A Three-way Classification with Game-theoretic N-Soft Sets for Handling Missing Ratings in Context-aware Recommender Systems

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***Abstract*—Context-aware recommender system (CARS) plays a vital role to paved way for improving traditional recommendation phenomena. A key issue in CARS is the selection of influencing contextual group which is most suitable for the recommendation of an item. In general, the selection of suitable contextual group in CARS is faced with challenges due to uncertainty in the classification of items with missing non-binary ratings. The underlying reaction of each user towards an item is implicitly assumed to be binary in nature because of which uncertainty occurs in the item classification. In particular, such a situation where ratings are missing for an item, make the exploitation of appropriate contextual group difficult for an item recommendation. In this article, we address the problem of the inappropriate classification of items into irrelevant contextual groups due to missing non-binary ratings. To this extent, we propose a three-way classification model using game-theoretic N-soft sets for improving the classification process by handling missing ratings. In particular, a game is formulated using game-theoretic N-soft sets to determine the effective threshold configuration used to induce three-way classification of items with missing non-binary ratings. Moreover, a thorough evaluation of our proposed model is carried out on the datasets of *LDOS-CoMoDa* and *InCarMusic*, where outcome signifies the effectiveness and performance of the proposed model.**

***Index Terms*—Three-way classification, Missing ratings, Game theory, N-Soft Sets, Context-aware Recommendation**

I. Introduction

It has been indicated that the consideration of contextual information leverage the performance of traditional recommender systems by utilizing the contextual factors under which the user has rated an item of interest [1]. For enticing the relevant recommendation, context-aware recommender system (CARS) efficiently performs a dynamic process by thoroughly analyzing the ratings explicitly provided by users to an item in a contextual scenario [2]. In general, a classification scheme is formulated in most of the time which categorize users into groups based upon their current contextual information [3]–[7]. The basic purpose behind this classification is to categorize the preferences of item consumption based upon the set of scenarios [8]. In particular, one of the major challenges faced by CARS is classification of an item with missing ratings into relevant contextual group [9]. The contribution of each users' opinion, in extant researches, is considered to be of binary (Like or Dislike) in nature during recommendation process [10]. But in fact, in daily life, non-binary opinion can be observed during evaluation of an item namely, Positive, Negative, or Missing rating [5], [11], [12]. Therefore, conflicting situation exists in exploitation of appropriate contextual group for an item due to the uncertain ratings on an item [5]. To this extent, the uncertain ratings in more general cases do not result in effective classification of an item in an appropriate contextual group based upon the non-binary opinion under realistic dynamic contexts [10].

The main contribution of this study is the alleviation of conflicting situation in classification due to missing ratings of an item. Although, several studies has suggested to consider the influence of contextual attributes different during recommendation process, while neglecting the categorization of non-binary decisive opinion ratings under different contexts [4]. To the best of authors' knowledge, however, no study has been publish yet to alleviate the conflicting situation due to missing ratings during classification process of items in CARS. In order to resolve the formulated problem, we proposed a three-way classification using Game-theoretic N-Soft Sets (*TWC-GTNS*). In many cases, however, it has been identified that the computation of effective threshold values in three-way classification is not practical [13]. To that purpose, we have formulated a game using novel combination of game theory and N-softs to compute the effective threshold configuration for inducing three-way classification. In view of the importance of non-binary ratings under different contexts in practical scenarios, N-Soft Set in conjunction with game theory, will represent the conflicting classification situation due to missing ratings on an item in the form of information systems. Consequently, three-way classification will utilize the thresholds from game theory to define the type of decisions for the categorization of items.

The rest of the paper is organized as follows: Section II presents extant research efforts related to our work. Section III demonstrates the fundamentals and limitations of three-way classification. Section IV describes the working of our proposed methodology. Subsequently, experimental setup and discussion on results is provided in Section V, followed by, the conclusion of paper and future directions in Section VI.

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II. Related Work

In this section, we have formulated the problem statement related to ignoring missing ratings in classification. In general, the extant researches consider a binary evaluation/ratings for an item which clearly reduce the efficiency of recommendation for practical real world dataset [5], [8]. As, in daily life, a binary evaluation is not always feasible especially in situations that are characterized by uncertainty and lack of appropriate rating on an item [5]. For the sake of improving quality of recommendation, various kind of approaches has been adopted in extant researches to exploit relevant contextual group [1], [8]. Typically, the most frequently adopted technique to exploit relevant contexts is through statistical methods by rating data [2], [9]. However, the findings of these research work lacks accuracy as statistical methods have their own limitations and fails to provide relevant results when dense rating are not available for all given items in each contextual scenario [4]. In addition, these two studies [2], [9] does not focus upon the selection of relevant contextual group and also ignore the inclusion of missing ratings over items. In particular, the process of recommendation is based upon the classification of the users in specific groups with common contextual attributes and ratings on similar item [3], [9]. The classification approach has been identified as the most appropriate technique for the exploitation of relevant context by associating the items with influencing contextual groups. Besides the traditional classification approaches such as kNN [3], [9], K-means [3], [4], we can cite some state-of-the-art advance approaches such as t-test approach which use statistical methodology to extract the influencing contextual group for an item [8], but it was not practical for a large set of contextual attributes [5]. Relatedly, a rough set based attribute reduction technique was introduced by authors in [11]. According to this study, the deferring decision for the evaluation of an item can be reduce to a positive or negative decision. However, rough sets lack parameterization property which results in limited decision rules for a large set of attributes [11]. To this end, the most suitable approach proposed by authors in [1] and [8] is the combination of rough sets and soft sets (Soft-Rough sets) for selecting minimal set of influencing contextual attributes in which a video/item is highly relevant. The major difference between these two studies is that in [1], only a hypothetical observation has been implemented using Soft-Rough sets over a small dummy data. However, to support the previously proven hypothesis, the authors in [8] has implemented the proposed approach over a well-known context-aware dataset ("LDOS-CoMoDa"). Both studies, however, ignore the missing ratings during the computation of minimal set of influencing contextual attributes and therefore concerned about the classification of items with missing ratings in different contexts. Relatedly, an approach based upon associative classifier has been developed by the authors in [12] to fill missing data in a database. However, the major challenge of this research work was the identification of appropriate classifier each time. The change in classifier values every time refers to the change in classification scheme.

Aided by the studies from literature, we proposed a cutting-edge three-way classification approach for improving classification of an item by considering non-binary missing evaluation. Our work addresses the wider and more generic problem of conflicting situation in appropriate item classification in relevant contextual group. The prior attempts by extant research works to handle conflict, motivates that improving of classification algorithm is not a suitable solution [8]. There is a need of novel methodology which can cope with the classification of items with uncertain opinions [5]. To that purpose, we proposed a novel Three-way classification (TWC) approach in which the selection of suitable threshold configuration will be achieve by fuse N-Soft Set theory with Game theory for handling conflict in classification. The basic motivation behind our proposed approach has been taken from [5] and [13]. In particular, Game-theoretic Rough sets were adopted in these articles to compute the appropriate thresholds for TWC. In comparison, our proposed approach is different from these studies in terms of thresholds computation for TWC. Our work mainly focus upon the conflicting situation handling, hence, representing conflicting condition is more suitable through soft sets [14]. However, in our case, we have used N-soft sets which produce a parameterized description of the item ratings which are neither binary nor continuous and hence it can represent the conflicting situation during classification of items with non-binary and missing evaluation. Such representation will be used by the game theory to compute threshold based on the degree of conflict. Consequently, three-way classification will utilize these thresholds to define the type of decisions for categorizing the items in relevant contextual groups.

III. Limitations of Three-way Classification

A three-way classification (TWC) decisions are constructed based on the ternary notations of acceptance to classify in a group, rejection from the classification, and deferring from a definite decision [10]. For instance, CARS obtain ratings on an item with sparsity caused due to the missing ratings. If there exist missing rating sparsity during classification process, the system should have to considered the missing ratings thereby deferring a final definite classification decision for a while. In these cases, a TWC method should be used for an added option of handling missing ratings [13].

Most commonly use of three-way decision making can be observed through Probabilistic Rough Set theory which provides a methodology for the classification of incomplete data on the basis of probability thresholds [11]. The formal upper approximation $\overline{apr}$ and lower approximation $\underline{apr}$ can be defined using a threshold (α, β), where $(0 \leqslant \beta < \alpha \leqslant 1)$ [13]. On the basis of probabilistic formal approximation of rough sets, the membership of an object y belonging to a universal set U based upon conditional probability $\mathcal{P}(\mathcal{X}|[y])$ will be in positive, negative, and boundary region using equations 1, 2, and 3, respectively: [15].

$$\begin{aligned} POS_{(\alpha,\beta)}(\mathcal{X}) =& \underline{apr}_{(\alpha,\beta)}(\mathcal{X}) \\ =& \{y \in U \mid \mathcal{P}(\mathcal{X}|[y]) \geqslant \alpha\}, \end{aligned} \tag{1}$$

$$
\begin{aligned}
NEG_{(\alpha,\beta)}(\mathcal{X}) =& \overline{apr}_{(\alpha,\beta)}(\mathcal{X}) \\
=& \{y \,\epsilon\, U \mid \mathcal{P}(\mathcal{X}|[y]) \leqslant \beta\},
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
BND_{(\alpha,\beta)}(\mathcal{X}) =& \overline{apr}_{(\alpha,\beta)}(\mathcal{X}) - \underline{apr}_{(\alpha,\beta)}(\mathcal{X}) \\
=& \{y \,\epsilon\, U \mid \beta < \mathcal{P}(\mathcal{X}|[y]) < \alpha\}.
\end{aligned} \tag{3}
$$

The justification of using probabilistic rough sets for deriving three-way decisions is that in a particular context, each of the three partially ordered sets with pair-wise disjoint regions interpret the level of acceptance, rejection, and deferring from a straighten decision using a threshold (α, β), where ($0 \leqslant \beta < \alpha \leqslant 1$) [10].

However, the state-of-the-art TWC decisions have a major concern related to the accuracy of the classification. It is surprising to note that the principle concern in TWC is the selection of suitable thresholds [10], [13]. As TWC can be viewed as an extension of rough set theory, the classification of an object into a certain group relay upon the selection of probabilities. The change in probability values can refer to the change in classification scheme. Hence, the selection of most appropriate threshold has been a major concern in standard TWC [13]. This problem can be understand through an example. Let we have a set of ratings *R* for different items *I*. To begin, the set *I* can be divided into set *A* and *B*, where set *A* contains items with no missing rating and set *B* contain items with missing ratings. In particular, it is assumed that since items of set *A* does not contain any deferring ratings, therefore the level of conflict will be low and any conventional classification algorithm can be employed for appropriate classification of such items. For classifying items with missing ratings, we can divide the set *A* into sets A_{comp} and A_{miss}, where A_{comp} represents the set of items with complete decisions. Likewise, A_{miss} represents a simulated missing decision item set constructed from *A*, where the rate of missing decision is kept equal to that of the missing decision rate in set *I*. This leads to the division of set *I* into complete decision and induced missing decisions which are randomly selected to be kept equal. Lastly, the items with missing decisions are being decided by three-way decision framework. In particular, the association of each item with a group is validated through three-way decision.

TABLE I: Sample dataset for classification of missing ratings

	c_1	c_2	c_3	c_4		c_1	c_2	c_3	c_4
I_1	7	4	6	5	I_{11}	4	6*	4	6
I_2	8	6	8	7*	I_{12}	5	8	7*	9
I_3	4*	7	9	10	I_{13}	6*	7	9	8
I_4	9	3	3	5	I_{14}	8	9*	3	7*
I_5	6	4*	4*	6	I_{15}	6	3	2*	3
I_6	7	7	6	2*	I_{16}	4*	3	3	2*
I_7	5*	4	3	6*	I_{17}	7	9	7	4*
I_8	7	8	6	3	I_{18}	9	2	8	6
I_9	5*	4	7	5	I_{19}	3	3	3*	8
I_{10}	6	3*	8	7	I_{20}	1	7	3	7

For the sake of simplicity and clarity, the Table I demonstrate the above defined example. Here we have 20 items where rows of the table corresponds to the items which are represented as I_1, I_2, I_3, $\cdots$, I_{20} and columns of the table corresponds to the four contextual groups which are represented as c_1, c_2, c_3, and c_4. To begin, the missing decision rate is randomly chosen for construction of new set from *A*. For a simple illustration, let we have four contextual groups obtained from the given contexts. If a scale of 3 is use for the evaluation of an item than a simple illustration of opinion computation for an item can be defined as: $\sum_{i=1}^{n} \mathrm{R}_{u_i}$, where R_{u_i} denotes the rating from each user towards an item and *n* represents total number of items. The missing opinions are assumed to be the values with asterisk "*" on top of them.

For applying TWC on missing opinions, the evaluation function f(g_i, I_i), where g_i denotes a group, I_i represents an item can be defined as:

$$
f(g_i, I_i) = \frac{Number\, of\, I_i\, neighbors\, belonging\, to\, g_i}{Total\, neighbors\, of\, I_i} \tag{4}
$$

It can be noted that above equation works on the basis of existing neighbors of an item which requires a certain distance measuring metric. So, the distance between two items I_1 and I_2 from Table I can be represented as

$$
\begin{aligned}
distance_{(2,1)} =& \sqrt{\sum_{i=1}^{N} (I_2^i - I_1^i)^2}, \\
=& \sqrt{(8-7)^2 + (6-4)^2 + (8-6)^2 + (* - 5)^2}, \\
=& \sqrt{(8-7)^2 + (6-4)^2 + (8-6)^2} = \sqrt{9} = 3.
\end{aligned}
$$

Where, an item i.e. I_1 represents the value of item 1 under *i*th contextual scenario. Likewise, the distance of item I_2 from all other items in set *A* are distance$_{(I_2,I_1)}$ = 3, distance$_{(I_2,I_3)}$ = 1.41, $\cdots$, distance$_{(I_2,I_{20})}$ = 8.70. By sorting these computed distances, we can find the nearest or closest neighboring items for each I_a. For instance, let us consider that we have found six nearest neighbor of I_2. It should be noted that for the sake of demonstrating the example, we have to consider this assumption. In particular, each of these 6 neighbors belongs to one of the four contextual groups. The computation of one distance is given below,

$$
f(g_1, I_2) = \frac{1}{6} = 0.166.
$$

In particular, the missing opinion value will be ignore during distance metric computing between objects. Once these evaluation functions are computed then each item will have some percentage probability belonging to a particular contextual group. For example, in above computation it has been noted that 16% neighbors of I_2 belongs to the contextual group g_1. Likewise, we can compute the other neighbors' distance. After this, we can use thresholds configuration (α, β) over the computed probabilities to classify each item in a particular group on the basis of probability. The classification, rejection, and deferring of an item from a group will be accomplish according to the concepts given in equation 1, 2, and 3, respectively. In general, if we consider the thresholds (α, β) to be (1, 0) than the item I_2 will be in partial of each of the contextual group due to the arising of deferring condition. Likewise, if we assume the thresholds (α, β) to be (0.4, 0.2) than the item I_2 will be definite decision of classification to contextual group g_3. Hence, it can be concluded that the configuration of thresholds matters a lot during TWC. Moreover, the classification is usually a tradeoff between effectiveness vs generality which can be defined as [13],

$$Effectiveness = \frac{Correctly\, classified\, items}{Total\, classified\, items}, \tag{6}$$

$$Generality = \frac{Total\, classified\, items}{Total\, items\, in\, set\, I}. \tag{7}$$

The effectiveness of classification refers to the fraction of correctly classified item by the total classified items. Likewise, the generality of classification can be defined as the fraction of total classified item by the total items in the set I [13]. In general, there exist a tradeoff between effectiveness and generality. Hence, modifying thresholds to improve generality can affect the effectiveness of classification. Likewise, improving effectiveness may decrease the generality of classification [5]. To that purpose, we proposed a novel TWC approach in which the selection of suitable threshold configuration will be achieve by the fuse of N-Soft Set theory with Game theory for handling missing ratings in classification of items.

IV. Proposed Methodology

In this section, we present the development of novel *Game-Theoretic N-Soft Sets* (GTNS) for computing thresholds to improve *Three-Way Classification* (TWC).

A. Three-way classification with Game Theoretic N-Soft Sets

Earlier in Section III, we demonstrated a link between the thresholds $<\alpha, \beta>$ and the properties of effectiveness and generality of the classification. In this section, a three-way classification model is given on the basis of GTNS for selection of optimal threshold configuration and improving the tradeoff between effectiveness and generality of classification.

1) Formulating a game using using N-Soft sets: The basic idea of fusing N-soft sets with game theory has been taken from the techniques given in [14] and [16]. In our case, the objective behind formulating a game is the selection of optimal threshold configurations for improving the tradeoff between effectiveness $\mathcal{E}$ and generality $\mathcal{G}$ of TWC for classifying missing ratings. Hence, the players of the game will be the effectiveness and generality of the classification. Hence, each player is a tuple such as $\mathcal{P}$ = ($\mathcal{E}$, $\mathcal{G}$). Each player choose a strategy in order to maximize the overall pay-off. Let U be a universal set of paramters, a_n denotes the set of n strategies for the players. Then a N-Soft Set $\mathcal{S}$ over U represented through a pair *($\mathcal{S}$, $\mathcal{T}$)* which can be defined as a triple where $\mathcal{T}$= *($\mathcal{F}$, a_n, U)*, and $\mathcal{F}$ represents a mapping function from a_n towards the possible subsets from $\mathcal{P}(\mathcal{U}) \; \forall \; \mathcal{P}(\mathcal{U}) \in U$.

2) Tradeoff Computation between Effectiveness and Generality: Based upon the previously formulated goal, the players of the game will be the effectiveness and generality of the classification. The effectiveness can be denoted by $\mathcal{E}$ and the generality can be denoted by $\mathcal{G}$. Hence, each player is a tuple such as $\mathcal{P}$ = ($\mathcal{E}$, $\mathcal{G}$). Each player choose a strategy in order to maximize the overall pay-off.

For a particular player $\mathcal{P}$ the selection of a behavior is called action a_t, the pay-off can be demonstrated as $\mu_{\mathcal{P},t}$ = $\mu(a_t)$. Whereas the pay-off or utility represents the benefit or cost as a result of action taken by a player. A simple pay-off function can be used to reflect the true gain based on the opposing players' action. As interesting as it may appears, an action pay-off is not only dependent upon the players' action, but also the opposing players' strategy. In general, the function μ_E is used to compute the pay-off using effectiveness measure. Likewise, the function μ_G is used to compute the pay-off using generality measure. A large value of these pay-off functions reflects the small size of boundary region as an indication that maximum objects are classified to definite groups. For that reason, suppose we have a player $\mathcal{P}$ taking several actions in a game. For the first turn, the player has executed an action t_1. Likewise, in second turn the player has executed an action t_2 from the set of strategies. To achieve the objective of maximizing effectiveness and making a tradeoff between effectiveness and generality, the pay-off of the performed actions should be $\mu_{E_{t_1}} \leqslant \mu_{E_{t_2}}$. However, for the opposite case if $\mu_{E_{t_1}} \geqslant \mu_{E_{t_2}}$, then the player has taken a poor action.

B. Handling missing values in CARS using TWC with GTNS

According to the proposed approach a game will be formulated to identify the most suitable configuration based upon the tradeoff between generality and effectiveness. Consequently, TWC will utilize the thresholds from game theory to define the type of decisions to categorize the missing rating in an appropriate polarity group. The working of our proposed algorithm *TWC-GTNS* for handling missing ratings in CARS is defined in algorithm 1 as follows:

1) Represent the given contextual attributes, items, and ratings from the dataset in the form of N-soft sets *($\mathcal{F}$, a_n, C)*, where $\mathcal{F}$ represents mapping function, a_n denotes the set with possible strategies and contextual groups is denoted by $\mathcal{C}$. Hence, for each contextual group there will be a set of possible strategies which will be choosen by the players of a game.
2) If detect a conflicting situation i.e. rating R = "*" in information system due to missing ratings than go to step 4, else go to step 3.
3) If there is no missing ratings on the items in a context, then all the ratings will definite classified into relevant polarity groups using any conventional algorithm.

4) Divide set of items into two groups *A* and *B*, where *A* denotes the set of items with complete ratings and *B* represents the set of ratings with missing ratings.
5) Apply any classical classification algorithm on set *A* for classifying the items into relevant contextual groups.
6) On the basis of missing ratings in *B*, divide set *A* into A_{comp} and A_{miss}, where A_{comp} contains no missing ratings and A_{miss} represents simulated missing values.
7) For the sake of classification of missing ratings, TWC is adopted. Compute the distance $\mathcal{D}$ of each missing value from the neighbour value. Then a game can be formulated to identify the most influencing configuration of thresholds for employing in TWC. After formulating the game, keep players $\mathcal{P}$ = $\mathcal{E}$, $\mathcal{G}$ and Compute the pay-off values of effectiveness and generality based upon the distance $\mathcal{D}$.
8) Generate a Pay-off table by computing the values of pay-off for each classification from above step. On the basis of pay-off table, learn optimal pay-off for computing the optimal thresholds, where a pay-off will be optimal when it will be greater than the previously computed pay-off.
9) Determine the configuration for the thresholds (α, β) from the above step where generality of the classification should be greater than the value of effectiveness or either $\alpha \leqslant$ 0.5 or $\beta \geqslant$ 0.5.
10) On the basis of computed configuration of thresholds, perform TWC of missing ratings into appropriate groups.
11) Lastly, after TWC, each contextual group will contain items that have been highly rated in that group with appropriate complete ratings.

Algorithm 1: Proposed Algorithm of TWC-GTNS

Input: $\mathcal{I} = Items$, a_n = Strategies, $\mathcal{C} = Contexts$
Output: Three-way classified missing-ratings

```
Compute (F, a_n, C)
Initialize S = (F, a_n, C), where S is N-Soft set
for each S={Ratings on an item} from (F, a_n, C); do
    Divide I into A and B.
    Apply any classification algorithm on A.
    Divide A into set A_comp and A_miss.
    Calculate pay-off using μ_E(α,β) and μ_G(α,β).
    Generate pay-off table using μ_E and μ_G.
    if (μ_E(α,β) ⩽ μ_G(α,β)) || (α ⩽ 0.5 or β ⩾ 0.5) then
        Compute distance D to neighboring ratings.
    else
        Try other possible actions from the pay-off table
    end if
    Apply TWC using computed configuration of (α, β).
end for
```

V. Experimental Results and Discussion

In this section, we illustrated the experiment for assessing the usefulness of our proposed approach.

A. Experimental Setup

In this section, we presented detailed experimental settings adopted during our evaluation process.

1) Datasets Description: For the sake of comparing our proposed methodology with the extant approaches, we required a contextually rich datasets. It has been indicated from several literature studies that the most contextually-rich dataset is *"LDOS-CoMoDa"*, followed by the dataset of *"InCarMusic"* [3], [8]. A brief description of these datasets is: *LDOS-CoMoDa* dataset contains 12 contextual attributes. The total number of users are 121 and 1232 items, followed by 2296 ratings on the items. Likewise, the dataset of *"InCarMusic"* contains 7 contextual attributes with information of 42 users and 139 items. However, this dataset is a collection of 4012 ratings.

2) Experimental Results: For the sake of handling missing ratings in the datasets of "LDOS-CoMoDa" and "InCarMusic" [3], we have applied our proposed algorithm of TWC-GTNS outlined in Section IV-B. In order to simulate the experiment for handling missing ratings, we have randomly removed rating values from different items in different contextual groups. In general, we have use percentage of missing ratings as 10%, 20%, 25%, 30%, and 40%. Likewise, we have compared the results from our TWC-GTNS on these missing rating percentages with the extreme threshold configurations (α, β) which are (0.5, 0.5) model and (1, 0). In the experiment, our proposed algorithm TWC-GTNS has objective to increase the pay-off value for the new action in each iteration. We initially started from the model (1, 0). By changing these thresholds in each iteration, we stopped the algorithm when generality $\geqslant$ effectiveness. Each experiment was conducted multiple times with different set of missing ratings and an average value of results is reported in the tables. Table II is constructed for illustrating the experimental results on "LDOS-CoMoDa" dataset. It can be indicated from the table that for fixed configuration of thresholds as (α, β) = (0.5, 0.5), the effectiveness of classification was 86% with an improved generality of 98%. Likewise, for (α, β) = (1, 0), the concluded effectiveness was 94% with a generality of 35%. In case of "LDOS-CoMoDa" dataset, the determined threshold configuration was (α, β) = (0.60 $\pm$ 0.03, 0.38 $\pm$ 0.02) with an effectiveness of 92.5% and generality of 90.5%. In comparison to the fixed threshold of (0.5, 0.5), the proposed TWC-GTNS improved effectiveness by upto 5.14% at an average decrease of 10% in generality. While, it can be observed in comparison to the fixed threshold of (1, 0), there is significant improvement in the generality by upto 45% at a decrease of 5.38% in effectiveness.

Experimental results on "InCarMusic" dataset are presented in Table III. It can be observed that for extreme threshold configuration (α, β) = (0.5, 0.5), the effectiveness was 86.6% with an average increase of 98.6% in generality. Similarly, the reported effectiveness with (α, β) = (1, 0) was 95% and the generality was 36%. Likewise, In case of "InCarMusic" dataset, the determined threshold configuration was (α, β) = (0.64 $\pm$ 0.02, 0.36 $\pm$ 0.02) with an effectiveness of 92.6%

TABLE II: Results for *"LDOS-CoMoDa"* dataset using proposed model *TWC-GTNS*

Missing Ratings %	*(α, β) = (0.5, 0.5)*		*(α, β) = (1, 0)*		***TWC-GTNS***		
	Effectiveness	Generality	Effectiveness	Generality	(α, β)	Effectiveness	Generality
10%	0.8773	0.9916	0.9913	0.4198	(0.59, 0.37)	0.9316	0.9108
20%	0.8516	0.9832	0.9631	0.4336	(0.59, 0.39)	0.9130	0.9019
25%	0.8498	0.9806	0.9564	0.4219	(0.58, 0.37)	0.8901	0.8704
30%	0.8381	0.9783	0.9433	0.4397	(0.61, 0.38)	0.8813	0.8627
40%	0.8219	0.9769	0.9106	0.4583	(0.62, 0.40)	0.8795	0.8644
Average	0.8477	0.9821	0.9529	0.4346	(0.60, 0.38)	0.8991	0.8801

TABLE III: Results for *"InCarMusic"* dataset using proposed model *TWC-GTNS*

Missing Ratings %	*(α, β) = (0.5, 0.5)*		*(α, β) = (1, 0)*		***TWC-GTNS***		
	Effectiveness	Generality	Effectiveness	Generality	(α, β)	Effectiveness	Generality
10%	0.8721	0.9961	0.9706	0.3369	(0.63, 0.35)	0.9429	0.9208
20%	0.8701	0.9910	0.9618	0.3517	(0.62, 0.36)	0.9386	0.9173
25%	0.8669	0.9819	0.9471	0.3629	(0.64, 0.37)	0.9217	0.9033
30%	0.8613	0.9801	0.9571	0.3699	(0.64, 0.36)	0.9166	0.8996
40%	0.8582	0.9787	0.9216	0.3771	(0.65, 0.48)	0.9086	0.8871
Average	0.8658	0.9856	0.9476	0.3597	(0.64, 0.36)	0.9257	0.9056

and generality of 90.6%. However, it can be noted that in comparison with (0.5, 0.5), there was an average improvement of 6% in effectiveness by TWC-GTNS, along with a mere cost decrease of 8% in generality. Lastly, in comparison with (1, 0), the proposed model improved generality by upto 55% at a cost of only 2.2% decrease in effectiveness. Hence, it can be indicated that the proposed TWC-GTNS is a suitable approach for missing ratings in CARS.

3) Evaluation Protocol and Measures: In order to simulate a realistic context-aware recommendation scenario in our evaluation, we have selected both of the complete datasets of "LDOS-CoMoDa" and "InCarMusic". By having maximum number of ratings in a dataset, we can compute the accuracy of the proposed model. Since, CARS requires an initial phase of training, we have adopted the approach proposed by [5]. Both of the datasets are divided into three subsets namely, training, testing, and validation. The training subset contains 50% of the dataset, testing subset contains 30%, where validation subset contains 20% of the dataset.

In particular, we have compared the proposed methodology to several state-of-the-art approaches which has been used for improving the accuracy of recommendation in CARS. The baseline compared are: SRS-CaVRS proposed in [8], RST based CARS proposed in [11], and DWR proposed by [17]. For the sake of comparison between the baselines and our proposed algorithm, we have use recall@n and DCG@n for evaluating the effectiveness of the recommendation results. Likewise, we have use RMSE and MAE for evaluating the performance of the proposed methodology in comparison with the baselines.

B. Discussion on Results

In this section, we assess the effectiveness and performance of the proposed model in terms of improving results of recommendations by handling missing ratings in comparison to the state-of-the-art approaches. To that purpose, we discuss the computed performance and effectiveness of the proposed model by answering the following questions:

- What is the overall effectiveness of TWC-GTNS for improving recommendation results in CARS?
- What is the overall performance of TWC-GTNS while generating influencing recommendation results?

In the remainder of this section, we address each of these questions in turn.

1) Effectiveness of the proposed approach: As an exercise to address the Q1, we computed the effectiveness of the proposed methodology by comparing it with three state-of-the-art baselines. The results obtained for recall@ n= 5, 10, 50, and 100 on both datasets are summarized in Figure 1 and Figure 2 and presented together in Table IV, where n represents the total number of items observation. The results of our proposed approach has significantly outperforms the existing approaches by improving recommendation quality. Considering both the dataset, the recall values are quite encouraging for our proposed methodology. The results of recommendation gets improve by 7% as indicated by the blue line in both datasets. For the strongest baseline of SRS-CaVRS, it has been observed that the recall values were near to our proposed approach for a smaller value of n. However, the recall values of our approach for both datasets increase in comparison to the baselines with an increase in number of observations. Hence, this behavior indicates that the effectiveness of recommendation using proposed approach will not decrease for a larger observations. Likewise, we further investigate the effectiveness using distributed cumulative gain (DCG). Figure 3(a) and (b) summarize the results of DCG for "LDOS-CoMoDa" and "InCarMusic", respectively. The results of DCG indicates that the gain for effectiveness of

TABLE IV: Results for *LDOS-CoMoDa* and *InCarMusic* on the *Test* subset using proposed model *TWC-GTNS*

Task	Algorithms	Dataset of LDOS-CoMoDa				Dataset of InCarMusic			
		Recall@n				Recall@n			
		n=5	n=10	n=50	n=100	n=5	n=10	n=50	n=100
Alleviating Data Sparsity	DWR	+5.31%(± 0.01)	+6.60%(± 0.01)	+7.37%(± 0.02)	+9.94%(± 0.02)	+8.06%(± 0.01)	+7.13%(± 0.01)	+5.83%(± 0.02)	+3.76%(± 0.02)
	RST	+13.69%(± 0.01)	+19.00%(± 0.01)	+21.41%(± 0.02)	+22.97%(± 0.02)	+16.11%(± 0.01)	+14.54%(± 0.01)	+11.76%(± 0.02)	+7.41%(± 0.02)
	SRS-CaVRS	+16.50%(± 0.01)	+26.50%(± 0.01)	+28.41%(± 0.02)	+30.97%(± 0.02)	+16.11%(± 0.01)	+14.54%(± 0.01)	+11.76%(± 0.02)	+7.41%(± 0.02)
	TWC-GTNS	**+28.50%(± 0.01)**	**+30.02%(± 0.01)**	**+35.57%(± 0.02)**	**+37.47%(± 0.02)**	**+25.38%(± 0.01)**	**+20.32%(± 0.01)**	**+17.90%(± 0.02)**	**+14.89%(± 0.02)**

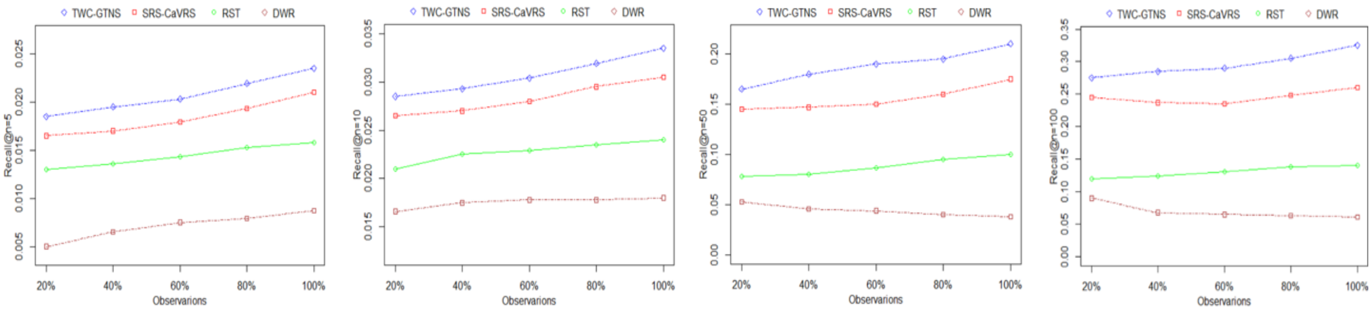


Fig. 1: Results of recall@n with n = 5, 10, 50, 100 of our approach *TWC-GTNS* on *LDOS-CoMoDa*

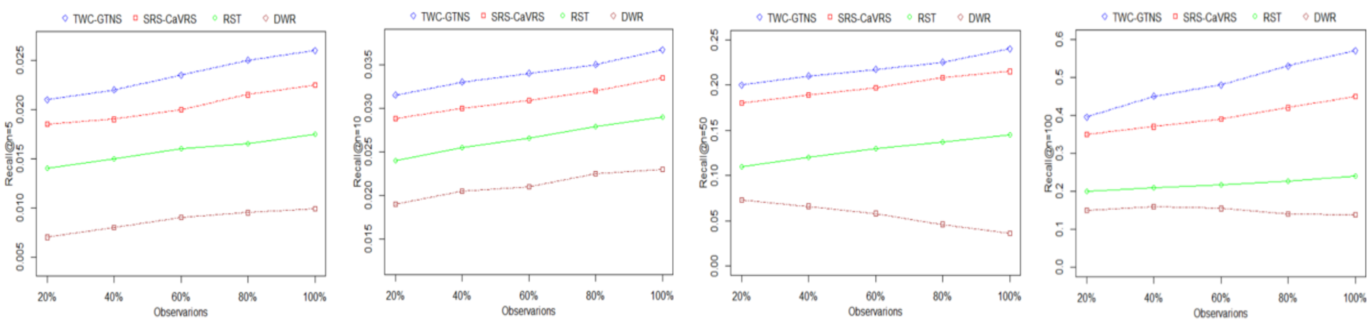


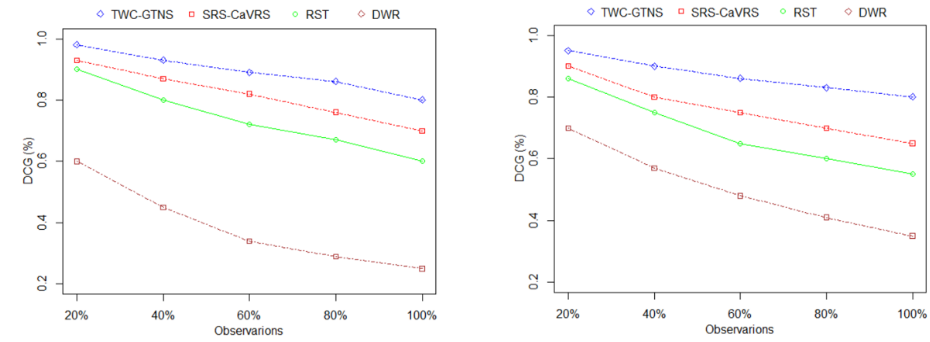
Fig. 2: Results of recall@n with n = 5, 10, 50, 100 of our approach *TWC-GTNS* on *InCarMusic*

recommendation through our approach have high comprehensive values and handling missing ratings using TWC-GTNS for improving CARS is potentially useful.

2) Performance of the proposed approach: In order to evaluate the performance of CARS through the proposed approach, we have used RMSE and MAE. Figure 4 represents the results of RMSE and MAE for both datasets using the three baselines along with proposed approach. A set of four bar is shown for each category of the considered dataset for both evaluations (RMSE and MAE). The grey bar represents the highest errors reported in the recommendation through DWR approach. Likewise, a significantly low error is reported by SRS-CaVRS in light grey bar. However, it can be observed from the graphs that least RMSE and MAE has been reported in case of our proposed methodology in white bar. This means that the overall uncertainty has significantly improved.

VI. Conclusion and Future Directions

A major concern in the application of CARS is the categorization of items into relevant contextual groups. We examine the role of handling missing ratings in the capacity of an essential component for selecting the influencing context to improve the recommendation quality. This paper considers the classification of items in the presence of uncertainty due to missing ratings. In order to handle the missing ratings, we proposed a three-way classification using game-theoretic N-soft sets (TWC-GTNS). Two properties of classification i.e. effectiveness and generality, are being focused using TWC. However, a key issue in TWC is the selection of suitable threshold configuration. To that purpose, we have formulated a game using novel game theory based on N-soft sets for determining effective thresholds from a non-binary ratings. Consequently, these thresholds are utilized by TWC to define the type of decisions for the categorization of items. In a thorough evaluation on a large datasets of *LDOS-CoMoDa* and *InCarMusic*, we showed the actual efficiency and performance of the proposed scheme in terms of improving the recommendation quality by handling missing ratings. The comparison of the TWC-GTNS results with three other baselines suggests that the proposed approach not only improves the tradeoff between the generality and effectiveness of the classification of missing ratings but also provides accurate recommendations in CARS. In particular, the consideration of different evaluation measures for quantifying the complexity of the proposed algorithm will open the door to extensive future researches.



(a) DCG results for LDOS-CoMoDa

(b) DCG results for InCarMusic

Fig. 3: Comparison of results of DCG for *LDOS-CoMoDa* and *InCarMusic* using *TWC-GTNS*

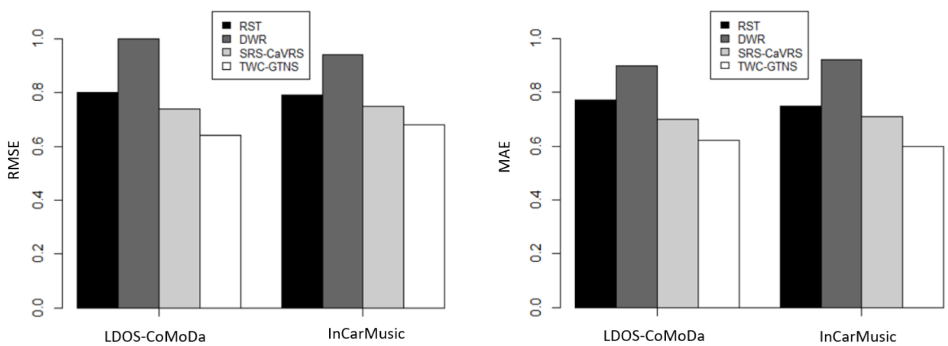


Fig. 4: Results of RMSE and MAE of our approach *TWC-GTNS* on *LDOS-CoMoDa* and *InCarMusic*

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