An interpretable fuzzy system in the on-line signature scalable verification

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***Abstract*—This paper proposes new original solutions for the use of interpretable flexible fuzzy systems for identity verification based on an on-line signature. Such solutions must be scalable because the verification of the identity of each user must be carried out independently of one another. In addition, a large number of system users limit the possibilities of iterative system learning. An important issue is the ability to interpret the system rules because it explains how the similarity of test signatures to reference signature templates is assessed. In this paper, we propose an approach that meets all of the above requirements and works effectively for the on-line signatures' database used in the simulations.**

***Index Terms*—flexible fuzzy system, interpretability, identity verification, on-line signature, IT systems security**

I. Introduction

On-line signature is a biometric attribute which is commonly used to verify the identity of an individual. It is acquired by using a digital input device, e.g. a graphic tablet. This kind of signature is described by signals which tend to change over time and which contain characteristic information about dynamics of the signing process, and can be very useful in the verification phase. Moreover, this biometric attribute is commonly accepted in the society, and hence it can be used in many areas of life.

Unfortunately, identity verification using on-line signature is a difficult process. This is due to the fact that the signatures of an individual are characterized by relatively high intra-class variance. Moreover, in the learning phase of the classifier false signatures created by qualified forgers (so-called skilled forgeries) are not available. We also cannot use genuine signatures of other users as forgeries because they differ too much from skilled forgeries and in this case, accuracy of the classifier would be too low. In addition, a large number of system users limits the possibilities of iterative learning. Due to this, we propose the use of an interpretable fuzzy system for on-line signature scalable verification. The proposed system is a one-class classifier and does not require the use of forged signatures in the learning phase.

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In the literature, we can find three main groups of methods for the on-line signature verification - global, local and regional ones [7], [9], [16]. In this paper we focus on the regional approach based on descriptors created in regions (partitions) of the signature. The descriptors are determined individually for each user in the learning phase of the system and on that basis a proper signature template is created. During the test phase, descriptors created from the test signature are compared to the template and identity verification is performed.

A. Motivation

The problem of the on-line signature verification is specific. This is primarily due to the fact that each user must be analyzed individually. Such analysis is performed in the so-called learning phase but forged signatures are not available in this phase. Due to the scalability of the approach, such analysis also should not include signatures of other users because in this case, the accuracy of the approach would be dependent on the number of signatures in the database. For this reason, using a properly designed fuzzy system to assess the similarity of on-line signatures seems to be a very interesting idea. The system is a non-linear one-class classifier. This means that in both the learning and testing phases, it does not take into account information about other users' signatures. The use of these signatures would definitely simplify the problem considered in this paper and allow the use of known classification methods and learning algorithms of fuzzy systems (e.g. gradient or population-based algorithms [6], [20], [24]). However, it would definitely limit the practical use of the proposed method. The assumptions adopted in this work are widely accepted in biometric applications because they allow for easy expansion of new users and new signatures in the biometric system (they ensure scalability). Our previous experience with such a system confirms this thesis (see e.g. [27]).

The solutions proposed in this paper have been developed on the basis of our experience regarding: dynamic signature verification (DSV), fuzzy systems (FSs) and their combinations. First of all, in our previous works [27]–[29] we did not choose the number of fuzzy rules in the system used for identity verification for each user. This is important from the point of view of the rule base interpretability, as it allows

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minimizing the number of rules and adjusting it individually to the specifics of the individual signatures of each user.

Secondly, we used a modified defuzzification formula in the fuzzy system for identity verification. This formula makes the number of rules in the rule base independent of the number of discretization points of the defuzzification formula. We proposed and used such solutions in our previous works [5], but we did not use them in the problem of identity verification.

B. Novel elements of the proposed approach

The elements of novelty described in this paper can be summarized as follows:

- We propose the use of a fuzzy system for assessing the similarity of test signatures to reference signatures acquired in the learning phase. It is performed independently for each user and scalable-independent of the number of users. The fuzzy system does not require learning, its rule base is selected taking into account the criteria of interpretability. The novel element proposed in this paper is the possibility of individual selection of the number of fuzzy system rules for each user. This solution has not been considered in our papers so far. At the same time, it is important that the architecture and method of determining the rule base of the system are consistent for all users, which greatly facilitates the implementation of the algorithm and the analysis of how the system works.
- We propose the use of a modified defuzzification formula in the fuzzy system to assess the similarity of test signatures to reference signatures. In this formula, the number of discretization points of a fuzzy inference from the rule base does not have to be equal to the number of fuzzy rules, as in the case of e.g. the standard center of area formula [23]. We used this approach in our previous works on fuzzy systems and their interpretability [3], [5], but it was not considered in relation to the specific problem of identity verification.
- We propose a clear, detailed and corrected notation of the identity verification algorithm based on the on-line signature. One of the features of this algorithm is that it divides each signature into P parts. Each part corresponds to a different time moment of signing. For example, in the case of $P = 3$, these parts correspond to the initial, middle and final moment of signing. Evaluation of the consistency of the signing process in these time moments allows us to determine their weights of importance. These weights are taken into account when comparing the shape of the test signature to the reference signatures - those parts of the test signature in which the user signs in a more stable way (with similar dynamics) are more important. Such notation has not been presented in our previous works.

Summary of main characteristics of the algorithms for the on-line signature verification based on the regional approach is presented in Table I. These features are denoted as follows: **f1** - Does the method divide the signature into the parts in order to increase the efficiency of signature verification accuracy? **f2** - Does the method focus on fast performance? **f3** - Does the method evaluate the stability of the signature in selected parts of the signature? **f4** - Does the method take into account the hierarchy of selected parts of the signature in the classification process? **f5** - Is the way of classification interpretable? **f6** - Does the method select the most important parts of the signature and does it verify the test signatures on their basis? **f7** - Does the method select the number of fuzzy rules of the system to assess the similarity of the signatures?

TABLE I
MAIN CHARACTERISTICS OF THE ALGORITHMS FOR THE ON-LINE SIGNATURE VERIFICATION BASED ON THE REGIONAL APPROACH.

Characteristics of the method	**f1**	**f2**	**f3**	**f4**	**f5**	**f6**	**f7**
Khan et al. [18]	**yes**	no	**yes**	no	no	no	no
Ibrahim et al. [16]	**yes**	no	**yes**	no	no	no	no
Fierrez et al. [10]	**yes**	no	no	no	no	no	no
Huang and Hong [15]	**yes**	no	**yes**	**yes**	no	no	no
Faúndez-Zanuy and Pascual-Gaspar [8]	**yes**	**yes**	no	no	no	no	no
Pascual-Gaspar et al. [21]	**yes**	**yes**	no	no	no	no	no
Cpałka and Zalasiński [4]	**yes**	no	**yes**	**yes**	**yes**	**yes**	no
Zalasiński and Cpałka [27]	**yes**	no	**yes**	**yes**	**yes**	**yes**	no
our method	**yes**	no	**yes**	**yes**	**yes**	**yes**	**yes**

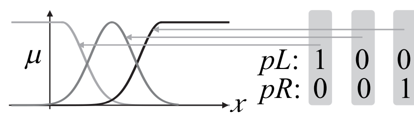

Fig. 1. Adopted interpretation of parameters $\{pL, pR\}$ of the Gaussian membership function (1).

C. Structure of the paper

The paper consists of five sections. Section II contains description of FSs for on-line signature verification. Section III presents a detailed description of the proposed algorithm. The simulations are described in Section IV. Finally, the conclusions are drawn and presented in Section V.

II. DESCRIPTION OF THE FS FOR ON-LINE SIGNATURES VERIFICATION

In this paper, we consider a Mamdani-type flexible fuzzy system [2]. The information on the implementation of its individual components is presented in Sections II-A - II-C.

A. Notation of fuzzy sets

Fuzzy sets of different types can be used in FSs. In this paper, we focus on the Gaussian sets. In our previous works we analyzed in detail interpretability criteria of fuzzy systems with the Gaussian function [5], which are briefly summarized in Section II-D.

A typical notation of the Gaussian function [23] does not allow us, for example, to model the case in which the value of the membership function above a certain limit value of a linguistic variable should have a constant value equal to 1. In this case, a different membership function can be used, e.g. the sigmoid-type. However, in this case, the consistency of the membership function type is not maintained and interpretation can be more difficult [5], [12]. Therefore, in this paper, we used a modified notation of the Gaussian function:

$$\mu_A\left(x, \overline{xA}, \overline{\sigma A}, pL, pR\right) = \max\left\{\begin{array}{c} \operatorname{sgn}\left(-x+\overline{xA}\right)-(1-pL), \\ \exp\left(-\left(\frac{x-\overline{xA}}{\overline{\sigma A}}\right)^2\right), \\ \operatorname{sgn}\left(+x-\overline{xA}\right)-(1-pR) \end{array}\right\}, \tag{1}$$

where $\left\{\overline{xA}, \overline{\sigma A}\right\}$ are the middle and width of the Gaussian function, and $\{pL, pR\}$ are the parameters indicating the way of saturating the function (see Fig. 1).

B. Notation of fuzzy rules

The considered FS works on the basis of the $nRules$ fuzzy rules $\{rule_1, rule_2, \ldots rule_{nRules}\}$. Each $rule_m$ has the following form:

$$rule_{(m)}: \left[\begin{array}{c} \textbf{IF}\left(\begin{array}{c} x_{p=1} \textbf{ is } A_{p=1,m} \\ \textbf{with } w_{p=1} \end{array}\right) \textbf{AND} \\ \left(\begin{array}{c} x_{p=2} \textbf{ is } A_{p=2,m} \\ \textbf{with } w_{p=2} \end{array}\right) \textbf{AND} \ldots \\ \left(\begin{array}{c} x_{p=P} \textbf{ is } A_{p=P,m} \\ \textbf{with } w_{p=P} \end{array}\right) \\ \textbf{THEN } (y \textbf{ is } B_m) \end{array}\right], \tag{2}$$

where $x_1, x_2, \ldots x_P$ are the inputs of the system, y is the output of the system, $\left\{A_{p=1,m}, A_{p=2,m}, \ldots, A_{p=P,m}\right\}$ are the input fuzzy sets of rule m, B_m is the output fuzzy set of rule m, $\left\{w_{p=1}, w_{p=2}, \ldots, w_{p=P}\right\}$ are the weights of input fuzzy sets ($w_p^A \in \langle 0, 1\rangle$). Please note that in the standard notation of fuzzy rules, parameters $w_{i,k}^A$ are not available. Their use increases the flexibility of rules notation and introduces the hierarchy of importance of input fuzzy sets. In our previous papers each input fuzzy set has its own weight of importance [4]. In this paper weights are common within the sets associated with the same inputs.

C. Notation of the fuzzy system

In our previous papers we presented different varieties of FSs and ways of deriving dependencies describing their output signals [4], [28], [29]. Since for the system proposed in this paper it is performed analogously, we decided not to present these derivations.

In the case of using the singleton type fuzzification and center of area method for defuzzification [23], the output signal $\bar{y}$ (in the paper we use one output) of the FS working on the basis of rules (2) has the following form:

$$\bar{y} = \frac{\sum\limits_{r=1}^{R} \bar{y}_r^{\text{def}} \cdot \mathop{S}\limits_{m=1}^{nRules}\left(T\left(\begin{array}{c} \tau_m(\bar{\mathbf{x}}), \\ \mu_{B_m}\left(\begin{array}{c} \bar{y}_r^{\text{def}}, \overline{xB}_m, \overline{\sigma B}, \\ pL_m, pR_m \end{array}\right) \end{array}\right)\right)}{\sum\limits_{r=1}^{R} \mathop{S}\limits_{m=1}^{nRules}\left(T\left(\begin{array}{c} \tau_m(\bar{\mathbf{x}}), \\ \mu_{B_m}\left(\begin{array}{c} \bar{y}_r^{\text{def}}, \overline{xB}_m, \overline{\sigma B}, \\ pL_m, pR_m \end{array}\right) \end{array}\right)\right)}, \tag{3}$$

where $\tau_m(\bar{\mathbf{x}})$ is the activation level of rule $rule_m$ determined as follows:

$$\tau_m(\bar{\mathbf{x}}) = \mathop{T^*}\limits_{p=1}^{P}\left(\mu_{A_{p,m}}\left(\begin{array}{c} x_p, \overline{xA}_{p,m}, \overline{\sigma A}_p, \\ pL_m, pR_m \end{array}\right); w_p\right). \tag{4}$$

In formulas (3) and (4) the following notation is used: $\bar{x}_1, \bar{x}_2, \ldots \bar{x}_n$ are the signals given to the inputs of the system, $\{\mu_{A_{p,m}}(\cdot), \mu_{B_m}(\cdot)\}$ are the membership functions of the fuzzy sets (parameter $\overline{\sigma B}$ is common to all rules), R is the number of discretization points of the FS in the defuzzification formula using the center of area method, $\bar{y}_r^{\text{def}}$ ($r = 1, 2, \ldots R$) are the points in which discretization of the fuzzy set being inference from the rule base is performed, $T(\cdot)$ is the t-norm [22] being the inference operator, $T^*(\cdot)$ is the t-norm with weights of arguments [22] being the aggregation operator of rules' predecessors (2), $S(\cdot)$ is the t-conorm being the aggregation operator of fuzzy inferences from rules (2). Operator $T^*(\cdot)$ with weights of arguments which we proposed in paper [22] in order to take into account importance of predecessors in the rule base. These operators were also used for reduction of the rule base [23]. The relationship between the triangular norms and their variants with weights of arguments is as follows:

$$\left\{\begin{array}{l} T^*(\mathbf{a}; \mathbf{w}) = \mathop{T}\limits_{i=1}^{n}\left(S\left(a_i, 1-w_i\right)\right) \\ S^*(\mathbf{a}; \mathbf{w}) = \mathop{S}\limits_{i=1}^{n}\left(T\left(a_i, w_i\right)\right), \end{array}\right. \tag{5}$$

where $a_i \in \langle 0, 1\rangle$ ($i = 1, 2, \ldots n$) are the arguments of operators of form (5), and $w_i \in \langle 0, 1\rangle$ are the weights of the arguments. It is easy to notice that when in dependencies (5), for example, triangular norms of the algebraic type are used, they take the following form:

$$\left\{\begin{array}{l} T^*\{\mathbf{a}; \mathbf{w}\} = \prod\limits_{i=1}^{n}\left(1+(a_i-1)\cdot w_i\right) \\ S^*\{\mathbf{a}; \mathbf{w}\} = 1-\prod\limits_{i=1}^{n}\left(1-a_i \cdot w_i\right). \end{array}\right. \tag{6}$$

The operators of form (5) meet the following conditions, which result from the boundary conditions for triangular norms:

$$\left\{\begin{array}{l} T^*(a_1, a_2; 1, 1) = T(a_1, a_2) \\ T^*(a_1, 0; w_1, w_2) = S(a_1, 1-w_1) \\ S^*(a_1, a_2; 1, 1) = S(a_1, a_2) \\ S^*(a_1, 0; w_1, w_2) = T(a_1, w_1). \end{array}\right. \tag{7}$$

More details about the operators of form (5) and their uses in FSs can be found in our earlier papers [4], [22]. The method of determining parameters of the FS (3) for the on-line signature scalable verification problem is described in Section III.

D. Interpretability of fuzzy system

Interpretability is a feature that makes it easier to understand the rule base (2) and how the FS (3) works. It can be ensured by taking into account the assumptions about the FS already at

TABLE II
SAMPLE CRITERIA FOR ASSESSING THE INTERPRETABILITY OF SYSTEM (3) WITH THE GAUSSIAN FUZZY SETS (1).

No.	Description
1	**The criterion for assessing the complexity of the system** takes into account the total number of fuzzy sets, fuzzy rules and discretization points relative to the maximum value. It assumes that a less complex system is easier to interpret.
2	**The criterion for assessing the location of neighboring fuzzy sets** takes into account intersection points of adjacent sets. It assumes that they should intersect at points x, for which $\mu(x) = 0.5$. This condition can be easily met by appropriate determination of value σ. For two adjacent Gaussian functions with the centers at points x_1 and x_2, this can be done by solving equation $\mu\left(0.5 \cdot (x_1 + x_2), x_1, \sigma\right) = 0.5$. Then, $\sigma = (x_2 - x_1) / \left(2 \cdot \sqrt{\log(2)}\right)$.
3	**The criterion for assessing the coverage of input data by input sets** determines for each set from the learning sequence the sum of its memberships to the input fuzzy sets. It assumes that the determined sum should be equal to 1.
4	**The criterion for assessing the activation of fuzzy rules** determines the activation level of each from $nRules$ rules for each set in the learning sequence. It assumes that only one from $\tau_m(\cdot)$ ($m = 1, 2, \ldots nRules$) should be close to 1, while the others should be close to 0. The rules that meet this condition are easier to analyze.
5	**The criterion for assessing the shape consistency** of fuzzy sets of form (1).

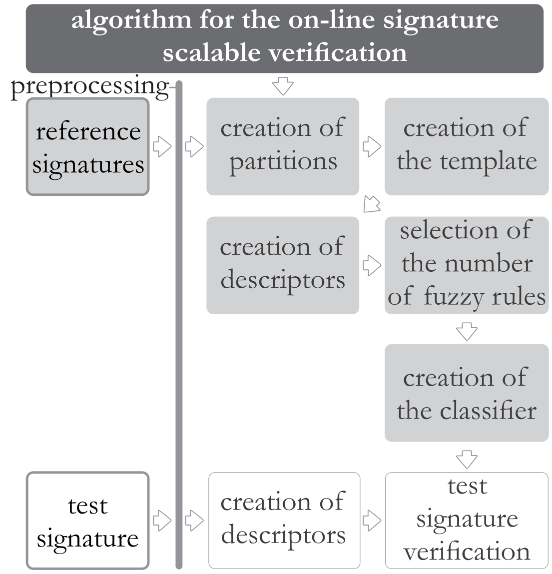

Fig. 2. General schema of the algorithm for the on-line signature scalable verification.

the stage of its construction (learning). In this paper we would like to: (a) ensure the readability of the rule base (fuzzy sets and rules) of system (3), (b) ensure consistency of system (3) components (e.g. unification: the shape of fuzzy sets, etc.), and (c) minimize the structure of system (3) without compromising its performance. In our previous papers [3], [19], [25] we used dedicated criteria to assess the interpretability of system (3). They are used for subjective evaluation of the readability of various system (3) components. For example, if they return a value from the range $\langle 0, 1 \rangle$, then it can be interpreted as follows: 0 means the most favorable value, and 1 - the most unfavorable value. However, the more important goal of the criteria is their use in the design process (selection of structure and parameters) of the fuzzy system (3). The considered criteria are presented in Table II and their detailed implementation can be found e.g. in [5]. We take into account the criteria mentioned in Table II in the application of the interpretable fuzzy system to the problem of scalable on-line signature verification.

III. DETAILED DESCRIPTION OF THE ALGORITHM

The on-line signature partitioning algorithm proposed in this paper uses the notation described in Appendix A. It works in two modes: the learning phase (Section III-A) and the test phase (Section III-B). Algorithms, described in Sections III-A and III-B are presented in Appendix. General idea of the algorithm is shown in Fig. 2.

A. Learning phase

At the beginning of the learning phase user i creates J reference signatures (Alg. 1, line 1) and a parameter describing the tolerance of the verification process is determined (Alg. 1, line 2). Introduction of parameter δ_i allows the algorithm, among others, to adjust the way of its operation to specific fields of its application and to take into account the trend of changes in the signature of each user occurring over time [27].

Next, the base signature with index $jBase$ is selected from all reference signatures (Alg. 1, line 3). It is one of the reference signatures created in the acquisition phase, whose distance from the remaining reference signatures is the smallest according to the adopted distance measure (e.g Euclidean). The remaining reference signatures, in the training phase, (Alg. 1, line 4) and the test signatures, in the test phase, (Alg. 5, line 3) are adjusted to the base signature in the standard normalization procedure [11]. It uses, among others, the Dynamic Time Warping algorithm [1].

After the normalization performed in the training phase, normalized trajectories of the base signature are stored in the database. They are used to normalize the test signatures in the test phase (Alg. 1, line 5).

Next, partitioning of the base signature $jBase$ for a fixed number of vertical sections equal for all the users $P \in [1, 3]$ is performed (Alg. 1, line 6 and Alg. 2). The maximum value of P has been limited because its increase causes excessive decomposition of the signatures and reduces the ability to interpret partitions.

In the partitioning procedure, determination of the vertical sections is performed (Alg. 2, lines 1-6). Variable $\mathbf{pv}_i$ stores indexes of assignment of points included in the base signature (the number of points is K_i) to the respective vertical sections. In turn, variable $\mathbf{kv}_i$ stores the number of points in each vertical section. Partitions are selected taking into account the base signature, which is the most representative of the reference signatures. Other reference signatures use it later in the learning phase.

After partitioning, in algorithm Alg. 1 determination of the shape templates for J reference signatures and parameters of the fuzzy classifier for the evaluation of the similarity of the reference and test signatures is performed (Alg. 1, line 7 and Alg. 4). In the first part of this procedure, the templates of the reference signatures' shapes are determined (Alg. 4, line 6). They are created independently for each signal describing the dynamics of signing (v and z) and each shape trajectory (x and y).

After the determination of the templates, they are compared to the reference signatures. On this basis, Euclidean distances between them are calculated (Alg. 4, lines 8 and 13). Values of these distances are important in assessing user i's way of signing. For the reference signatures that are similar (as they should be), the distance values are close to zero. Then, the tolerance for evaluating the test signatures of the users claiming to be i is small. If the reference signatures are less similar, then the tolerance of evaluating the test signatures of the users claiming to be user i must be higher, although this is unfavorable from the point of view of the verification process effectiveness.

The determined distances, which are a measure of the heterogeneity of the reference signatures, are the basis for determining the parameters of system (3) for assessing the similarity of the test signatures to the reference signatures. They are determined individually for each user. These parameters are the boundaries of inclusion of the reference signatures in partitions (Alg. 4, line 15), weights of importance of partitions (Alg. 4, line 17), parameters of fuzzy sets (Alg. 4, lines: 19, 21, 28, 30), additional parameters of the shapes of fuzzy sets (Alg. 4, lines: 26 and 27; see Fig. 1), and discretization points of system (3) (Alg. 4, lines: 31-33). The number of points R increases the precision of defuzzification by the center of area method and is an algorithm parameter (common for all users).

In order to determine the values of weights, it is necessary to determine the standard deviation for the borders of inclusion of the reference signatures in partitions (Alg. 4, line 16). The value of the weight of importance should be directly proportional to the value of the similarity of the reference signatures shape in the partition. Therefore, small weight values relate to the partitions associated with the signature fragments clearly differing from each other within reference signatures.

After the determination of the system parameters for assessing the similarity of the test signatures to the reference signatures, the parameters are saved to the database for use in the test phase (Alg. 4, line 34).

The last step in the learning phase is the selection of the number of fuzzy rules of system (3) for user i (Alg. 1, line 8) and storing it in the database (Alg. 1, line 9). The procedure for selecting the number of rules is performed in Algorithm 3. At the beginning of this algorithm, the number of rules changes within the allowable range (Alg. 3, line 1). For the number of rules indicated by m, all reference signatures, treated as the test signatures, are used sequentially as input values of the fuzzy system (Alg. 3, line 3). At the same time, system (3) responses are summed (Alg. 3, line 7) and average sum $SumTmp$ is determined (Alg. 3, line 9). The goal of the system is the fuzzy determination of the similarity of the signatures, therefore the number of rules is chosen for which average sum $SumTmp$ has the lowest value (Alg. 3, lines 10-18).

B. Test phase

At the beginning of the test phase, the test signature of the user and information about his/her potential identity are acquired (Alg. 5, line 1). The selected user will be further marked with index i. Next, the reference signatures' parameters of user i are read from the database (Alg. 5, line 2).

Later in the verification phase, the shape and length of the test signature are normalized taking into account the base signature of the user i (Alg. 5, line 3). This is implemented in the same way as in the learning phase. After this step, the test signature is represented by a set of normalized trajectories: $\mathbf{xtst}_i^{\{v\}}$, $\mathbf{ytst}_i^{\{v\}}$, $\mathbf{xtst}_i^{\{z\}}$, and $\mathbf{ytst}_i^{\{z\}}$.

After the normalization, distances $dtst_{i,p}^{\{s,a\}}$ between normalized trajectories of the test signature and templates of the reference signature of user i are determined (Alg. 5, lines 4-14). In practice the distances are used as input values of system (3) in order to assess the similarity of the test signature to the reference signatures. Therefore, the notation of fuzzy rules (2) has the following form:

$$rule_m : \left[\begin{array}{c} \textbf{IF} \left(\begin{array}{c} dtst_{i,p=1}^{\{s=v,a=x\}} \textbf{ is } A_{i,p=1,m}^{\{s=v,a=x\}} \\ \textbf{with } w_{i,p=1}^{\{s=v,a=x\}} \end{array} \right) \textbf{AND} \\ \left(\begin{array}{c} dtst_{i,p=2}^{\{s=v,a=x\}} \textbf{ is } A_{i,p=2,m}^{\{s=v,a=x\}} \\ \textbf{with } w_{i,p=2}^{\{s=v,a=x\}} \end{array} \right) \textbf{AND} \ldots \\ \left(\begin{array}{c} dtst_{i,p=P}^{\{s=z,a=y\}} \textbf{ is } A_{i,p=P,m=nRules}^{\{s=z,a=y\}} \\ \textbf{with } w_{i,p=P}^{\{s=z,a=y\}} \end{array} \right) \\ \textbf{THEN } (y_i \textbf{ is } B_m) \end{array} \right]. \quad (8)$$

Notation (8) also takes into account the parameters determined in Algorithm 4. Formula (3) can be written in the same way.

During the signatures verification (Alg. 5, line 16) coefficient $cth \in [0, 1]$ is used. Its value is common for all users of a biometric system and it is usually close to 0.5 (this value was adopted in the simulations). The use of this coefficient allows us to eliminate disproportions between the FAR and FRR coefficients (see e.g. [26]) and to adapt the system operation to the expectations arising from the area of its application.

IV. Simulations

In this section we present the results of the simulations performed using the authorial test environment implemented in C# programming language. In the simulations we used commercial DS2 Signature database distributed by the BioSecure Association [13]. It contains signatures of 210 users. The signatures were acquired in two sessions using a digitizing tablet. Each session contains 15 genuine signatures and 10 skilled forgeries per person.

A. The course of the simulations

In the simulations the following assumptions were adopted: (a) During the training phase 5 genuine signatures of each signer from session number one were used. During the test phase 10 genuine signatures and 10 forged signatures of each signer from session number two were used. (b) Number of partitions $P = [2, 3, 4]$. (c) Maximum number of rules $nRulesMax = 4$. (d) The test was carried out five times for all signers, taking into account different number of partitions, with the use of randomly chosen training and test signatures. (e) The simulation results were evaluated using FAR (False Acceptance Rate), FRR (False Rejection Rate), and EER (Equal Error Rate), which are commonly known in biometrics [17].

B. Simulation results

We implemented and performed the simulations in accordance with the assumptions presented in IV-A. In Table III we present the results obtained by the method proposed in this paper in comparison to the methods of other authors and our previous methods for the dynamic signature verification based on the regional approach (in this table the results of our method were obtained for $P = 2$). Moreover, we present information about the obtained classification results for each variant of the number of partitions (see Table IV) and the percentage value of the number of users for which a given number of fuzzy rules was selected (see Table V).

TABLE III
COMPARISON OF THE ACCURACY OF DIFFERENT METHODS FOR THE SIGNATURE VERIFICATION FOR THE BIOSECURE DATABASE. THE BEST RESULT IS PRESENTED IN BOLD.

Method	FAR	FRR	EER
Methods of other authors [14]	-	-	3.48-30.13%
Cpałka et al. [4]	3.36%	3.30%	3.33%
Zalasiński, Cpałka [27]	2.77%	3.50%	3.14%
Our method	**2.16%**	**2.50%**	**2.33%**

TABLE IV
COMPARISON OF THE ACCURACY OF OUR METHOD FOR A DIFFERENT NUMBER OF PARTITIONS P. THE BEST RESULT IS GIVEN IN BOLD.

Number of partitions P	FAR	FRR	EER
2	**2.16%**	**2.50%**	**2.33%**
3	3.03%	3.27%	3.15%
4	4.13%	4.67%	4.40%

TABLE V
PERCENTAGE VALUE OF THE NUMBER OF USERS FOR WHICH A GIVEN NUMBER OF FUZZY RULES $nRules$ WAS SELECTED. THE BEST RESULT IN TERMS OF INTERPRETABILITY IS GIVEN IN BOLD.

Number of fuzzy rules $nRules$	Percentage number of users
2	**72.86%**
3	24.76%
4	2.38%

The proposed algorithm can be evaluated as follows:

- It received the highest accuracy when the signature was divided into 2 partitions, associated with the initial and final moments of signing (see Table IV). Moreover, we can see that increasing the number of partitions does not result in increasing of the method accuracy.
- It received the highest accuracy in comparison to the methods of other authors and our previous methods based on signature partitioning (see Table III). In this case, the key role was played by the mechanism for selecting the number of fuzzy rules, which has not been implemented previously.
- It most often selects 2 fuzzy rules for the flexible neuro-fuzzy one-class classifier (see Table V); however, for some users a different number of rules is selected. It can mean that some users are characterized by a special way of signing, which can be better analyzed when taking into account a larger number of fuzzy rules. Moreover, increasing the number of rules should not increase the accuracy of the verification because 4 rules were selected only for 2.38% of the users in the database. A small number of the selected rules also simplifies interpretability of the system.

C. Weaknesses of the proposed approach

The weaknesses of the proposed algorithm are: a) its sensitivity to changes of a handwritten signature occurring over a very long period (this is a characteristic of most biometric methods based on the behavioural attributesits), b) dependence of its accuracy on the number of genuine signatures available at the stage of the learning phase, c) greater number of parameters stored in the database in comparison to our previous methods based on signature partitioning.

V. CONCLUSIONS

In this paper we have proposed an interpretable flexible fuzzy system for on-line signature scalable verification. The scalability of our solution makes the verification process for each user independent from one another. The proposed system is a type of a one-class classifier, so the use of forged signatures is not necessary in the learning phase of the system. The use of a novel mechanism for selection of the number of the fuzzy rules increases the accuracy of the proposed classifier in comparison to other methods presented in the literature.

Our future plan includes implementation of this mechanism in the classifier determined on the basis of the predicted values of signature descriptors.

APPENDIX

A. Adopted notation

The algorithm proposed in this paper is divided into four parts: Alg. 1 - Alg. 5. The following variables are used in its notation:

- $\mathbf{x}_{i,j=jBase} = [x_{i,j=jBase,1}, \ldots, x_{i,j=jBase,K_i}]$ and $\mathbf{y}_{i,j=jBase} = [y_{i,j=jBase,1}, \ldots, y_{i,j=jBase,K_i}]$ - normalized trajectories describing the shape of the reference base signature of user i; j is an index of the signature, $jBase$

is an index of the base signature, K_i is the number of the base signature discretization points.

- $\mathbf{v}_{i,j=jBase} = [v_{i,j=jBase,1}, \ldots, v_{i,j=jBase,K_i}]$ and $\mathbf{z}_{i,j=jBase} = [z_{i,j=jBase,1}, \ldots, z_{i,j=jBase,K_i}]$ - normalized signals describing the dynamics of the reference base signature of user i (pen velocity v and pen pressure z).
- $\mathbf{X}_i^{\{v\}} = [\mathbf{x}_{i,1}^{\{v\}}, \ldots, \mathbf{x}_{i,J}^{\{v\}}]$ ($\mathbf{x}_{i,j}^{\{v\}} = [x_{i,j,1}^{\{v\}}, \ldots, x_{i,j,K_i}^{\{v\}}]$), $\mathbf{Y}_i^{\{v\}} = [\mathbf{y}_{i,1}^{\{v\}}, \ldots, \mathbf{y}_{i,J}^{\{v\}}]$ ($\mathbf{y}_{i,j}^{\{v\}} = [y_{i,j,1}^{\{v\}}, \ldots, y_{i,j,K_i}^{\{v\}}]$), $\mathbf{X}_i^{\{z\}}$, and $\mathbf{Y}_i^{\{z\}}$ - trajectories describing shape of the reference signatures of user i normalized on the basis of his/her base signature $jBase$ (signals $\mathbf{x}_{i,j=jBase}$, $\mathbf{y}_{i,j=jBase}$, $\mathbf{v}_{i,j=jBase}$, and $\mathbf{z}_{i,j=jBase}$).
- $\mathbf{xtst}_i^{\{v\}} = [xtst_{i,1}^{\{v\}}, \ldots, xtst_{i,K_i}^{\{v\}}]$, $\mathbf{ytst}_i^{\{v\}} = [ytst_{i,1}^{\{v\}}, \ldots, ytst_{i,K_i}^{\{v\}}]$, $\mathbf{xtst}_i^{\{z\}}$, and $\mathbf{ytst}_i^{\{z\}}$ - trajectories describing the shape of the test signature of the user claiming to be user i normalized on the basis of base signature $jBase$ of user i (signals $\mathbf{x}_{i,j=jBase}$, $\mathbf{y}_{i,j=jBase}$, $\mathbf{v}_{i,j=jBase}$, and $\mathbf{z}_{i,j=jBase}$).
- $\mathbf{pv}_i = [pv_{i,1}, \ldots, pv_{i,K_i}]$ - indicators of the shape trajectory membership to vertical sections.
- $\mathbf{kv}_i = [kv_{i,1}, \ldots, kv_{i,P}]$ - the number of points in the vertical sections; P is the number of vertical sections.
- $\mathbf{tc}_i^{\{s,a\}} = [tc_{i,1}^{\{s,a\}}, \ldots, tc_{i,K_i}^{\{s,a\}}]$ - shape templates of the reference signatures determined for normalized shape trajectories $a \in \{x, y\}$, where x is a horizontal shape trajectory and y is a vertical shape trajectory.
- $\mathbf{d}_{i,j}^{\{s,a\}} = [d_{i,j,1}^{\{s,a\}}, \ldots, d_{i,j,P}^{\{s,a\}}]$ - descriptors of reference signature j of user i.
- $\mathbf{sd}_i^{\{s,a\}} = [sd_{i,1}^{\{s,a\}}, \ldots, sd_{i,P}^{\{s,a\}}]$ - values of the standard deviation of the descriptors of the reference signatures of user i.
- $\mathbf{w}_i^{\{s,a\}} = [w_{i,1}^{\{s,a\}}, \ldots, w_{i,P}^{\{s,a\}}]$ - weights of partitions.
- $\mathbf{dmax}_i^{\{s,a\}} = [dmax_{i,1}^{\{s,a\}}, \ldots, dmax_{i,P}^{\{s,a\}}]$ - boundaries of inclusion of the reference signatures in partitions.
- $\overline{\mathbf{XA}}_{i,p}^{\{s,a\}} = [\overline{\mathbf{xA}}_{i,1}^{\{s,a\}}, \ldots, \overline{\mathbf{xA}}_{i,P}^{\{s,a\}}]$ ($\overline{\mathbf{xA}}_{i,p}^{\{s,a\}} = [\overline{xA}_{i,p,1}^{\{s,a\}}, \ldots, \overline{xA}_{i,p,nRules_i}^{\{s,a\}}]$) - parameters of the centers of the input Gaussian fuzzy sets of the system to assess the similarity of the signature of the user claiming to be user i to the reference signatures of user i.
- $\overline{\sigma\mathbf{A}}_i^{\{s,a\}} = [\overline{\sigma A}_{i,1}^{\{s,a\}}, \ldots, \overline{\sigma A}_{i,P}^{\{s,a\}}]$ - parameters of the widths of the input Gaussian fuzzy sets of the system to assess the similarity of the signature of the user claiming to be user i to the reference signatures of user i.
- $\overline{\mathbf{xB}} = [\overline{xB}_1, \ldots, \overline{xB}_{nRules_i}]$ - parameters of the centers of the output Gaussian fuzzy sets.
- $\overline{\sigma B}$-parameters of the widths of the output Gaussian fuzzy sets.
- $\mathbf{pL} = [pL_1, \ldots, pL_{nRules_i}]$ and $\mathbf{pR} = [pR_1, \ldots, pR_{nRules_i}]$ - parameters indicating the way of saturating the Gaussian function (Fig. 1)

B. Algorithms used in the learning phase

Algorithm 1 Learning phase for user i

get $J \geq 1$ reference signatures
get parameter $\delta_i > 0$ describing the tolerance of the verification process
determine the base signature (determine $jBase \in [1, J]$) represented by reference trajectories $\mathbf{v}_{i,j=jBase}$ and $\mathbf{z}_{i,j=jBase}$
normalize the shape and length J of the reference signatures of user i on the basis of his/her base signature $jBase$ (trajectories $\mathbf{x}_{i,j=jBase}$, $\mathbf{y}_{i,j=jBase}$, $\mathbf{v}_{i,j=jBase}$, and $\mathbf{z}_{i,j=jBase}$) - determine $\mathbf{X}_i^{\{v\}}$, $\mathbf{Y}_i^{\{v\}}$, $\mathbf{X}_i^{\{z\}}$, and $\mathbf{Y}_i^{\{z\}}$
store the reference trajectories of base signature $jBase$: $\mathbf{x}_{i,j=jBase}$, $\mathbf{y}_{i,j=jBase}$, $\mathbf{v}_{i,j=jBase}$, and $\mathbf{z}_{i,j=jBase}$
perform partitioning of base signature $jBase$ for P vertical sections (Algorithm 2)
determine the shape templates for reference signatures J and parameters of the fuzzy classifier for evaluating the similarity of the signatures (Algorithm 4)
select the number of rules $nRules_i \in \{2, 3, \ldots, nRulesMax\}$ of system (3) for user i (Algorithm 3)
save the number of rules $nRules_i$ of system (3) for user i

Algorithm 2 Partitioning of base signature $jBase$ for vertical sections P for user i

$\mathbf{kv}_i := \mathbf{0}$
for $k := 1$ **to** K_i **do**
$$pv_{i,k} := \begin{cases} 1 \text{ for } 0 < k \leq \text{int}\left(\frac{K_i}{P}\right) \\ 2 \text{ for int}\left(\frac{K_i}{P}\right) < k \leq \text{int}\left(\frac{2 \cdot K_i}{P}\right) \\ \vdots \\ P \text{ for int}\left(\frac{(P-1) \cdot K_i}{P}\right) < k \leq K_i \end{cases}$$
$kv_{i,p=pv_{i,k}} += 1$
end for k
store in the database: $\mathbf{pv}_i$, $\mathbf{kv}_i$

Algorithm 3 Selection of the number of rules $nRules_i \in \{2, 3, \ldots, nRulesMax\}$ for system (3) for user i

for $m := 2$ **to** $nRulesMax$ **do** $\triangleright$ selection of $nRules_i$
$SumTmp := 0$
for $j := 1$ **to** J **do**
$\mathbf{xtst}_i^{\{v\}} := \mathbf{x}_{i,j}^{\{v\}}$; $\mathbf{ytst}_i^{\{v\}} := \mathbf{y}_{i,j}^{\{v\}}$
$\mathbf{xtst}_i^{\{z\}} := \mathbf{x}_{i,j}^{\{z\}}$; $\mathbf{ytst}_i^{\{z\}} := \mathbf{y}_{i,j}^{\{z\}}$
determine $\bar{y}_i$ of fuzzy system (3) (Algorithm 5)
$SumTmp += \bar{y}_i$ $\triangleright$ eq. (3)
end for j
$SumTmp := \frac{1}{J} \cdot SumTmp$
if $m == 2$ **then**
$SumTmpMax := SumTmp$
$nRules_i := m$
else
if $SumTmp > SumTmpMax$ **then**
$SumTmpMax := SumTmp$
$nRules_i := m$
end if
end if
end for m

Algorithm 4 Determination of the templates of the reference signatures' shape and parameters of fuzzy classifier (3) used for evaluating similarity of the signatures for user i

for each a **in** $\{x, y\}$ **do**
for each s **in** $\{v, z\}$ **do**
$\mathbf{tc}_i^{\{s,a\}} := \mathbf{0}$
$\mathbf{d}_{i,j}^{\{s,a\}} := \mathbf{0}$; $\mathbf{sd}_i^{\{s,a\}} := \mathbf{0}$; $\mathbf{dmax}_i^{\{s,a\}} := \mathbf{0}$
for $k := 1$ **to** K_i **do**
$tc_{i,k}^{\{s,a\}} := \frac{1}{J} \cdot \sum_{j:=1}^{J} a_{i,j,k}^{\{s\}}$
for $j := 1$ **to** J **do**
$d_{i,j,p=pv_{i,k}}^{\{s,a\}} += \left(a_{i,j,k}^{\{s\}} - tc_{i,k}^{\{s,a\}}\right)^2$
end for j
end for k
for $p := 1$ **to** P **do**
for $j := 1$ **to** J **do**
$d_{i,j,p}^{\{s,a\}} := \sqrt{d_{i,j,p}^{\{s,a\}}}$;
end for j
$dmax_{i,p}^{\{s,a\}} := \delta_i \cdot \max_{j:=1,\ldots,J} \left\{ d_{i,j,p}^{\{s,a\}} \right\}$
$sd_{i,p}^{\{s,a\}} += \frac{\sum_{j:=1}^{J} \sqrt{\left(\frac{1}{J} \cdot \sum_{j:=1}^{J} d_{i,j,p}^{\{s,a\}} - d_{i,j,p}^{\{s,a\}}\right)^2}}{\sqrt{J}}$
$w_{i,p}^{\{s,a\}} := 1 - \frac{sd_{i,p}^{\{s,a\}} \cdot \frac{1}{J} \cdot \sum_{j:=1}^{J} d_{i,j,p}^{\{s,a\}}}{\max_{p2:=1,2,\ldots,P} \left\{ sd_{i,p2}^{\{s,a\}} \cdot \frac{1}{J} \cdot \sum_{j:=1}^{J} d_{i,j,p}^{\{s,a\}} \right\}}$
for $m := 1$ **to** $nRules_i$ **do**
$\overline{xA}_{i,p,m}^{\{s,a\}} := \frac{(m-1) \cdot dmax_{i,p}^{\{s,a\}}}{nRules_i - 1}$
end for m
$\overline{\sigma A}_{i,p}^{\{s,a\}} := \frac{\overline{xA}_{i,p,2}^{\{s,a\}} - \overline{xA}_{i,p,1}^{\{s,a\}}}{2 \cdot \sqrt{\log\left(\frac{1}{0.5}\right)}}$
end for p
end for s
end for a
for $m := 1$ **to** $nRules_i$ **do**
$pL_m := \begin{cases} 1 \text{ for } m == 1 \\ 0 \text{ otherwise} \end{cases}$
$pR_m := \begin{cases} 1 \text{ for } m == nRules_i \\ 0 \text{ otherwise} \end{cases}$
$\overline{xB}_m := \frac{m-1}{nRules_i - 1}$
end for m
$\overline{\sigma B} := \frac{\overline{xB}_2 - \overline{xB}_1}{2 \cdot \sqrt{\log\left(\frac{1}{0.5}\right)}}$
for $r := 1$ **to** R **do** $\triangleright$ discretization points of system (3)
$\bar{y}_r^{\text{def}} := \frac{r-1}{R-1}$
end for r
store in the database: $\mathbf{tc}_i^{\{s,a\}}$, $\mathbf{w}_i^{\{s,a\}}$, $\overline{\mathbf{XA}}_{i,p}^{\{s,a\}}$, $\overline{\sigma \mathbf{A}}_i^{\{s,a\}}$, $\overline{\mathbf{xB}}$, $\overline{\sigma B}$, $\mathbf{pL}$, and $\mathbf{pR}$

C. Algorithms used in the test phase

Algorithm 5 Verification phase of the signature of the user claiming to be user i for selected partitions of the reference signatures of user i

get a test signature and index of user i
load from the database: $\mathbf{tc}_i^{\{s,a\}}$, $\mathbf{w}_i^{\{s,a\}}$, $\overline{\mathbf{XA}}_{i,p}^{\{s,a\}}$, $\overline{\sigma\mathbf{A}}_i^{\{s,a\}}$, $\overline{\mathbf{xB}}$, $\overline{\sigma B}$, $\mathbf{pL}$, and $\mathbf{pR}$
normalize the shape and length of the test signature on the basis of base signature $jBase$ of user i-use $\mathbf{a}_{i,j=jBase}$, $\mathbf{s}_{i,j=jBase}$ and determine $\mathbf{atst}_i^{\{s\}}$
for each a **in** $\{x, y\}$ **do**
  for each s **in** $\{v, z\}$ **do**
    $\mathbf{dtst}_i^{\{s,a\}} := \mathbf{0}$
    for $k := 1$ **to** K_i **do**
      $dtst_{i,p=pv_{i,k}}^{\{s,a\}} += \left(atst_{i,k}^{\{s\}} - tc_{i,k}^{\{s,a\}}\right)^2$
    end for k
    for $p := 1$ **to** P **do**
      $dtst_{i,p}^{\{s,a\}} := \sqrt{dtst_{i,p}^{\{s,a\}}}$
    end for p
  end for s
end for a
determine output signal $\bar{y}_i$ of system (3) for the test signature
if $\bar{y}_i > cth$ **then**
  the test signature was created by user i (it is genuine)
else
  the test signature was not created by user i
end if

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