A Fuzzy Approach for Texture-based Segmentation

Pedro Manuel Martínez-Jiménez
Department of Automation, Electronics and Computer Architecture and Networks
University of Cádiz
Cádiz, Spain
pedromartinez@decsai.ugr.es

Jesús Chamorro-Martínez
Department of Computer Science and Artificial Intelligence
University of Granada
Granada, Spain
jesus@decsai.ugr.es

Belén Prados-Suárez
Department of Computer Languages and Systems
University of Granada
Granada, Spain
belenps@ugr.es

***Abstract*—In this paper, we propose a fuzzy approach for texture-based segmentation, where both the obtained regions and the texture features are fuzzy. This way, we have considered the imprecision related to the texture concept, as well as the imprecision associated with the boundaries between regions in the images. With regard to the texture descriptors, we propose to define them on the basis of the perceptual properties of texture. Since these properties are imprecise by nature, they are modelled by means of fuzzy sets defined on the domain of some of the most representative measures of each property. With regard to the segmentation technique, a fuzzy path-based segmentation is proposed. In this approach, fuzzy connectivity is used to measure the relationship between any pair of pixels. Thus, given a set of seed points, fuzzy regions are obtained on the basis of the connectivity between each seed point and the rest of pixels in the image. Finally, this proposal is applied to obtain fuzzy segmentations from real images.**

***Index Terms*—Image analysis, texture modelling, fuzzy segmentation**

I. Introduction

Image segmentation is one of the most important tasks in computer vision, since it is the starting point for a great variety of image analysis applications, such as medical imaging [1], [2], object identification and classification [3], [4], content-based image retrieval [5], etc. Image segmentation techniques split an image into several regions in such way that pixels in the same region share certain characteristics (usually associated with color or/and texture). A large number of approaches to the segmentation problem can be found in the literature, including pixel based techniques [6]–[8], frontier based techniques [9], [10], and region based techniques [11], [12].

Usually, in real images, there are no abrupt changes in the characteristics from one region to another, but the boundaries between regions are imprecise and blurred. Thus, in the segmentation task, it is difficult to assign a region to the pixels in these boundaries. In order to face this problem, fuzzy segmentation techniques, that model each region in the image through a fuzzy set, have been proposed [13]–[16]. In those approaches, the concept of "region" is extended to "fuzzy region", where each pixel in the image can be assigned to several regions with different membership degrees.

In most of segmentation techniques (fuzzy or not), the image characteristics that are usually employed are related with the features of color and/or texture, depending on the specific problem to be solved. In this paper, we will focus our work in fuzzy segmentation based on texture features.

Texture is one of the most difficult low level feature to characterize, since it is an imprecise and abstract concept. Although there is not an accurate definition for the concept of texture, it is usually described by human according to some intuitive perceptual properties, like *coarseness*, *directionality*, *contrast*, *line-likeness* or *regularity* [17], [18]. This properties are also imprecise concepts, because, except in extreme cases, we cannot set a precise threshold between textures that strictly accomplish a property and textures that do not, but the accomplishment of the property is gradual in nature. In our approach, we propose to describe texture on the basis of these properties, and, in order to incorporate the uncertainty associated to them, fuzzy representation models will be employed.

From our knowledge, all the texture-based fuzzy segmentation techniques in the literature try to model texture by means of crisp characteristics, without taking into account the own imprecision related to it [19]–[22]. Thus, since the descriptors used in these approaches do not provide a textural representation interpretable by humans, the obtained segmentation may not be in accordance with human perception.

In this paper, we propose a texture-based fuzzy segmentation approach, which uses texture features that are also fuzzy. On the one hand, as has been commented above, we propose to describe texture on the basis of its perceptual properties. First, these properties are modelled by means of fuzzy sets defined on the domain of some of the most representative measures of each property [23], and then, fuzzy descriptors are defined using the proposed representation models. On the other hand, we have proposed a path-based fuzzy segmentation technique, which incorporate spatial information related to adjacency between pixels. In this technique, fuzzy connectivity

This work has been partially supported by the Spanish Ministry of Science, Innovation and Universities and the European Regional Development Fund - ERDF (Fondo Europeo de Desarrollo Regional - FEDER) under project PGC2018-096156-B-I00 *Recuperación y Descripción de Imágenes mediante Lenguaje Natural usando Técnicas de Aprendizaje Profundo y Computación Flexible*.

is used to measure the relationship between any pair of pixels. Thus, given a set of seed points, fuzzy regions are obtained on the basis of the connectivity between each seed point and the rest of pixels in the image [13], [24], [25]. Thus, in our approach, we have considered the imprecision related to the texture concept, as well as the imprecision associated with the boundaries between regions in the images.

The rest of the paper is organized as follows. Section II introduces the methodology proposed to obtain the fuzzy texture descriptors, while in section III we describe the proposed fuzzy segmentation that uses these characteristics. In section IV some results obtained by applying this segmentation technique are shown. Finally, section V summarizes the main conclusions and future works.

II. Texture Features

In this section, we describe the methodology proposed to obtain the texture features employed in the fuzzy segmentation shown in section III. As has been commented above, these features are based on the perceptual properties of texture. In particular, coarseness, contrast and directionality, that are considered the three most fundamental properties in texture analysis [18], will be employed. In our approach, we propose to obtain models to represent these texture properties, and, on the basis of these models, texture descriptors associated to a pixel or region of image will be defined. Thus, two questions need to be faced: firstly, how to model the perceptual properties and, secondly, how to define the texture descriptors.

With regard to the texture models, as mentioned in the above section, the perceptual properties are imprecise by nature, what suggests the use of representation models that incorporate the uncertainty associated to them. For this modelling, we propose to use the methodology introduced in our previous work [23], where texture properties are modelled by means of fuzzy sets defined on the domain of some of the most representative measures of each property. In that work, we propose to model a texture property $p \in \mathcal{P} = \{$*fineness*, *contrast*,*directionality*$\}$ as a fuzzy set $\mathcal{T}_k^p$ defined on the domain of a computational measure F_k^p of this property[1]. The membership function[2] of this fuzzy set will be defined as

$$\mathcal{T}_k^p : \mathbb{R} \to [0,1] \tag{1}$$

of the form

$$\mathcal{T}_k^p(x; a_n \dots a_0, \alpha, \beta) = \begin{cases} 0 & x < \alpha, \\ poly(x; a_n \dots a_0) & \alpha \le x \le \beta, \\ 1 & x > \beta \end{cases} \tag{2}$$

with $poly(x; a_n \dots a_0)$ being a polynomial function

$$poly(x; a_n \dots a_0) = a_n x^n + \dots + a_1 x^1 + a_0 \tag{3}$$

[1]In [23], we employ a set of representative measures of each property, obtaining the corresponding fuzzy set $\mathcal{T}_k^p$ for each one.

[2]To simplify the notation, as it is usual in the scope of fuzzy sets, we will use the same notation $\mathcal{T}_k^p$ for the fuzzy set and for the membership function that defines it.

TABLE I
PARAMETER VALUES OF THE MEMBERSHIP FUNCTION $\mathcal{T}_k^p$ CORRESPONDING TO THE MEASURE WITH THE LOWEST FITTING ERROR FOR EACH PROPERTY

	Fineness (Amadasun)	Contrast (Tamura)	Directionality (Tamura)
a_3	-6.6128	1.6877	648.14
a_2	9.4901	-3.9536	-1792.31
a_1	-6.4835	3.8763	1657.35
a_0	1.8707	-0.5728	-511.969
α	0.1727	0.1775	0.8594
β	0.5858	0.9620	0.9865

In [23], we proposed to obtain this model by using a perceptually-based approach that relates the computational measures with the human perception of the property. For this purpose, two questions need to be faced: firstly, how to obtain the data about the “human perception” of the property and, secondly, how to fit these data with the measures in order to obtain the membership function. To get information about the human perception of a texture property, a set of images covering different presence degrees of this property was gathered. These images were used to collect, by means of a poll, human assessments about the perceived presence of the property.

To obtain the membership function $\mathcal{T}_k^p$ for a given measure F_k^p, a robust fitting method was applied in order to obtain suitable functions relating the values of the measure calculated for each image with the presence degree of the property perceived by humans. The detailed description of the analyzed computational measures, the set of images, the pool and the robust fitting method are described in detail in [23]. The parameter values ($a_n \dots a_0$, α and β) of the membership function $\mathcal{T}_k^p$ corresponding to the measure with the lowest fitting error for each property are shown in Table I.

With regard to the texture descriptors, we propose to define them on the basis of the obtained models. In particular, since these models are fuzzy, we will define a *fuzzy texture descriptor* as a level-two fuzzy set as follows:

$$FTD = \sum_{p \in \mathcal{P}} v^p / p \tag{4}$$

where $v^p \in [0,1]$ is the degree of fulfillment of the property $p \in \mathcal{P}$. Thus, we can obtain the fuzzy texture descriptor associated to a pixel by considering a window centered on this pixel, and measuring the degree of fulfillment of the texture properties by means of the proposed fuzzy sets $\mathcal{T}_k^p$.

III. Path-based Fuzzy Segmentation

As has been commented in section I, in our approach a path-based fuzzy segmentation is proposed, which incorporate spatial information related to adjacency between pixels (unlike the clustering-based methods, where each pixel is classified without considering its neighbourhood). Thus, in this type of techniques, a fuzzy region is defined as a set of *resemblant* and *connected* pixels. Hence, in order to obtain a fuzzy segmentation of an image, first we need to define the (inherently fuzzy) concepts of *resemblance* and *connectivity*.

In section III-A, a fuzzy resemblance relation between neighbour pixels is defined by using the fuzzy resemblance relation between their corresponding texture descriptors. In section III-B, the connectivity between any pair of pixels is defined on the basis of the homogeneity degree of the most homogeneous path joining them [26]. Section III-C describes a measure to obtain the homogeneity degree of a path. Finally, given a set of seed points, fuzzy regions are obtained on the basis of the connectivity between each seed point and the rest of pixels in the image [26], [27]. Thus, in section III-D, we define a measure to obtain the degree in which each pixel in the image belongs to each region.

A. Fuzzy Resemblance between Texture Descriptors

In image segmentation, regions consist of a set of connected pixels whose features are resemblant, i.e., not different. In our approach, the resemblance between pixel features will be measured on the basis of the resemblance between the fuzzy texture descriptors defined in the previous section. In particular, we propose to calculate it by means of the *Generalized Resemblance between Fuzzy Sets* introduced in [28], which is based on the concept of double inclusion.

Let FTD_i and FTD_j be two fuzzy texture descriptors defined over a finite reference universe of fuzzy sets $\mathcal{P}$, let S be a similarity relation defined over the elements of $\mathcal{P}$, let $\otimes_1$ and $\otimes_2$ be two t-norms, and let J be a fuzzy implication operator. The resemblance degree between FTD_i and FTD_j is calculated as follows:

$$\begin{aligned}&\mathcal{FR}_{S,\otimes_1,\otimes_2,J}(FTD_i,FTD_j) =\\ &\otimes_1(\Theta_{S,\otimes_2,J}(FTD_j,FTD_i),\Theta_{S,\otimes_2,J}(FTD_i,FTD_j))\end{aligned} \quad (5)$$

with $\Theta_{S,\otimes_2,J}(FTD_j,FTD_i)$ being the inclusion degree of FTD_i in FTD_j driven by the similarity relation S, the t-norm $\otimes_2$ and the fuzzy implication operator J.

In our approach, we propose to calculate this inclusion degree as follows:

$$\Theta_{S,\otimes_2,J}(FTD_j,FTD_i) = \min_{\widetilde{x}\in\mathcal{P}} \max_{\widetilde{y}\in\mathcal{P}} \theta^{i,j}_{S,\otimes_2,J}(\widetilde{x},\widetilde{y}) \quad (6)$$

where

$$\theta^{i,j}_{S,\otimes_2,J}(\widetilde{x},\widetilde{y}) = \otimes_2(J(FTD_i(\widetilde{x}),FTD_j(\widetilde{y})),S(\widetilde{x},\widetilde{y})) \quad (7)$$

In this paper, we will use the minimum as t-norm, both in $\otimes_1$ and $\otimes_2$, the similarity relation S defined in equation (8) and the Goguen operator [29] as implication operator J, since it verifies all the axioms Sinha-Dougherty [30].

$$S(\widetilde{x},\widetilde{y}) = \begin{cases} 1 & \text{if } \widetilde{x}=\widetilde{y} \\ 0 & \text{otherwise}\end{cases} \quad (8)$$

B. Fuzzy Connectivity between Pixels

In fuzzy path-based image segmentation, the notion of fuzzy connectivity of two pixels indicates to which degree those pixels belong to a group of the topologically connected pixels with resemblant features. Thus, the fuzzy connectivity between two pixels is not measured by using directly the resemblance between their feature vectors, but by using information about the homogeneity of the paths joining them. Before defining the proposed measure of fuzzy connectivity, we have to introduce some previous definitions.

Definition III.1. *A path between two pixels p and q is a sequence*

$$\pi_{pq} = (r_1, r_2, \ldots, r_k) \quad (9)$$

where $k \geq 1$, such that $r_1 = p$, $r_k = q$ and r_i is adjacent to r_{i+1} $\forall i \in \{1,\ldots,k-1\}$.

Let Π_{pq} be the set of possible paths linking the pixels p and q, let $\mathcal{Q}(\pi_{pq})$ be the set of pixels in the path π_{pq}, and let π^{st}_{pq} be the subpath of π_{pq} that connects r and s with $r,s \in \mathcal{Q}(\pi_{pq})$, and r appearing before s. Also, let $\pi^{-1}_{pq} = \{p_n,\ldots,p_1\}$ be the reverse of path $\pi_{pq} = \{p_1,\ldots,p_n\}$.

Definition III.2. *The fuzzy homogeneity of a path $\pi_{pq} \in \Pi_{pq}$ is defined as a function*

$$homo : \Pi_{pq} \to [0,1] \quad (10)$$

calculated on the basis of the resemblances between consecutive points on the path.

To measure the resemblances between consecutive points, the relation $\mathcal{FR}$ will be used. In section III-C we shall study desirable properties of $homo$ and we shall propose a candidate function.

Taking into account the $homo$ function, we define the optimum path between p and q, $\hat{\pi}_{pq}$, as the path that links both points with maximum homogeneity, in the following way:

Definition III.3. *The optimum path between p and q is:*

$$\hat{\pi}_{pq} = \arg\max_{\pi_{pq}\in\Pi_{pq}} \{homo(\pi_{pq})\} \quad (11)$$

Based on this optimum path, we can get the measure of the connectivity between two pixels as follows:

Definition III.4. *The fuzzy connectivity between two pixels p and q is the homogeneity of the optimum path from p to q:*

$$conn(p,q) = homo(\hat{\pi}_{pq}) \quad (12)$$

Let us remark that the homogeneity measure defined in (12) uses topographic information (paths linking the pixels) and resemblance between pixel features.

C. Measuring the Homogeneity of a Path

Given a path $\pi_{pq} = (r_1,\ldots,r_n)$, our aim is to obtain a function $homo(\pi_{pq})$ measuring the homogeneity of the path π_{pq}. For simplicity, let us define a resemblance relation

$\mathcal{PR}$ between neighbour pixels, induced by the relation $\mathcal{FR}$ between their corresponding features, in the following way:

$$\mathcal{PR}(p,q) = \mathcal{FR}(FTD_p, FTD_q) \tag{13}$$

Hence, it seems natural to define $homo(\pi_{pq})$ as an aggregation of the resemblances between consecutive points in the path π_{pq}, i.e., $homo(\pi_{pq}) = Aggr(ReSet(\pi_{pq}))$, where $ReSet$ is the following bag (multiset) of values:

$$ReSet(\pi_{pq}) = \{\mathcal{PR}(r_k, r_{k+1}) \mid r_k, r_{k+1} \in \mathcal{Q}(\pi_{pq})\}$$

In order to choose the aggregation function $Aggr$, first we study the set of properties that the $homo$ function should verify. We propose the following minimal set of properties for $homo$:

1) Let $\pi_{pq} = p, q$ be a path consisting of two adjacent pixels. Then $homo(\pi_{pq}) = \mathcal{PR}(p,q)$. As a consequence, if $\pi_{pp} = p, p$ then $homo(\pi_{pp}) = 1$.
2) The homogeneity of a path should be less or equal than the resemblance between consecutive pixels in the path, i.e., $homo(\pi_{pq}) \leq \min ReSet(\pi_{pq})$. The rationale behind this property is that a path is completely homogeneous if all the possible pairs of consecutive pixels are resemblant. Hence, the homogeneity of the path has an upper bound in the minimum value of resemblance between pairs of consecutive pixels.
3) Monotony: $homo(\pi_{pq}^{rs}) \geq homo(\pi_{pq})$.
4) Let π_{pq} and $\pi_{p'q'}$ be two paths such that $ReSet(\pi_{pq}) = ReSet(\pi_{p'q'})$. Then $homo(\pi_{pq}) = homo(\pi_{p'q'})$. In particular, $homo(\pi_{pq}) = homo(\pi_{pq}^{-1})$.

These properties suggest the use of a t-norm to aggregate the resemblances between consecutive pixels into the final homogeneity of the whole path, i.e., we propose to define the aggregation function as

$$Aggr(ReSet(\pi_{pq})) = \bigwedge ReSet(\pi_{pq}) \tag{14}$$

were $\bigwedge$ is a t-norm.

It is trivial to show that this function satisfy all the properties we have required. In our approach, we propose to use the Dubois-Prade's parametric t-norm:

$$I(a,b) = \frac{ab}{\max(a,b,\alpha)} \tag{15}$$

In the experiments shown in section IV, we have used this t-norm with a parameter value of $\alpha = 0.995$.

D. Membership Function for Fuzzy Regions

In section III-B we have introduced the use of paths to measure the fuzzy connectivity between any pair of pixels. Now this connectivity will be used to obtain fuzzy regions in an image by means of the growing region algorithm proposed in [31]. In particular, since in this approach a region is defined as a fuzzy subset of connected pixels, we need to define a measure that indicates the degree in which each pixel in the image belongs to each region. Under the assumption that a fuzzy region $\widetilde{R}_s$ has a representative seed point r_s, we introduce the following membership function associated to each region:

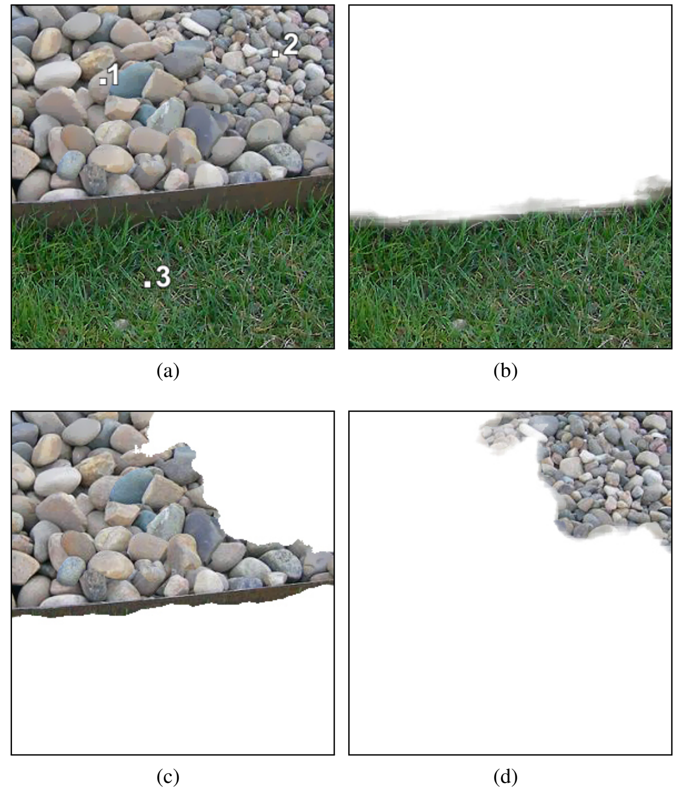


Fig. 1. Fuzzy segmentation of the image shown in (a), using the seed points marked as 1, 2, and 3. (b)(c)(d) Fuzzy regions obtained with the seed points 1, 2 and 3, respectively.

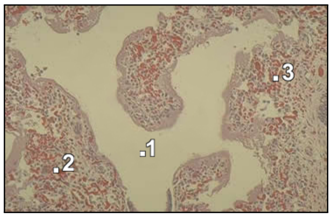
Definition III.5. *We define the membership degree $\mu_{\widetilde{R}_s}(p)$ of a pixel p to a fuzzy region $\widetilde{R}_s$ as:*

$$\mu_{\widetilde{R}_s}(p) = conn(p, r_s) \tag{16}$$

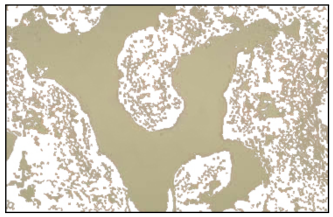
Using equation (16) we can calculate the membership degree of every pixel p to each region $\widetilde{R}_s$. This allows us to obtain a set of fuzzy regions $\widetilde{\Psi} = \{\widetilde{R}_1, \widetilde{R}_2, \ldots, \widetilde{R}_m\}$ from a set of seed points $\Psi = \{r_1, r_2, \ldots, r_m\}$. An algorithm to calculate $\widetilde{\Psi}$ is proposed in [31] with a computational complexity of $O(mn)$, where n is the number of pixels and m is the number of seeds.

IV. Results

In this section, we show the results obtained by applying the proposed fuzzy segmentation to several experiments with real images. For the first one, we have used the image shown in Figure 1(a), where we can see three different textures, corresponding to the big stones, the grass, and the gravel. It can be noticed that these textures may be perceived with a different presence degree of the fineness property: the region of stones can be considered as a coarse texture, the grass is a fine texture,

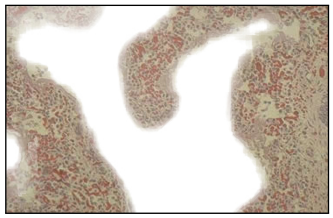


(a)



(b)

(c)



(d)

Fig. 2. Segmentation of the microscopic image shown in (a), in order to separate the uniform light region from the rest of tissue. (b) Segmentation using color information. (c)(d) Fuzzy texture-based segmentation proposed in this paper.

while the gravel is perceived as an intermediate coarseness texture. Thus, using the methodology proposed in section II, the fuzzy texture descriptors obtained for the pixels in these regions will have a different membership degree associated to the fineness property.

In order to apply our segmentation approach, we have placed one seed point in each texture, marked as 1, 2 and 3 in Figure 1(a). Figures 1(b)-(d) show the fuzzy regions obtained for the seed points 1, 2 and 3, respectively. In these images, the membership degree of each pixel to the corresponding fuzzy region is used as the opacity of its color, i.e. this degree is mapped as the inverse of the pixels transparency. Thus, pixels with membership degree 1 have the same color as in the original image, and they become more transparent as this degree decreases, with fully transparent (white) pixels for those with 0 degree. It can be noticed that each texture has been segmented in a different fuzzy region. In addition, it can be seen that pixels that belong clearly to a texture have high membership degrees to the corresponding region, while pixels on the boundaries of different textures have less degree, as it was expected.

For the second experiment, we have used the image shown in Figure 2(a), which is a microscopic image of a premature infant's lung tissue affected by the hyaline membrane disease. In this case, we want to separate the collapsed alveoli presents in the image, that corresponds to the uniform light region, from the rest of lung tissue. Since both regions share colors, a segmentation using color features is not possible, as can be seen in Figure 2(b), so texture information is needed for extracting the uniform areas. Figure 2(c) shows the fuzzy region obtained for the seed point marked as 1 in the original image, while Figure 2(d) shows the fuzzy regions obtained for the seed points 2 and 3 (the combination of both regions). It can be noticed that in this case the collapsed alveoli can be easily extracted from the rest of tissue.

(a)

(b)

(c)

Fig. 3. Fuzzy segmentation of the image shown in (a), using the seed points marked as 1 and 2. (b)(c) Fuzzy regions obtained with the seed points 1 and 2, respectively.

For the third experiment, we have considered the image shown in Figure 3(a), where we can see two tires with different wear levels. The tire on the right has deep grooves, while the one on the left has an irregular wear in the center and on one side. In this case, we want to segment the image according to the wear level of tires. Since physical textures with different relief in nature may be perceived as textures with different contrast in the image (due to the illumination effect), the proposed fuzzy texture-based segmentation can be applied for this purpose. Figures 3(b)-(c) show the fuzzy regions obtained for the seed points 1 and 2, respectively, and it can be noticed that tires have been segmented appropriately.

V. Conclusions

In this paper we have proposed a texture-based segmentation, where both the obtained regions and the texture features are fuzzy. On the one hand, we have described texture on the basis of its most important perceptual properties: fineness, contrast and directionality. Since these properties are imprecise by nature, they have been modelled by means of fuzzy sets defined on the domain of some of the most representative measures of each property. Then, fuzzy descriptors have been defined using the proposed representation models. On the other hand, we have proposed a path-based fuzzy segmentation technique, where fuzzy connectivity has been used to measure the relationship between any pair of pixels. Given a set of seed points, fuzzy regions have been obtained on the basis of the connectivity between each seed point and the rest of pixels in the image. Thus, in our approach, we have considered the imprecision related to the texture concept, as well as the imprecision associated with the boundaries between regions in the images. As has been shown in the experiments of section IV, the obtained fuzzy regions match what a human would expect.

As a future work we will study a fuzzy segmentation technique based on the combination of fuzzy color and fuzzy texture features. In this case, a degree of resemblance between neighbour pixels combining both type of descriptors must be defined.

REFERENCES

[1] E. Smistad, T. L. Falch, M. Bozorgi, A. C. Elster, and F. Lindseth, "Medical image segmentation on gpus – a comprehensive review," *Medical Image Analysis*, vol. 20, no. 1, pp. 1 – 18, 2015.

[2] G. Wang, W. Li, M. A. Zuluaga, R. Pratt, P. A. Patel, M. Aertsen, T. Doel, A. L. David, J. Deprest, S. Ourselin, and T. Vercauteren, "Interactive medical image segmentation using deep learning with image-specific fine tuning," *IEEE Transactions on Medical Imaging*, vol. 37, no. 7, pp. 1562–1573, July 2018.

[3] N. Audebert, B. Le Saux, and S. Lefèvre, "Segment-before-detect: Vehicle detection and classification through semantic segmentation of aerial images," *Remote Sensing*, vol. 9, no. 4, 2017.

[4] R. Ptucha, A. Bhat, A. Kuppusamy, and S. E. Lyshevski, "Object recognition, identification and classification for intelligent surveillance and reconnaissance platforms," in *Signal Processing, Sensor/Information Fusion, and Target Recognition XXVIII*, I. Kadar, E. P. Blasch, and L. L. Grewe, Eds., vol. 11018, International Society for Optics and Photonics. SPIE, 2019, pp. 189 – 200.

[5] A. Alzu'bi, A. Amira, and N. Ramzan, "Semantic content-based image retrieval: A comprehensive study," *Journal of Visual Communication and Image Representation*, vol. 32, pp. 20 – 54, 2015.

[6] J. S. Sevak, A. D. Kapadia, J. B. Chavda, A. Shah, and M. Rahevar, "Survey on semantic image segmentation techniques," in *2017 International Conference on Intelligent Sustainable Systems (ICISS)*, Dec 2017, pp. 306–313.

[7] Z. Tian, L. Liu, Z. Zhang, and B. Fei, "Superpixel-based segmentation for 3d prostate mr images," *IEEE Transactions on Medical Imaging*, vol. 35, no. 3, pp. 791–801, March 2016.

[8] G. Yan, J. F. Mas, B. H. P. Maathuis, Z. Xiangmin, and P. M. V. Dijk, "Comparison of pixel-based and object-oriented image classification approaches—a case study in a coal fire area, wuda, inner mongolia, china," *International Journal of Remote Sensing*, vol. 27, no. 18, pp. 4039–4055, 2006.

[9] R. S. Rodrigues, J. F. Morgado, and A. J. Gomes, "A contour-based segmentation algorithm for triangle meshes in 3d space," *Computers & Graphics*, vol. 49, pp. 24 – 35, 2015.

[10] Q. D. Vu, S. Graham, T. Kurc, M. N. N. To, M. Shaban, T. Qaiser, N. A. Koohbanani, S. A. Khurram, J. Kalpathy-Cramer, T. Zhao, R. Gupta, J. T. Kwak, N. Rajpoot, J. Saltz, and K. Farahani, "Methods for segmentation and classification of digital microscopy tissue images," *Frontiers in Bioengineering and Biotechnology*, vol. 7, p. 53, 2019.

[11] S. Yuheng and Y. Hao, "Image segmentation algorithms overview," *CoRR*, vol. abs/1707.02051, 2017.

[12] N. M. Zaitoun and M. J. Aqel, "Survey on image segmentation techniques," *Procedia Computer Science*, vol. 65, pp. 797 – 806, 2015, international Conference on Communications, management, and Information technology (ICCMIT'2015).

[13] H. D. Cheng and J. Li, "Fuzzy homogeneity and scale-space approach to color image segmentation," *Pattern Recognition*, vol. 36, no. 7, pp. 1545–1562, Jul. 2003.

[14] T. H. Farag, W. A. Hassan, H. A. Ayad, A. S. AlBahussain, U. A. Badawi, and M. K. Alsmadi, "Extended absolute fuzzy connectedness segmentation algorithm utilizing region and boundary-based information," *Arabian Journal for Science and Engineering*, vol. 42, no. 8, pp. 3573–3583, Aug 2017.

[15] L. Feng, H. Li, Y. Gao, and Y. Zhang, "A color image segmentation method based on region salient color and fuzzy c-means algorithm," *Circuits, Systems, and Signal Processing*, Apr 2019.

[16] A. Touil and K. Kalti, "Iterative fuzzy segmentation for an accurate delimitation of the breast region," *Computer Methods and Programs in Biomedicine*, vol. 132, pp. 137 – 147, 2016.

[17] A. R. Rao and G. L. Lohse, "Identifying high level features of texture perception," *Graphical Models and Image Processing*, vol. 55, no. 3, pp. 218–233, 1993.

[18] H. Tamura, S. Mori, and T. Yamawaki, "Textural features corresponding to visual perception," *IEEE Trans. on Systems, Man and Cybernetics*, vol. 8, pp. 460–473, 1978.

[19] S. K. Choy, S. Y. Lam, K. W. Yu, W. Y. Lee, and K. T. Leung, "Fuzzy model-based clustering and its application in image segmentation," *Pattern Recognition*, vol. 68, pp. 141 – 157, 2017.

[20] R. Gao and H. Wu, "Agricultural image target segmentation based on fuzzy set," *Optik*, vol. 126, no. 24, pp. 5320 – 5324, 2015.

[21] H. Min, W. Jia, X.-F. Wang, Y. Zhao, R.-X. Hu, Y.-T. Luo, F. Xue, and J.-T. Lu, "An intensity-texture model based level set method for image segmentation," *Pattern Recognition*, vol. 48, no. 4, pp. 1547 – 1562, 2015.

[22] P. Qian, K. Zhao, Y. Jiang, K.-H. Su, Z. Deng, S. Wang, and R. F. Muzic, "Knowledge-leveraged transfer fuzzy c-means for texture image segmentation with self-adaptive cluster prototype matching," *Knowledge-Based Systems*, vol. 130, pp. 33 – 50, 2017.

[23] J. Chamorro-Martínez, P. Martínez-Jiménez, J. Soto-Hidalgo, and B. Prados-Suarez, "Perception-based fuzzy sets for visual texture modelling," *Soft Computing*, vol. 18, no. 12, pp. 2485–2499, 2014.

[24] Y. Guo, Y. Akbulut, A. Şengür, R. Xia, and F. Smarandache, "An efficient image segmentation algorithm using neutrosophic graph cut," *Symmetry*, vol. 9, no. 9, p. 185, Sep 2017.

[25] S. A. Nadeem, E. A. Hoffman, and P. K. Saha, "Novel theory and algorithms of path-gradient and their application to image segmentation," in *2017 Ninth International Conference on Advances in Pattern Recognition (ICAPR)*, Dec 2017, pp. 1–6.

[26] J. K. Udupa and S. Samarasekera, "Fuzzy connectedness and object definition: Theory, algorithms, and applications in image segmentation," *Graphical Models and Image Processing*, vol. 58, no. 3, pp. 246 – 261, 1996.

[27] B. M. Carvalho, C. J. Gau, G. T. Herman, and T. Y. Kong, "Algorithms for fuzzy segmentation," *Pattern Analysis & Applications*, vol. 2, no. 1, pp. 73–81, Apr 1999.

[28] N. Marín, J. Medina, O. Pons, D. Sánchez, and M. Vila, "Complex object comparison in a fuzzy context," *Information and Software Technology*, vol. 45, no. 7, pp. 431–444, 2003.

[29] M. Baczynski and B. Jayaram, *Fuzzy Implications*, 1st ed. Springer Publishing Company, Incorporated, 2008.

[30] D. Sinha and E. Dougherty, "Fuzzification of set inclusion: Theory and applications," *Fuzzy Sets and Systems*, vol. 55, no. 1, pp. 15 – 42, 1993.

[31] J. Chamorro-Martínez, D. Sánchez, B. Prados-Suarez, and E. Galán-Perales, "Fuzzy homogeneity measures for path-based colour image segmentation," *International Journal of Intelligent Systems Technologies and Applications*, vol. 1, no. 3/4, pp. 301–318, 2006.