An Interval Type-2 Fuzzy Dynamic Approach To Replacement of Server Equipment

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***Abstract*— Equipment replacement decision which aims to find the best time to retire an old system is a key element in the planning process. Replacement scenarios consider the life span associated to each equipment category and the replacement of the obsolete equipment by an equivalent during the remaining life span after its obsolescence. This multi-stage decision-making problem can be solved by dynamic programming, but technical features that change over the years, unpredictable economy and time can cause uncertainty in prices. By combining classical dynamic programming with fuzzy set theory, this model can be revised. The purpose of this paper is to develop an optimal equipment replacement policy using combined interval type-2 fuzzy set and dynamic programming for the first time. The proposed methodology is applied to server equipment replacement problem.**

Keywords*— server equipment replacement, fuzzy dynamic programming, interval type-2 fuzzy sets, replacement analysis

I. Introduction

The demand for renewal is attribute to factors such as age, worn out, lack of economic sufficiency, or technical inadequacy. In this case, engineers can apply different scenarios such as the use of old machine after maintenance or to purchase a new machine. The basic problem concerns a type of machines which deteriorate with age, and make decision about when to replace the incumbent machine, when to replace its replacement so as to minimize the total cost during the next N years.

In economic terms, the management will have to evaluate whether he wants to continue with the existing equipment which was purchased maybe few years back and it has still some life or to purchase new equipment with certainly some added cost. But this added cost could be offset by the lower operating costs of the new equipment and also the life which ahead.

There are globally referred to as the “forces of retirement” and these forces include the following:

- Physical Deterioration
- Technological Obsolescence
- Functional Obsolescence
- Legal Obsolescence
- Style/Aesthetic Obsolescence
- Economic Obsolescence

In addition to the problems developing on the machine over the years, there are some factors that are effective in the decision to renew. The technical features that change over the years may be insufficient, there may be fluctuations in the economy and money may lose its value over time. Technical, economic and time-related uncertainties can be used to express the price. In order to express such uncertainties and use them in the decision-making problem, the fuzzy set theory developed by Zadeh [1] in 1965. The dynamic programming model used in the renewal analysis can be revised to cover these uncertainties. Thus, instead of the crisp numbers used in the classical dynamic programming model, decisions can be made with ambiguous expressions created depending on the situation. The contribution of this paper is the development an optimal equipment replacement policy using combined interval type-2 fuzzy set and dynamic programming for the first time. The proposed methodology is applied to server equipment replacement problem.

The paper’s overview is as follows. Section II illustrates literature highlights. Section III presents proposed methodology, which is formalized as dynamic programming with interval type-2 fuzzy numbers. Server equipment replacement is selected as the application and dynamic approach with interval type-2 fuzzy numbers is applied to decide replacement policy in Section IV. Finally, the comments and discussion are given in the last section.

II. Literature Review

The principles of dynamic programming are used in studying certain types of sequential decision problems. A sequential decision problem may be characterised as one in which each decision affects future decisions.

The concepts of a “state”, a “stage”, and the principle of optimality are necessary in explaining the structure of dynamic programming. State variables are those values which completely describe the instantaneous situation of the process. The process may be viewed as a planning horizon composed of finite or infinite stages. The designation of the stage is arbitrary, and depends upon the convenience of the planner and the intricacies of the problem. The stage of the system may be defined as certain points in time where planners make decisions.

Another important concept in dynamic programming is the “decision vector”. At each stage the planner makes a decision by choosing between available alternatives or policies, which involve a transformation in state variables. Each transformation is associated with a certain amount of cost or reward for any given activity . The number of policies open to the planner are dependent upon several factors and may vary from state to state. A rational decision maker will choose the policy which maximizes the objective function of the system [2].

Replacement is required in systems such as machines, tools, vehicles, capital assets and others and it may arise as a

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result of technological advancement, deterioration in efficiencies of these items over their life span, or it may be due to temporary or complete failure of these systems. The machinery replacement is seen as the act of finding the adequate moment to change equipment in use, based on the analysis of a criterion or of a decision criteria group [3].

Aronson and Aronofsky [4] gives two basic reasons for equipment replacement. The first being degradation or deterioration and obsolescence and the second is the complete or partial failure that may occur in the original unit or units which in turn forces the decision of immediate replacement or repair of single or group units. Similarly, Sen et al. [5] identify some situations when replacement can take place. They include: when an old item has failed and does not work at all or the item is expected to fail soon; when an existing item deteriorates and works badly and also needs expensive maintenance; and when a better model of an item has been developed. Consequently, there are several approaches for resolving equipment replacement problems. These include total average cost, equivalent annual cost, differential equations, capital budgeting/cash flow, dynamic programming, Markovian processes and shortest path methods [4].

Putterman [6] captures dynamic programming as a procedure for finding optimal policies for sequential decision problems. Adda and Cooper [7] simply refer to dynamic programming as the process of solving sequential optimization problems where one needs to find the best decisions one after another. On their part, Gupta and Hira [8] view dynamic programming as a mathematical tool used to simplify decision problems by breaking such decisions down into a sequence of decision steps over time.

Dynamic programming has also been described as a method that in general solves optimization problems that involve making a sequence of decisions by determining, for each decision, subproblems that can be solved in like fashion, such that an optimal solution of the original problem can be found from optimal solutions of the sub-problems" [9]. It is important to note that a common thread runs through these definitions. The common thread is that dynamic programming deals with sequential decision processes that entail dividing the problem to be solved into smaller problems known as sub-problems or stages. The sub-problems are solved one after the other, so that the answers to these small problems are used to solve the larger ones in order that the overall solution is optimal in relation to the original problem. Cooper and Cooper [10] enumerate the main elements associated with a dynamic programming problem to include stages, states, decisions, transformations and returns. In the literature, problems that can be solved by dynamic programming exhibit the following properties:

- The problem can be decomposed into sub-problems or stages and a decision is be made in each stage.
- Each stage has a number of possible states.
- The decision in each stage is to transform the current state into a state associated with the next stage.
- The Policy (or the best sequence of decisions) at any stage is independent of the decisions made at prior stages.
- There is a recursive relationship which identifies the optimal decision at stage t, given that stage t + 1 has already been solved.
- The history of the system must have no influence on its future behaviour.

The advantages of using dynamic programming in equipment replacement problems are that few constraints are placed on the function. It is flexible, and it has the ability to generate solutions quickly as well as optimize them within a wider range of options. Other advantages cited by some authors include the determination of absolute (global) maxima or minima and its ability to handle non-linear and discontinuous functions [11,12].

Recently, Tarawneh et al. [13] worked on field evaluation and behavior of the soil improved using dynamic replacement. They propose a methodology to estimate the settlement of the soil improved using dynamic replacement. Wang and Nguyen [14] propose a solution to technology replacement policy and capacity plan of resources. They had capacity planning with technology replacement by stochastic dynamic programming. They solved the problem by a pattern search-genetic algorithm to maximize the expected net present profit over a finite time horizon. Fan et al. [15] used a stochastic dynamic programming approach for the equipment replacement optimization under uncertainty. They developed the Bellman approach and implemented it to solving the equipment replacement optimization dynamic programming problem. Hsu et al. [16] applied a dynamic programming approach to aircraft replacement scheduling. They developed a stochastic dynamic programming model to optimize airline decisions regarding purchasing, leasing, or disposing of aircraft over time. Moghaddam and Usher [17] presented mathematical models and a solution approach to determine the optimal preventive maintenance schedules for a repairable and maintainable series system of components with an increasing rate of occurrence of failure.

Huirne et al. [18] introduced a stochastic dynamic programming model, which runs on a personal computer to determine the economic optimal replacement policy in swine breeding herds. Kececioglu and Sun [19] proposed a general discrete-time dynamic programming model for the opportunistic replacement policy and its application to ball-bearing systems. Ding et al. [20] studied on a dynamic approach for emergency decision making based on prospecttheory with interval-valued Pythagorean fuzzy linguistic variables.

Chaudhuri and Suresh [21] developed An algorithm for maintenance and replacement policy using fuzzy set theory. Their model can be used for making a maintenance and replacement policy for a finite time horizon. Cardoso and Gomide [22] studied on Newspaper demand prediction and replacement model based on fuzzy clustering and rules. Their aim is to predict newspaper demand as accurately as possible to meet customer need and decrease loss, the number of newspaper offered but not sold. Popova and Wu [23] used renewal reward processes with fuzzy rewards and their applications to T-age replacement policies. Zhang et al. [24] stressed about fuzzy age-dependent replacement policy and simultaneous perturbation stochastic approximation algorithm based-on fuzzy simulation. Tolga et al. [25] select an operating system using fuzzy replacement analysis and analytic hierarchy process. Their aim is creating an Operating

System selection framework for decision makers by using fuzzy expression on economic part of the decision process. Chang [26] presented a fuzzy methodology for replacement of equipment. He modeled addible market and cost effects from the replacements against the counterpart fuzzily and interactively in addition to the equipment deterioration.

III. METHODOLOGY

In this section, the preliminaries and definitions of the proposed method with interval type-2 fuzzy numbers [27] are given.

A. Preliminaries

A type-2 fuzzy set ($\tilde{\tilde{A}}$) in the universe of discourse X can be presented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$ as follows;

$$\tilde{\tilde{A}} = \left\{\left((x,u), \mu_{\tilde{\tilde{A}}}(x,u)\right) \middle| \ \forall x \in X, \forall u \in J_x \subseteq [0,1], \ 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1 \right\} \quad (1)$$

The type-2 fuzzy set $\tilde{\tilde{A}}$ can also be presented as follows [28]:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u) \quad (2)$$

where $J_x \subseteq [0,1]$ and $\iint$ denote union over all admissible x and u.

Interval type-2 fuzzy set [29] is a special case of this definition where $\mu_{\tilde{\tilde{A}}}(x,u) = 1$. Based on the definition, trapezoidal interval type-2 fuzzy set is represented as follows [28]:

$$\tilde{\tilde{A}}_i = (\tilde{A}_i^U; \tilde{A}_i^L) = \left(\begin{pmatrix} a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; \\ H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U) \end{pmatrix}, \begin{pmatrix} a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; \\ H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L) \end{pmatrix}\right) \quad (3)$$

where $\tilde{A}_i^U$ and $\tilde{A}_i^L$ are type-1 fuzzy sets; a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L *and* a_{i4}^L are the references points of the interval type-2 fuzzy set $\tilde{\tilde{A}}_i$. $H_j(\tilde{A}_i^U)$ shows the membership value of the element $a_{j(j+1)}^U$ in the upper trapezoidal membership function $(\tilde{A}_i^U)$, $1 \leq j \leq 2$. $H_j(\tilde{A}_i^L)$ denotes the membership value of the element $a_{j(j+1)}^L$ in the lower trapezoidal membership function $\tilde{A}_i^L$, $1 \leq j \leq 2$ [30]. Figure 1 represents a trapezoidal interval type-2 fuzzy set.

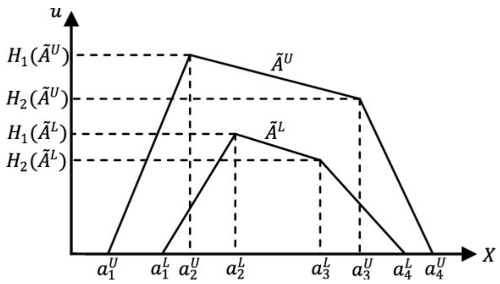

Fig. 1. Illustration of trapezoidal interval type-2 fuzzy set

Assume k is a crisp number and $\tilde{\tilde{A}}_1$, $\tilde{\tilde{A}}_2$ interval type-2 fuzzy sets as given in the following:

$$\tilde{\tilde{A}}_1 = \left(\begin{pmatrix} a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; \\ H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U) \end{pmatrix}, \begin{pmatrix} a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; \\ H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L) \end{pmatrix}\right)$$

$$\tilde{\tilde{A}}_2 = \left(\begin{pmatrix} a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; \\ H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U) \end{pmatrix}, \begin{pmatrix} a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; \\ H_1(\tilde{A}_2^L), H_2(\tilde{A}_2^L) \end{pmatrix}\right)$$

The arithmetic operations given by Chen and Lee [30] are follows:

Addition: $\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 = ((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \min\left(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)\right), \min\left(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U)\right)), (a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; \min\left(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)\right), \min\left(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U)\right)))$ (4)

Substraction: $\tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 = ((a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; \min\left(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)\right), \min\left(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U)\right)), (a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L; \min\left(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)\right), \min\left(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U)\right)))$ (5)

Multiplication: $\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 = ((a_{11}^U * a_{21}^U, a_{12}^U * a_{22}^U, a_{13}^U * a_{23}^U, a_{14}^U * a_{24}^U; \min\left(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)\right), \min\left(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U)\right)), (a_{11}^L * a_{21}^L, a_{12}^L * a_{22}^L, a_{13}^L * a_{23}^L, a_{14}^L * a_{24}^L; \min\left(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)\right), \min\left(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U)\right)))$ (6)

Multiplication with a crisp number:

$$k\tilde{\tilde{A}}_1 = \left(\left(k * a_{11}^U, k * a_{12}^U, k * a_{13}^U, k * a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)\right), (k * a_{11}^L, k * a_{12}^L, k * a_{13}^L, k * a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L))\right) \quad (7)$$

Division:

$$\frac{\tilde{\tilde{a}}_{ij}}{\tilde{\tilde{b}}_{ij}} = \left(\left(\frac{a_1^U}{b_4^U}, \frac{a_2^U}{b_3^U}, \frac{a_3^U}{b_2^U}, \frac{a_4^U}{b_1^U}; \min\left(H_1(a^U); H_1(b^U)\right), \min\left(H_2(a^U); H_2(b^U)\right)\right), \left(\frac{a_1^L}{b_4^L}, \frac{a_2^L}{b_3^L}, \frac{a_3^L}{b_2^L}, \frac{a_4^L}{b_1^L}; \min\left(H_1(a^L); H_1(b^L)\right), \min\left(H_2(a^L); H_2(b^L)\right)\right)\right) \quad (8)$$

Division by crisp number:

$$\frac{\tilde{\tilde{A}}_1}{k} = \left(\left(\frac{1}{k} * a_{11}^U, \frac{1}{k} * a_{12}^U, \frac{1}{k} * a_{13}^U, \frac{1}{k} * a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)\right), \left(\frac{1}{k} * a_{11}^L, \frac{1}{k} * a_{12}^L, \frac{1}{k} * a_{13}^L, \frac{1}{k} * a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)\right)\right)$$

where *k>0.* (9)

B. Proposed Model

A dynamic programming model for machine replacement problem consists of revenue, operating cost and salvage value [31]. The year is represented by *"i" (i=1,2,…,n)* and the State at Stage-(i) is the age of machine at the beginning of the i^{th} year. The recursive equation in fuzzy environment is as follows:

$$\tilde{f}_i(t) = max \begin{cases} DTtrT(\tilde{r}(t) - \tilde{c}(t) + \tilde{f}_{i+1}(t)) & \text{if keep,} \\ DTtrT(\tilde{r}(0) + \tilde{s}(t) - \tilde{c}(0) - \tilde{p} + \tilde{f}_{i+1}(t1)) & \text{if replace.} \end{cases} \quad (10)$$

The boundary condition is

$$\tilde{f}_{iN+1}(t) = \tilde{s}(t), \ \forall t. \quad (11)$$

where,

$\tilde{f}_i(t)$ = maximum net income for years (i=1,2,…,n),

$\tilde{r}(t)$ = yearly revenue,

$\tilde{s}(t)$ = salvage value,

$\tilde{c}(t)$ = operating cost,

$\tilde{p}$ = acquisition cost of new machine.

Defuzzifying and comparing fuzzy costs to decide “keep” and “replace”, we use the DTtrT method [32] as follows:

$$\mathrm{DTtrT} = \frac{\frac{(u_u - l_u)+(\beta_u m_{1u} - l_u)+(\alpha_u m_{2u} - l_u)}{4} + l_u + \frac{(u_L - l_L)+(\beta_L m_{1L} - l_L)+(\alpha_L m_{2L} - l_L)}{4} + l_L}{2} \tag{12}$$

The alternatives at Stage-(i), call for either keeping or replacing the machine at the beginning of the year.

IV. Application

A university wants to replace its 4-years-old servers. All computers have the same components and they are bought at the same time. The decision makers apply the interval type-2 fuzzy dynamic programming replacement analysis method for their computers over the next 5 years.

The cost of new machine is \$(200K, 215K, 230K, 250K; 1, 1)(207K, 220K, 225K, 238K; 0.8, 0.8). To determine the revenue of the computer, we look at the annual saving achieved by computer capacity, speed, maintenance and repair costs and electricity costs. Thus, annual savings that increase productivity is about \$(30K, 40K, 50K, 60K; 1, 1) (33K, 42K, 47K, 57K; 0.8, 0.8). This is the revenue of new computer every year this number decreases.

The operating costs include the cost of electricity, maintenance, repair and updating of machines. As a machine ages, it slows down, consumes more electricity and the maintenance cost increases. The new machine costs \$(300, 400, 550, 650; 1, 1)(350, 425, 485, 575; 0.8,0.8) per year. The salvage value of one year old machine is \$(180, 195, 205, 220; 1, 1)(185, 198, 202, 213; 0.8,0.8).

The revenues, operating costs and salvage values by age are given in Table I. According to the regulation of the university, the machines which are 8 years old must be replaced.

To implement dynamic programming, “keep” and “replace” decisions are made according to the Eq.(10)-(11).

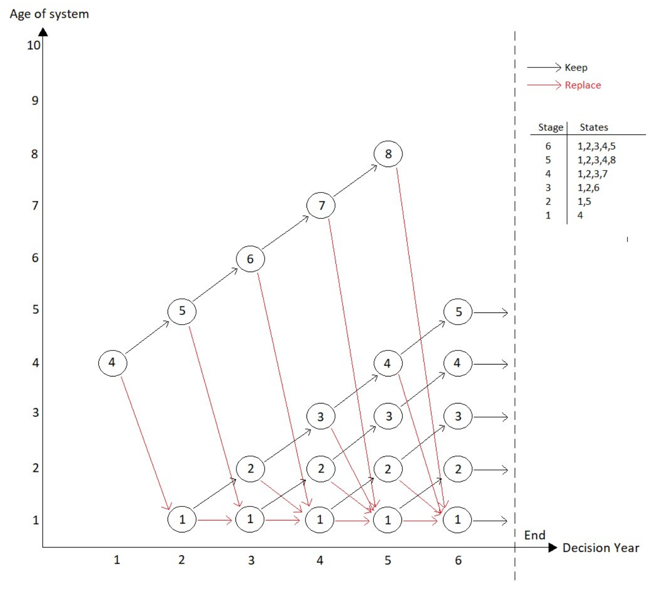

Fig. 2. Network diagram of the server replacement problem

Figure 2 summarizes the network diagram of the computer replacement problem.

Table II-III-IV-V-VI and VII in Appendix present results for various alternatives of interval type-2 fuzzy dynamic programming problem.

- At Stage-1, original 4 years old equipment unit has to be replaced. This means that the server computer at the beginning of the next year would be one year old.
- At Stage-2, the equipment unit must be kept, so machine would be two years old.
- At Stage-3, two years old machine can be replaced or kept. So, there is two ways of the same cost.
  - *Strategy 1:* If university decide to “replace” at Stage-3, machine would be one year old at Stage-4, so it must be kept and at Stage 5 it would be kept as well.
  - *Strategy 2:* If university decide to “keep” at Stage-3, machine would be three years old at Stage-4, so it must be replaced and at Stage 5 it must be kept. Therefore, machine would be two years old at the end of the next 5 years.

Thus, the maximum net income is \$(44.5K, 210.6K, 368.3K, 512.8K; 1,1)(121.525K, 253.84K, 328.07K, 456.55K; 0.8,0.8) and the optimal replacement strategy is either R-K-R-K-K / R-K-K-R-K.

Figure 3 summarizes the network diagram of two optimal results for the server replacement problem.

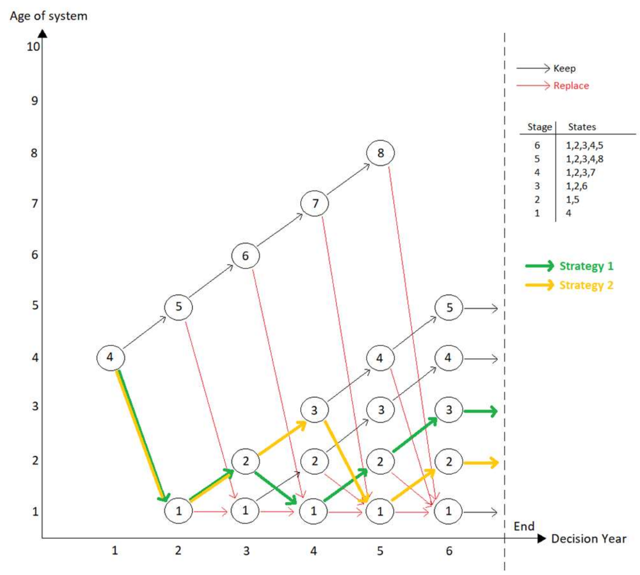

Fig. 3. Network diagram of two optimal results for the server replacement problem

V. Conclusion and Future Work

This study demonstrates a new version fuzzy dynamic programming approach with interval type-2 fuzzy numbers in replacement analysis. This article contributes to the literature by applying an interval type-2 fuzzy dynamic programming to equipment replacement problem. A numerical example for university server equipment is proposed to satisfy the actual methodology. Unlike classical dynamic programming, prices are expressed by taking into account the change in technical characteristics, the unpredictability of the economy and the value of money in years. The multi-stage decision-making approach has been revised with intervel type-2 fuzzy set. Thus, more detailed expressions have been used, rather than the crisp numbers used in the classical model. Based on the given cost in interval type-2 fuzzy numbers, two strategies emerge for replacement decision with the same net income.

In future studies, other fuzzy types can be examined and can be applied to different types of equipment.

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APPENDIX

TABLE I. THE REVENUES, OPERATING COSTS AND SALVAGE VALUES BY AGE

Age (t)	Revenue $\tilde{r}(t)$	Operating Cost $\tilde{c}(t)$	Salvage Value $\tilde{s}(t)$
0	(30,40,50,60;1,1) (33,42,47,57;0.8,0.8)	(0.3,0.4,0.55,0.65;1,1) (0.35,0.425,0.485,0.875;0.8,0.8)	X[a]
1	(29,37,48,55;1,1) (31,39,45,53;0.8,0.8)	(0.65,0.75,0.875,0.95;1,1) (0.7,0.825,0.85,0.875;0.8,0.8)	(180,195,205,220;1,1) (185,198,202,213;0.8,0.8)
2	(27,35,45,52;1,1) (30,38,43,50;0.8,0.8)	(1.1,1.3,1.45,1.6;1,1) (1.25,1.375,1.375,1.45;0.8,0.8)	(165,185,200,210;1,1) (175,190,199,205;0.8,0.8)
3	(24,31,44,50;1,1) (27,33,42,47;0.8,0.8)	(1.645,1.9,2.1,2.35;1,1) (1.75,2.165,2.05,2.265;0.8,0.8)	(155,170,180,195;1,1) (163,174,177,189;0.8,0.8)
4	(20,28,41,47;1,1) (20,27,35,42;0.8,0.8)	(2.15,2.4,2.5,2.675;1,1) (2.3,2.65,2.375,2.57;0.8,0.8)	(140,145,165,185;1,1) (143,150,160,180;0.8,0.8)
5	(17,24,38,45;1,1) (33,42,47,57;0.8,0.8)	(2.87,3.0,3.15,3.3;1,1) (2.95,3.125,3.125,3.225;0.8,0.8)	(123,130,150,170;1,1) (127,140,144,155;0.8,0.8)
6	(14,20,34,41;1,1) (17,23,31,38;0.8,0.8)	(3.145,3.4,3.575,3.75;1,1) (3.3,3.5,3.55,3.575;0.8,0.8)	(84,95,113,127;1,1) (90,103,105,120;0.8,0.8)
7	(10,16,29,38;1,1) (13,18,26,34;0.8,0.8)	(3.315,3.6,3.845,4.05;1,1) (3.475,3.725,3.775,3.95;0.8,0.8)	(63,70,87,102;1,1) (65,75,84,96;0.8,0.8)
8	(7,13,26,35;1,1) (9,15,23,31;0.8,0.8)	(3.975,4.1,4.35,4..55;1,1) (4.0,4.275,4.3,4.375;0.8,0.8)	(44,50,57,65;1,1) (47,52,55,62;0.8,0.8)

[a] X means that there is no salvage value for a computer less than one year old.

TABLE II. STAGE-6

t	$f_6(t)$	*Decision*
0	(30,40,50,60;1,1) (33,42,47,57;0.8,0.8)	-
1	(29,37,48,55;1,1) (31,39,45,53;0.8,0.8)	-
2	(27,35,45,52;1,1) (30,38,43,50;0.8,0.8)	-
3	(24,31,44,50;1,1) (27,33,42,47;0.8,0.8)	-
4	(20,28,41,47;1,1) (20,27,35,42;0.8,0.8)	-
5	(17,24,38,45;1,1) (33,42,47,57;0.8,0.8)	-
6	(14,20,34,41;1,1) (17,23,31,38;0.8,0.8)	-
7	(10,16,29,38;1,1) (13,18,26,34;0.8,0.8)	-
8	(7,13,26,35;1,1) (9,15,23,31;0.8,0.8)	-

TABLE III. STAGE-5

t	*Keep* $\tilde{r}(t)-\tilde{c}(t)-\tilde{f}_6(t+1)$	*DtTrt*	*Replace* $\tilde{r}(0)+\tilde{s}(t)-\tilde{c}(0)-\tilde{p}+\tilde{f}_6(1)$	*DtTrt*	$f_5(t)$	*Decision*
1	(193.05, 221.125, 247.25, 264.35;1,1) (205.125,228.15,243.175,257.3;0.8,0.8)	882,63	(139.35,199.45,244.6,299.7;1,1) (164.425,212.515,230.575,275.65;0.8,0.8)	838,8235	(193.05, 221.125, 247.25, 264.35;1,1) (205.125,228.15,243.175,257.3;0.8,0.8)	K
2	(180.4,203.55,223.7,245.9;1,1) (191.55,210.625,218.625,237.75;0.8,0.8)	813,125	(124.35,189.45,239.6,289.7;1,1) (154.425,204.515,227.575,267.65;0.8,0.8)	805,4235	(180.4,203.55,223.7,245.9;1,1) (191.55,210.625,218.625,237.75;0.8,0.8)	K
3	(161.65,173.9,207.1,233.355;1,1) (167.735,180.95,199.835,225.25;0.8,0.8)	736,809	(114.35,174.45,219.6,274.7;1,1) (142.425,188.515,205.575,251.65;0.8,0.8)	746,2235	(161.65,173.9,207.1,233.355;1,1) (167.735,180.95,199.835,225.25;0.8,0.8)	R
4	(140.325,155.5,188.6,214.85;1,1) (146.43,167.735,179.35,196.7;0.8,0.8)	659,993	(99.35,149.45,204.6,264.7;1,1) (122.425,164.515,188.575,242.65;0.8,0.8)	682,8235	(140.325,155.5,188.6,214.85;1,1) (146.43,167.735,179.35,196.7;0.8,0.8)	R
8	Must replace	-	(3.35,54.45,96.6,144.7;1,1) (26.425,66.515,83.575,124.65;0.8,0.8)	285,1235	(3.35,54.45,96.6,144.7;1,1) (26.425,66.515,83.575,124.65;0.8,0.8)	R

TABLE IV. STAGE-4

t	*Keep* $\tilde{r}(t)-\tilde{c}(t)-\tilde{f}_5(t+1)$	*DtTrt*	*Replace* $\tilde{r}(0)+\tilde{s}(t)-\tilde{c}(0)-\tilde{p}+\tilde{f}_5(1)$	*DtTrt*	$f_4(t)$	*Decision*
1	(208.45,239.675,270.95,300.25;1,1) (221.675,248.775,262.8,290.05;0.8,0.8)	970,155	(152.4,225.575,286.85,344.05;1,1) (184.55,242.665,271.75,319.95;0.8,0.8)	962,4535	(208.45,239.675,270.95,300.25;1,1) (221.675,248.775,262.8,290.05;0.8,0.8)	K
2	(139.75,208,263.3,325.6;1,1) (170.975,225.14,247.2,300.4;0.8,0.8)	892,9485	(137.4,215.575,281.85,334.05;1,1) (174.55,234.665,268.75,311.95;0.8,0.8)	929,0535	(137.4,215.575,281.85,334.05;1,1) (174.55,234.665,268.75,311.95;0.8,0.8)	R
3	(121,178.35,246.7,313.055,52;1,1) (147.16,195.465,228.41,287.9;0.8,0.8)	816,6325	(127.4,200.575,261.85,319.05;1,1) (162.55,218.665,246.75,295.95;0.8,0.8)	869,8535	(127.4,200.575,261.85,319.05;1,1) (162.55,218.665,246.75,295.95;0.8,0.8)	R
7	(9.3,66.605,4,122,179,385;1,1) (35.475,80.74,105.85,155.175;0.8,0.8)	358,606	(35.4,100.575,168.85,226.05;1,1) (64.55,119.665,153.75,202.95;0.8,0.8)	508,5535	(35.4,100.575,168.85,226.05;1,1) (64.55,119.665,153.75,202.95;0.8,0.8)	R

TABLE V. STAGE-3

t	*Keep* $\tilde{r}(t) - \tilde{c}(t) - \tilde{f}_4(t+1)$	$DtTrt$	*Replace* $\tilde{r}(0) + \tilde{s}(t) - \tilde{c}(0) - \tilde{p} + \tilde{f}_4(1)$	$DtTrt$	$f_3(t)$	*Decision*
1	(165.45,251.7,329.1,388.4;1,1) (204.675,272.815,312.925,364.25;0.8,0.8)	1086,084	(167.8,244.125,310.55,379.95;1,1) (201.1,263.29,291.375,352.7;0.8,0.8)	1049,979	(165.45,251.7,329.1,388.4;1,1) (204.675,272.815,312.925,364.25;0.8,0.8)	K
2	(152.8,234.125,305.55,369.95;1,1) (191.1,255.29,288.375,344.7;0.8,0.8)	1016,579	(152.8,234.125,310.55,379.95;1,1) (191.1,255.29,288.375,344.7;0.8,0.8)	1016,579	(152.8,234.125,310.55,379.95;1,1) (191.1,255.29,288.375,344.7;0.8,0.8)	K/R
6	(45.65,117.0,199.45,263.905;1,1) (77.975,139.115,181.25,237.65;0.8,0.8)	598,961	(71.8,144.125,218.55,286.95;1,1) (106.1,168.29,194.375,259.7;0.8,0.8)	688,6785	(71.8,144.125,218.55,286.95;1,1) (106.1,168.29,194.375,259.7;0.8,0.8)	R

TABLE VI. STAGE-2

t	*Keep* $\tilde{r}(t) - \tilde{c}(t) - \tilde{f}_3(t+1)$	$DtTrt$	*Replace* $\tilde{r}(0) + \tilde{s}(t) - \tilde{c}(0) - \tilde{p} + \tilde{f}_3(1)$	$DtTrt$	$f_2(t)$	*Decision*
1	(232,250.675,267.375,273.15;1,1) (229.825,243.525,267.465,283.4;0.8,0.8)	1173,609	(124.8,256.15,368.7,468.1;1,1) (184.1,287.33,341.5,426.9;0.8,0.8)	1165,907	(232,250.675,267.375,273.15;1,1) (229.825,243.525,267.465,283.4;0.8,0.8)	K
5	(85.5,164.975,253.55,329.08;1,1) (122.875,192.165,226.25,298.75;0.8,0.8)	794,731	(67.8,191.15,313.7,418.1;1,1) (126.1,229.33,283.5,368.9;0.8,0.8)	948,007	(67.8,191.15,313.7,418.1;1,1) (126.1,229.33,283.5,368.9;0.8,0.8)	R

TABLE VII. STAGE-1

t	*Keep* $\tilde{r}(t) - \tilde{c}(t) - \tilde{f}_2(t+1)$	DtTrt	*Replace* $\tilde{r}(0) + \tilde{s}(t) - \tilde{c}(0) - \tilde{p} + \tilde{f}_2(1)$	$DtTrt$	$f_1(t)$	*Decision*
4	(85.125,216.65,352.3,462.95;1,1) (145.53,256.955,318.85,410.6;0.8,0.8)	1066,9	(44.15,210.6,368.3,512.8;1,1) (121.525,253.845,328.075,456.55;0.8,0.8)	1097,432	(44.15,210.6,368.3,512.8;1,1) (121.525,253.845,328.075,456.55;0.8,0.8)	R