Intuitionistic Fuzzy PROMETHEE II Technique for Multi-criteria Decision Making Problems Based on Distance and Similarity measures

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***Abstract*—In multi-criteria decision making (MCDM) methods, if decision makers (DMs) are not able to treat the precise data in order to define their preferences, the intuitionistic fuzzy set (IFS) theory enables them. Therefore, the IFS attributes are connected with the degree of membership and non-membership functions. In this work we propose a new version of the intuitionistic fuzzy PROMETHEE II (IF-PROMETHEE II) method aiming and solving the MCDM problems. A distance and similarity measures are employed to measure the deviations between alternatives on intuitionistic fuzzy sets. We propose to apply the distance and the similarity measure between alternatives to determine the preference matrix. Then, a ranking algorithm is applied to indicate the order of superiority of alternatives. Finally, a practical example is provided for an application of organization evaluations.**

***Index Terms*—Intuitionistic fuzzy sets (IFSs), Multi-criteria decision making, PROMETHEE II, Intuitionistic Fuzzy Distance Measures, Intuitionistic Fuzzy Similarity Measures.**

I. Introduction

Different life problems can have many solutions (alternatives) and can be resolved based on different criteria (attributes). Hence, numerous methods of multi-criteria Decision Making have been proposed since 1971 and for each method many versions can be found.

Multi-criteria decision making methods are applied in medical diagnosis [1], [2] and engineering systems [3], [4]. Methods of MCDM are applied in crisp domain, then extended to be used with fuzzy sets and their generalizations such as intuitionistic fuzzy sets (IFSs) [5]–[7] interval valued fuzzy sets, type-2 fuzzy sets, etc. PROMETHEE is presented in literature with many versions. The first version deals with crisp values where information is certain and complete. The other versions deal with fuzzy sets type-1 and its generalizations where information is incomplete, imprecise and uncertain.PROMETHEE is very known methods, defined using intuitionistic fuzzy sets in [8]–[11]. The combination of the PROMETHEE method and the fuzzy set theory is applied to many applications in addition to energy planning [9], [12]. Rani and Jain [8] developed the intuitionistic PROMETHEE technique for multi-criteria decision making problems based on entropy measure. Murat et al. (2015) used PROMETHEE I and PROMETHEE II to evaluate performance in schools. PROMETHEE method uses preference function to rank alternatives. The measures of similarity, which have the same role of distance measures, are rarely used. Some researches proposed or applied similarity measures. Safari et al. [13], [14] exposed a decision making method using a similarity measure. [15] exposed an intuitionistic fuzzy similarity measure and applied it on TOPSIS method for air-conditioning system selection problem. [7], [16], [17] presented a resolution of decision making problems using similarity measures between intuitionistic fuzzy sets and between interval valued intuitionistic fuzzy sets.

In addition, methods of weight are important tools to give importance to some criteria among others. Hence, to find the appropriate alternative. Methods of weight are neglected using intuitionistic fuzzy sets, only three methods are found in literature [18], [19], [20] and [21].

Our aim is to present and compare some distance measures used in PROMETHEE II under intuitionistic fuzzy environment and to propose the application of similarity measures in preference function instead of distance measures used in literature [11], [22]. The remaining of this paper is organized as follows: In section 2, some preliminaries about IFSs, distance measures and similarity measures are presented. In section 3, PROMETHEE method under intuitionistic fuzzy sets is exposed. In section 4, PROMETHEE approach using intuitionistic fuzzy sets and similarity measures is proposed. In section 5, the proposed approach is applied to assessment and evaluations of some organizations and in section 6 a conclusion is deduced.

II. Preliminaries

In this section, We present basic definitions of intuitionistic fuzzy sets and similarity measures between IFSs proposed in literature.

A. Definition of Intuitionistic fuzzy sets (IFSs)

IFSs are introduced by Atanassov [23], [24] who defined a membership degree μ, a non-membership degree ν and a

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degree of hesitation π of an element $x \in A$ where A is a set in the discourse universe X.

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|x \in X\} \quad (1)$$

with the conditions: $0 \leq \mu_A(x) \leq 1$, $0 \leq \nu_A(x) \leq 1$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$
and $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$

III. Intuitionistic Fuzzy Distance Measures Proposed in Literature

In this section, we present some distance measures, existing in literature, between tow IFSs A and B.

A. Definition 1

Atanassov [24] proposed the following distance measures:

- Hamming Distance

$$dH1(A, B) = \frac{1}{2n}\sum_{i=1}^{n}[|\mu_A - \mu_B| + |\nu_A - \nu_B|] \quad (2)$$

- Euclidean Distance

$$dE(A, B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}(\mu_A - \mu_B)^2 + (\nu_A - \nu_B)^2} \quad (3)$$

- [24] present these distances using the max operator

$$J_1(A, B) = \max_i |\mu_A(x_i) - \mu_B(x_i)| \quad (4)$$

Where $i = 1, 2, \ldots, n$ and J_1 defines the Hamming distance between fuzzy sets

$$J_2(A, B) = \max_i |\nu_A(x_i) - \nu_B(x_i)| \quad (5)$$

Where $i = 1, 2, \ldots, n$

$$J(A, B) = \frac{1}{2}(J_1(A, B) + J_2(A, B)) \quad (6)$$

B. Definition 2

Szmidt and Kacprzyk [25] interpreted the precedent measures (2, 3) by a geometric perspective. Thus, they concluded that the hesitation degree must be taken into consideration to compute euclidean and hamming distances and proposed these measures. We recall that the hesitation degree is computed by this formula: $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ with $0 \leq \pi_A(x) \leq 1$.

- Hamming Distance

$$dh2(A, B) = \frac{1}{2n}\sum_{i=1}^{n}[|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|] \quad (7)$$

- Euclidean Distance

$$q_{IFS}(A, B) = \frac{1}{\sqrt{2n}} \times \sqrt{\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi(A, B))^2]} \quad (8)$$

Where $\pi(A, B) = \pi_A(x_i) - \pi_B(x_i)$

C. Definition 3

In the real space $\mathbb{R}$, for any two intervals $A = [a_1, a_2]$ and $B = [b_1, b_2]$, the Hausdorff distance $H(A, B)$ is defined as:

$$Hd(A, B) = \max\{|a_1 - b_1|, |a_2 - b_2|\} \quad (9)$$

Based on the definition of $Hd(A, B)$, Hung and Yang [26] proposed Hausdorff distance IFSs A and B defined in [0,1] by this formula:

$$d_{Hd}(A, B) = \frac{1}{n}\sum_{i=1}^{n} max(|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|) \quad (10)$$

Using the $d_{Hd}(A, B)$ definition, the authors proposed some similarity measures between IFSs in [26].

D. Definition 4

Grzegorzewski [27] proposed the following formulas for Euclidean and Hamming distances for IFSs based on Hausdorff metric.

- Hamming Distance

$$d_h(A, B) = \frac{1}{n}\sum_{i=1}^{n} max (|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|) \quad (11)$$

- Euclidean Distance

$$dE_h(A, B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} max\,(\mu(A, B)^2, \nu(A, B)^2)} \quad (12)$$

Where $\mu(A, B) = \mu_A(x_i) - \mu_B(x_i)$and $\nu(A, B) = \nu_A(x_i) - \nu_B(x_i)$

E. Definition 5

Wang and Xin [28] proposed the following distance measures:

$$d_1(A, B) = \frac{1}{n}\sum_{i=1}^{n}[\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{4} + \frac{\max(|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|)}{2}] \quad (13)$$

IV. Similarity Measures between Intuitionistic Fuzzy Sets from Literature

Some existing similarity measures between tow IFSs A and B of n elements are presented in table I [29].

Notation The following notations are used to simplify the formulas of similarity measures:
$\mu_A^i = \mu_A(x_i)$ and $\mu_B^i = \mu_B(x_i)$
$\nu_A^i = \nu_A(x_i)$ and $\nu_B^i = \nu_B(x_i)$
$\Delta_\mu^i = \mu_A^i - \mu_B^i$, $\Delta_\nu^i = \nu_A^i - \nu_B^i$ and $\Delta_\pi^i = \pi_A^i - \pi_B^i$

TABLE I
SIMILARITY MEASURES BETWEEN IFSS

Authors	IFSMs	
Park et al. [30]	$S_P = 1 - \frac{1}{n}\sum_{i=1}^{n} \lvert\Delta_\pi^i\rvert$	(1)
Mitchell [31]	$S_{mod}(A,B) = \frac{1}{2}\left(\varphi_\mu(A,B) + \varphi_\Phi(A,B)\right)$(14) where $\varphi_\mu(A,B)$ and $\varphi_\Phi(A,B)$ denote respectively the similarity measures between the low membership functions μ_A and μ_B and the high membership functions $\Phi_A = 1 - \nu_A$ and $\Phi_B = 1 - \nu_B$ as follows: $\varphi_\mu(A,B) = S_d^p(\mu_A(x_i), \mu_B(x_i)) = 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} \lvert\Delta_\mu^i\rvert^p}$ $\varphi_\Phi(A,B) = S_d^p(\Phi_A(x_i), \Phi_B(x_i)) = 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} \lvert\Delta_\nu^i\rvert^p}$	
Liang and Shi [32]	$S_e^p(A,B) = 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} \left(\varphi_{\mu_{AB}}(x_i) + \varphi_{\nu_{AB}}(x_i)\right)^p}$ where $\varphi_{\mu_{AB}}(x_i) = \frac{\lvert\Delta_\mu^i\rvert}{2}$ and $\varphi_{\nu_{AB}}(x_i) = \left\lvert \frac{1-\nu_A(x_i)}{2} - \frac{1-\nu_B(x_i)}{2} \right\rvert$	(15)
Zhang and Fu [33]	$S_{ZF}(A,B) = 1 - \frac{1}{2n}\sum_{i=1}^{n} \left(\lvert\delta_A(x_i) - \delta_B(x_i)\rvert + \lvert\alpha_A(x_i) - \alpha_B(x_i)\rvert\right)$ where: $\delta_A(x_i) = \mu_A(x_i) + \pi_A(x_i)\mu_A(x)$ and $\alpha_A(x_i) = \nu_A(x_i) + \pi_A(x_i)\nu_A(x_i)$	(16)
Hung and Yang [34]:	$S_{w1}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{min(\mu_A(x_i),\mu_B(x_i)) + min(\nu_A(x_i),\nu_B(x_i))}{max(\mu_A(x_i),\mu_B(x_i)) + max(\nu_A(x_i),\nu_B(x_i))}$	(17)
	$S_{pk3}(A,B) = 1 - \frac{\sum_{i=1}^{n} \lvert\mu_A(x_i) - \mu_B(x_i)\rvert + \lvert\nu_A(x_i) - \nu_B(x_i)\rvert}{\sum_{i=1}^{n} (\lvert\mu_A(x_i) + \mu_B(x_i)\rvert + \lvert\nu_A(x_i) + \nu_B(x_i)\rvert)}$	(18)
Chu et al. [35]	$S_{C_2}^p = 1 - \left[\sum_{i=1}^{n} w_j \left(\delta_1\{\frac{\lvert\Delta_\mu^i\rvert}{max(\mu_A^i,\mu_B^i)}\}^p + \delta_2\{\frac{\lvert\Delta_\nu^i\rvert}{max(\nu_A^i,\nu_B^i)}\}^p + \delta_3\{\frac{\lvert\Delta_\pi^i\rvert}{max(\pi_A^i,\pi_B^i)}\}^p\right)\right]^{\frac{1}{p}}$ It is considered that $\frac{0}{0} = 0$	(19)
Luo and Ren [36]	$S_R = 1 - \frac{1}{3n}\sum_{i=1}^{n} \left(\lvert\mu_A^2(x_i) - \mu_B^2(x_i)\rvert + \lvert\nu_A^2(x_i) - \nu_B^2(x_i)\rvert + \lvert m_A^2(x_i) - m_B^2(x_i)\rvert\right)$ $m_A(x_i) = \frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}$ and $m_B(x_i) = \frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}$	(20)
Hung et al. [37]	$S_l^p(A,B) = \frac{2^{\frac{1}{p}} - L_p(A,B)}{2^{\frac{1}{p}}}$ where $L_p(A,B) = \frac{1}{n}\sum_{i=1}^{n} d_p(I_A, I_B)$ and $d_p(I_A(x_i), I_B(x_i)) = (\lvert\mu_A - \mu_B\rvert^p + \lvert\nu_A - \nu_B\rvert^p)^{1/p},\ p \geq 1$	(21)
Fan et al. [38]	$S_L(A,B) = 1 - \frac{\sum_{i=1}^{n} \lvert S_A(x_i) - S_B(x_i)\rvert}{4n} - \frac{\sum_{i=1}^{n} \lvert\mu_A - \mu_B\rvert + \lvert\nu_A - \nu_B\rvert}{4n}$ where $S_A(x_i) = \mu_A - \nu_A$ and $S_B(x_i) = \mu_B - \nu_B$	(22)
Hong et al. [39]	$S_H(A,B) = 1 - \frac{\sum_{i=1}^{n} \lvert\mu_A - \mu_B\rvert + \lvert\nu_A - \nu_B\rvert}{2n}$	(23)
Chen et al. [40]	$S_C(A,B) = 1 - \frac{\lvert 2(\mu_A(x_i) - \mu_B(x_i)) - (\nu_A(x_i) - \nu_B(x_i))\rvert}{3} \times \left(1 - \frac{\pi_A(x_i) - \pi_B(x_i)}{2}\right) - \frac{\lvert 2(\nu_A(x_i) - \nu_B(x_i)) - (\mu_A(x_i) - \mu_B(x_i))\rvert}{3} \times \left(\frac{\pi_A(x_i) - \pi_B(x_i)}{2}\right)$	(24)

V. PRESENTATION OF DIFFERENT VERSIONS OF PROMETHEE

The Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) method [41] uses pairwise comparisons and outranking relationships to choose the best alternatives. The final selection is based on the positive and negative preference of each alternative. The positive preference indicates how an alternative is outranking all the other alternatives and the negative preference flow indicates an alternative is outranked by all the other alternatives. Many versions are presented in literature, PROMETHEE I obtains partial ranking, PROMETHEE II provides a complete ranking. The ranking generated by PROMETHEE I is partial because it does not compare conflicting actions. On the other hand, PROMETHEE II ranks alternatives according to the net flow, which equals to the balance of the positive and the negative preference flows. An alternative with a higher net flow is better.

A. PROMETHEE I and II Methods From Literature

The PROMETHEE method was introduced by Vincke (1985) and Brans et al. [41]. This method induces the preferential function to describe the preference difference between pairs of alternatives on each criterion and describes the preference difference from the point of view of decision makers. The values obtained by this function range from 0 to 1. The bigger the function's value is, the difference of the preference becomes larger. When the value is zero, there is no preferential difference between pair of alternatives. When the value is one, an alternative is strictly outranking the others. In the following, The PROMETHEE I and PROMETHEE II method are presented.

- Step1: Determine a multi-criteria preference index as:

$$H(A_i, A_k) = \sum_{j=1}^{n} p_j(A_i, A_k) w_j \tag{25}$$

  Where w_j is the weight of each criterion. Sometimes weights are assigned by decision makers. $p_j(A_i, A_k)$ determines the preference functions defined for each criterion. It translates the difference between the evaluations obtained by two alternatives into a preference degree from zero to one. In order to facilitate the selection of a specific preference function, authors in [42] proposed six basic types of function:

  - Usual criterion

$$p_1(A_i, A_k) = \begin{cases} 0 & \forall d \leq 0 \\ 1 & \forall d > 0 \end{cases}$$

– Quasi criterion

$$p_2(A_i, A_k) = \begin{cases} 0 & \forall d \leq q_k \\ 1 & \forall d \geq q_k \end{cases}$$

– Criteria with linear preference

$$p_3(A_i, A_k) = \begin{cases} 0 & d \leq 0 \\ d/p_k & 0 \leq d \leq p_k \\ 1 & d \geq p_k \end{cases}$$

– Level criterion

$$p_4(A_i, A_k) = \begin{cases} 0 & d \leq q_k \\ 0.5 & q_k < d \leq p_k \\ 1 & d > p_k \end{cases}$$

where p:preference threshold and q:indifference threshold

– Indifference criterion

$$p_5(A_i, A_k) = \begin{cases} 0 & d \leq q_k \\ \frac{d-q_k}{p_k-d-q_k} & q_k \leq d \leq p_k \\ 1 & d \geq p_k \end{cases}$$

– Gaussian criterion.

$$p_6(A_i, A_k) = \begin{cases} 0 & d < 0 \\ 1 - exp(-d^2/2\sigma^2) & d \geq 0 \end{cases}$$

For each criterion, the value of an indifference threshold q, the value of a strict preference threshold p, and the value of an intermediate value between p and q, s, has to be fixed [43]. In each case, these parameters have a clear significance for the decision-maker. For two alternatives A_i and A_k, the decision maker should select one type of preference functions. This index values obtained $H(A_i, A_k)$ between 0 and 1, and represents the global intensity of preference between the couples of alternatives.

- Step 2: Determination of deviations based on pairwise comparisons

$$d_j(a, b) = g_j(a) - g_j(b) \tag{26}$$

where $d_j(a, b)$ denotes the difference between the evaluations of alternatives a and b on each criterion.

- Step 3: Application of the preference function:

$$p_j(a, b) = F_j[d_j(a, b)], j = 1, \ldots, k \tag{27}$$

where $p_j(a, b)$ is a preference function applied for alternatives a and b. There are six types of preference functions and the decision makers can select one of them.

- Step 4: Obtaining the preference order In this step, ranking alternatives can be made partially or completely. Partial ranking is obtained using PROMETHEE I, and complete ranking is performed using PROMETHEE II.

1) Partial ranking of the alternatives: PROMETHEE I:

$$\phi^+(A_i) = \frac{1}{m-1} \sum_{x \in A} H(A_i, A_k) \tag{28}$$

$$\phi^-(A_i) = \frac{1}{m-1} \sum_{x \in A} H(A_k, A_i) \tag{29}$$

$\phi^+(A_i)$ represents the positive outranking flow (or leaving flow). It expresses how much each alternative dominates all the others. $\phi^-(A_i)$ represents the negative outranking flow (or entering flow). It expresses how much each alternative is dominated by all the others.

- The smaller $\phi^-(A_i)$, the better the alternative. $\phi^-(A_i)$ represents the weakness of A_i. The latter is preferred to A_k when $\phi^+(A_i) \geq \phi^+(A_k)$, $\phi^-(A_i) \leq \phi^-(A_k)$, and at least one of the inequalities holds as a strict inequality.
- A_i and A_k are indifferent when $\phi^+(A_i) = \phi^+(A_k)$ and $\phi^-(A_i) = \phi^-(A_k)$.
- A_i and A_k are incomparable otherwise.
  In this partial ranking some couples of alternatives are comparable, some others are not. Thus, we need to calculate the net outranking flow in the following step.

2) Complete Ranking of the alternatives: PROMETHEE II :

$$\Phi(A_i) = \Phi^+(A_i) - \Phi^-(A_i) \tag{30}$$

Where $\Phi(A_i)$ denotes the net outranking flow for each alternative. The PROMETHEE II completes ranking as:

– A_i is preferred to A_k when $\Phi(A_i) > \Phi(A_k)$
– A_i and A_k are indifferent when $\Phi(A_i) = \Phi(A_k)$.

All alternatives are now comparable, the alternative with the highest $\Phi(A_i)$ can be considered as the best one. A considerable part of information will be lost by taking the difference of positive and negative outranking flows. This information can be useful in concrete applications for decision making.

B. Proposed PROMETHEE II Method Based on distance Measures Between IFs

In this section, the IF-PROMETHEE method is developed and applied to solve MCDM problems in intuitionistic fuzzy environment. The proposed intuitionistic fuzzy PROMETHEE II has the following steps:

- Step 1: For multi-criteria decision making problem, generate a set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$ and a set of criteria $C = \{C_1, C_2, \ldots, C_n\}$. In this step, the evaluation values of the alternatives over the criteria are intuitionistic fuzzy values.
- Step 2: Determine the weights (w_j) of criteria
  There are many definition of weights in literature [19], [44]. In this work, the Intuitionistic Fuzzy standard deviation (IF-SD) [19] is applied as:

$$\delta_j = \sqrt{S(\mu_{ij}) + S(\nu_{ij})} \tag{31}$$

Where:

$$S(\mu_{ij}) = \sum_{i=1}^{m} \frac{(\mu_{ij}(C_j) - \bar{\mu_j}(Cj))^2}{m}$$

$$S(\nu_{ij}) = \sum_{i=1}^{m} \frac{(\nu_{ij}(C_j) - \bar{\nu_j}(Cj))^2}{m}$$

Where

$$W_j = \frac{\sigma_j}{\sum_{k=1}^{m} \sigma_j} \quad (32)$$

where $j = 1, \ldots, n$

- Step 3: Determine the deviations $dj(A_i, A_k)$ based on pairwise comparisons between i and k alternatives with respect to criterion j.
  We propose to use different distance measures (8), (12) and (13) to determine the deviations d between alternatives under the IFSs as:
  - For benefit criteria

$$d(A_{ij}, A_{kj}) = \begin{cases} \sqrt{\frac{1}{2n}\sum_{i=1}^{n} (\mu_A - \mu_B)^2 + (\nu_A - \nu_B)^2} \\ \quad if \quad A_{ij} \geq A_{kj} \\ 0 \quad \text{otherwise} \end{cases}$$

  - For cost criteria

$$d(A_{ij}, A_{kj}) = \begin{cases} \sqrt{\frac{1}{2n}\sum_{i=1}^{n} (\mu_A - \mu_B)^2 + (\nu_A - \nu_B)^2} \\ \quad if \quad A_{ij} \leq A_{kj} \\ 0 \quad \text{otherwise} \end{cases}$$

  where $i \neq k, i, k = 1, 2, \ldots, m$

- Step 4: Calculate the preference function $Pj(A_i, A_k)$ over the j^{th} criterion.
  There are six main types of preference functions in section V-A. We propose to use Gaussian criterion with distance measure (8),(11) and (10).

$$p(A_{ij}, A_{kj}) = \begin{cases} 0 & d < 0 \\ 1 - exp(-d^2/2\sigma^2) & d \geq 0 \end{cases}$$

- Step 5: Calculate the Preferences Index
  The preference index used the distances measures (8),(11) and (10) witch are integrated in preference function $Pj(A_i, A_k)$ in step 4. We compute the preference index $Hd(x_i, x_k)$ using the criterion weighted value is given as follows:

$$Hd(x_i, x_k) = \sum_{j=1}^{n} w_j \times p_j(A_i, A_k), i, k = 1, 2, \ldots, m \quad (33)$$

  where w_j (32) are weights associated with each criteria.
- Step 6: Determine the leaving flow and entering flow
  We compute the leaving flow $\Phi^+(x_i)$ and entering flow $\Phi^-(x_i)$ using the the preference index $Hd(x_i, x_k)$ (33):

$$\Phi^+(x_i) = \frac{1}{m-1} \sum_{x \in A} Hd(x_i, x_k), i = 1, 2, \ldots, m \quad (34)$$

  and

$$\Phi^-(x_i) = \frac{1}{m-1} \sum_{x \in A} Hd(x_k, x_i), i = 1, 2, \ldots, m \quad (35)$$

- Step 7: Ranking the alternatives according the net flow (36).

$$\Phi(A_i) = \Phi^+(A_i) - \Phi^-(A_i) \quad (36)$$

C. Proposed PROMETHEE II Method Based on Similarity Measures between IFs

- Step 1-2: Same as steps of PROMETHEE II based on distance Measures, section V-B.
- Step 3: We propose to use two similarity measures S_C (24) and S_H (23) to determine the deviations dS between alternatives defined by:
  - For benefit criteria

$$dS(A_{ij}, A_{kj}) = \begin{cases} 1 - \frac{|2(\mu_A(x_i) - \mu_B(x_i)) - (\nu_A(x_i) - \nu_B(x_i))|}{3} \\ \times \left(1 - \frac{\pi_A(x_i) - \pi_B(x_i)}{2}\right) \\ - \frac{|2(\nu_A(x_i) - \nu_B(x_i)) - (\mu_A(x_i) - \mu_B(x_i))|}{3} \\ \times \left(\frac{\pi_A(x_i) - \pi_B(x_i)}{2}\right) \quad if \quad A_{ij} \geq A_{kj} \\ 0 \quad \text{otherwise} \end{cases}$$

  - For cost criteria

$$dS(A_{ij}, A_{kj}) = \begin{cases} 1 - \frac{|2(\mu_A(x_i) - \mu_B(x_i)) - (\nu_A(x_i) - \nu_B(x_i))|}{3} \\ \times \left(1 - \frac{\pi_A(x_i) - \pi_B(x_i)}{2}\right) \\ - \frac{|2(\nu_A(x_i) - \nu_B(x_i)) - (\mu_A(x_i) - \mu_B(x_i))|}{3} \\ \times \left(\frac{\pi_A(x_i) - \pi_B(x_i)}{2}\right) \quad if \quad A_{ij} \leq A_{kj} \\ 0 \quad \text{otherwise} \end{cases}$$

  where $i \neq k, i, k = 1, 2, \ldots, m$

- Step 4: We propose to calculate the preference function $P_j(A_i, A_k)$ over the j^{th} criterion using similarity measures dS instead of distance measure d used in step 4 section V-B .

$$p(A_{ij}, A_{kj}) = \begin{cases} 0 & dS < 0 \\ 1 - exp(-dS^2/2\sigma^2) & dS \geq 0 \end{cases}$$

- Step 5: Calculate the Preferences Index
  We compute the preference index $Hs(x_i, x_k)$ using the criterion weighted value is given as follows:

$$Hs(x_i, x_k) = \sum_{j=1}^{n} w_j \times p_j(A_i, A_k), i, k = 1, 2, \ldots, m \quad (37)$$

  where w_j (32) are weights associated with each criteria.
- Step 6: Determine the leaving flow and entering flow
  We compute the leaving flow $\Phi^+(x_i)$ and entering flow $\Phi^-(x_i)$ using the the preference index $Hs(x_i, x_k)$ (37):

$$\Phi^+(x_i) = \frac{1}{m-1} \sum_{x \in A} Hs(x_i, x_k), i = 1, 2, \ldots, m \quad (38)$$

  and

$$\Phi^-(x_i) = \frac{1}{m-1} \sum_{x \in A} Hs(x_k, x_i), i = 1, 2, \ldots, m \quad (39)$$

- Step 7: same as step in section V-B.

D. ILLUSTRATIVE EXAMPLES

The proposed methods: IF-PROMETHEE II using distance measures and IF-PROMETHEE II using similarity measures are applied to MCDM problems from literature [45]. The latter consists to select the best assessment and evaluations of some organizations considering four alternatives: Bajaj Steel (A_1), H.D.F.C. Bank (A_2), Tata Steel (A_3) and Infotech Enterprises (A_4) are assessed for their performance on the basis of following five benefit criteria $(c_1, c_2, c_3, c_4, c_5)$:

- c_1: Earnings per share(EPS).
- c_2: Face value.
- c_3: P/C (Put Call) Ratio.
- c_4: Dividend.
- c_5: P/E (Price to earnings) ratio

The following intuitionistic fuzzy sets decision making matrix (40) presents the relationship between criteria and alternatives of data set:

$$\begin{array}{c} \\ A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{array}{c} \begin{array}{ccc} C1 & C2 & C3 \end{array} \\ \begin{pmatrix} [0.23, 0.587] & [0.61, 0.2] & [0.192, 0.63] \\ [0.26, 0.554] & [0.2, 0.61] & [0.63, 0.192] \\ [0.62, 0.197] & [0.61, 0.2] & [0.259, 0.56] \\ [0.197, 0.62] & [0.36, 0.454] & [0.337, 0.484] \end{pmatrix} \end{array} \tag{40}$$

$$\begin{array}{c} \\ A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{array}{c} \begin{array}{cc} C4 & C5 \end{array} \\ \begin{pmatrix} [0.22, 0.75] & [0.196, 0.62] \\ [0.094, 0.875] & [0.62, 0.196] \\ [0.31, 0.66] & [0.227, 0.59] \\ [0.15, 0.82] & [0.332, 0.50] \end{pmatrix} \end{array}$$

E. Application of PROMETHEE II Method Using Different Distance Measures

- *Step 1:* Use decision matrix (40) provided by [8] to evaluate four organizations.
- *Step 2:* Compute the weights of criteria using Intuitionistic Fuzzy standard deviation (IF-SD) [19]:
  $w = \{0.1879, 0.2717, 0.2113, 0.1230, 0.2061\}$.
- *Step 3:* Determination of deviation by pairwise comparison
  This step involves the calculation of distance measures between alternatives by pairwise comparison. The deviations are obtained using different distance formulas (8),(11) and (10) given in step 3 in section V-B. The obtained results are shown in table II.
- *Step 3:* Calculate the preference index.
  Calculation of the preference index of each organization takes into account the criteria weight. It is also a value to show the degree of preference of each organization over another one. The index is calculated using (33). The preference index is presented in Table III.
- *Step 4:* Determine the positive and negative outranking flows of each organization (PROMETHEE I partial ranking).
  (a) Leaving flow and entering flow of organizations
  Positive outranking flow $\phi^+(a)$ shows the domination degree of the organization over other ones. In contrast, negative outranking flow $\phi^-(a)$ shows the domination degree of other organizations over current one. formulas (28) and (29) are used to calculate these two flows. Leaving flow and entering flow of organizations are shown in table IV.
  (b) Determine the net flow value (PROMETHEE II) for each organization.
  Net flow values $\phi(A)$ are calculated to avoid incomparability. Equation (36) is used to complete the calculation of net outranking flow. It is presented in table V.

TABLE II
DEVIATIONS BETWEEN ALTERNATIVES USING DISTANCE MEASURE (11) WITH RESPECT TO CRITERIA Cj

Alternatives	C_1	C_2	C_3	C_4	C_5
A1,A2	0.9688	0.5900	0.5620	0.8743	0.5760
A1,A3	0.6100	1.0000	0.9318	0.9100	0.9694
A1,A4	0.9670	0.7484	0.8546	0.9300	0.8764
A2,A1	0.9688	0.5900	0.5620	0.8743	0.5760
A2,A3	0.6412	0.5900	0.6302	0.7843	0.6066
A2,A4	0.9358	0.8416	0.7074	0.9443	0.6996
A3,A1	0.6100	1.0000	0.9318	0.9100	0.9694
A3,A2	0.6412	0.5900	0.6302	0.7843	0.6066
A3,A4	0.5770	0.7484	0.9228	0.8400	0.9070
A4,A1	0.9670	0.7484	0.8546	0.9300	0.8764
A4,A2	0.9358	0.8416	0.7074	0.9443	0.6996
A4,A3	0.5770	0.7484	0.9228	0.8400	0.9070

TABLE III
PREFERENCE INDEX VALUE

Alternatives	$A1$	$A2$	$A3$	A_4
A1	0	0.2609	0.0807	0.1032
A2	0.0627	0	0.2511	0.2866
A3	0.1356	0.3062	0	0.0905
A4	0.1583	0.1715	0.1684	0

TABLE IV
PROMETHEE I FLOW

Alternatives	$\phi^+(a)$	$\phi^-(a)$
A1	0.4448	0.3566
A2	0.6003	0.7386
A3	0.5323	0.5002
A4	0.4982	0.4803

TABLE V
NET FLOW VALUE OF ORGANIZATIONS

Alternatives	**Net flow** $\phi(A)$	**rank**
A_1	0.3566	4
A_2	0.7386	1
A_3	0.5002	2
A_4	0.4803	3

The ranking of the organizations is arranged in descending order of net flow value. The best organization is the one having the highest net flow value, $\phi(a)$. By using PROMETHEE II

TABLE VI
NET FLOW VALUE OF ORGANIZATIONS USING DISTANCE MEASURES APPLIED TO AN EXAMPLE OF MCDM PROBLEM

Distance Measure	Net flow $\phi(A_i)$				Sorted Alternatives
	A_1	A_2	A_3	A_4	
q_{IFS} (8)	0.3566	0.7386	0.5002	0.4803	$A_2 > A_3 > A_4 > A_1$
dE_h (12)	0.5183	0.3188	0.3374	0.1366	$A_1 > A_3 > A_2 > A_4$
d_{Hd} (10)	1.5218	1.2398	0.9349	1.0700	$A_1 > A_2 > A_4 > A_3$

(complete ranking) method, organization A_1 (Bajaj Steel) is selected as the best alternative using distance measures dE_h (12) and d_{Hd} (10). However using formulas q_{IFS} (8) the best alternative is A_2

The comparison between distances measures considering the ranking lists for the IF-PROMETHEE II method, demonstrates the influence of distance measure in MCDM problem resolution.

F. Application of PROMETHEE II Method Using Different Similarity Measures

TABLE VII
NET FLOW VALUES OF ORGANIZATIONS USING SIMILARITY MEASURES APPLIED TO AN EXAMPLE OF MCDM PROBLEM

similarity Measure	Net flow,$\phi(A)$				Sorted Alternatives
	A_1	A_2	A_3	A_4	
S_C (24)	2.4428	2.1370	2.3273	2.4719	$A_2 < A_3 < A_1 < A_4$
S_H (23)	-3.7308	-3.8247	-4.1538	-4.6635	$A_4 < A_3 < A_2 < A_1$

The ranking of the organizations is arranged in ascending order of net flow value. The results of IF-PROMETHEE II using similarity measures are presented in table VII. It can be seen that alternative A_2 is ranked first using similarity measure S_C (24). However using S_H (23) A_4 is ranked first.

Note. It should be noted that similarity and distance measures are dual concepts, when one increases, the other decreases. Because of this the maximum of net flow should be taken when a similarity measure is applied instead of its minimum when a distance measure is applied for more information see [14].

The results of ranking of the four organizations by IF-PROMETHEE II (IF-PROME II) using similarity and distance measures are presented in table VIII in addition of those of some methods from literature. The latter obtained A_3 as better alternative and A_1 or A_2 second best alternatives. However, by IF-PROMETHEE II using distance dE_h and d_{Hd}, the best alternative is A_1 followed by A_3 or A_2 and using distance q_{IFS} the best alternative is A_2 followed by $A3$. Moreover, using similarity measure S_C, the obtained results show that the appropriate alternative is A_2 followed by A_3 and using the similarity measure S_H, the appropriate alternative is A_4 followed by A_3. It can be concluded that appropriate alternative can be A_2, A_1 or A_3 as they are ranked first or second by all methods. It should be noted that the differences of results obtained by the proposed methods and other ones presented in table VIII implies the important influence of similarity and distance measures in PROMETHEE II method.

TABLE VIII
COMPARISON WITH EXISTING TECHNIQUES

Technique	Ranking	Optimal choice
IF-TOPSIS [45]	$A_3 > A_1 > A_4 > A_2$	A_3
IF-TOPSIS [46]	$A_3 > A_1 > A_4 > A_2$	A_3
IF-PROMETHEE [8]	$A_3 > A_2 > A_1 > A_4$	A_3
IF-PROM II using q_{IFS}	$A_2 > A_3 > A_4 > A_1$	A_2
IF-PROME II using dE_h	$A_1 > A_3 > A_2 > A_4$	A_1
IF-PROME II using d_{Hd}	$A_1 > A_2 > A_4 > A_3$	A_1
IF-PROME II using S_C	$A_2 < A_3 < A_1 < A_4$	A_2
IF-PROME II using S_H	$A_4 < A_3 < A_2 < A_1$	A_4

VI. CONCLUSION

In this study, we presented several intuitionistic distances and similarity measures proposed in literature, also we detailed PROMETHE II and IF-PROMETHEE II method from literature. the approach IF-PROMETHEE II is developed for multiple criteria decision making problems using distance and similarity measures for computation deviations between alternatives It should be noted that the difference between the proposed methods and other methods presented in literature can be caused by the impact of distance and similarity measures on the ranking of alternatives. This approach is applied for selecting the best organizations of investment. As perspective, possibility measures will be applied to PROMETHEE [47], [48]. In addition, other decision making methods will be studied and compared with proposed ones.

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