Predictive Control based on Fuzzy Optimization for Multi-Room HVAC Systems

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***Abstract*—A model predictive control strategy (MPC) based on fuzzy optimization is proposed in this work for a multi-room heating, ventilation and air conditioning (HVAC) system. The proposed strategy aims to minimize energy consumption, while requiring different thermal conditions for each room. For this system, the combination of MPC and fuzzy optimization arises as a suitable control strategy, due to the benefits given by the use of fuzzy constraints for the managment of thermal comfort.The soft constraint scheme provided by the fuzzy optimization allows to reduce the power consumption of HVAC systems. This is achieved by allowing some constraint violations in specific cases where the proper operation of the system is not compromised. In this context, the main contribution of this work is the introduction of a new framework where the thermal requirements of several rooms can be managed by fuzzy constraints, which are handled as additional terms in the objective function of the MPC. The optimization problem of the MPC is nonlinear, and it is solved with a suitable particle swarm optimization (PSO) method. Simulations results show the effectiveness of the proposed controller to reduce the energy consumption compared with a classical MPC implementation, while maintaining constraint satisfaction in appropriate levels.**

***Index Terms*—Model predictive control, Fuzzy optimization, Building climate systems**

I. Introduction

The building sector is responsible of nearly 40% of the total energy consumption in developed countries [1]. Because of this, the improvement of energy efficiency in buildings is highlighted as an important goal of climatization systems, from environmental, social and economic points of view. In this context, the objective of this work is to develop a control strategy for the optimal operation of heating, ventilation and air conditioning (HVAC) systems, focusing on minimizing energy consumption, while maintaining the thermal comfort conditions for the occupants of the different rooms. This problem has been tackled by several works using predictive control strategies (e.g. [2]–[6]) and fuzzy control strategies (e.g. [7]–[9]), showing that both, fuzzy control and model predictive control (MPC) are effective approaches to control HVAC systems. Of these, MPC strategies are naturally suited for HVAC systems: they seek an optimal operation of the HVAC system, in terms of energy consumption, which is achieved by solving an optimization problem that considers thermal comfort requirements as system constraints.

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Building temperature models usually depend on stochastic variables related to weather and room occupancy, which act as external disturbances with predicted values that are unknown and uncertain. These disturbances may provoke problems with constraint compliance, thus leading to the use of robust control strategies [10], [11] that enforce constraints for the worst case predictions of the external disturbances. However, thermal comfort requirements are not hard constraints that must necessarily be met the entire time, thus making these formulations overly conservative. Therefore, some works have considered allowing a certain degree of constraint violations, which enables a reduction of energy consumption. This relaxation has been implemented for HVAC systems with stochastic MPC formulations that use scenarios or the probability functions of the external disturbances (e.g. [12]–[14]), and with robust MPC based on interval fuzzy models [6]. The allowed margin of constraint violation is related with the probability function of the constraint compliance in stochastic MPC formulations, and with the coverage level of fuzzy intervals in the case of robust MPC of [6]. In the same line of these strategies, a fuzzy optimization scheme that enables soft constraints is considered this work in the formulation of the MPC controller.

The idea of fuzzy optimization is originally presented by [15]. The objective function and constraints have their own membership functions, but the system constraints are not handled as constraints in the optimization. Instead, they are added to the objective function with their own fuzzy membership functions. This enables a soft constraint approach, as a certain margin of constraint violations can be permitted according to the definition of the associated membership functions.

Fuzzy optimization has been used in the control of several kinds of systems, such as solar power plants [16], for reactive power and voltage control in power systems [17], [18] and for wastewater treatment processes control [19]. It was also used for path planning problems [20], [21], generation scheduling of energy systems [22], [23] water management [24], [25] and for a parking lot operation [26].

From the HVAC systems point of view, the application of fuzzy optimization is not new. A previous implementation is shown in [8], where that method is used to control the air temperature, CO_2 and humidity, and is shown to be effective when the controller focuses on a single room. In this context, this work focuses the design of a controller for a multi-room HVAC system that has the typical objective of minimizing the energy consumption, while aiming to maintain (potentially) different thermal conditions for different rooms. This is motivated by the fact that some types of rooms might have different temperature requirements despite being inside the same building (e.g. reception hall, laboratories, etc.).

In order to achieve the goal of this work, a MPC strategy based on fuzzy optimization is proposed, where the novelty lies in the use of fuzzy constraints for the thermal comfort handling in multiple rooms simultaneously. The resulting optimization problem is solved with particle swarm optimization (PSO), while neural networks are used for the disturbance forecasting. Simulation results and a comparative analysis are shown for the MPC with fuzzy optimization and the classical MPC defined by hard constraints, both implemented for performing a reference tracking.

This paper starts with a brief explanation of the model predictive control strategy (MPC) in Section II, which also describes how fuzzy optimization is included in the formulation of the controller. Next, Section III presents a description of the multi-room HVAC model, which is the system to be controlled in this work, and the control problem statement. Finally, simulation results are shown in Section IV, where a comparative analysis is performed between the classical MPC and the proposed MPC based on fuzzy optimization.

II. Model predictive control based on Fuzzy Optimization

MPC is a control strategy that solves an optimization problem, where the future behavior of the system is predicted by a process model and a sequence of future control actions that are chosen to minimize a performance index.

In a general MPC formulation, a typical dynamic system

$$x(k+1) = f\left(x(k), u(k), \omega(k)\right), \tag{1}$$

is considered to be controlled. In (1), $x(k) \in \mathbb{R}^{n_x}$ is the state of the system, $u(k) \in \mathbb{R}^{n_u}$ is the input vector, and $\omega(k) \in \mathbb{R}^{n_\omega}$ is a disturbance signal. The system is assumed to be subject to linear constraints, which are expressed in the form

$$Fx(k) + Gu(k) \leq h, \tag{2}$$

where $F \in \mathbb{R}^{n_c \times n_x}$, $G \in \mathbb{R}^{n_c \times n_u}$ and $h \in \mathbb{R}^{n_c}$, are the elements that define the n_c constraints.

In this framework, the sequence of future control actions over a finite prediction horizon N, is computed by solving the following optimization problem:

$$\begin{aligned}
\min_{\vec{x},\vec{u}} \quad & J(\vec{x},\vec{u}) \\
s.t. \quad & F\hat{x}(k+j) + Gu(k+j) \leq h, \\
& \hat{x}(k+j+1) = \hat{f}\left(\hat{x}(k+j), u(k+j), \hat{\omega}(k+j)\right), \\
& j = 0, ..., N-1, \\
& \hat{x}(k) = x(k),
\end{aligned} \tag{3}$$

where $\vec{x} = \left[\hat{x}(k), \ldots, \hat{x}(k+N)\right]^T$ is the vector of predicted states and $\vec{u} = \left[u(k), \ldots, u(k+N-1)\right]^T$ is the vector of future control actions, which are the decision variables in the optimization problem. Also in (3), $\hat{f}$ is the identified predictive model which tries to emulate the behavior of (1), $\hat{\omega}(k+j)$ are the predicted disturbances given by an autoregressive model (e.g. a trained neural network model) and $J(\vec{x},\vec{u})$ is the objective function. The optimization problem is solved with a suitable algorithm and from the solution obtained, only the first element of the optimal sequence of control actions is applied to the system [27]. Then, in order to introduce feedback, the process is repeated for each instant k during the operation of the controlled system, solving the same optimization, but with updated values of the system state and disturbance predictions.

Here, combination of predictive control with fuzzy optimization, as previously done in [16], [28], [29] and [30], is proposed in order to handle some restrictions of the system as soft constraints in the formulation of (3). The fuzzy optimization consists in the use of a new objective function J_f defined by several fuzzy membership functions. Some of them are associated to the n_{obj} objectives of the controller (which come from the original objective function $J(\vec{x},\vec{u})$). Additionally, those rows of the system constraints (2) that do not necessarily need to be met all the time, are not included as hard constraints in the optimization, and instead are treated as soft constrains by using n_{sc} membership functions in J_f. With a little abuse of language, we will refer to each of these membership functions as soft constraints, even if each membership function can be associated with more than one row of (2). Thus, the new objective function J_f is defined as

$$\begin{aligned}
J_f(\vec{x},\vec{u}) = \sum_{j=0}^{N} \Bigg\{ & \sum_{i=1}^{n_{obj}} \left[\mu_J^i(\hat{x}(k+j), u(k+j)\right]^p \\
& + \sum_{i=1}^{n_{sc}} \left[\mu_c^i\left(\hat{x}(k+j), u(k+j)\right)\right]^p \Bigg\},
\end{aligned} \tag{4}$$

where $\mu_J^i(\hat{x}(k+j), u(k+j))$ are the membership functions associated to the n_{obj} control objectives, $\mu_c^i(\hat{x}(k+j), u(k+j))$ are the membership functions associated to the n_{sc} soft constraints, and $p \in (0, \infty)$ is a parameter that adjusts the degree of fuzzy aggregation. ; a greater value of p represent a harder aggregation of the different fuzzy membership functions [16].

The compliance of the different control objectives and soft constraints is handled by the maximization of the fuzzy membership functions included in (4). Consequently, the shapes defined for $\mu_J(\hat{x}(k+j), u(k+j))$ and $\mu_c(\hat{x}(k+j), u(k+j))$ are relevant in order to obtain the desirable results. For example, Fig. 1 shows how a constraint for the state x, originally specified by two rows of (2) defining a lower and upper bound, is handled by the shape of a single membership function μ_c.

As shown in Fig. 1, $\mu_c(x)$ establishes bounds for the state, such that $x \in [x_2, x_3]$ is the desirable result that have assigned the maximum value for the membership function, i.e $\mu_c(x) = 1$. Additionally, the ranges $[x_1, x_2)$ and $(x_3, x_4]$ are also considered tolerable results, so in these cases the membership function comply $0 \leq \mu_c(x) \leq 1$.

The definition of the n_{obj} control objectives and the n_{sc} soft constraints follows the idea showed in Fig. 1. The maximum values for the membership functions $\mu_J(\hat{x}(k+j), u(k+j))$ and $\mu_c(\hat{x}(k+j), u(k+j))$ are assigned when $(\hat{x}(k+j), u(k+j))$ are desirable results. On the other hand, values in the range $(0, 1)$ are used for $\mu_J(\hat{x}(k+j), u(k+j))$ and $\mu_c(\hat{x}(k+j), u(k+j))$ when the results are acceptable but not ideal.

Taking into account this strategy for the management of control objectives and soft constraints, the optimization (3) can be rewritten in the form

$$\begin{aligned} \max_{\vec{x},\vec{u}} \quad & J_f(\vec{x}, \vec{u}) \\ s.t. \quad & g(\hat{x}(k+j), u(k+j)) \leq h_g \\ & \hat{x}(k+j+1) = \hat{f}\left(\hat{x}(k+j), u(k+j), \hat{\omega}(k+j)\right) \\ & j = 0, ..., N-1 \\ & \hat{x}(k) = x(k), \end{aligned} \tag{5}$$

where $J_f(\vec{x}, \vec{u})$ is the objective function presented in (4) and the inequality defined by $g(\hat{x}(k+j), u(k+j))$ and h_g, represents the constraints from (2) that cannot be included in $J_f(\vec{x}, \vec{u})$ (e.g. the hard constraint defined for $u(k+j)$ that is given by the limits of the actuators). Due to the nonlinear behavior of the membership functions, is important to note that (5) must be solved by a suitable nonlinear solver.

The formulation presented for this model predictive control based on fuzzy optimization (hereinafter denoted as FO-MPC) is represented by the block diagram shown in Fig. 2

In section III we describe the dynamics of an multi-room HVAC model which is used for testing the performance of the FO-MPC implementation.

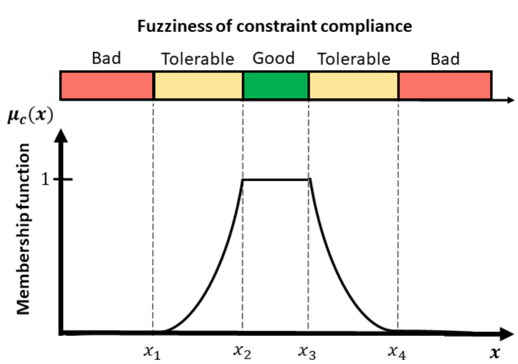


Fig. 1. Constraint handled by a fuzzy membership function.

III. Case study: HVAC System

In this section, a model of a Heating, Ventilation and Air Conditioning Systems (HVAC), as shown in [1], is presented as a case study of this work.

Under some simplifications, the dynamic of each room temperature can be represented by a two state model based in a resistive-capacitive circuit analogy. Thus, the equation which describes the dynamic of each room is written as:

$$\begin{aligned} C_1^j \dot{T}_1^j =& \dot{m}_s^j c_p (T_s^j - T_1^j) + (T_2^j - T_1^j)/R^j + \\ & \sum_{i \in \mathcal{N}^j} (T_1^i - T_1^j)/R_{ij} + (T_a - T_1^j)/R_a^j + P_d^j, \end{aligned} \tag{6}$$

$$C_2^j \dot{T}_2^j = (T_1^j - T_2^j)/R^j, \tag{7}$$

$$T_s^j = \delta \frac{\sum_{i\in\nu} \dot{m}_s^i T^i}{\sum_{i\in\nu} \dot{m}_s^i} + (1-\delta) T_a - \Delta T_c + \Delta T_h^j, \tag{8}$$

where T_1^j represents the air temperature (which has fast dynamics), T_2^j is the temperature of the furniture and walls (has slow dynamics), and T_s^j is the air supply temperature, all of them denoted for room j. In this model, C_i^j and c_p^j are the thermal capacitance and the specific heat capacity of room air, respectively. On the other hand, R^j, R_{ij} and R_a^j are the heat resistances of the model, where R^j is associated to the heat exchange between T_1^j and T_2^j, R_{ij} correspond to the exchange between different rooms (denoted by the indexes i and j) and R_a^j represents the resistance of the room with respect to the ambient temperature of the exterior of the building. In (6), $\mathcal{N}^j$ denotes the neighborhood of room j, i.e. the set of rooms that are physically connected by at least one wall with room j, while ν in (8) represents the set of all rooms to be climatized.

The manipulated variables are the air mass flow rate of each room $\dot{m}_s^i$, the temperature differences imposed by the cooling and heating coils (ΔT_c and ΔT_h^j respectively), and the air recirculation rate δ, which for our MPC implementation is considered as a constant parameter $\delta = 0.5$. The control objective is to keep the air temperatures T_1^j within a certain comfort range and minimize the control energy. The ambient temperature T_a and the internal load P_d^j, related to occupancy of the room j, are the external disturbances. Fig. 3 presents a diagram showing the different variables considered in the model of a multi-room HVAC system.

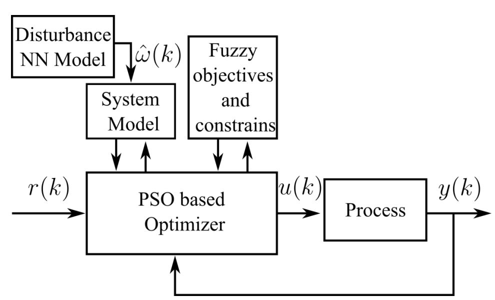


Fig. 2. Block diagram of the FO-MPC implementation.

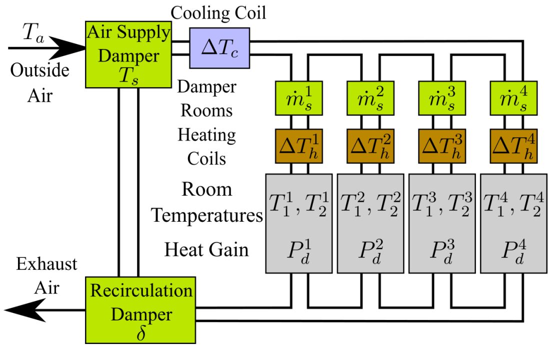


Fig. 3. HVAC system diagram.

The power consumption of the whole HVAC system presented in Fig. 3 is given by

$$\begin{aligned} P = \quad & \kappa_f \left(\sum_{i \in \nu} \dot{m}_s^i \right)^2 + \frac{c_p}{\eta_c} \left(\sum_{i \in \nu} \dot{m}_s^i \right) \Delta T_c \\ & + \frac{c_p}{\eta_h} \left(\sum_{i \in \nu} \dot{m}_s^i \Delta T_h^i \right) \end{aligned} \tag{9}$$

where κ_f, η_c and η_h are constants related to the efficiency of the power consumption of each actuator of the system [2].

In this work, four rooms are air-conditioned with the HVAC system. The arrangement of the rooms follows the layout shown in Fig. 4, where each thermal zone is exposed to an unique interaction with the other rooms. Each zone has a dimension of 20[m] $\times$ 30[m] $\times$ 3.5[m], sharing the same thermal capacitance and resistance with the other rooms, thus $C_1^j = 9.163 \cdot 10^6$ [K/W], $C_2^j = 1.694 \cdot 10^7$ [K/W] and $R^j = 1.7 \cdot 10^{-3}$ [K/W] for $j = 1, \ldots, 4$. The other heat resistances considered in the HVAC model have the following values: $R_{12} = 2.88 \cdot 10^{-2}$ [K/W], $R_{23} = R_{34} = 1.92 \cdot 10^{-2}$ [K/W], $R_a^1 = R_a^4 = 1.4 \cdot 10^{-3}$ [K/W] and $R_a^2 = R_a^3 = 1.5 \cdot 10^{-3}$ [K/W]. The parameters that define the power consumption in (9) are $\kappa_f = 65[\mathrm{W} \cdot \mathrm{s}^2/\mathrm{kg}^2]$, $\eta_c = 4$, $\eta_h = 0.9$ and $c_p = 1012[\mathrm{J/kg} \cdot \mathrm{K}]$. Finally, the usual maximum occupancy for each room is 40 people.

The MPC controller will use the Euler discretization of the model (6)-(8), with a sampling time $\Delta t = 600$ [s]:

$$\begin{aligned} C_1^j T_1^j(k+1) =& C_1^j T_1^j(k) + \Delta t \dot{m}_s^j(k) c_p (T_s^j(k) - T_1^j(k)) \\ & + \Delta t \frac{(T_2^j(k) - T_1^j(k))}{R^j} + \\ & \Delta t \sum_{i \in N^j} \frac{(T_1^i(k) - T_1^j(k))}{R_{ij}} \\ & + \Delta t \frac{(T_a(k) - T_1^j(k))}{R_a^j} + \Delta t P_d^j, \end{aligned} \tag{10}$$

$$C_2^j T_2^j(k+1) = C_2^j T_2^j(k) + \Delta t \frac{(T_1^j(k) - T_2^j(k))}{R^j}, \tag{11}$$

$$\begin{aligned} T_s^j(k) =& \delta \frac{\left(\sum_{i \in \nu} \dot{m}_s^i(k) T_1^i(k)\right)}{\left(\sum_{i \in \nu} \dot{m}_s^i(k)\right)} + (1-\delta) T_a(k) \\ & - \Delta T_c(k) + \Delta T_h^j(k). \end{aligned} \tag{12}$$

The main goal of the controller is the minimization of the power consumption while maintaining the temperature of each room within an interval around certain references values. This objective is implemented using the following room temperature constraint

$$r^j - 1.5 \leq T_1^j \leq r^j + 1.5 \tag{13}$$

where r^j is the reference value of the room j and the maximum deviation allowed around r^j is $1.5^o C$.

Based on this, the FO-MPC solves a similar optimization problem as in (5), using the objective function:

$$\begin{aligned} J_f = \Bigg\{ & \sum_{i=1}^{4} \sum_{j=1}^{N} \left[\mu_e \left(\hat{T}_1^i(k+j) - r^i(k+j) \right) \right]^2 \\ & + \left[\mu_u \left(\sum_{j=1}^{N} \hat{P}(k+j) \right) \right]^2 \Bigg\}, \end{aligned} \tag{14}$$

where μ_e and μ_u are the membership function associated to (13) and the prediction of power consumption over the prediction horizon $N = 6$, respectively. The functions $\mu_e(x)$ and $\mu_u(x)$ are defined for this case study as:

$$\varphi(x) = \begin{cases} 3 - 1.8|x| & \text{if } |x| \leq 1.5 \\ 0.3158 - 0.0105|x| & \text{if } 1.5 < |x| \end{cases}, \tag{15}$$

$$\mu_e(x) = \begin{cases} 0 & \text{if } \varphi(x) \leq 0 \\ \varphi(x) & \text{if } 0 \leq \varphi(x) \leq 1 \\ 1 & \text{if } 1 < \varphi(x) \end{cases}, \tag{16}$$

$$\mu_u(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{(3.3488 \cdot 10^6)} & \text{if } 0 < x \leq 3.3488 \cdot 10^6 \\ 0 & \text{if } 3.3488 \cdot 10^6 < x \end{cases}, \tag{17}$$

resulting in the shapes shown in Fig. 5.

Given the definition of $\mu_e(x)$ in (16), the use of it in (14) implements (13) as a soft constraint in FO-MPC.

The references r^i for temperature T_1^i of each room i are:

$$\left. \begin{aligned} r^1(k+j) &= 15°C \\ r^2(k+j) &= 21°C \\ r^3(k+j) &= 23°C \\ r^4(k+j) &= 27°C \end{aligned} \right\} \quad \textit{from 7 am to 9 pm}, \tag{18}$$

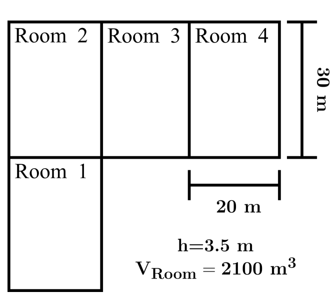
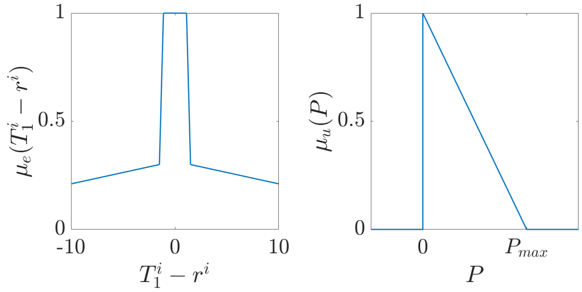


Fig. 4. Room layout for the experiment.



(a) Temperature deviation (b) Power consumption

Fig. 5. Membership functions.

and no references are considered during night hours. At these times constraint (13) is not enforced, and the respective membership functions $\mu_e(x)$ are replaced by null values.

On the other hand, FO-MPC must consider as hard constraints the actuators limits like the air mass flow limitations for the air supply fan and maximum temperature changes in ΔT_h^i and ΔT_c. These constraints are defined as follows:

$$0.005 \text{ [kg/s]} \leq \dot{m}_s^i \leq 5 \text{ [kg/s]}, \tag{19}$$

$$0 \text{ °C} \leq \Delta T_h^i \leq 13 \text{ °C}, \tag{20}$$

$$0 \text{ °C} \leq \Delta T_c \leq 13 \text{ °C}. \tag{21}$$

Due to the nonlinearities in the definition of the membership functions (μ_e and μ_u) and the system model, FO-MPC has to be implemented with a dedicated solver for nonlinear optimization. In this work, the Particle Swarm Optimization (PSO) algorithm is chosen for solving (5), where $g(\cdot) \leq h_g$ represents the hard constraints (19)-(21). For the disturbance signals, experimental data collected from a weather station located in La Araucanía Region, Chile, in 2016 is used as the ambient temperature T_a and the internal occupancy P_d^j is simulated with the Markov chain algorithm proposed in [31], assigning a heat generation per person of 150 [W].

The results of FO-MPC are compared with those obtained with a classical MPC in order to analyze the benefits of the proposed control strategy. In this case study, the classical MPC solves (3), using the following objective function:

$$J = \sum_{i=1}^{4}\sum_{j=1}^{N} \lambda_e \left(\hat{T}_1^i(k+j) - r^i(k+j)\right)^2 + \lambda_p \sum_{j=1}^{N} \hat{P}(k+j)$$
$$+ \, exp\left\{\eta\left[\left(\hat{T}_1^i(k+j) - r^i(k+j)\right)^2 - e_{max}^2\right]\right\}, \tag{22}$$

and considering (19)-(21) as the hard constraints of the system. In the objective function (22), λ_e, λ_p are weight parameters related to error tracking and power consumption. As done with the FO-MPC implementation, the optimization problem of the classical MPC is solved with PSO algorithm. The classical formulation of the MPC would normally include (13) as a hard constraint in the optimization problem. However, since PSO is used to solve the optimization, this constraint is actually implemented with an additional term in (22) given by an exponential penalty function; this term strongly penalizes occurrences where the tracking error is greater than the ideal maximum defined value. Thus, η is a positive parameter and e_{max} is the maximum allowed deviation around the reference, i.e. $e_{max} = 1.5°C$. Here, the inclusion of this penalty term in (22) was considered due to the difficulties presented by the PSO algorithm when trying to handle this condition in a similar way as done with the other system constraints.

Next section presents the simulation results of the disturbance modeling and the comparison between the implementation of FO-MPC and MPC, both during the operation of a multi-room HVAC system model.

IV. Simulation Results

A. Disturbance modeling

The external disturbances that affect the system dynamics must be modeled first in order to implement an MPC strategy for the HVAC system. In this work, feedforward Neural Networks are used predict future values of the disturbances, the ambient temperature and occupancy of the rooms, which are necessary for obtaining the future temperatures of the rooms in the optimization problem (5).

Experimental data collected from a weather station located in La Araucanía Region, Chile, is used for training and validation of neural networks for the modeling of the ambient temperature T_a. Here, a feedforward single-layer perceptron is trained, using 5 regressors of the ambient temperature as inputs of the model, and 10 neurons in the hidden layer. The behaviors of the predictions for 6 steps ahead and the corresponding future measurements are shown in Fig. 6. Also, the Root Mean Square Error (RMSE) of the predictions is included as a performance metric of the trained model.

As it can be seen from Fig. 6 and Table I, the trained model results with a low prediction error for the ambient temperature within the prediction horizon of the controller (RMSE less than 1 when using up to $N = 6$ prediction steps). This is satisfactory enough to consider the use of this trained neural network as the predictive model of the disturbance.

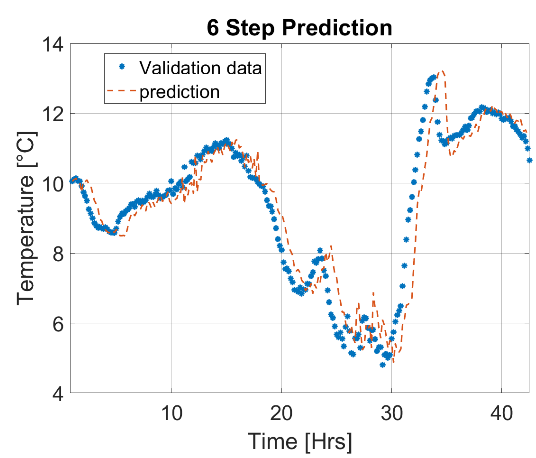


Fig. 6. Prediction of ambient temperature 6 step ahead.

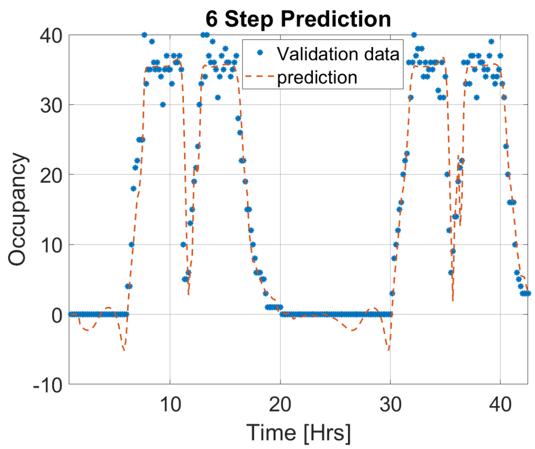


Fig. 7. Prediction of occupancy level 6 step ahead.

On the other hand, for the modeling of the internal occupancy of the rooms, artificial data previously generated by the use of a Markov chain based algorithm, as proposed in [31], is considered for training and validation of the neural network model. Similar to the case of the ambient temperature, a feed-forward single-layer perceptron is trained, using 5 regressors of the occupancy level as inputs of the model, and 10 neurons in the hidden layer. The results of the predictions obtained 6 step ahead and the values of the RMSE are included in Fig. 7 and Table II, respectively.

As it is observed in Fig. 7, the neural network model gives impossible outputs as predictions (specifically, negative values of occupancy), for steps predictions greater than one. In order to compensate this, a saturation at the output of the predictive model is considered, making the values bounded to [0,40]. On the other hand, the results of RMSE in Table II show that the predictive model is good enough for being used in the MPC implementation, because maintains a low error within the prediction horizon of the controller (RMSE less than 4 when using up to $N = 6$ prediction steps).

Finally, the disturbance signals corresponding to the internal heat gain of each room P_d^j, are calculated from the occupancy values assuming a heat generation per person of 150 [W].

B. MPC implementation

As previously mentioned, two MPC strategies are implemented and compared in this paper. One is the MPC based on fuzzy optimization (FO-MPC), and the other is the classical MPC implementation. In this work, the optimization problems defined for both controllers are solved with PSO, and both use the same predictive model (10)-(12) for the room temperatures. The simulation spans 2 days of data outside the training set.

The results are shown in Tables III and IV, where the energy consumption, the average computation time and the constraint satisfaction level of each room are presented. Also, Fig.8, 9, 10 and 11 present the graphical comparison between the temperature behavior under the effect of both controllers, for each room considered in the HVAC system.

It is observed from Table IV that FO-MPC obtains higher levels of constraint satisfaction than classical MPC. The FO-MPC implementation reached satisfaction levels above 99% for the first three rooms, which are slightly higher than the levels obtained by the classical MPC. However, in the fourth room (which has the reference of 27°C) both controllers had a lower satisfaction percentage of the room temperature constraint, and classical MPC had a higher satisfaction level than FO-MPC by a margin of 0.5%. These results are due to the fact that FO-MPC can handle better the temperature constraints for the first three rooms, thanks to the use of soft constraints that allowed a greater temperature deviation in the fourth room, which have the most demanding temperature requirement. Additionally, the satisfaction levels obtained for all rooms confirm that both implemented controllers are appropriate to be applied in this multi-room HVAC case study, because they manage to comply the constraints in more than 90% of the simulated cases.

A notable result is the behavior presented by room 1, shown in Fig, 8, where the room temperature suffers a big overshot just before the activation of the constraints (at 7 am). This is caused by the fact that the other rooms have higher temperature requirements, so the recirculation air that is distributed to all rooms will get hotter. Then, due to the energy minimization, the controller decides to activate the cooling coil for the temperature regulation of the first room later as possible.

FO On the other hand, Table III shows that FO-MPC consumes less energy than the classical MPC. This behaviour is the main advantage identified for FO-MPC, and can be

TABLE I
PREDICTION ERROR FOR AMBIENT TEMPERATURE MODELING

	Steps ahead		
Metric	1 step	6 step	12 step
RMSE	0.2187	0.7440	1.2410

TABLE II
PREDICTION ERROR FOR OCCUPANCY LEVEL MODELING

	Steps ahead		
Metric	1 step	6 steps	12 steps
RMSE	1.6092	3.0437	28.2981

TABLE III
SIMULATION EFFICIENCY METRICS

Controller	Energy consumption [kWh]	Computation Time [s]
FO-MPC	2135.6	5.0795
MPC	2335.8	3.4712

TABLE IV
CONSTRAINT SATISFACTION LEVEL

Controller	Room 1	Room 2	Room 3	Room 4
FO-MPC	100 %	99.96 %	99.26 %	91.78 %
MPC	95.46 %	96.13 %	98.65 %	92.26 %

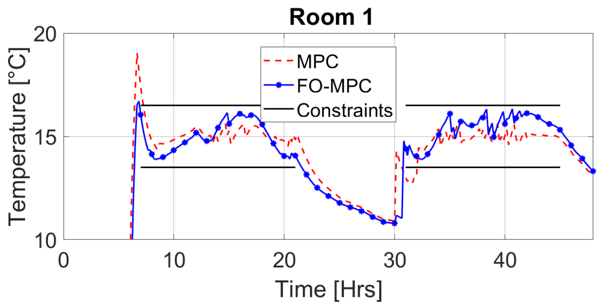


Fig. 8. Simulation in Room 1.

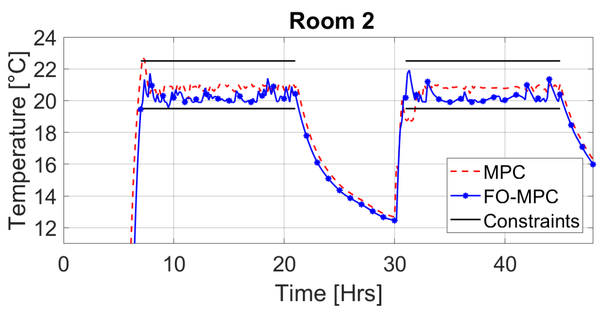


Fig. 9. Simulation in Room 2.

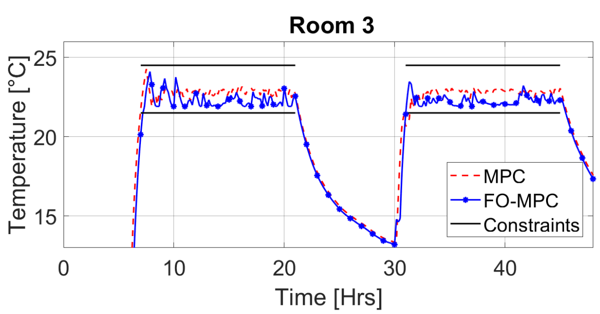


Fig. 10. Simulation in Room 3.

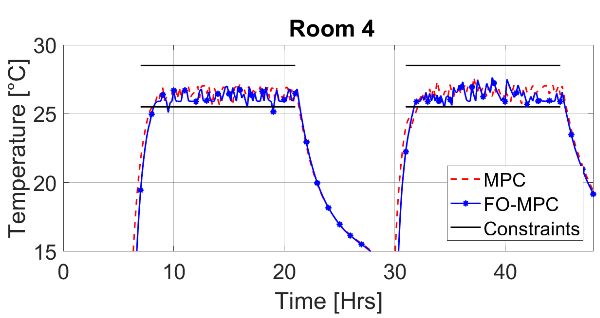


Fig. 11. Simulation in Room 4.

explained due to the fact that its objective function is more permissive with the temperature constraint than that used in the classical MPC. Due to that, FO-MPC was able to obtain lower temperatures in rooms 2-4. This way, the control actions given by FO-MPC are less strong than those provided by classic implementation of MPC, resulting in the lower power consumption observed.

Finally, also from Table III it can be observed that FO-MPC had a higher computational time than the classical MPC. This is due to the higher complexity that means the handling of its objective function. However, this increment is not significant enough and remains small when compared with the sampling time of the HVAC system (10 minutes).

These results show that FO-MPC is suitable for a multi-room HVAC system with a multiple reference situation, where some actuators are used by all rooms (for example, the cooling coil). Another observation that can be made at this point is that both controllers did not reach 100% of compliance with the original constraints for room temperature (the deviation has to be less than 1.5°C around the reference). This illustrates the difficulty of trying to comply these conditions for multiple rooms, which can provoke problems of infeasibility in the hypothetical case of using an MPC designed with other types of optimizers that only consider hard constraints.

V. Conclusions

This work presents a MPC strategy that uses the idea of fuzzy optimization, in order to control a multi-room HVAC system. The main motivation for doing this is the fact that some conditions of the HVAC are not hard restrictions, thus the controller may allow to violate those constraints by a small margin, without compromising the operation of the system (e.g. the comfort region for the room temperature). With the inclusion of fuzzy optimization in the MPC, these conditions can be treated as soft constraints, handled by membership functions in the objective function.

Simulation results show that the use of the new objective function given by the fuzzy optimization managed to reduce the energy consumption with respect to the classical MPC implementation, while the controller tried to satisfy the requirements of four different rooms. That was due to the fact that the new objective function resulted more permissive in terms of the temperature constraints than that used in the classical MPC, which allowed a greater flexibility in controlling the rooms with higher temperatures requirements.

On the other hand, despite the fact that FO-MPC obtained higher constraint satisfaction levels than the classical MPC, both controllers failed to comply the system conditions in a 100% of the cases. Here, it can be emphasized that this situation would be a serious problem if the MPC had been implemented using another optimizer with hard constraints instead of fuzzy optimization with PSO. This is due to the fact that, in the hypothetical case of using the alternative solver defined only by hard constraints, the imposition of the system conditions can provoke infeasibility in the optimization problem, which in turn can cause bad responses in the controller.

Additionally, the computation time of FO-MPC remains short enough for application (i.e. the average computational time is still considerable lower than the sample time of ten minutes).

In summary, simulation results show that the proposed FO-MPC is better suited than classical MPC to control the temperature of a multi-room HVAC system, when the controllers face a multiple references situation (i.e. each room has different temperature requirements).

REFERENCES

[1] Y. Ma, G. Anderson, and F. Borrelli, "A distributed predictive control approach to building temperature regulation," in *Proceedings of the 2011 American Control Conference*, June 2011, pp. 2089–2094.

[2] A. Kelman and F. Borrelli, "Bilinear model predictive control of a hvac system using sequential quadratic programming," *IFAC Proceedings Volumes*, vol. 44, no. 1, pp. 9869–9874, 2011.

[3] Y. Ma, A. Kelman, A. Daly, and F. Borrelli, "Predictive control for energy efficient buildings with thermal storage: Modeling, stimulation, and experiments," *IEEE Control Systems Magazine*, vol. 32, no. 1, pp. 44–64, Feb 2012.

[4] M. Maasoumy and A. Sangiovanni-Vincentelli, "Optimal control of building hvac systems in the presence of imperfect predictions," in *ASME 2012 5th Annual Dynamic Systems and Control Conference joint with the JSME 2012 11th Motion and Vibration Conference*. American Society of Mechanical Engineers, 2012, pp. 257–266.

[5] A. Parisio, M. Molinari, D. Varagnolo, and K. H. Johansson, "A scenario-based predictive control approach to building hvac management systems," in *2013 IEEE International Conference on Automation Science and Engineering (CASE)*, Aug 2013, pp. 428–435.

[6] O. Cartagena, D. Muñoz-Carpintero, and D. Sáez, "A robust predictive control strategy for building hvac systems based on interval fuzzy models," in *2018 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, July 2018, pp. 1–8.

[7] F. Naqvi, S. Iqbal, and Q. Ain, "Energy optimization in smart buildings using intelligent fuzzy ventilation," in *2019 International Conference on Electrical, Communication, and Computer Engineering (ICECCE)*, July 2019, pp. 1–7.

[8] T. Bernard and H. B. Kuntze, "Multi-objective optimization of building climate control systems using fuzzy-logic," in *1999 European Control Conference (ECC)*, Aug 1999, pp. 2572–2577.

[9] K. E. M. Reena, A. T. Mathew, and L. Jacob, "Real-time occupancy based hvac control using interval type-2 fuzzy logic system in intelligent buildings," in *2017 12th IEEE Conference on Industrial Electronics and Applications (ICIEA)*, June 2017, pp. 455–460.

[10] R. Gondhalekar, F. Oldewurtel, and C. N. Jones, "Least-restrictive robust mpc of periodic affine systems with application to building climate control," in *49th IEEE Conference on Decision and Control (CDC)*, 2010, pp. 5257–5263.

[11] X. Xu, S. Wang, and G. Huang, "Robust mpc for temperature control of air-conditioning systems concerning on constraints and multitype uncertainties," *Building Services Engineering Research and Technology*, vol. 31, no. 1, pp. 39–55, 2010.

[12] Y. Ma, J. Matuško, and F. Borrelli, "Stochastic model predictive control for building hvac systems: Complexity and conservatism," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 1, pp. 101–116, Jan 2015.

[13] F. Oldewurtel, C. N. Jones, A. Parisio, and M. Morari, "Stochastic model predictive control for building climate control," *IEEE Transactions on Control Systems Technology*, vol. 22, no. 3, pp. 1198–1205, May 2014.

[14] D. Muñoz-Carpintero, G. Hu, and C. J. Spanos, "Stochastic model predictive control with adaptive constraint tightening for non-conservative chance constraints satisfaction," *Automatica*, vol. 96, pp. 32 – 39, 2018.

[15] R. E. Bellman and L. A. Zadeh, "Decision-making in a fuzzy environment," *Management Science*, vol. 17, no. 4, pp. B–141–B–164, 1970.

[16] A. Flores, D. Saez, J. Araya, M. Berenguel, and A. Cipriano, "Fuzzy predictive control of a solar power plant," *IEEE Transactions on Fuzzy Systems*, vol. 13, no. 1, pp. 58–68, Feb 2005.

[17] W. Zhang and Y. Liu, "Multi-objective reactive power and voltage control based on fuzzy optimization strategy and fuzzy adaptive particle swarm," *International Journal of Electrical Power & Energy Systems*, vol. 30, no. 9, pp. 525 – 532, 2008.

[18] R.-H. Liang, Y.-K. Chen, and Y.-T. Chen, "Volt/var control in a distribution system by a fuzzy optimization approach," *International Journal of Electrical Power & Energy Systems*, vol. 33, no. 2, pp. 278 – 287, 2011.

[19] A. E. S. Choi, C. C. M. Futalan, and J.-J. Yee, "Fuzzy optimization for the removal of uranium from mine water using batch electrocoagulation: A case study," *Nuclear Engineering and Technology*, 2019.

[20] Y. Zheng and B. Liu, "Fuzzy vehicle routing model with credibility measure and its hybrid intelligent algorithm," *Applied Mathematics and Computation*, vol. 176, no. 2, pp. 673 – 683, 2006.

[21] J. Du, X. Li, L. Yu, R. Dan, and J. Zhou, "Multi-depot vehicle routing problem for hazardous materials transportation: A fuzzy bilevel programming," *Information Sciences*, vol. 399, pp. 201 – 218, 2017.

[22] R. Liang and J. Liao, "A fuzzy-optimization approach for generation scheduling with wind and solar energy systems," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 1665–1674, Nov 2007.

[23] A. Derghal, N. Goléa, and N. Essounbouli, "An interval fuzzy optimization-based technique to optimal generation scheduling with load uncertainty," *IFAC-PapersOnLine*, vol. 49, no. 12, pp. 1122 – 1127, 2016, 8th IFAC Conference on Manufacturing Modelling, Management and Control MIM 2016.

[24] G. Fu, "A fuzzy optimization method for multicriteria decision making: An application to reservoir flood control operation," *Expert Systems with Applications*, vol. 34, no. 1, pp. 145 – 149, 2008.

[25] M. S. Taskhiri, R. R. Tan, and A. S. Chiu, "Emergy-based fuzzy optimization approach for water reuse in an eco-industrial park," *Resources, Conservation and Recycling*, vol. 55, no. 7, pp. 730 – 737, 2011.

[26] S. Faddel, A. T. Al-Awami, and M. Abido, "Fuzzy optimization for the operation of electric vehicle parking lots," *Electric Power Systems Research*, vol. 145, pp. 166 – 174, 2017.

[27] B. Kouvaritakis and M. Cannon, *Model Predictive Control: Classical, Robust and Stochastic*. Springer International Publishing, 2016.

[28] Shaoyuan Li and Yugeng Xi, "Generalized predictive control with fuzzy soft constraints," in *Ninth IEEE International Conference on Fuzzy Systems. FUZZ- IEEE 2000 (Cat. No.00CH37063)*, vol. 1, May 2000, pp. 411–416 vol.1.

[29] C. Arnold, T. Aissa, and S. Lambeck, "Using uncertain set points in predictive functional control," in *2013 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, July 2013, pp. 1–8.

[30] T. Aissa, C. Arnold, and S. Lambeck, "Fuzzy set points predicitve functional control for implicit regulator calculation of mimo-systems," in *2015 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, Aug 2015, pp. 1–8.

[31] C. Wang, D. Yan, and Y. Jiang, "A novel approach for building occupancy simulation," *Building Simulation*, vol. 4, no. 2, pp. 149–167, Jun 2011.