Using SAT/SMT Solvers for Efficiently Tuning Fuzzy Logic Programs

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Abstract—During the last years we have developed advanced tools for tuning fuzzy logic programs devoted to facilitate the selection of the more appropriate set of weights and fuzzy connectives used in programs rules. Designing accurate techniques for automating these tasks is very useful for programmers, even when they are time consuming. In order to increase its performance, in this paper we make use of powerful and well-known SAT/SMT solvers for improving our original approaches. Inspired by some previous experiences we have acquired in this setting, whose impact is growing in many modern software tools, we show some representative experiments (related to circuit validation and linear regression) and benchmarks which illustrate the significant advantages enjoyed by the new empowered method.

Index Terms—Fuzzy Logic Programming, SAT/SMT, Tuning, Software for Soft Computing

I. INTRODUCTION

Research on SAT (Boolean Satisfiability) and SMT (Satisfiability Modulo Theories) [9], [17] represents a successful and large tradition in the development of highly efficient automatic theorem provers for classic logic with a wide range of practical applications. There also exist attempts for covering fuzzy logics, as occurs with the approaches presented in [7], [30]. Moreover, if automatic theorem proving supposes both a starting point for the foundations of logic programming as well as one of its important application fields [16], [26], in [6], [10] we showed some preliminary guidelines about how fuzzy logic programming can face the automatic proving of fuzzy theorems by making use of the “Fuzzy Logic Programming Environment for Research”, FLOPER in brief, developed in our research group.1 We have successfully used this tool for implementing real soft computing applications connecting with cloud computing [27]–[29], the semantic web [2]–[5] and, more recently, neural networks [21].

One step beyond, the main goal of the present paper is to make use of SMT solvers for reinforcing some tuning techniques we have recently implemented on this platform [20], [22]. The main reason for implementing our approach with MALP is the fact that, to the best of our knowledge, this is the only fuzzy logic programming language for which there exist tuning techniques already available.2

1The online version of the system is available at http://dectau.uclm.es/malp/sandbook
2See [1]—as well as [25]— for analyzing a wide list of modern fuzzy software systems including the FLOPER system.

In this paper we focus on the so-called multi-adjoint logic programming approach, MALP in brief [14], [19], a powerful and promising approach in the area of fuzzy logic programming. In this framework, a program is a set of “weighted” rules whose bodies contain atoms connected by fuzzy connectives defined on a concrete lattice of truth degrees. Consider, for instance, the following MALP rule: “good(X) ←prod @aver(nice(X), cheap(X)) with 0.8”, where aggregator @aver is typically defined as @aver(x1, x2) ≜ (x1 + x2)/2. Therefore, the rule specifies that X is good—with a truth degree of 0.8—whenever X be nice and cheap enough. Assuming that X is nice and cheap with, e.g., truth degrees n and c, respectively, then X is good with a truth degree of 0.8 + ((n + c)/2).

To solve a MALP goal, i.e., a query to the system plus a substitution (initially the empty substitution, denoted by id), a generalization of the classical modus ponens inference rule called admissible steps (→ AS), are systematically applied on atoms in a similar way to classical resolution steps in pure logic programming, thus returning a state composed by a computed substitution together with an expression where all atoms have been exploited. Next, this expression is interpreted under a given lattice by means of interpretive steps (→ IS), hence returning a pair (truth degree; substitution), called fuzzy computed answer (fca), which is the fuzzy counterpart of the classical notion of computed answer used in pure logic programming (see [14], [19] for details).

When specifying a MALP program, it might sometimes be difficult to assign weights—truth degrees—to program rules, as well as to determine the right connectives. In order to overcome this drawback, in [20] we have recently introduced a symbolic extension of MALP programs called symbolic multi-adjoint logic programming, sMALP in brief. Here, we can write rules containing symbolic weights and symbolic connectives, i.e., truth degrees and operators—denoted as “#label”—which are not defined on its associated multi-adjoint lattice. In order to evaluate these programs, we introduce a symbolic operational semantics that delays the evaluation of symbolic expressions. Therefore, a symbolic fuzzy computed answer—called sfca—could now include symbolic (unknown) truth values and connectives.

The approach is correct in the sense that using the symbolic semantics and then replacing the unknown values and connectives by concrete ones gives the same result as replacing these
values and connectives in the original sMALP program and, then, applying the concrete semantics on the resulting MALP program. sMALP programs can be used to tune a program w.r.t. a given set of test cases, thus easing what is considered the most difficult part of the process: the specification of the right weights and connectives for each rule.

The SAT/SMT-based tuning technique we introduce in this paper makes use of one of the most popular solvers nowadays, i.e. Z3 [8], [23], according to the following three main tasks:

1) Firstly, the lattice of truth degrees of the sMALP program to be tuned is translated to Z3 syntax.
2) Next, the set of sfca’s obtained after partially evaluating the test cases introduced a priori by the user are also automatically coded as a Z3 formula.
3) Finally, the Z3 solver is launched in order to minimize the deviation of the solutions w.r.t. the set of test cases while checking the satisfiability of such formula.

The structure of this paper is as follows. In Section II we recast from [20], [22] the original tuning method we have recently introduced in the FLOPER system. While Section III explains the new SAT/SMT-based reinforcement applied on such technique, Section IV illustrates the benefits of using the empowered tuning process focusing on two particular domains (circuit validation and linear regression). Finally, in Section V we show our conclusions and provide some lines for future research.

II. Tuning Symbolic Fuzzy Logic Programs

Let’s now summarize the automated technique for tuning multi-adjoint logic programs using sMALP programs that we initially presented in [20] and next implemented in our online tool in [22]. We firstly introduce the running example of this section.

Example 1: At the bottom of Figure 1, we specify the lattice ([0, 1], ≤) loaded by default in our freely accessible tool. In general, lattices are described by means of a set of PROLOG clauses where the definition of the following predicates is mandatory: member/1 and members/1, that identify the elements of the lattice; bot/1 and top/1 stand for the infimum and supremum elements of the lattice; and finally leq/2, that implements the ordering relation. Connectives are defined as predicates whose meaning is given by a number of clauses. The name of a predicate has the form and_label, or_label or agr_label depending on whether it implements a conjunction, a disjunction or an aggregator, where label is an identifier of that particular connective. The arity of the predicate is n + 1, where n is the arity of the connective that it implements, so its last parameter is a variable to be unified with the truth value resulting of its evaluation.

Moreover, at the top of Figure 1, we can see a sMALP program loaded in the system. Here, we consider a travel guide that offers information about three restaurants, named attica, celler and gaggan, where each one of them is featured by three factors: the restaurant services, the quality of its food, and the price, denoted by predicates service, food and price, respectively. We assume that all weights can be easily ob-
The following definition formalizes the algorithms followed to the program, which in our running example coincides to the so-called Gödel, Łukasiewicz and aggregator symbols \( fca \). Obviously, the larger the domain of values and connectives does not contain symbolic constants. For instance, in our running example we can introduce the following three test cases: 0.75 \( \rightarrow \) good_restaurant(attica), 0.8 \( \rightarrow \) good_restaurant(celler) and 0.9 \( \rightarrow \) good_restaurant(gaggan). Then, users simply need to click on the Tune program button for proceeding with the tuning process. The precision of the technique depends on the set of symbolic substitutions considered at tuning time. So, for assigning values to the symbolic constants, our tool takes into account all the truth values defined on a membership predicate (which in our case is declared as members([0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]) as well as the set of connectives defined in the lattice associated to the program, which in our running example coincides with the three conjunction and disjunction connectives based on the so-called Product, Gödel and Łukasiewicz logics. Obviously, the larger the domain of values and connectives is, the more precise the results are. For tuning an sMALP program, we have implemented three methods, which exhibit different run-times (but they obviously produce the same significant solutions and such improvements can be applied to both the basic and symbolic tuning methods.

### Algorithms for tuning sMALP programs

**Input:** an sMALP program \( \mathcal{P} \) and a number of (expected) test cases \( (Q_i, \langle v_i; \theta_i \rangle)^3 \), where \( Q_i \) is a goal and \( \langle v_i; \theta_i \rangle \) is its expected fca for \( i = 1, \ldots, k \).

**Output:** symbolic substitution \( \Theta \).

**Basic method:**
1. Consider a finite number of possible symbolic substitutions for \( \text{sym}(\mathcal{P}) \), say \( \Theta_1, \ldots, \Theta_n, n > 0 \).
2. For each \( j \in \{1, \ldots, n\} \), compute \( \langle Q_i; \theta_i \rangle \rightarrow^* \langle v_i,j; \theta_i \rangle \) in \( \mathcal{P}\Theta_j \), for \( i = 1, \ldots, k \).
3. Return the symbolic substitution \( \Theta_j \) that minimizes \( \sum_{i=1}^{k} \text{distance}(v_i, v_i,j) \).

**Symbolic method:**
1. For each test case \( (Q_i, \langle v_i; \theta_i \rangle) \), compute the sfca \( \langle Q_i,j; \theta_i \rangle \) of \( \langle Q_i, id \rangle \) in \( \mathcal{P} \).
2. Consider a finite number of possible symbolic substitutions for \( \text{sym}(\mathcal{P}) \), say \( \Theta_1, \ldots, \Theta_n, n > 0 \).
3. For each \( j \in \{1, \ldots, n\} \), compute \( \langle Q_i, \Theta_j \rangle \rightarrow^*_S \langle v_i,j, \theta_i \rangle \), for \( i = 1, \ldots, k \).
4. Return the symbolic substitution \( \Theta_j \) that minimizes \( \sum_{i=1}^{k} \text{distance}(v_i, v_i,j) \).

As seen in Figure 2, the system also reports the processing time required by each method and offers an option for applying the best symbolic substitution to the original sMALP program in order to show the final, tuned MALP program.

**Figure 2.** Screenshot of the online tool after completing a tuning process.

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3 For readability, we usually write the simplified form \( v_i \rightarrow Q_i \) by assuming that goal \( Q_i \) is a ground goal, that is, a goal without variables and thus, substitution \( \theta_i \) is the identity -empty- substitution.
Example 2: In our case, the best substitution is \( \Theta = \{ \#s_1/\prod, \#s_2/@\text{aver}, \#s_3/0.3 \} \) with a deviation of 0.05. By applying the symbolic substitution, we obtain a MALP program without symbolic constants, which can be executed w.r.t. a goal like \text{good\_restaurant}(X), thus obtaining the following three fca’s: \( <0.856, X/\text{attica}>, <0.943, X/\text{celler}> \) and \( <0.949, X/\text{gaggan}> \).

III. SAT/SMT FOR IMPROVING TUNING TECHNIQUES

Boolean satisfiability (SAT) is the problem of checking if a propositional logic formula can ever evaluate to true [17]. Satisfiability modulo theories (SMT) generalizes boolean satisfiability by adding equality reasoning, arithmetic, quantifiers, and other useful first-order theories. An SMT solver is a tool for deciding the satisfiability of formulas in these theories. In this work, we have integrated the FLOPER system with Z3 [23], an SMT solver from Microsoft Research, with the aim of improving the performance of the tuning techniques previously implemented for manipulating MALP programs. Z3 supports the SMT-LIB [8] standard, which includes a scripting language that defines a textual interface for SMT solvers. In this section we describe how the MALP system translates a tuning problem into an SMT-LIB script that Z3 can solve in order to find the best symbolic substitution for a symbolic fuzzy logic program, given a set of test cases.

As seen in Example 1, MALP associates to each fuzzy program a lattice of truth degrees. In order to perform the tuning process with Z3, it is necessary to translate the lattice associated with the program to SMT-LIB function declarations. Note that the predicate \text{members}/1 is no longer necessary since Z3 will search for the most appropriate values from the underlying domain.

After partially executing the goal of each test case in the FLOPER environment, the resulting sMALP expressions not containing atoms are also translated to SMT-LIB, storing in a variable \text{deviation!} the sum of the distances between the sfca’s and the expected truth degrees of the corresponding test cases, thus minimizing this variable in Z3, as follows:

```lisp
(assert (= deviation! (+ (lat!distance td_1 expr_1) (lat!distance td_2 expr_2) ... (lat!distance td_n expr_n))))
(minimize deviation!)
(check-sat!)
(get-model)
```

Example 3: The tuning process with Z3 shown in Figure 4 produces the following SMT-LIB script, where the symbolic connectives \( \#|s_1 \) and \( \#@s_2 \) are represented as strings \text{sym!or!2!s1} and \text{sym!agr!2!s2}, respectively, and the symbolic value \( \#s_3 \) is represented as a real number \text{sym!td!0!s3}:

```lisp
(define-fun lat!member ((x Real)) Bool
  (and (= x 0.0) (= x 1.0)))

(define-fun lat!distance ((x Real) (y Real)) Real
  (abs (- y x)))

(define-fun lat!or!luka2 ((x Real) (y Real)) Real
  (min (+ * x y)))

(define-fun lat!or!prod2 ((x Real) (y Real)) Real
  (max x y))

(define-fun lat!and!luka2 ((x Real) (y Real)) Real
  (max (+ (* x y) 1)))

(define-fun lat!and!prod2 ((x Real) (y Real)) Real
  (min x y))

(define-fun lat!agrt!every1 ((x Real) Real
  (/ (+ x y) 2))

(define-fun lat!agrt!very1 ((x Real) Real
  (* x))

(define-fun dom!sym!and!land2 ((s String)) Bool
  (or (= s "and\_godel") (= s "and\_luka") (= s "and\_prod")))

(define-fun dom!sym!or!luka2 ((s String)) Bool
  (or (= s "or\_godel") (= s "or\_luka") (= s "or\_prod")))

(define-fun dom!sym!agrt!1 ((s String) Bool
  (or (= s "agr\_very")))

(define-fun dom!sym!agrt!2 ((s String)) Bool
  (or (= s "agr\_ever")))

(define-fun call!sym!and!land2 ((s String) (x Real) (y Real)) Real
  (ite (= s "and\_godel")
    (lat!and!godel2 x y)
    (ite (= s "and\_luka")
      (lat!and!luka2 x y)
      (lat!and!prod2 x y))))

(define-fun call!sym!or!luka2 ((s String) (x Real) (y Real)) Real
  (ite (= s "or\_godel")
    (lat!lor!godel2 x y)
    (ite (= s "or\_luka")
      (lat!lor!luka2 x y)
      (lat!lor!prod2 x y))))

(define-fun call!sym!agrt!1 ((s String) (x Real) Real
  (lat!agrt!very1 x))

(define-fun call!sym!agrt!2 ((s String) (x Real) Real
  (lat!agrt!every1 x))
```

Figure 3. Lattice \([0,1], \leq\) in SMT-LIB.
The Z3 system produces the following output, whose model is interpreted by the FLOPER environment in order to generate the symbolic substitution \( \Theta = \{\#s1/\text{prod}, \#s3/0.2428571428, \#s2/\text{aver}\} \) with a deviation of 0.044:

\[
\text{sat} \quad \text{(model)}
\]

\[
\text{(define-fun sym!or!2!s1 () String "or_prod")}
\]

\[
\text{(define-fun sym!deviation! () Real 0.044)}
\]

\[
\text{(define-fun sym!agr!2!s2 () String "agr_aver")}
\]

IV. EXAMPLES

In this section we illustrate the use and benefits achieved on the afore mentioned SAT/SMT-based symbolic tuning technique by focusing on two well-known scenarios (circuit validation and linear regression), but it is important to indicate that the technique can be applied to any real world software application coded in MALP with the FLOPER environment.

A. Combinational equivalence checking

The problem of checking the equivalence of combinational circuits is an essential circuit design task. The simplest form of equivalence checking addresses combinational circuits. Let \( C_A \) and \( C_B \) denote two combinational circuits, both with inputs \( x_1, \ldots, x_n \) and both with \( m \) outputs, \( y_1, \ldots, y_m \) and \( C_B \) with outputs \( w_1, \ldots, w_m \). The function implemented by each one of the two circuits is defined as follows: \( f_A : \{0,1\}^n \rightarrow \{0,1\}^m \) and \( f_B : \{0,1\}^n \rightarrow \{0,1\}^m \).

Let \( x \in \{0,1\}^n \) and define \( f_A(x) = (f_{A,1}(x), \ldots, f_{A,m}(x)) \) and \( f_B(x) = (f_{B,1}(x), \ldots, f_{B,m}(x)) \). The two circuits are not equivalent if the following condition holds:

\[
\exists x \in \{0,1\}^n \quad \exists 1 \leq i \leq m \quad f_{A,i}(x) \neq f_{B,i}(x)
\]

which can be represented as the following satisfiability problem [18]:

\[
\bigwedge_{i=1}^{n} (f_{A,i}(x) \oplus f_{B,i}(x)) = 1
\]

The resulting satisfiability problem is illustrated in Figure 5, and it is referred to as a miter [11]. From these results, it is easy to encode in CNF the problem of verifying the equivalence of two combinational circuits and, therefore, it is capable of being tuned as a fuzzy logic program.

We will encode the combinational circuits in MALP as predicates of arity 2, where the first argument is a list containing the inputs, and the second one is a list containing the outputs. To check the equivalence between two circuits, we will implement the miter represented in Figure 5 as a predicate miter/3 that takes two circuits (two atoms representing the name of the predicate of each circuit) and an input list, and checks if any of the outputs of both circuits is different for the input provided. miter/3 is evaluated with a truth degree of false when all outputs are equal for the given input, or with a truth degree of true in any other case.

```prolog
zip_xor([],[],[]).
zip_xor([X|Xs],[Y|Ys],[@xor(X,Y)|Zs]) :-
    zip_xor(Xs,Ys,Zs).
fold_or([],false).
fold_or([X|Xs],',[\('(X,Ys) :-
    fold_or(Xs,Ys).
miter(Ca,Cb,Xs) :-
call(Ca,Xs,Ys),
call(Cb,Xs,Ws),
zip_xor(Ws,Xs,XOR),
fold_or(XOR,OR),
OR.
```

In order to check the equivalence between two circuits \( C_A \) and \( C_B \) with \( n \) inputs using the tuning technique, the system only needs a test case with the following shape:

\[
\text{true} \rightarrow \text{miter}(C_A, C_B, [\#x1, \ldots, \#xm]).
\]
This test case tells the system that we want to find a combination of inputs \((x_1,\ldots,x_m)\) for \(C_A\) and \(C_B\) such that the output of the miter/3 predicate is true, that is, such that some of the outputs \(y_i\) of both circuits are different. If the circuits are equivalent, the system will not be able to find such assignment of values, and it will return an arbitrary symbolic substitution with a deviation of 1.0. Otherwise, it will return a symbolic substitution for which one of the outputs is different in both circuits, with a deviation of 0.0.

**Example 4:** Let \(a/2\) and \(b/2\) be the MALP predicates representing the combinational circuits shown in Figure 6.

\[
a([X0,X1,X2,X3], [Y0,Y1]) :-
\text{truth}_degree(
\text{not}(\text{not}(\text{not}(\text{not}(X0, \text{not}(\text{not}(\text{not}(%(X1,X2), \text{not}(%(X2,X3) ))))}, Y0),
\text{truth}_degree(\text{not}(\text{not}(\text{not}(X2,X3)), Y1).
\]

\[
b([X0,X1,X2,X3], [Y0,Y1]) :-
\text{truth}_degree(
\text{not}(\text{not}(\text{not}(X0, '1')(%(X1,X2), \text{not}(%(X2,X3))))), Y0),
\text{truth}_degree(\text{not}(\text{not}(\text{not}(\text{not}(X2,X3), X3)), Y1).
\]

We run the tuning process with this program and with the following test case:

\[
\text{true} \rightarrow \text{miter}(a, b, \text{[[X0, , X1, , X2, , X3]]}.
\]

The MALP environment gives the following output, which demonstrates that the circuits are not equivalent, since the deviation of the best symbolic substitution found is 1.0:

\[
\{\text{X0/false, X1/false, X2/false, X3/false}\}
\text{deviation: 1.0}
\]

**Example 5:** Let us now introduce a third combinational circuit, not equivalent to the previous two ones, represented in MALP by the following predicate \(c/2\):

\[
c([X0,X1,X2,X3], [Y0,Y1]) :-
\text{truth}_degree(\text{not}(\text{not}(\text{not}(X0, X1))), Y0),
\text{truth}_degree(\text{not}(\text{not}(\text{not}(X2,X3)), Y1).
\]

We run the tuning process to check the equivalence of the circuits \(a/2\) and \(c/2\). In this case, the system finds a combination of the inputs, \((0,0,0,1)\), for which both circuits produce different outputs, since the deviation is 0.0:

\[
\{\text{X0/false, X1/false, X2/false, X3/true}\}
\text{deviation: 0.0}
\]

For this combination, circuit \(a/2\) produces the outputs \((1,1)\), while circuit \(c/2\) produces the outputs \((1,0)\).

Table I summarizes the averages of execution time in milliseconds\(^4\) associated to the tuning algorithms when checking the equivalence of several combinational circuits by varying the number of inputs. It is worth mentioning that, apart from the fact that the tuning method based on Z3 is always better than the one not using it, as wanted, it also slightly increases its execution time when the number of inputs grow, while the MALP method (not using Z3) largely reduces its performance in an exponential way w.r.t. the number of input signals.

**B. Linear regression**

Linear regression is a linear approach for modelling the relationship between a dependent variable and one or more explanatory variables. Data consist of \(n\) observations on a dependent variable \(Y\) and \(p\) explanatory variables, \(X_1,\ldots,X_p\). The relationship between \(Y\) an \(X_1,\ldots,X_p\) is formulated as a linear model:

\[
Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p + \varepsilon
\]

\(^4\)Each cell contains the average after 100 runs using a desktop computer equipped with an AMD Opteron™ processor @ 1593 MHz and 2.00 GB RAM.
where $\beta_0, \beta_1, \ldots, \beta_p$ are constants referred to as the regression coefficients and $\varepsilon$ is a random disturbance [12].

We will express the regression model as a MALP program with a single rule that defines Equation 1 by combining the connectives $\text{add}(x, y) = x + y$ and $\text{prod}(x, y) = xy$ of the real lattice, where parameters $\beta_i$ are symbolic constants:

$$y(X_1, \ldots, X_p) \leftarrow \#b0 \mid \text{add}(\#b1 \& \text{prod} X1) \mid \text{add} \ldots \mid \text{add}(\#bp \& \text{prod} Xp).$$

This predicate takes $p$ explanatory variables as inputs and is evaluated with a truth degree which is a linear combination of these inputs. To find the parameters $\beta_i$ that best fit the data, we will tune this program by entering a test case for each sample in the data set, where the value of the dependent variable will be the expected truth degree of the test case.

**Example 6:** A popular, classical study described in [24], measured the frequency (the number of wing vibrations per second) of chirps made by a ground cricket, at various ground temperatures. The resulting data is shown in Table II, where the first column represents the temperature in Celsius scale, and the second one represents the noise in decibels. We want to analyse the relationship between the temperature and the noise level generated by the crickets. Since the data set has only one explanatory variable, i.e. temperature, the program will have two symbolic constants:

$$\text{chirps}(\text{Temp}) \leftarrow \#b0 \mid \text{add}(\#b1 \& \text{prod} \text{Temp}).$$

Each sample of the data set shown in Table II becomes a test case, thus obtaining the following set of test cases:

<table>
<thead>
<tr>
<th>Temp</th>
<th>Noise</th>
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<tbody>
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<table>
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<td>88.6</td>
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Regression models by varying the number of explanatory variables and the number of test cases.

### Table II

**Data set of Example 6.**

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<tbody>
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V. **Conclusions and Future Work**

In this paper we have collected from [20], [22] our initial formulation of a symbolic extension of fuzzy logic programs belonging to the so-called multi-adjoint logic programming approach, as well as some tuning techniques useful for tailoring $\text{sMALP}$ programs (also providing an online tool freely available via URL [http://dectau.uclm.es/malp/sandbox](http://dectau.uclm.es/malp/sandbox)). Next, inspired by our previous works [10] and [6], where we proposed two techniques for evaluating propositional fuzzy formulae with the $\text{FLOPER}$ system in an alternative way than fuzzy SAT/SMT methods, we have focused on improving the former tuning techniques with the use of powerful SAT/SMT solvers.

Even when embedding the use of such solvers inside the core of our initial symbolic tuning algorithm has required some translation procedures for adapting the syntax of several MALP components (lattice of truth degrees, symbolic fuzzy computed answers, etc.) to the Z3 notation, the resulting performance of the new method has been highly improved.
Our benchmarks have revealed significant advantages in several aspects and, what is more important, although we have focused only on two well-known scenarios (circuit validation and linear regression) the technique is applicable to any other real world application coded in MALP with \texttt{FLOPER}.

We are nowadays facing more involved non-linear regression problems [13]. Other pending task for the near future consists in exploring the synergies between our tuning approach and machine learning strategies and, since in [15] we have designed a new fuzzy language extending MALP with similarity relations, we also plan to enrich the present implementation of the SAT/SMT-based symbolic tuning technique to cope with \texttt{FASIL\_MC} programs managing similarities.

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\section*{References}


