Autonomous Driving of Truck-Trailer Mobile Robots with Linear-Fuzzy Control for Trajectory Following

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Abstract
The modeling and autonomous control of truck-trailer mobile robots for trajectory following are addressed. The robot kinematical model is analyzed and used for designing a positioning control system based on linear controllers integrated in a fuzzy-logic approach. The design takes into account both positioning performance and jack-knife avoidance. The results of robot positioning control are extended to trajectory following for which a novel strategy is proposed applicable to general shape desired trajectories. The effectiveness of the proposed methods are verified for linear, circular and sinusoidal trajectories where the mobile robot converges to the desired trajectories, avoiding jack-knife positions and with bounded values of input steering angle.
Keywords. Fuzzy control, Linear-fuzzy integration, Trailer-type robot, Trajectory Following

I. INTRODUCTION

Autonomous truck-trailer mobile robots are used in diverse fields in industry given their advantages in delivery and transportation applications. They can accomplish transportation tasks in a faster and cheaper way compared to multiple individual mobile robots. Their transportation capacity increases with the number of trailers pulled or pushed by a truck moving forward or backward.

However, truck-trailer mobile robots configure a complex, nonlinear, unstable, underactuated and nonholonomic system difficult to control, especially when moving backwards, which have led to an intensive research work for analyzing their motion characteristics and autonomous control. The most of work has been based on robot kinematical model valid when the robot moves at low speeds without wheels side-slipping. In this condition, the robot motion is determined only by geometrical considerations independent of masses, inertias and road friction forces.

Diverse control strategies have been proposed to make the truck-trailer robot autonomously moves describing desired trajectories in complex environments. Approximate linearization and feedback linearization of kinematic model equations were used in [1] to [3] for designing stabilizing controllers for robot positioning applicable to a limited range of operating conditions. Chained representation of robot kinematical equations have been used in [4] and [5] for designing nonlinear controllers based on feedback linearization and backstepping techniques for positioning and path tracking control. The differentially flat structure of mobile robots has been used for designing controllers in [6] and [7].

Fuzzy logic have been used in [8] and [9] to propose diverse control strategies based on human driver experience expressed through linguistic rules. Neural networks have been applied in [10] and [11] for training connectionist controllers based on static or dynamic learning algorithms. Other techniques based on sliding mode control and their integration with neural networks and fuzzy systems, has been proposed in [12] and [13]. The control schemes have been applied to robot positioning, backing up, linear and nonlinear trajectory following, path planning, parallel parking, jack-knife avoidance, robots formation among other control objectives.

II. PROBLEM DEFINITION AND CONTROL STRATEGY

The problem to be solved is the designing of an autonomous control system for the positioning and trajectory following of a truck-trailer mobile robot. The positioning control problem is shown in Figure 1: the mobile robot, starting from arbitrary initial positions, should achieve the desired position without colliding with obstacles around the goal position.

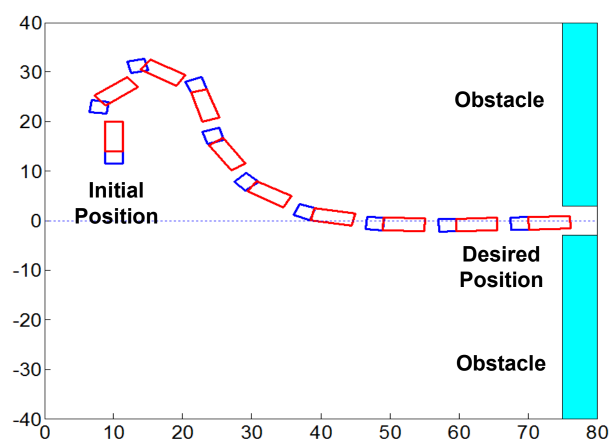

Figure 1. Mobile robot positioning control problem.

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Figure 2 shows a truck-trailer vehicle consisting of a truck with front steering wheels and traction wheels, and a passive trailer with support rear wheels. The trailer is articulated to the truck at the midpoint of the traction axis and it is pulled or pushed by the truck as it moves forward or backward.

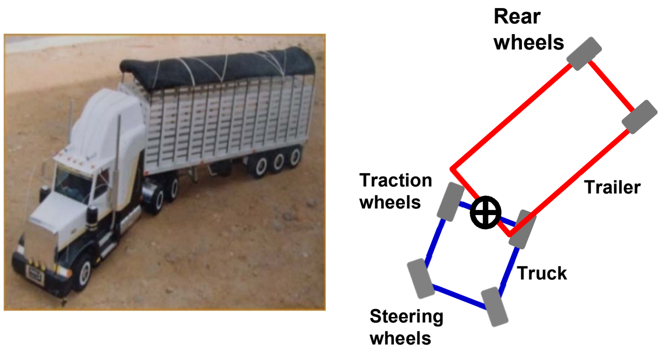


Figure 2. Truck-trailer mobile robot.

Assuming that left and right wheels move in a similar pattern, the truck-trailer robot can be modeled as a two articulated bars as it is shown in Figure 3. Coordinates (x, y) represent the position of trailer rear wheel, θ_1 and θ_2 are the angles of truck and trailer respect to X axis, θ_{12} is the angle of truck respect to trailer, δ is the steering angle, v is the robot backward speed, and L_1 and L_2 are the lengths of truck and trailer, respectively. Counter clockwise angles are positive.

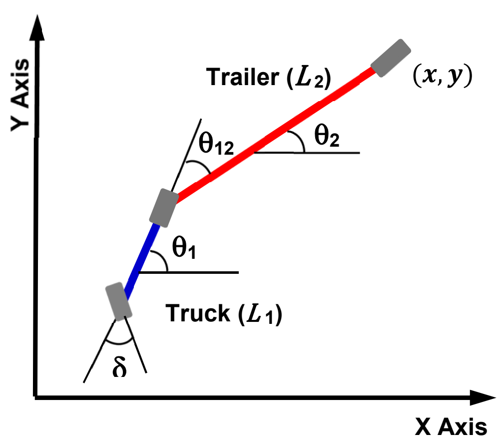


Figure 3. Two-bars model of truck-trailer robot.

Considering the robot moves at low speeds, it can be assumed that wheels do not side-slip so that lineal velocities of traction and rear wheels are aligned to truck and trailer axis, respectively. Under this consideration, the truck-trailer mobile robot model is given by the following equations:

$$\dot{x} = v\cos\theta_{12}\cos\theta_2 \qquad \text{(1)}$$

$$\dot{y} = v\cos\theta_{12}\sin\theta_2 \qquad \text{(2)}$$

$$\dot{\theta}_1 = -\frac{v}{L_1}\tan\delta \qquad \text{(3)}$$

$$\dot{\theta}_2 = -\frac{v}{L_2}\sin\theta_{12} \qquad \text{(4)}$$

The truck-trailer angle θ_{12} is:

$$\theta_{12} = \theta_1 - \theta_2 \qquad \text{(5)}$$

and from equations (3), (4) and (5) the equation of $\dot{\theta}_{12}$ is obtained:

$$\dot{\theta}_{12} = \frac{v}{L_2}\sin\theta_{12} - \frac{v}{L_1}\tan\delta \qquad \text{(6)}$$

Considering the traction wheels moves at constant backward speed (v=constant) and defining the state vector $\mathbf{x}$ and control vector u as:

$$\mathbf{x} = [\, y \;\; \theta_2 \;\; \theta_{12} \,]^{\mathrm{T}} \qquad \text{(7)}$$

$$\mathrm{u} = \tan\delta \qquad \text{(8)}$$

equations (2), (4) and (6) can be represented by the following affine nonlinear state-space equation:

$$\dot{\mathbf{x}} = \boldsymbol{f}(\mathbf{x}) + \boldsymbol{g}(\mathbf{x})\mathrm{u} \qquad \text{(9)}$$

The no inclusion of coordinate x in state vector $\mathbf{x}$ simplifies the controller design process without affecting the robot positioning and trajectory following capacity as far as it moves only forward or only backward.

To achieve the goal position without colliding with obstacles, it has been proposed a control strategy in which the robot prioritizes the achievement of line y^*=0 with horizontal inclination θ_2^*=0º, θ_{12}^*=0º and, afterwards the robot moves straightforward to the goal position as it is shown in Figure 1. With this strategy, coordinate x is not required for control and it is applicable if there is enough space between the robot initial position and the goal position. Considering only three variables $(y, \theta_2, \theta_{12})$ the positioning control problem turns to be a stabilization problem.

Linearizing equations (2), (4) and (6) around the desired angles θ_2^*=0° and θ_{12}^*=0°, the following linear state-space equation is obtained:

$$\begin{bmatrix} \dot{y} \\ \dot{\theta}_2 \\ \dot{\theta}_{12} \end{bmatrix} = \begin{bmatrix} 0 & v & v \\ 0 & v & -v/L_2 \\ 0 & v & v/L_2 \end{bmatrix} \begin{bmatrix} y \\ \theta_2 \\ \theta_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -v/L_2 \end{bmatrix} \tan\delta \quad \text{............... (10)}$$

A full-state stabilizing control law for the linear system is given by:

$$\tan\delta = -k_1 y - k_2\theta_2 - k_3\theta_{12} \quad \text{............... (11)}$$

where coefficients k_1, k_2 and k_3 are properly chosen so that the closed loop linear system is stable. This control law is only valid around θ_{12}=0 and it is not guaranteed it will stabilize the mobile robot for other angles θ_{12} in the range -90° to +90°, where the extreme values correspond to jack-knife positions. To solve this problem, a rule-based fuzzy control will be applied: the range of variation of θ_{12} will be partitioned in three parts corresponding to each jack-knife position (around θ_{12} = -90° and around θ_{12} = +90°), and to the linearized model (around θ_{12} = 0°). For each of these partitions a simple linear controller is designed, and afterwards, the three controllers are integrated in a fuzzy-logic approach as a weighted sum of their outputs (weights given by the membership values of each partition).

Figure 4 shows the partitions and membership functions of angle θ_{12} in the range -90° to +90° (*Negative Big, Zero, Positive Big*). As it was stated, control law (11) is valid when θ_{12}=*Zero*, but it does not apply for other linguistic values of θ_{12}. Then, simple controllers will be designed for the other two partitions.

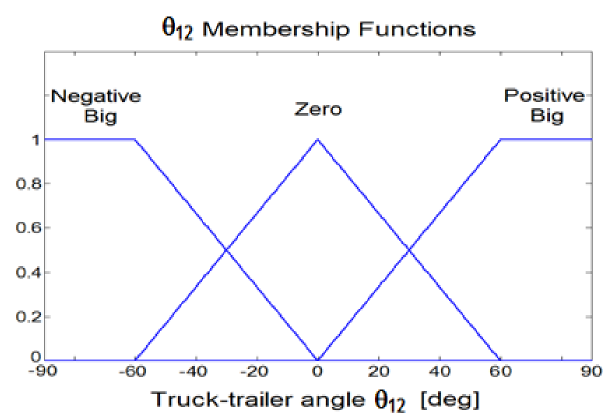


Figure 4. Partitions and membership functions of truck-trailer angle θ_{12}.

From equation (6), it is noted that there is an inverse relationship between $\dot{\theta}_{12}$ and $\tan(\delta)$: if $\tan(\delta)$ increases, $\dot{\theta}_{12}$ decreases and vice versa. Also, it is clear that one of the positioning control objectives is to keep truck-trailer angle θ_{12} at small values in order to avoid jack-knife positions. To do that, the following fuzzy reasoning is applied: if θ_{12} is *Positive Big*, $\dot{\theta}_{12}$ should be negative for bringing θ_{12} toward zero and, to achieve that, $\tan(\delta)$ should be positive. Similarly, when θ_{12} is *Negative Big*, $\dot{\theta}_{12}$ should be positive for bringing θ_{12} toward zero and, to achieve that, $\tan(\delta)$ should be negative. This reasoning is summarized in the following fuzzy rules:

IF θ_{12} = *Positive Big* THEN $\delta = \delta_{max}$ (positive)
IF θ_{12} = *Zero* THEN δ = Equation (11)
IF θ_{12} = *Negative Big* THEN $\delta = \delta_{min}$ (negative)
.................................(12)

It is important to point out that the proposed control strategy does not only focus on attainment of the desired position but also on the avoidance of jack-knife positions (angle θ_{12} close to +90° or -90°). This fuzzy control law was applied to the mobile robot to achieve different desired positions starting from arbitrary initial positions. Figure 1 and Figure 5 show the trajectories of the mobile robot from two different initial positions to the goal position x^*=80, y^*=0, θ_2^*=0°, θ_{12}^*=0°. In both cases, the robot is able to asymptotically achieve the goal position without colliding with obstacles around. The steering angle δ was bounded to the range from δ_{min} = -30° to δ_{max} = +30°.

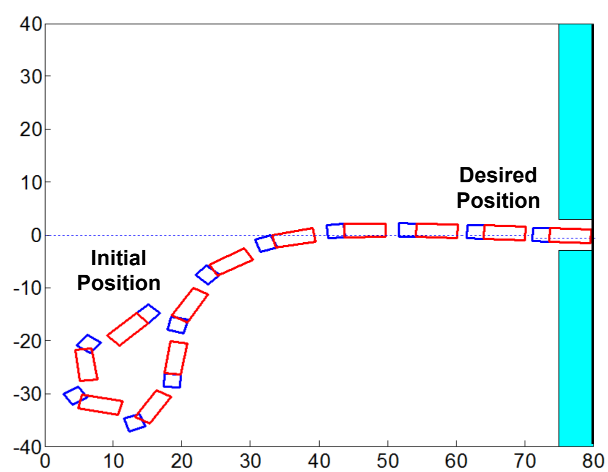


Figure 5. Trajectory of mobile robot starting from initial position (x=10, y=-20, θ_2=-135°, θ_{12}=0°) and achieving the goal position at y^*=0.

The control law (11) can be easily modified to make the robot achieves other fixed goal positions as follows:

$$\tan\delta - \tan\delta^* = -k_1(y - y^*) - k_2(\theta_2 - \theta_2^*) - k_3(\theta_{12} - \theta_{12}^*) \quad \text{.................. (13)}$$

where y^*, θ_2^* and θ_{12}^* represent the desired position of the mobile robot, and δ^* is the corresponding steering angle. It is important to coherently set the desired values of y^*, θ_2^*, θ_{12}^* and δ^* to physically realizable values to attain consistent robot responses. Figure 6 shows the trajectory of the mobile robot from an arbitrary initial position to goal position x^*=80, y^*=20, θ_2^*=0°, θ_{12}^*=0° with δ^*=0° which represents a consistent and physically attainable robot position at convergence.

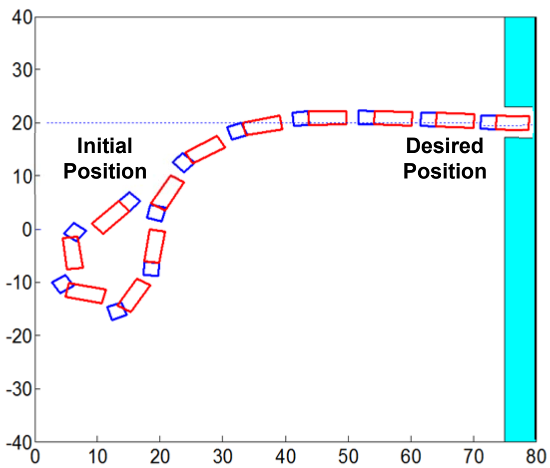


Figure 6. Trajectory of mobile robot starting from initial position (x=10, y=0, θ_2=-135°, θ_{12}=0°) and achieving the goal position at y^*=20.

It is important to note that, since v is constant, when the robot converges to line $y=y^*$ and moves along it toward the goal position, the value of $\dot{x}$ equals to v. It is equivalent to state that coordinate x is proportional to time and behaves as an independent variable.

III. TRAJECTORY FOLLOWING

The controller given by equation (13) can be applied to the mobile robot for following any desired trajectory. To do that, instantaneous proper desired values of y^*, θ_2^*, θ_{12}^* and δ^* should be determined based on the robot kinematical equations and the geometry of the desired trajectory. The desired values will be found using the *perpendicular desired position methodology* explained afterwards.

3.1. Perpendicular desired position

In this approach, a perpendicular line is drawn from the present robot position coordinates (x, y) to the desired trajectory. The coordinate y of the intersection point represents the instantaneous desired coordinate y^*, and the angle of the tangent to the desired trajectory in the intersection point represents the desired trailer inclination angle θ_2^*. Using these values, the desired values θ_{12}^* and δ^* can be obtained from the robot kinematical equations and the desired trajectory equation. It is important to note that the perpendicular line between the robot present position and the desired trajectory represents the instantaneous minimum distance between them. This methodology is applicable when the computation of the intersection point is not cumbersome and it is unique. In the following, the methodology will be explained for linear and circular desired trajectories.

3.1.1. Linear desired trajectory

Figure 7 shows the linear trajectory control problem. Point P represents the robot instantaneous position given by coordinates (x, y) and line AB, with inclination angle α, represents the trajectory to be followed. The equation of line AB is:

$$y^* = ax + b \quad \text{................................ (14)}$$

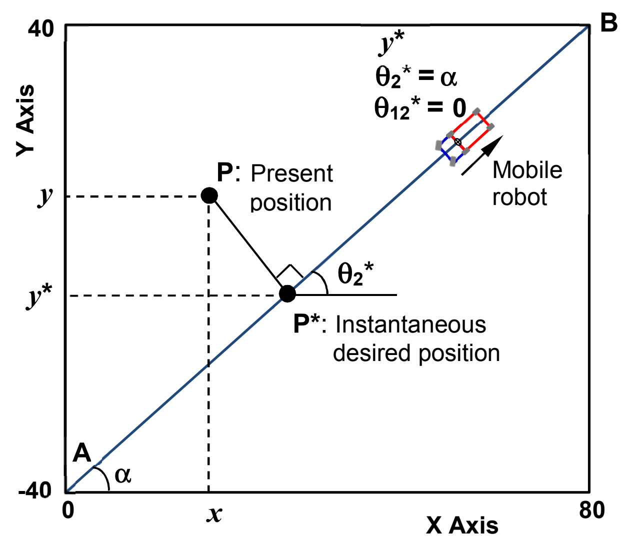


Figure 7. Linear trajectory following. P* represents the instantaneous desired position for computing coordinate y^* and inclination angle θ_2^*.

Drawing a perpendicular line from point P to line AB, point P* is obtained whose coordinate y^* represents the instantaneous desired value of robot coordinate y. By geometrical relationships, the value of y^* is given by:

$$y^{*}= \frac{a\,x+a^2 y+b}{1+a^2} \qquad \text{.............................. (15)}$$

Considering that the tangent to line AB at point P* is the same line, the desired angle θ_2^* is equal to the line inclination angle α. Also, considering that truck and trailer should be aligned to line AB, it is clear that the desired value of truck-trailer angle θ_{12}^*=0°, and the desired value of steering angle δ^*=0°. Figure 8 (a) and (b) show the trajectory of the mobile robot following a linear trajectory with inclination angle α=45° given by the equation:

$$y^* = x - 40 \qquad \text{............................ (16)}$$

starting from two different initial positions. In both cases, the robot asymptotically converges to the desired linear trajectory without steady-state error which verifies the effectiveness of the proposed control strategy.

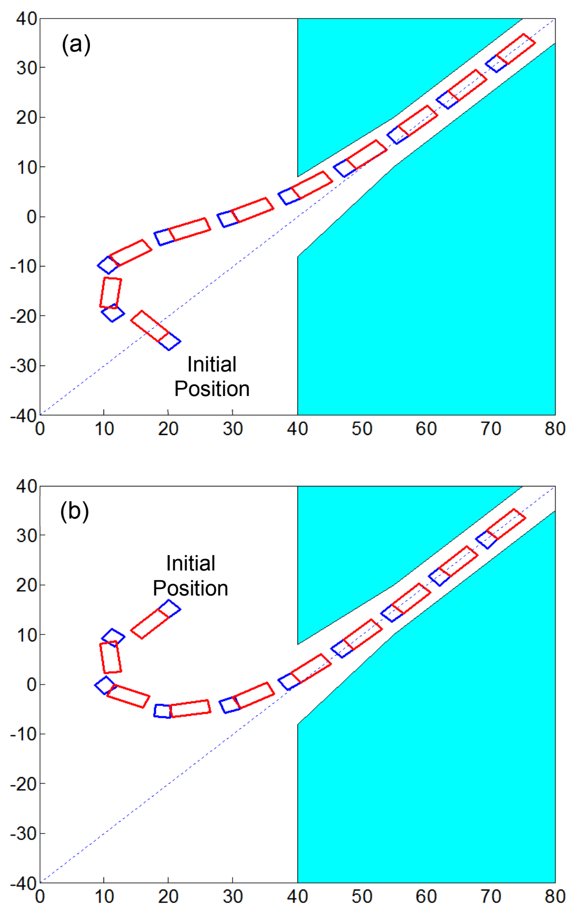


Figure 8. Trajectories of mobile robot from two different initial positions converging into the desired linear trajectory.
Initial position (a) x=15, y=-20, θ_2=135°, θ_{12}=0°
(b) x=15, y=10, θ_2=135°, θ_{12}=0°

Similarly as the positioning control problem presented in the previous section, since velocity v is constant, when the robot converges to the desired linear trajectory and moves along it, the value of $\dot{x}$ equals to $v \cos \alpha$ which is constant. It is equivalent to state that coordinate x is proportional to time and behaves as an independent variable.

3.1.2. Circular Desired Trajectory

Figure 9 shows the circular trajectory control problem. Point P represents the robot instantaneous position given by coordinates (x, y) and the circular line represents the trajectory to be followed. The equation of the circular path with center C and coordinates (x_C, y_C), and radius R is:

$$y = \sqrt{R^2 - (x - x_c)^2} + y_c \qquad (x_c - R) \le x \le x_c \qquad \text{........................(17)}$$

Drawing a perpendicular line from point P to the circular path, point P* is obtained whose coordinate y^* represents the instantaneous desired value of robot coordinate y.

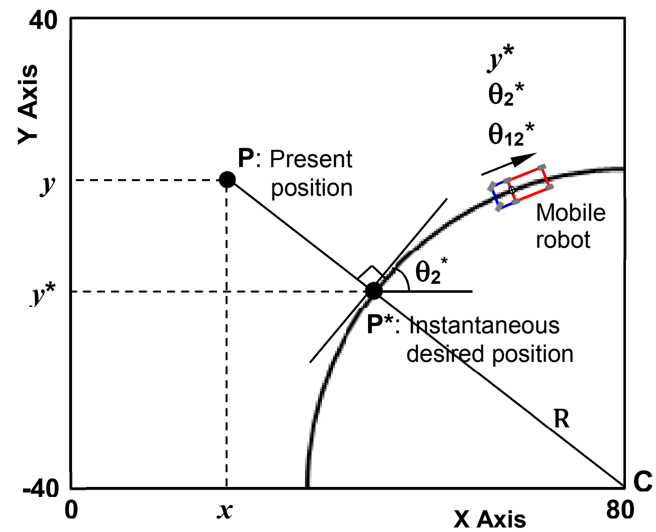


Figure 9. Circular trajectory following. P* represents the instantaneous desired position for computing coordinate y^* and inclination angle θ_2^*.

The perpendicular line also passes by the center C of the circular path, and the distance PP* represents the minimum distance from P to the circular path. By geometrical relationships, the value of y^* is given by:

$$y^* = \frac{R(y-y_c)}{\sqrt{(x-x_c)^2+(y-y_c)^2}} - y_c \qquad \text{............ (18)}$$

The inclination angle θ_2^* of the tangent to the circular trajectory at point P* represents the desired instantaneous inclination angle of the trailer and can be determined by geometrical relationships as:

$$\theta_2^* = \tan\left(\frac{x_c - x}{y - y_c}\right) \quad \text{..................... (19)}$$

Differentiating equation (19) with respect to time and replacing the expressions of $\dot{x}$ and $\dot{y}$ given by equations (1) and (2) the expression of the desired truck-trailer angle θ_{12}^* is obtained as:

$$\theta_{12}^* = \operatorname{atan}\left(\frac{L_2}{R}\right) \quad \text{..................... (20)}$$

It is noted that the value of θ_{12}^* is constant and does not depend on robot coordinates or angles. This result is expected considering the truck and trailer relative position required to describe circular trajectories, and it is the same as the result presented in [7].

Finally, considering that $\dot{\theta}_{12}^*=0$ and replacing the values of θ_{12}^* and $\dot{\theta}_{12}^*$ in equation (6), the value of δ^* is obtained which is also constant as it is expected for circular trajectories:

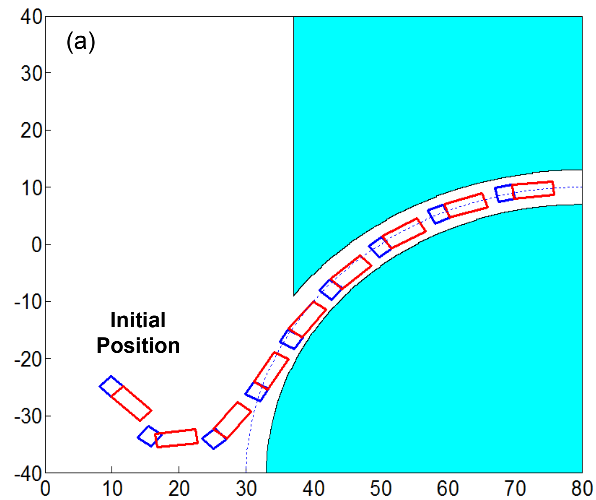
$$\delta^* = \operatorname{atan}\left(\frac{L_1}{\sqrt{L_2^2 + R^2}}\right) \quad \text{..................... (21)}$$

Figure 10 (a) and (b) show the trajectory of the mobile robot following a circular trajectory with center in point (80,-40) and radius R=50, starting from two different initial positions. In both cases, the robot asymptotically converges to the desired circular trajectory without steady-state error which verifies the effectiveness of the proposed control strategy.

IV. EFFECT OF FUZZY PARTITIONS

As it was presented in Section II, the proposed controller integrates three partitions of truck-trailer angle θ_{12}: *Negative Big, Zero* and *Positive Big.*

Considering that the approximate linearized model of equation (10) is defined for partition *Zero* (small values of θ_{12}), the range of this partition plays an important role on the control performance. This effect will be analyzed through two fuzzy controllers, Fuzzy 1 and Fuzzy 2, whose partitions and membership functions are shown in Figure 11: controller Fuzzy 1 with a wider range of partition *Zero* and Controller Fuzzy 2 with a narrower partition.



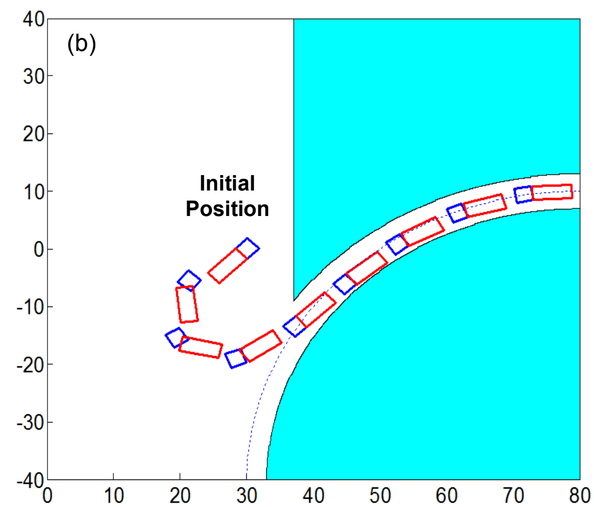


Figure 10. Trajectories of mobile robot from two different initial positions converging into the circular desired trajectory.
Initial position (a) x=15, y=-30, θ_2=-45°, θ_{12}=0°
(b) x=25, y=-5, θ_2=-135°, θ_{12}=0°

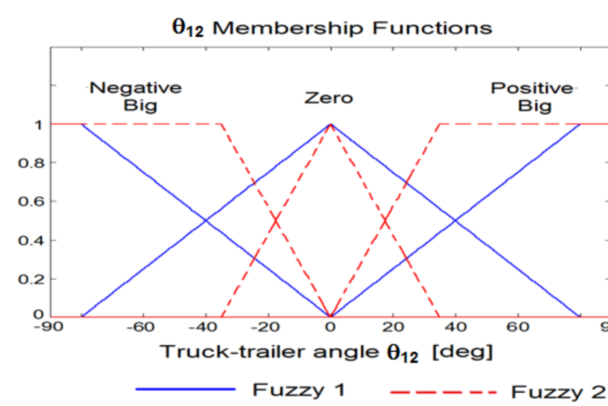


Figure 11. Partitions and membership functions of truck-trailer angle θ_{12} for controllers Fuzzy 1 y Fuzzy 2.

Figure 12 shows the trajectory of the mobile robot for both controllers starting from the same initial position and moving toward the same fixed desired position. Although both controllers are able to conduct the mobile robot to the goal position, the robot with controller Fuzzy 1 converges faster to the desired coordinate $y^*=0$ and describes a trajectory with smaller turning radius. These results are explained by the fact that extreme partitions *Negative Big* and *Positive Big*, having lower membership values in controller Fuzzy 1, impose lesser restrictions on truck-trailer angle θ_{12}, which results in higher values but without reaching unwanted jack-knife positions.

Figure 13 shows the time response of truck inclination angle θ_1, trailer inclination angle θ_2, truck-trailer angle θ_{12}, and steering angle δ corresponding to the trajectories showed in Figure 12. As it is expected, the response for controller Fuzzy 1 converges faster than controller Fuzzy 2 and with higher values of truck-trailer angle θ_{12}, and steering angle δ. These results validate the coherence of the proposed control strategy.

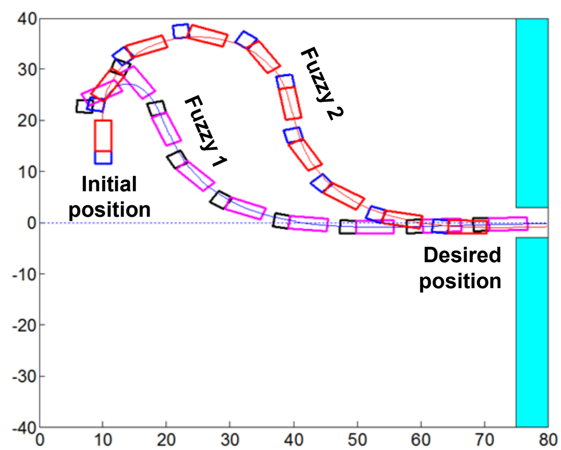


Figure 12. Trajectories of mobile robot with controllers Fuzzy1 and Fuzzy 2 starting from the same initial position: x=10, y=20, θ_2=90°, θ_{12}=0°

V. CONCLUSIONS

The kinematical model and nonholonomic constraints of truck-trailer mobile robots have been derived and analyzed. A novel control strategy integrating linear controllers in a fuzzy logic approach has been proposed, assuring the robot achieve goal positions avoiding jack-knifing. Two strategies were proposed for trajectory following, one of them being applicable for general shape desired paths. The effectiveness of the trajectory following control strategies have been verified for linear, circular and sinusoidal trajectories where the mobile robot

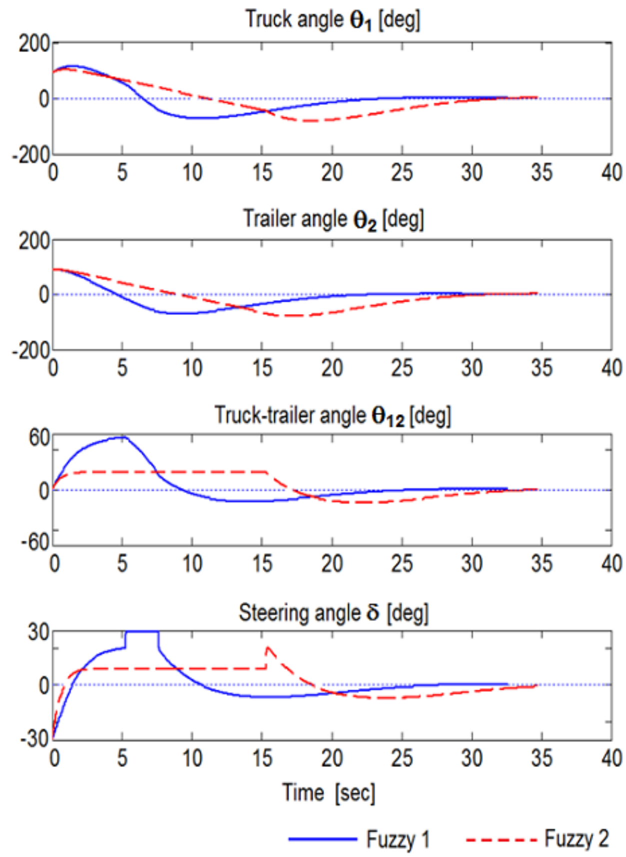


Figure 13. Time response of truck inclination angle θ_1, trailer inclination angle θ_2, truck-trailer angle θ_{12} and steering angle δ for controllers Fuzzy 1 and Fuzzy 2.

converges to the desired trajectories with bounded values of the steering angle. The effect of the size of truck-trailer angle partitions in fuzzy control was analyzed and it was found that a wider range of the central partition results in faster convergence of the mobile robot at the expense of higher values of truck-trailer angle θ_{12} and steering angle δ.

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