A Design Approach for General Type-2 Fuzzy Logic Controllers with an Online Scheduling Mechanism

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***Abstract*— This paper proposes a systematic approach to solve the design problem of General Type-2 (GT2) Fuzzy Logic Controllers (FLCs) with an online scheduling mechanism for performance enhancements. We firstly suggest constructing the GT2-FLC over its baseline type-1 and interval type-2 FLCs, and then to tune a single design parameter which defines the shape of the secondary membership functions. We present how the shape of the secondary membership function changes with respect to the design parameter and show resulting effect on the control surface generation. The presented comparative analysis on the control surfaces show that aggressive and smooth control surfaces can be easily generated by tuning the design parameter. We suggest tuning the design parameter by providing a tradeoff between robustness and performance of the control system. Also, to achieve satisfactory control performances for various steady-state points, we propose an online scheduling mechanism that tunes the design parameter with respect to the operating points. We perform a simulation study on a nonlinear system to validate our analyses and proposed design methods. The simulation results show that GT2-FLC has a potential to improve overall system performances in comparison to its type-1 and interval type-2 counterparts, while the developed scheduling mechanism provides an opportunity to achieve satisfactory results for various operating points.**

Keywords—General type-2 fuzzy logic controllers, general type-2 fuzzy sets, design method, online scheduling mechanism

I. INTRODUCTION

Fuzzy Logic Controllers (FLCs) have been successfully implemented for various control engineering problems [1-21]. It is shown that especially Interval Type-2 (IT2) FLCs are capable to achieve better control performances than their Type-1 (T1) counterparts [4-14]. Moreover, as their Control Surfaces (CSs) are usually smooth around the steady state, IT2-FLCs are potentially more robust than their T1 counterparts [4-6, 9]. These potential improvements of IT2-FLCs mainly occur due to the extra degree of freedom provided by the Footprint of Uncertainty (FOU) in their antecedent Membership Functions (MFs) that are defined with IT2 Fuzzy Sets (FSs). As shown in [8-14], a proper tuning of the FOU size provides an advantage to the IT2-FLC over its T1 baseline, as various CSs (in terms of aggressiveness and smoothness) can be generated by tuning its FOU Design Parameters (DPs).

In the last decade, researchers have also given attention to General Type-2 (GT2) FLCs. It has been shown that GT2-FLCs have further potentials to outperform their IT2 and T1 counterparts [1, 2, 15-23]. This is due to the fact that GT2-FLCs use and process GT2-FSs which have T1-FSs in their Secondary MFs (SMFs) instead of interval FSs [1, 2]. Thus, GT2-FLCs have more DPs to be tuned in comparison to their T1 and IT2 counterparts [16, 22]. Although the internal structure of GT2-FLCs is relatively more complex than T1 and IT2 FLCs, via the zSlices or α-plane representations, the output of GT2-FLCs can be easily defined as an aggregation of various T1 and IT2 FLCs [1]. In order to provide explanations about the potential performance improvements of GT2-FLCs in comparison to IT2, T1 and non-fuzzy counterparts, the rule and novelty partitions of GT2-FLCs were examined in [23]. Moreover in [16], the effects of the size and shape of the SMFs on the control performance were examined. Although, the design of GT2-FSs is more complex than IT2-FSs due to the difficulty of tuning their DPs, it is shown that GT2-FLCs can provide a tradeoff between the robustness and performance by tuning the size and shape of the SMFs [16].

In this paper, we present a design approach for GT2-FLCs and an online Scheduling Mechanism (SM) to enhance the performance of the GT2-FLC. We first examine the internal structure of the GT2-FLC after presenting the ones for T1 and IT2 FLCs. In the design of the GT2-FLC, we consider the same structure as its IT2 counterpart (i.e. same rule base, consequents and antecedent MFs at primary level) and we employ trapezoid T1-FSs as the SMF of the GT2-FSs. To accomplish the SMFs design easily, we propose a novel parameterization to define trapezoid SMFs via a single DP, which is the only DP of the GT2-FLC. We show that the proposed method also provides the opportunity to transform SMFs into the widely used FSs (e.g. crisp, interval and triangular) via the single DP. Then, we investigate the effect of the DP on the CS generation, by comparing the resulting CSs of the GT2-FLCs for various DPs with T1 and IT2 counterparts. It is clearly shown that the GT2-FLC can achieve various CSs in terms of aggressiveness and smoothness, which provides a design flexibility to GT2-FLCs. Based on our comparative analysis, we suggest designing GT2-FLCs by providing a tradeoff between performance and robustness. Moreover, we also propose an online SM to tune the DP with respect to the steady state Operating Points (OPs) to enhance the system performance. The proposed SM changes the DP in online manner, and thus the CS of the GT2-FLC is tuned according to the steady state OPs. Finally, we present a

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comparative simulation study on a nonlinear system in order to evaluate and validate our proposed design recommendations/ methods for GT2-FLCs.

II. General Type-2 Fuzzy Logic Controllers

Here, we present the internal structure of the PID-type GT2-FLC along with its T1 and IT2 fuzzy counterparts. We then present a structural design suggestion to construct the GT2-FLC.

A PID type FLC is constructed by selecting its inputs as the error (e) and the change of error (Δe); and its output as the control signal (u) [7-10]. In this PID controller structure, K_e and K_d are the input Scaling Factors (SFs) that normalize the inputs to the universe of discourse of antecedent MFs ($E = K_e e$ and $\Delta E = K_d \Delta e$, respectively), while the FLC output (U) is rescaled by the output SFs (K_a and K_b) to the control signal as [7-10]:

$$u = K_a + K_b \int U dt \tag{1}$$

A. Internal Structure of the T1 and IT2 FLCs

We prefer to construct T1 and IT2 FLCs composed of $N = 3x3$ ($n = 1,2,\ldots,9$) rules as shown in Table I. The inputs, $x_1 = e$ and $x_2 = \Delta e$, are partitioned with $I = 3$ $(i = 1,2,3)$ and $K = 3$ $(k = 1,2,3)$ MFs, respectively. The rule structure of the T1-FLC employing T1-FSs (the bold lines in Fig. 1) $A_{j,i}$ $(j = 1,2)$ is as follows:

$$R_n\text{: IF } x_1 \text{ is } A_{1,i} \text{ AND } x_2 \text{ is } A_{2,k} \text{ THEN } y \text{ is } C_n \tag{2}$$

while the rule structure of the IT2-FLC employing IT2-FSs is

$$R_n\text{: IF } x_1 \text{ is } \tilde{A}_{1,i} \text{ AND } x_2 \text{ is } \tilde{A}_{2,k} \text{ THEN } y \text{ is } C_n \tag{3}$$

Here, C_n is a crisp consequent MF. Note that, as IT2-FSs are employed, it holds that $\mu_{\tilde{A}}(x,u) = 1$ for $\forall u \in J_x \subseteq [0,1]$ where J_x is the primary MF [1].

TABLE I. The Rule Base of T1 and IT2 FLCs

x_2 \ x_1	$^aA_{1,1}$ or $^b\tilde{A}_{1,1}$	$^aA_{1,2}$ or $^b\tilde{A}_{1,2}$	$^aA_{1,3}$ or $^b\tilde{A}_{1,3}$
$^aA_{2,1}$ or $^b\tilde{A}_{2,1}$	$C_1 = -1$	$C_2 = -0.8$	$C_3 = 0$
$^aA_{2,2}$ or $^b\tilde{A}_{2,2}$	$C_4 = -0.8$	$C_5 = 0$	$C_6 = 0.8$
$^aA_{2,3}$ or $^b\tilde{A}_{2,3}$	$C_7 = 0$	$C_8 = 0.8$	$C_9 = 1$

[a] Defined for the T1-FLC. [b] Defined for IT2-FLC.

The output of the T1-FLC is defined as follows:

$$y_{T1} = \frac{\sum_{n=1}^{N} f_n C_n}{\sum_{n=1}^{N} f_n} \tag{4}$$

where firing strength f_n is defined as:

$$f_n = \mu_{A_{1,i}} \times \mu_{A_{2,k}} \tag{5}$$

Here, $\times$ indicates the product t-norm operation and $\mu_{A_{j,i}}$ is defined for an input x_j as follows:

$$\mu_{A_{j,i}} = \begin{cases} \dfrac{x_j - c_{j,i-1}}{c_{j,i} - c_{j,i-1}}, & x_j \in [c_{j,i-1}, c_{j,i}] \\ \dfrac{c_{j,i+1} - x_j}{c_{j,i+1} - c_{j,i}}, & x_j \in [c_{j,i}, c_{j,i+1}] \end{cases} \tag{6}$$

In this study, we set $c_{j,i}$ as $c_{j,1} = -1$, $c_{j,2}$=0, and $c_{j,3}$=1.

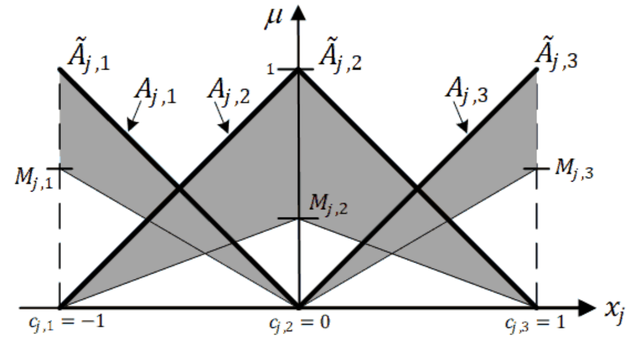

Fig. 1. Illustration of antecedent MFs of the T1 and IT2 FLCs

The output of the IT2-FLC is defined as [1]:

$$y_{IT2} = \left(\underline{y}_{IT2} + \overline{y}_{IT2}\right)/2 \tag{7}$$

where $\underline{y}_{IT2}$ and $\overline{y}_{IT2}$ are defined as follows:

$$\underline{y}_{IT2} = \frac{\sum_{n=1}^{L} \overline{f_n} C_n + \sum_{n=L+1}^{N} \underline{f_n} C_n}{\sum_{n=1}^{L} \overline{f_n} + \sum_{n=L+1}^{N} \underline{f_n}} \tag{8}$$

$$\overline{y}_{IT2} = \frac{\sum_{n=1}^{R} \underline{f_n} C_n + \sum_{n=R+1}^{N} \overline{f_n} C_n}{\sum_{n=1}^{R} \underline{f_n} + \sum_{n=R+1}^{N} \overline{f_n}} \tag{9}$$

where L and R are the switching points that are found by the KM algorithms [1], $\overline{f_n}$ and $\underline{f_n}$ denote upper and lower bounds of firing strengths, respectively; and are defined as:

$$\overline{f_n} = \overline{\mu}_{\tilde{A}_{1,i}} \times \overline{\mu}_{\tilde{A}_{2,k}} \quad \underline{f_n} = \underline{\mu}_{\tilde{A}_{1,i}} \times \underline{\mu}_{\tilde{A}_{2,k}} \tag{10}$$

where $\overline{\mu}_{\tilde{A}_{j,i}}$ is the Upper MF (UMF) defined as in (6), and $\underline{\mu}_{\tilde{A}_{j,i}}$ is the Lower MF (LMF) which are defined as follows:

$$\overline{\mu}_{\tilde{A}_{j,i}} = \mu_{A_{j,i}} \quad \underline{\mu}_{\tilde{A}_{j,i}} = \overline{\mu}_{\tilde{A}_{j,i}} M_{j,i} \tag{11}$$

Here $M_{j,i}$ is the height of the LMFs, which is the DP of IT2-FLCs that defines the size of FOU [12-14].

B. Internal Structure of the GT2-FLC

The rule structure of the GT2-FLC is identical to its baseline T1 and IT2 counterparts. Therefore, the GT2-FLC has the same rule base (as given in (3)) and the same MFs at primary level as its IT2 fuzzy counterpart. The fundamental difference between an IT2-FLC and a GT2-FLC is that GT2-FLCs use and process T1-FSs as SMFs instead of an interval set [1, 16].

The output of a GT2-FLC is defined as [1]:

$$y_{GT2} = \left(\sum_{p=1}^{P} y_{GT2}^{\alpha_p} \alpha_p\right) \Big/ \left(\sum_{p=1}^{P} \alpha_p\right) \tag{12}$$

where $y_{GT2}^{\alpha_p}$ is the output of IT2-FLC (or T1-FLC) associated with an α-plane α_p (α-T2-FLC), P $(p = 1,\ldots P)$ is the total number of α-planes (excluding $\alpha_0 = 0$) and $\alpha_p = p/P$ is the value of an α-plane. This representation gives opportunity to define the output of GT2-FLC (y_{GT2}) as an aggregation of α-plane outputs ($y_{GT2}^{\alpha_p}$), which are principally the outputs of T1-FLC (y_{T1}) and IT2-FLCs (y_{IT2}) [1, 16].

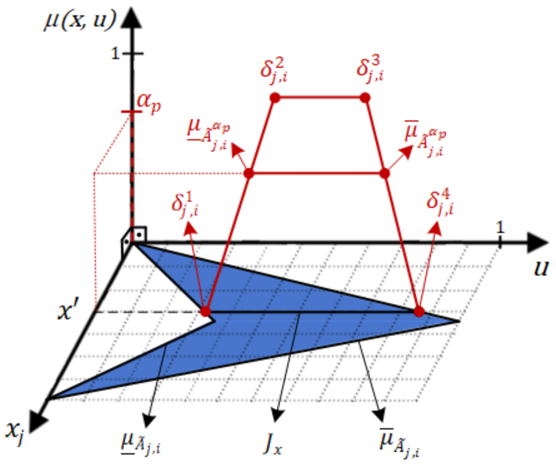

Fig. 2. Illustration of a GT2-FS with $\boldsymbol{\alpha}$-planes

In this study, we use trapezoid T1-FSs to define the SMF of the GT2-FS as shown in Fig. 2. Thus, the Lower MF (LMF) and the Upper MF (UMF)of an α-T2-FLC are then defined as:

$$\underline{\mu}_{\tilde{A}_{j,i}^{\alpha_p}} = \underline{\mu}_{\tilde{A}_{j,i}} + \left(\overline{\mu}_{\tilde{A}_{j,i}} - \underline{\mu}_{\tilde{A}_{j,i}}\right)\left(\delta_{j,i}^1 + \alpha_p\left(\delta_{j,i}^2 - \delta_{j,i}^1\right)\right) \quad (13)$$

$$\overline{\mu}_{\tilde{A}_{j,i}^{\alpha_p}} = \overline{\mu}_{\tilde{A}_{j,i}} - \left(\overline{\mu}_{\tilde{A}_{j,i}} - \underline{\mu}_{\tilde{A}_{j,i}}\right)\left(1 - \delta_{j,i}^4 + \alpha_p\left(\delta_{j,i}^4 - \delta_{j,i}^3\right)\right) \quad (14)$$

where $\delta_{j,i}^t$ $(t = 1,2,3,4)$ defines the shape and size of the SMF. Although trapezoid T1-FSs provide more design flexibility in defining the shape and support of the SMFs, the design is relatively complex as there are 4 parameters to be tuned. Therefore, to reduce the design complexity, we propose a simple parameter mapping as follows:

$$\begin{aligned} \delta_{j,i}^1 &= \min\left(\max\left(\theta_{j,i} - 1, 0\right), 1\right) \\ \delta_{j,i}^2 &= \min\left(\max\left(\theta_{j,i}, 0\right), 1\right) \\ \delta_{j,i}^3 &= \min\left(\max\left(\theta_{j,i} + 1, 0\right), 1\right) \\ \delta_{j,i}^4 &= \min\left(\max\left(\theta_{j,i} + 2, 0\right), 1\right) \end{aligned} \quad (15)$$

where $\theta_{j,i} \in [-2, 2]$ is the single DP of the SMF. The effect of $\theta_{j,i}$ on the shape of trapezoid SMF is shown in Fig. 3. It can be seen that various SMFs can be generated with the new DP.

As the GT2-FLC is constructed over its baseline T1 and IT2 FLCs, it is possible to calculate the membership grades of each α-T2-FLC as given in (13) and (14) via the ones of the baseline T1 and IT2 FLCs Then, once the corresponding firing intervals of each α-T2-FLC are calculated, the $y_{GT2}^{\alpha_p}$ outputs can be obtained to calculate the crisp output y_{GT2} given in (12).

In the design of GT2-FLCs, we assume that the baseline tuning parameters (primary MFs of antecedent IT2-FSs and consequent MFs) and structural settings (rule base, aggregation / union operators, type-reduction) are set and fixed. Thus, the DPs of GT2-FLCs are the total number of α-planes (P) and the DPs of the trapezoid SMFs.

It can be observed from (13-14) that the membership degrees of α-T2-FLC are defined with respect to the shape of the SMFs which directly influences the resulting value of $y_{GT2}^{\alpha_p}$. Therefore, we suggest handling the DPs $\left(\theta_{j,i}\right)$ that define the shape of the SMF as the main DPs of the GT2-FLCs.

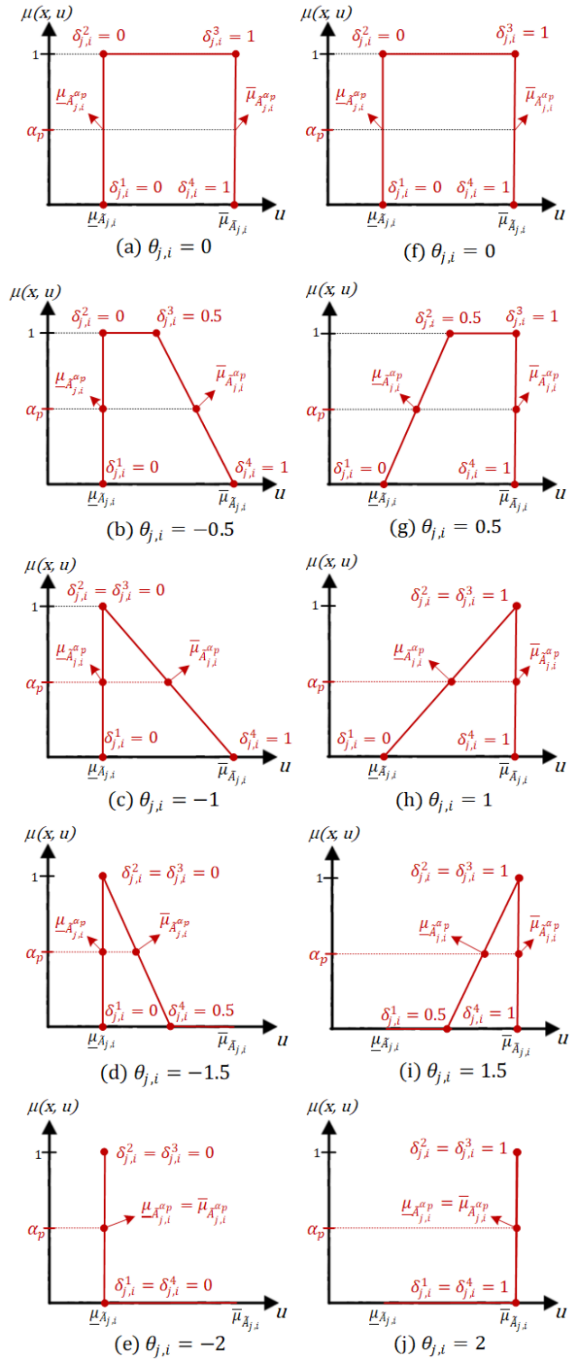

Fig. 3. Illustration of the trapezoid SMF with respect to θ value

We prefer to define the same SMF for each antecedent GT2-FS, and thus the design of the GT2-FLC is accomplished with a single parameter θ $\left(\theta_{j,i} = \theta \ \forall i, j\right)$. Note that, in the rest of the paper, we set and fix P as $P = 10$ since we focus only on the effect of the DP of the SMF in this study.

III. Control Surface Analyses of GT2-FLCs

In this section, we investigate the effect of the DP (θ) on the CS generation of GT2-FLCs. Let us firstly show how the structure of GT2-FLC changes with respect to the DP θ. It can be observed from:

- Fig. 3a that the trapezoid SMF reduces to an interval set as

$\underline{\mu}_{\tilde{A}_{j,i}^{\alpha_p}} = \underline{\mu}_{\tilde{A}_{j,i}^{\alpha_0}}$ and $\overline{\mu}_{\tilde{A}_{j,i}^{\alpha_p}} = \overline{\mu}_{\tilde{A}_{j,i}^{\alpha_0}}, \forall \alpha_p$ for $\theta = 0$. Here, the GT2-FLC reduces to its baseline IT2-FLC as follows:

$$y_{GT2}\Big|_{\theta=0} = y_{IT2} \tag{16}$$

- Fig. 3e that the trapezoid SMF reduces to a crisp set, since $\underline{\mu}_{\tilde{A}_{j,i}^{\alpha_p}} = \overline{\mu}_{\tilde{A}_{j,i}^{\alpha_p}} = \underline{\mu}_{\tilde{A}_{j,i}^{\alpha_0}}, \forall \alpha_p$ for $\theta = -2$. Here, the GT2-FLC transforms to a T1-FLC that only uses the LMFs of the IT2-FSs as follows:

$$y_{GT2}\Big|_{\theta=-2} = \frac{\sum_{n=1}^{N} \underline{f_n} C_n}{\sum_{n=1}^{N} \underline{f_n}} \tag{17}$$

- Fig. 3j that the trapezoid SMF reduces to a crisp set, since $\underline{\mu}_{\tilde{A}_{j,i}^{\alpha_p}} = \overline{\mu}_{\tilde{A}_{j,i}^{\alpha_p}} = \overline{\mu}_{\tilde{A}_{j,i}^{\alpha_0}} \; \forall \alpha_p$ for $\theta = 2$. Here, the GT2-FLC reduces to a T1-FLC that only uses the UMFs of the IT2-FSs, which is the baseline T1-FLC as follows:

$$y_{GT2}\Big|_{\theta=2} = y_{T1} \tag{18}$$

It can be concluded that by simply varying θ various T1 and IT2 FLCs can be obtained.

Now, let us analyze the CSs of GT2-FLCs for $\theta \in \{-2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2\}$ that results in the SMFs depicted in Fig. 3. As the GT2-FLC is constructed over its baseline T1 and IT2 FLCs, we firstly need to define the T1 and IT2 FLCs. We defined the T1-FLC with an aggressive CS (as given in Table I) and the IT2-FLC with a smooth CS by setting its FOU DPs as $M_{1,1} = 0.2$, $M_{1,2} = 0.9$, $M_{1,3} = 0.2$, $M_{2,1} = 0.2$, $M_{2,2} = 0.9$, and $M_{2,3} = 0.2$. The design of IT2-FLC is accomplished via the guidelines presented in [12-14].

The resulting CSs of the GT2-FLCs for various θ values are given in Fig. 4. However, since the analyses of the CSs given in Fig. 4 is not straightforward, we also present the differences between the CSs ($y_{GT2} - y_{T1}$ and $y_{GT2} - y_{IT2}$) by varying the DP in Fig. 5 and Fig. 6, respectively.

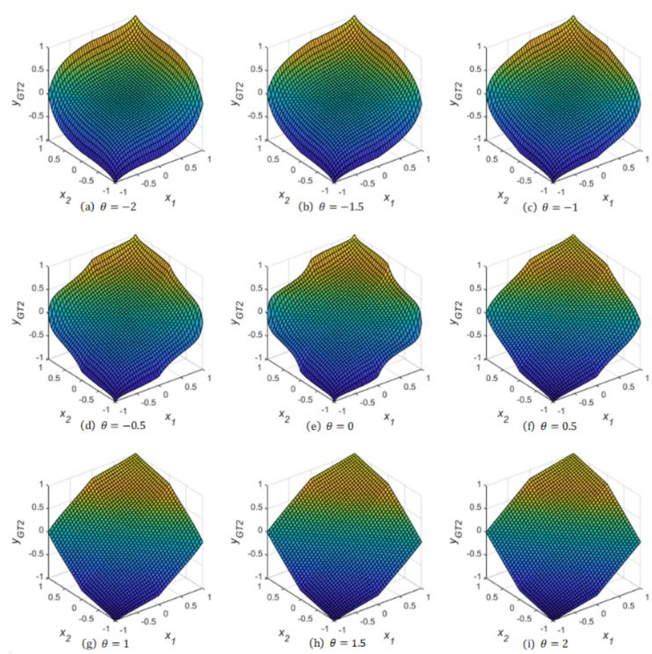

Fig. 4 Illustration of the resulting CSs of the GT2-FLCs

It can be clearly seen that the DP (θ), which defines the shape of the SMF, also defines the resulting shape of the CS in terms of aggressiveness and smoothness. It can be observed that, as the value of θ decreases from 2 to -2, the resulting CS becomes smoother, thus a potentially more robust controller can be obtained. When the value of θ increases from 0 to 2, the resulting CS transforms from its smooth baseline IT2-FLC into its aggressive baseline T1-FLC. Thus, tuning the DP (θ) might be an efficient way to design GT2-FLCs as the properties of the baseline FLCs can be preserved. It is worth to emphasize that the DP $\theta \in [-2, 2]$ provides not only design simplicity as only baseline T1 and IT2 FLCs are needed, but also a convenient design flexibility, as various CSs can be generated by tuning θ.

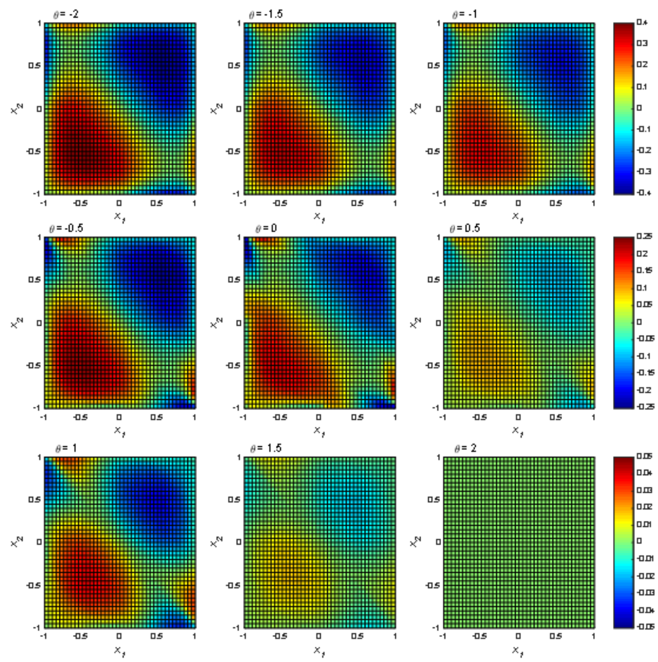

Fig. 5. Illustration of the differences between CSs: $y_{GT2} - y_{T1}$

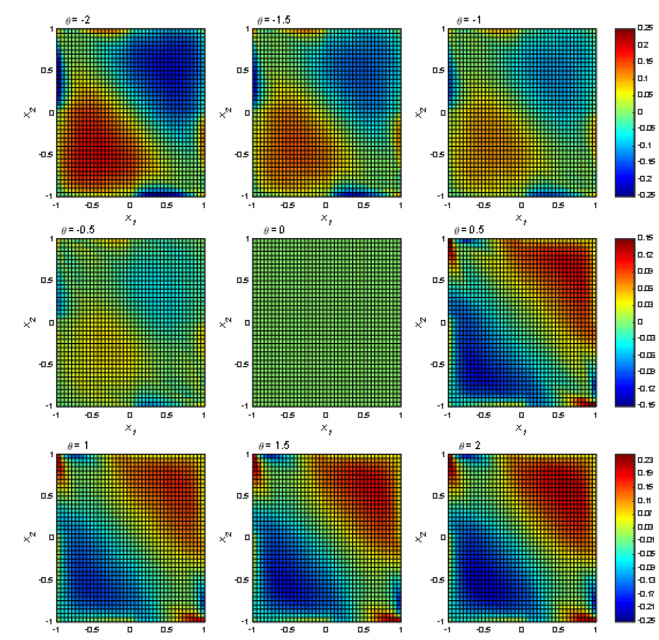

Fig. 6 Illustration of the differences between CSs: $y_{GT2} - y_{IT2}$

IV. Design of the CSs of GT2-FLCs

In the design of the GT2-FLCs, as it is usually done in the fuzzy control literature [4, 6, 9], we suggest first designing of baseline T1 and IT2 FLCs with aggressive and smooth CSs, respectively and then tuning the DP of GT2-FLC.

The following observations can be employed in tuning the DP(θ) of GT2-FLC:

- Decreasing θ from 2 to 0, converts the CS of GT2-FLC from its T1-FLC baseline to its IT2-FLC counterpart. Thus, the control performance and robustness of GT2-FLC will potentially lie between the baseline T1-FLC and IT2-FLC.
- Decreasing θ from 0 to -2, the CS of GT2-FLC transforms from its baseline IT2-FLC counterpart to a T1-FLC that only uses the LMFs as given in (17). Thus, designing a GT2-FLC with a $\theta \in [-2,0)$ gives the opportunity to design more robust GT2-FLCs than its baseline IT2-FLC.

We suggest one to tune the DP (θ) by providing a tradeoff between robustness (i.e. like IT2-FLC) and performance (i.e. like T1-FLC).

The resulting control system performance improvements of the GT2-FLCs for a fixed θ value heavily depends on the OP in which the system is controlled, especially for nonlinear systems. The control system performance might be optimal/ satisfactory at a particular OP where the GT2-FLC is tuned, yet its performance might degrade for other OPs. This is due to the fact that the dynamics of nonlinear systems might change with respect to OPs. This problem is usually solved with gain-scheduled controllers that basically schedule a collection of controllers, designed at various OPs, with respect to OPs [24]. Thus, we propose the following SM for GT2-FLCs to tune the DP with respect to steady-state OPs as follows:

$$\theta_r = f_r(r) \tag{19}$$

where $f_r(r)$ defines a linear interpolation that calculates the value of $\theta = \theta_r$ with respect to the steady-state OP (r). The design of the online SM is accomplished by defining DP ($\boldsymbol{\theta_r} = \{\theta_1, \theta_2, \dots \theta_k\}$) for each steady state OP ($\boldsymbol{r} = \{r_1, r_2, \dots r_k\}$).

V. Simulation Results

Here, we present comparative simulation results to show the efficiency of the proposed design approaches and online SM. We handled the following nonlinear benchmark system [25]:

$$d^2y/dt^2 + dy/dt + 0.25\, y^2 = u(t-0.5) \tag{20}$$

It can be interpreted from (20) that the system gain varies with respect to the output of the system (y), and thus the nonlinear process results with different dynamics at different OPs. Thus, we took account 4 steady state OPs $\boldsymbol{r} = \{0.6, 0.8, 1, 1.2\}$. To analyze the resulting performances of the FLCs, we provide the following control system performance measures; Settling Time (T_s), Rise Time (T_r) and Overshoot ($OS\%$).

In the design of the T1 and IT2 FLCs, we followed the design guidelines/suggestions presented in [13, 14] in terms of T_s, T_r, and $OS\%$. Hence, the SFs of PID type FLCs were handled as the DPs of the T1-FLC and set as $K_e = 1$, $K_{\text{d}} = 0.8$, $K_a = 0.15$ and $K_b = 0.12$ with respect to the OP change from r_3 to r_4. The IT2-FLC was designed for the OP change from r_1 to r_2 and its FOU DPs were set as $M_{1,1}$=0.02, $M_{1,2}$=0.9, $M_{1,3}$=0.02, $M_{2,2}$=0.3, $M_{2,2}$=0.8, and $M_{2,3}$=0.3.

We designed two GT2-FLCs; one with a fixed DP value and one with the online SM. In the design of the GT2-FLC with a fixed DP, we took account the OP variation from r_2 to r_3 and the fixed DP is selected as $\theta = 0.1$ to reduce T_s and T_r with an acceptable compromise of $OS\%$. In the design of GT2-FLC with the SM (GT2-FLC-SM), we defined the DP values as $\boldsymbol{\theta_r} = \{-1.2, -0.8, 0.1, 0.4\}$ for $\boldsymbol{r}$.

The comparative simulation results are given in Fig. 7 and the performance measures are tabulated in Table II. The results clearly demonstrated that an acceptable tradeoff between performance and robustness can be achieved by simply tuning the DP θ. Moreover, the proposed SM provides an opportunity to enhance the overall control performance since the DP (θ) is tuned in online manner with respect to the OP. For example, the GT2-FLC-SM resulted in a robust system response for the OP r_1, as it achieved the lowest $OS\%$ value without oscillations but the highest T_r value at the same time. For the OP r_4, the SM tuned the θ value from 0.1 to 0.4 to increase the aggressiveness of CSs which speeds up the system response. Thus, as given in Table II, the resulting T_r value is smaller than its IT2 and GT2 fuzzy counterparts. For the OP r_2 where the baseline IT2-FLC was designed, the GT2-FLC-SM was also able to reduce the T_s and OS% values by decreasing θ value to -0.8. It is concluded that the proposed online SM is an effective way to tune the DP with respect to OPs to enhance the control system performance.

VI. Conclusion

In this paper, we presented a systematic design approach for GT2-FLCs and an online SM to enhance the performance of the GT2-FLC. We suggested constructing the GT2-FLC over its baseline T1 and IT2 FLCs and to tune only a single DP which defines the shape and size of its SMFs. We first investigated the effect of the DP of the GT2-FLC on the CS generation and showed that both aggressive and smooth CSs can be generated by tuning the DP θ. We suggested to accomplish the GT2-FLC design by providing an acceptable tradeoff between robustness and performance. We also proposed an online SM that tunes the DP in online manner in order to end up with satisfactory control performances in various OPs. The simulation results clearly showed that the proposed design approach and the online SM are highly efficient to design GT2-FLCs.

TABLE II. The Performance Measures of the T1, IT2 and GT2 FLCs

	OP-1			OP-2			OP-3			OP-4		
	T_r (s)	***T_s (s)***	***OS%***	***T_r***	***T_s (s)***	***OS%***	***T_r (s)***	***T_s (s)***	***OS%***	***T_r (s)***	***T_s (s)***	***OS%***
T1-FLC	1.538	14.98	34.35	1.587	11.78	25.90	1.648	10.90	18.05	1.721	8.256	10.99
IT2-FLC	1.962	10.98	21.67	2.095	9.967	11.75	2.268	9.456	3.380	2.552	9.490	0.0
GT2-FLC	1.776	10.86	26.38	1.869	9.916	16.37	1.983	9.335	7.915	2.143	9.212	0.514
GT2-FLC-SM	2.221	12.96	15.08	2.189	7.484	8.196	1.983	9.335	7.915	1.863	8.880	6.241

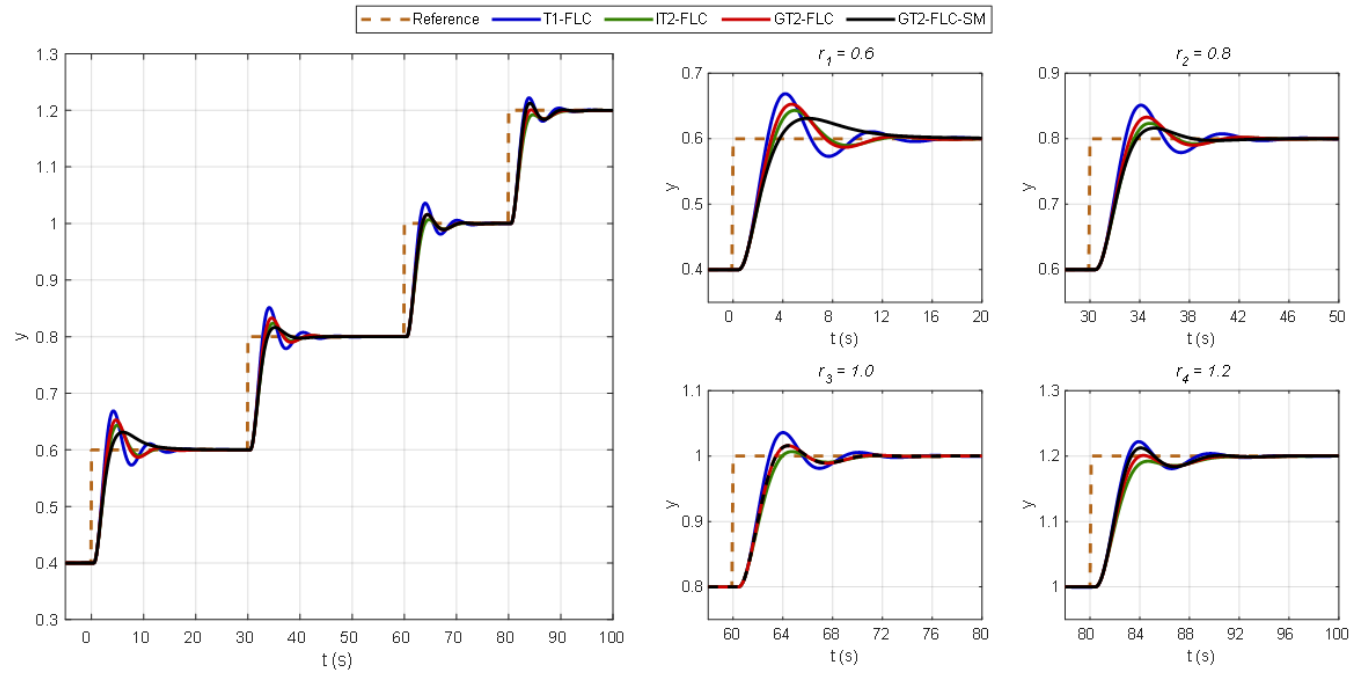

Fig. 7 Illustration of the performances of the PID type T1-FLC, IT2-FLC, GT2-FLC and GT2-FLC-SM

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