is strictly QB for all $\bar{w}_k \in \mathbb{E}_w$, $k \geq 0$, if $V_k > 1 \implies V_{k+1} - V_k < 0$ for any $\bar{w}_k \in \mathbb{E}_w$.

Definition 2: A set $\mathbb{E}$ is a positively invariant one for (18) and for all $\bar{w}_k \in \mathbb{E}_w$ if $\bar{e}_k \in \mathbb{E}$ implies $\bar{e}_{k+1} \in \mathbb{E}$ for any $\bar{w}_k \in \mathbb{E}_w$.

Note that the system (18) can be perceived as a single-delay one, and hence, it is proposed to define the invariant set as follows:

$$\mathbb{E} = \left\{(\bar{e}_k, \bar{e}_{k-1}) : \bar{e}_k^T P\bar{e}_k + \bar{e}_{k-1}^T R\bar{e}_{k-1} \leq 1\right\}. \tag{20}$$

Indeed, as it can be observed in [27], [28], [34], in a delay-free case (20) is defined with $R = 0$. Subsequently, it is necessary to define the ellipsoidal domain of $\bar{w}_k$, which is assumed to have the following form:

$$\mathbb{E}_w = \left\{\bar{w}_k : \bar{w}_k^T Q_w \bar{w}_k \leq 1\right\}, \quad Q_w \succ 0, \tag{21}$$

where Q_w is a known matrix shaping the ellipsoidal domain of $\bar{w}_k$. A practical way for determining this matrix is provided in the following authors' paper [36]. The above considerations allow to state the following theorem:

Theorem 1: The system described by (18) is strictly QB for all $\bar{w}_k \in \mathbb{E}_w$, if three exist matrices $P \succ 0$, $R \succ 0$, N and a scalar $\gamma, \beta \in (0,1)$, $\gamma + \beta < 1$, satisfying the following inequality:

$$\begin{bmatrix} R-(1-\gamma-\beta)P & 0 & 0 & 0 & \bar{A}^T(v_k)P - \bar{C}^T N^T \\ 0 & -(1-\gamma-\beta)R & 0 & 0 & \bar{B}^T(v_k)N^T \\ 0 & 0 & -\gamma Q_w & 0 & \bar{W}_1^T P \\ 0 & 0 & 0 & -\beta Q_w & \bar{W}_3^T N^T \\ P\bar{A}(v_k) - N\bar{C} & N\bar{B}(v_k) & P\bar{W}_1 & N\bar{W}_3 & -P \end{bmatrix} \prec 0. \tag{22}$$

Proof 1: Applying definition 1 as well as the facts that $\bar{w}_k^T Q_w \bar{w}_k \leq 1$ and $\bar{w}_{k-1}^T Q_w \bar{w}_{k-1} \leq 1$, it is possible to show that

$$\bar{w}_{k-1}^T Q_w \bar{w}_{k-1} < \bar{e}_k^T P\bar{e}_k + \bar{e}_{k-1}^T R\bar{e}_{k-1}, \tag{23}$$
$$\bar{w}_k^T Q_w \bar{w}_k < \bar{e}_k^T P\bar{e}_k + \bar{e}_{k-1}^T R\bar{e}_{k-1}, \tag{24}$$

Consequently, using the estimation error (18) and representing $\bar{v}_k = \begin{bmatrix}\bar{e}_k^T & \bar{e}_{k-1}^T & \bar{w}_k^T & \bar{w}_{k-1}^T\end{bmatrix}^T$, it can be shown that

$$\bar{v}_k^T \begin{bmatrix} A_1^T(v_k)PA_1(v_k) + R - P & A_1^T(v_k)PA_2(v_k) & A_1^T(v_k)P\bar{W}_1 & A_1^T(v_k)P\tilde{W}_1 \\ A_2^T(v_k)PA_1(v_k) & A_2^T(v_k)PA_2(v_k) - R & A_2^T(v_k)P\bar{W}_1 & A_2^T(v_k)P\tilde{W}_1 \\ \bar{W}_1^T PA_1(v_k) & \bar{W}_1^T(v_k)PA_2(v_k) & \bar{W}_1^T P\bar{W}_1 & \bar{W}_1^T P\tilde{W}_1 \\ \tilde{W}_1^T PA_1(v_k) & \tilde{W}_1^T PA_2(v_k) & \tilde{W}_1^T P\bar{W}_1 & \tilde{W}_1^T P\tilde{W}_1 \end{bmatrix} \bar{v}_k \prec 0. \tag{25}$$

From (23) and (24) it is obvious that for $\gamma > 0$ and for $\beta > 0$:

$$\gamma\bar{v}_k^T \begin{bmatrix} -P & 0 & 0 & 0 \\ 0 & -R & 0 & 0 \\ 0 & 0 & Q_w & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \bar{v}_k \prec 0, \tag{26}$$

$$\beta\bar{v}_k^T \begin{bmatrix} -P & 0 & 0 & 0 \\ 0 & -R & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_w \end{bmatrix} \bar{v}_k \prec 0. \tag{27}$$

Then, applying an S-procedure [37] to (25)–(27) the following result can be achieved

$$\bar{v}_k^T \begin{bmatrix} A_1^T(v_k)PA_1(v_k) + R - P + \gamma P + \beta P & A_1^T(v_k)PA_2(v_k) & A_1^T P\bar{W}_1 & A_1^T P\tilde{W}_1 \\ A_2^T(v_k)PA_1(v_k) & A_2^T(v_k)PA_2(v_k) - R + \gamma R + \beta R & A_2^T P\bar{W}_1 & A_2^T P\tilde{W}_1 \\ \bar{W}_1^T PA_1(v_k) & \bar{W}_1^T PA_2(v_k) & -\gamma Q_w + \bar{W}_1^T P\bar{W}_1 & \bar{W}_1^T P\tilde{W}_1 \\ \tilde{W}_1^T PA_1(v_k) & \tilde{W}_1^T PA_2(v_k) & \tilde{W}_1^T P\bar{W}_1 & \tilde{W}_1^T P\tilde{W}_1 - \beta Q_w \end{bmatrix} \bar{v}_k \prec 0. \tag{28}$$

Subsequently, applying Schur complements to (28) and then multiplying left- and right-side by diag $(\boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{P})$ gives

$$\begin{bmatrix} -\boldsymbol{P}+\boldsymbol{R}+\gamma\boldsymbol{P}+\beta\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_1^T(\boldsymbol{v}_k)\boldsymbol{P} \\ \boldsymbol{0} & -\boldsymbol{R}+\gamma\boldsymbol{R}+\beta\boldsymbol{R} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_2^T(\boldsymbol{v}_k)\boldsymbol{P} \\ \boldsymbol{0} & \boldsymbol{0} & -\gamma\boldsymbol{Q}_w & \boldsymbol{0} & \bar{\boldsymbol{W}}_1^T\boldsymbol{P} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\beta\boldsymbol{Q}_w & \tilde{\boldsymbol{W}}_1^T\boldsymbol{P} \\ \boldsymbol{P}\boldsymbol{A}_1(\boldsymbol{v}_k) & \boldsymbol{P}\boldsymbol{A}_2(\boldsymbol{v}_k) & \boldsymbol{P}\bar{\boldsymbol{W}}_1 & \boldsymbol{P}\tilde{\boldsymbol{W}}_1 & -\boldsymbol{P} \end{bmatrix} \prec 0. \tag{29}$$

Then, substituting:

$$\boldsymbol{P}\boldsymbol{A}_1(\boldsymbol{v}_k) = \boldsymbol{P}\bar{\boldsymbol{A}}(\boldsymbol{v}_k) - \boldsymbol{P}\bar{\boldsymbol{K}}\bar{\boldsymbol{C}} = \boldsymbol{P}\bar{\boldsymbol{A}}(\boldsymbol{v}_k) - \boldsymbol{N}\bar{\boldsymbol{C}}, \tag{30}$$

$$\boldsymbol{P}\boldsymbol{A}_2(\boldsymbol{v}_k) = \boldsymbol{P}\bar{\boldsymbol{K}}\bar{\boldsymbol{B}}(\boldsymbol{v}_k) = \boldsymbol{N}\bar{\boldsymbol{B}}(\boldsymbol{v}_k), \tag{31}$$

$$\boldsymbol{P}\bar{\boldsymbol{W}}_1 = \boldsymbol{P}\bar{\boldsymbol{W}}_1 - \boldsymbol{P}\bar{\boldsymbol{K}}\bar{\boldsymbol{W}}_2 = \boldsymbol{P}\bar{\boldsymbol{W}}_1 - \boldsymbol{N}\bar{\boldsymbol{W}}_2, \tag{32}$$

$$\boldsymbol{P}\tilde{\boldsymbol{W}}_1 = \boldsymbol{P}\bar{\boldsymbol{K}}\bar{\boldsymbol{W}}_3 = \boldsymbol{N}\bar{\boldsymbol{W}}_3, \tag{33}$$

leads to

$$\begin{bmatrix} \boldsymbol{R}-\boldsymbol{P}+\gamma\boldsymbol{P}+\beta\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \bar{\boldsymbol{A}}^T(\boldsymbol{v}_k)\boldsymbol{P}-\bar{\boldsymbol{C}}^T\boldsymbol{N}^T \\ \boldsymbol{0} & -\boldsymbol{R}+\gamma\boldsymbol{R}+\beta\boldsymbol{R} & \boldsymbol{0} & \boldsymbol{0} & \bar{\boldsymbol{B}}^T(\boldsymbol{v}_k)\boldsymbol{N}^T \\ \boldsymbol{0} & \boldsymbol{0} & -\gamma\boldsymbol{Q}_w & \boldsymbol{0} & \bar{\boldsymbol{W}}_1^T\boldsymbol{P} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\beta\boldsymbol{Q}_w & \bar{\boldsymbol{W}}_3^T\boldsymbol{N}^T \\ \boldsymbol{P}\bar{\boldsymbol{A}}(\boldsymbol{v}_k)-\boldsymbol{N}\bar{\boldsymbol{C}} & \boldsymbol{N}\bar{\boldsymbol{B}}(\boldsymbol{v}_k) & \boldsymbol{P}\bar{\boldsymbol{W}}_1 & \boldsymbol{N}\bar{\boldsymbol{W}}_3 & -\boldsymbol{P} \end{bmatrix} \prec 0. \tag{34}$$

However, to guarantee the solution feasibility, $\boldsymbol{R}-\boldsymbol{P}+\gamma\boldsymbol{P}+\beta\boldsymbol{P}$ in (34) is needed to be negative definite. Due to this fact it can be noticed that $(1-\gamma-\beta)\boldsymbol{P} \succ 0$, and hence, it should be $\gamma+\beta < 1$, which concludes the proof. ■

Finally, (22) can be easily formulated as an adequate set of M linear matrix inequalities corresponding to all vertices shaping (2). As a result, the problem is simplified to set $\alpha, \beta > 0$ and solve the set of LMIs (22) and obtaining the estimator matrices:

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K} \\ \boldsymbol{F} \end{bmatrix} = \boldsymbol{P}^{-1}\boldsymbol{N}. \tag{35}$$

Note that γ and β can be obtained by generating a solution grid formed with $\alpha > 0$ $\beta > 0$, $\alpha + \beta < 1$ for which the above mentioned LMIs are to be solved.

III. Determination of estimation error bounds

The section aims at determining the so-called uncertainty intervals that correspond to the fault and state maximum and minimum bounds, which are consistent with the available input-output data. Indeed, the estimator (8)–(10) provides point estimates of the sensor, actuator, and state of the system, respectively. However, from the fault diagnosis viewpoint, uncertainty intervals are a fundamental tool for undertaking decisions about faults [27], [36], [38]. To settle such an important problem, let us provide the following theorem.

***Theorem** 2:* If (18) is strictly QB for all $\bar{\boldsymbol{w}}_k \in \mathbb{E}_w$ then the uncertainty interval of the estimation error is:

$$-s_{i,k} \le \bar{\boldsymbol{e}}_{i,k} \le s_{i,k}, \tag{36}$$

$$s_{i,k} = \left(\eta_k(\gamma, \beta, \boldsymbol{s}_{k-1})\boldsymbol{c}_i^T\boldsymbol{P}^{-1}\boldsymbol{c}_i\right)^{\frac{1}{2}}, \tag{37}$$

$$\eta_k(\gamma, \beta, \boldsymbol{s}_{k-1}) = \zeta_k(\gamma, \beta) - \lambda_{\min}(\boldsymbol{R})\|\boldsymbol{s}_{k-1}\|, \tag{38}$$

where $\boldsymbol{c}_i$ is the ith column of an identity matrix while $\lambda_{\min}(\boldsymbol{R})$ stands for the smallest eigenvalue of $\boldsymbol{R}$.

***Proof** 2:* From (28) it can be deduced that:

$$V_{k+1} - V_k \le \gamma(\bar{\boldsymbol{w}}_k^T\boldsymbol{Q}_w\bar{\boldsymbol{w}}_k - V_k) + \beta(\bar{\boldsymbol{w}}_{k-1}^T\boldsymbol{Q}_w\bar{\boldsymbol{w}}_{k-1} - V_k), \tag{39}$$

and hence, bearing in mind $\bar{\boldsymbol{w}}_k^T\boldsymbol{Q}_w\bar{\boldsymbol{w}}_k \le 1$ and $\bar{\boldsymbol{w}}_{k-1}^T\boldsymbol{Q}_w\bar{\boldsymbol{w}}_{k-1} \le 1$, it can be written as:

$$V_{k+1} \le \gamma + \beta + (1-\gamma-\beta)V_k,$$

Thus, following [34], by the induction it can be shown that:

$$V_k \le \zeta_k(\gamma, \beta), \tag{40}$$

where:

$$\zeta_k(\gamma, \beta) = (1-\gamma-\beta)^k V_0 + 1 - (1-\gamma-\beta)^k \tag{41}$$

Finally, from (40) and (19), it can be shown that:

$$\bar{\boldsymbol{e}}_k^T\boldsymbol{P}\bar{\boldsymbol{e}}_k \le \zeta_k(\gamma, \beta) - \bar{\boldsymbol{e}}_{k-1}^T\boldsymbol{R}\bar{\boldsymbol{e}}_{k-1} \tag{42}$$

Finally, by applying the Rayleigh quotient, it can be concluded that:

$$\bar{\boldsymbol{e}}_k^T\boldsymbol{P}\bar{\boldsymbol{e}}_k \le \eta_k(\gamma, \beta, \boldsymbol{s}_{k-1}) = \zeta_k(\gamma, \beta) - \lambda_{\min}(\boldsymbol{R})\|\boldsymbol{s}_{k-1}\|,$$

which completes the proof. ■

Having the estimation error bounds (36) is is straightforward to derive the uncertainty intervals for the unknown states and faults, which can be realized according to the approach proposed in [27], [28].

IV. Experimental verification

In order to verify the correctness of the proposed estimator, it was implemented with the nonlinear laboratory three-tank system given in Fig. 1. A specific description of this system along with its nonlinear model and transformation into T–S form can be found in [32]. Due to the lack of space, it is omitted in this paper. In the experiment, the fault scenario had the following shape:

$$\boldsymbol{f}_{a,k} = \begin{cases} -0.27 \cdot \boldsymbol{u}_k & 7000 \le k \le 11000 \\ 0 & \text{otherwise} \end{cases}, \tag{43}$$

$$\boldsymbol{f}_{s,k} = \begin{cases} \boldsymbol{y}_k - 0.1 & 6000 \le k \le 9000 \\ 0 & \text{otherwise} \end{cases}, \tag{44}$$

along with the sensor fault distribution matrix

$$\boldsymbol{C}_f = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T, \tag{45}$$

Fig. 1. Laboratory three-tank system

which exhibits the 27% loss of effectiveness of the pump in the actuator fault case and the incorrect sensor readings in the second tank in the case of a sensor fault. In Figs. 2–4, a response of the system is presented. It can be deduced that the states were identified correctly in spite of the unappealing presence of actuator-sensor faults. Moreover, the state estimates follow the real state even if there was an incorrect measurement given from the sensor in the second tank. The estimated values in the initial phase coincide to the real ones very quickly. Figures 5 and 6 show the actuator and senor faults, respectively, along with their estimates. They were estimated in a very good quality. The states as well as senor-actuator faults are overbounded by the bounds developed in Section III, which guarantee that the real and estimated values are inside these bounds.

V. Conclusions

The paper investigated the issue of the state and actuator-sensor fault estimation T–S systems under the presence of ellipsoidal-bounded uncertainties. The stability of the estimator is guaranteed by the so-called QB approach, which ensures that the process and measurement uncertainties are bounded within an ellipsoidal set. The design problem is simplified into solving a set of LMIs. Therefore, estimation error bounds were developed, which form the so-called uncertainty intervals. They can be used to assess the performance of the proposed scheme as well as to obtain a reliable fault diagnosis. Indeed, instead of using a single point estimate of a fault, its uncertainty interval can be used. Thus, more reliable decisions

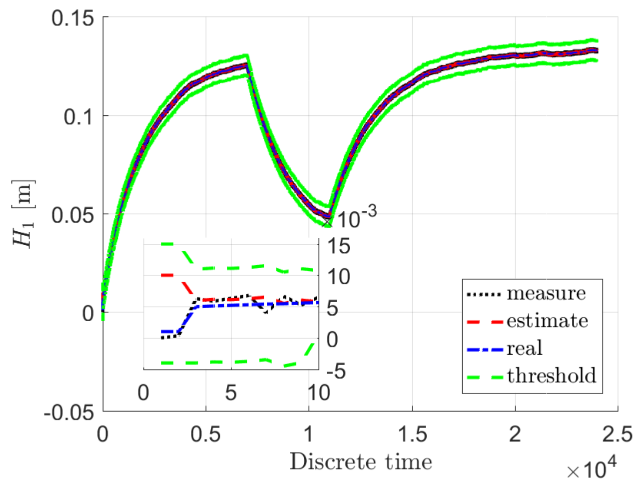

Fig. 2. The response of the system: first tank

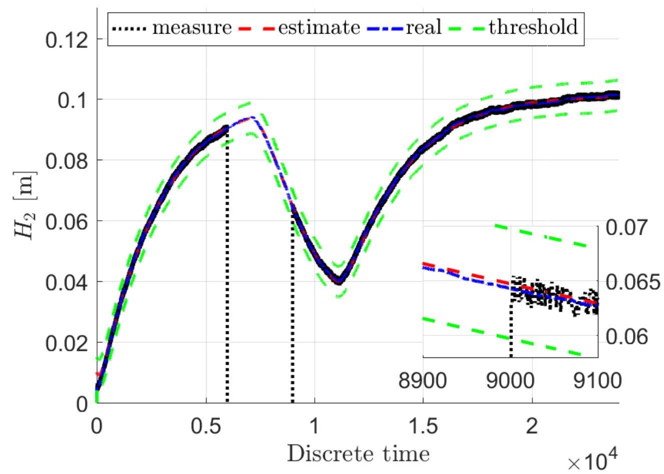

Fig. 3. The response of the system: second tank

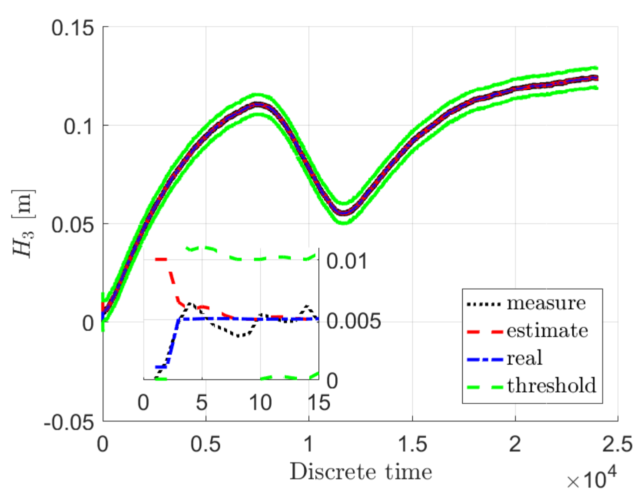

Fig. 4. The response of the system: third tank

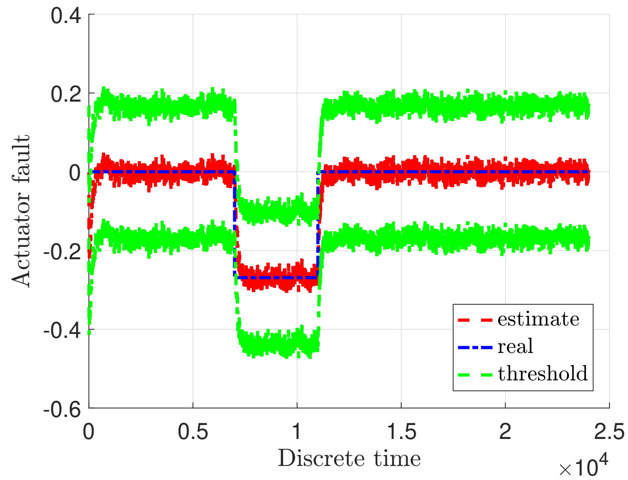

Fig. 5. Actuator fault

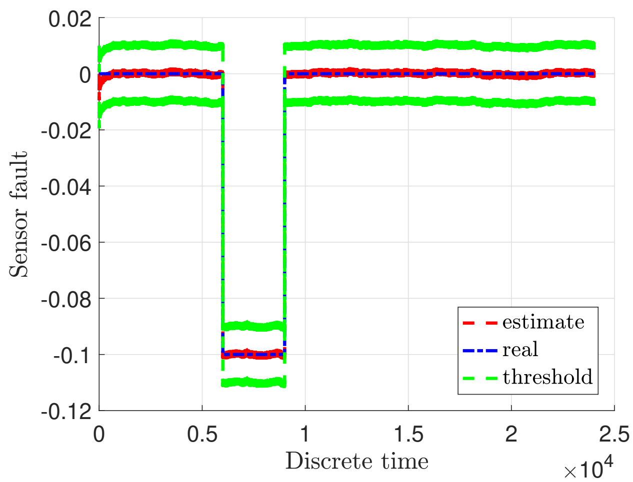

Fig. 6. Sensor fault

can be undertaken. This recommends to use the proposed estimator in the prospective integrated fault-tolerant scheme.

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