Interval-valued Intuitionistic Fuzzy TOPSIS method for Supplier Selection Problem

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***Abstract*—In this paper, Interval-valued intuitionistic fuzzy set (IVIFS) is exploited to propose a generalization of fuzzy TOPSIS method. We have termed the proposed TOPSIS method as Interval-valued intuitionistic fuzzy TOPSIS method (IVIFS-TOPSIS). Here, the IVIFS-TOPSIS handles the Supplier Selection problem in which linguistic variables based criteria description is given. The best supplier obtained by IVIFS-TOPSIS is in concurrence with the other well-known fuzzy TOPSIS methods. It is possible to use IVIFS-TOPSIS over other types of linguistic variable multi-criteria decision making (MCDM) problems.**

***Index Terms*—Interval-valued intuitionistic fuzzy sets; Multi-criteria Decision Making; Supplier selection; correlation coefficient; TOPSIS**

I. Introduction

Fuzzy set [1] proposed by Zadeh is an extension of classical set. Over the decades, different variants of fuzzy sets have been proposed in literature. Intuitionistic fuzzy sets (IFSs) [2] introduced by Atanassov, is one of the popular extensions of fuzzy set (see [54]- [57]). The intuitionistic fuzzy set consists of three components, namely membership, non-membership and hesitancy, respectively. If hesitancy is zero in the universe of discourse, then IFS reduces to fuzzy set. It is proven (see [3], [4]) that IFS captures vague/uncertain and incomplete information in a better fashion in comparison to fuzzy set. The interval-valued intuitionistic fuzzy set (IVIFS) [5]−[6] was introduced by Atanassov to generalize the IFS. The correlation and the correlation coefficient proposed by Bustince et al. [7], interval-valued intuitionistic fuzzy averaging (IIFWA) operator, similarity measures and distance measures introduced by Xu et al. [8], [9], [10] and topological study [11] of IVIFS enriched its theoretical aspects. It is useful to study [12], [13], [14] to get an idea of other types of correlation and correlation coefficients of IVIFSs. The IVIFS has applications in the field of decision making [15], [46], [47]; pattern recognition [16], [48], [49]; medical diagnosis [17], [50], [51]; operator theory [18], [52], [53] etc.

The decision making gets complicated while handling multi-criteria decision making (MCDM) problems (See [19], [20]). Supplier selection is a well-known MCDM research problem. Owing to the complexities involved in the Supplier selection problem (SSP), here decisions are taken by a group of experts in place of individuals. The main task of the SSP is to select the best supplier based on collective choice of experts from the given suppliers with each supplier satisfying certain criteria in its own manner. The prominent criteria used in SSP are cost, quality, delivery performance, reliability etc. (See [24]−[28]). Various types of solution approaches are used for dealing the SSP (See [29]−[34]).

The Technique for order preference by similarity to ideal solution (TOPSIS) developed by Hwang and Yoon [45] is an acclaimed MCDM problem solving method. It evaluates the performance of alternatives through the similarity with the support of ideal solution. This technique in [33], suggests the best alternative is the one which is nearest to the positive-ideal solution and farthest from the negative-ideal solution. Positive-ideal solution comprises of all the best values and Negative-ideal comprises of all the worst values among all the given alternatives. In MCDM problems, normally TOPSIS method is applied, but because of the its inefficiency to counter the uncertainty incorporated in experts perception, it receives criticism too. In classical TOPSIS, assessment of experts are assigned a numerical value [34] although in real world, this is seldom the case. Perception of experts are entirely uncertain and hence, instead of assigning numerical values to the assessment of experts, linguistic estimations are used. In order to deal with linguistic estimators, fuzzy TOPSIS methods were introduced. The IVIFS based methods are also used for dealing MCDM problems (See [35], [36], [37]).

It is usually found that membership and non-membership values of IFS are estimated without the help of precisely driven membership and non-membership functions. For dealing IVIFS, it is sufficient to estimate membership and non-membership intervals. Hence, derivation of IVIFS is simpler than that of IFS. Since, IVIFS and IFS are theoretically same, therefore IVIFS is not studied that much. In the paper, we have a mathematical procedure to construct IVIFS from IFS. We have also given a way through which unique weights of IFS can be transformed into weight intervals. These weight intervals are used in the proposal of Interval-valued intuitionistic fuzzy TOPSIS (IVIFS-TOPSIS) method. The IVIFS-TOPSIS is used to deal with the SSP. The

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results obtained by IVIFS-TOPSIS is the same as distance measure based intuitionistic fuzzy set TOPSIS (IFS-TOPSIS) method, but is better than correlation based IFS-TOPSIS methods.

The rest of the paper is organised as follows: Section II recalls basic concepts about IVIFSs. Section III provides equivalence between IFS and IVIFS. It also introduces interval-valued intuitionistic fuzzy TOPSIS method with the help of distance measure. In Section IV, proposed method is demonstrated with the help of supplier selection problem. Finally, Section V contains conclusion of the study.

II. Basic Concepts

Definition 2.1: **Intuitionistic Fuzzy Set** [2]
An intuitionistic fuzzy set (IFS) A in X is of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}. \tag{1}$$

Here $\mu_A : X \to$ [0,1] and $\nu_A : X \to$ [0,1] simultaneously assigns the degree of membership value and degree of non-membership value respectively to each element $x \in X$ with respect to A if

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \tag{2}$$

The third parameter of IFS, known as hesitation degree is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Definition 2.2: **Algebra of IFSs** [38]
Suppose $x, y \in X$ are IFSs such that, $x = (\mu_x, \nu_x)$ and $y = (\mu_y, \nu_y)$ then, we have the following:

(i) $x \oplus y = (\mu_x + \mu_y - \mu_x\mu_y, \nu_x\nu_y)$

(ii) $x \ominus y = \begin{cases} \left(\frac{\mu_x-\mu_y}{1-\mu_y}, \frac{\nu_x}{\nu_y}\right), & \mu_x \geq \mu_y, \\ & \nu_x \leq \nu_y, \\ & \nu_y \geq 0, \\ & \nu_x\pi_2 \leq \nu_y\pi_1 \\ (0,1), & otherwise \end{cases}$

(iii) $x \otimes y = (\mu_x\mu_y, \nu_x + \nu_y - \nu_x\nu_y)$

(iv) $\lambda x = (1 - (1-\mu_x)^\lambda, \nu_x^\lambda)$

(v) $x^\lambda = (\mu_x^\lambda, 1 - (1-\nu_x)^\lambda)$

Definition 2.3: **Interval-valued Intuitionistic Fuzzy Sets** [5]
An Interval-valued intuitionistic fuzzy set (IVIFS) A in X is of the form

$$A = \{\langle x, M_A(x), N_A(x)\rangle | x \in X\}. \tag{3}$$

where, $M_A : X \to$ [0,1] and $N_A : X \to$ [0,1] with the condition

$$0 \leq \text{Sup } M_A(x) + \text{Sup } N_A(x) \leq 1. \tag{4}$$

where M_A, N_A are intervals with $M_A = [\mu_A^L, \mu_A^U]$ and $N_A = [\nu_A^L, \nu_A^U]$ stands for the intervals of degree of membership and degree of non-membership respectively; μ_A^L, μ_A^U denotes the lower and upper membership values respectively and ν_A^L, ν_A^U denotes the lower and upper non-membership values respectively, of the element $x \in X$. The third parameter of IVIFS, known as hesitation degree is defined as:
$\pi_A(x) = [1 - \mu_A^U(x) - \nu_A^U(x), 1 - \mu_A^L(x) - \nu_A^L(x)]$.
For any $x \in X$, if we get $\mu_A^L = \mu_A^U$ and $\nu_A^L = \nu_A^U$, then A can be easily converted into IFS.

Definition 2.4: **Algebra of Intervals**
Suppose x, y are Intervals. Let $x = [a, b]$, $y = [c, d]$ we define the following:

(i) $x + y = [a + c, b + d]$

(ii) $x - y = [a - c, b - d]$

(iii) $x \times y = [ac, bd]$

(iv) $\lambda x = [\lambda a, \lambda b]$

Definition 2.5: **Algebra of IVIFSs** [39]
Suppose $x, y, z \in X$ are IVIFSs. Let $x = \big([a_1, b_1], [c_1, d_1]\big)$, $y = \big([a_2, b_2], [c_2, d_2]\big)$ and $z = \big([a, b], [c, d]\big)$.
with $*$ = min and $\diamond$ = max, we have the following:

(i) $x \oplus y = \big([a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2], [c_1c_2, d_1d_2]\big)$

(ii) $x \ominus y$=$\big\{[*(a_1, a_2), *(b_1, b_2)], [\diamond(c_1, c_2), \diamond(d_1, d_2)]\big\}$

(iii) $x \otimes y = \big([a_1a_2, b_1b_2], [c_1 + c_2 - c_1c_2, d_1 + d_2 - d_1d_2]\big)$

(iv) $\lambda z = \big([1 - (1-a)^\lambda, 1 - (1-b)^\lambda], [c^\lambda, d^\lambda]\big)$

(v) $z^\lambda = \big([a^\lambda, b^\lambda)], [1 - (1-c)^\lambda, 1 - (1-d)^\lambda]\big)$

Definition 2.6: **Distance Measure between IVIFSs** [40]
Let $D : IVIFS(X) \times IVIFS(X) \to$ [0,1]. $D(A, B)$ measures distance between IVIFSs A and B, if it satisfies the following properties:

1) $0 \leq D(A, B) \leq 1$.
2) $D(A, B)$ = 0 if and only if A = B.
3) $D(A, B)$ = $D(B, A)$.
4) If $A \subseteq B \subseteq C$, where A,B,$C \in IVIFSs(X)$, then $D(A, C) \geq D(A, B)$ and $D(A, C) \geq D(B, C)$.

For IVIFSs A and B, relation between distance measure $D(A, B)$ and their respective similarity measure $S(A, B)$ is given in [41] as:

$$S(A, B) = 1 - D(A, B) \tag{5}$$

The normalized Euclidean distance [45] for IVIFS A and B is as follows:

$$D(A, B) = \Bigg(\frac{1}{4n}\sum_{i=1}^{n}\big[\big(\mu_A^L - \mu_B^L\big)^2 + \big(\mu_A^U - \mu_B^U\big)^2 + \big(\nu_A^L - \nu_B^L\big)^2 + \big(\nu_A^U - \nu_B^U\big)^2\big]\Bigg)^{\frac{1}{2}} \tag{6}$$

III. Main Results:Proposed Interval-valued intuitionistic fuzzy TOPSIS method

A. Relationship between Intuitionistic fuzzy sets and Interval-valued intuitionistic fuzzy sets

It is a well known result that every IVIFS is an IFS [7] . For the consistency of our study, we have given mathematical elobaration (See Corollary 3.1 and Theorem 3.2) .

Theorem 3.1: Let $I' = [\mu^L, \mu^U]$ and $I'' = [\nu^L, \nu^U]$ are the two weight intervals satisfying the following conditions:

$$\mu^L + \nu^L \leq 1, \tag{7}$$
$$\mu^U + \nu^U \leq 1. \tag{8}$$

Here universe of discourse contains IFS. So its each element has membership interval $M' = [\mu, 1-\nu]$ and non-membership interval $N' = [\nu, 1-\mu]$.
Then $T' = I'M' + I''N'$ is less than equal to 1.

Proof 3.1: Using Def.(2.4) we have:

$$I'M' = [\mu^L\mu \ , \ \mu^U(1-\nu)] \tag{9}$$
$$I''N' = [\nu^L\nu \ , \ \nu^U(1-\mu)] \tag{10}$$

Then, by adding Eqn.(9) and Eqn.(10) with help of Def.(2.4)

$$I'M' + I''N' = [\mu^L\mu + \nu^L\nu \ , \ \mu^U(1-\nu) + \nu^U(1-\mu)]$$

$$\begin{aligned} &\text{As, } 0 \leq \mu^L, \nu^L \leq 1 \ , \ 0 \leq \mu \leq 1, \\ &0 \leq \nu \leq 1 \text{ and using Eqn.(7) we have,} \\ &0 \leq \mu^L\mu + \nu^L\nu \leq \mu^L + \nu^L \leq 1 \\ &\mu^L\mu + \nu^L\nu \leq 1. \end{aligned}$$

$$\begin{aligned} &\text{Similarly, } 0 \leq \mu^U, \nu^U \leq 1 \ , \ 0 \leq 1-\nu \leq 1, \\ &0 \leq 1-\mu \leq 1 \text{ and using Eqn.(8) we have,} \\ &0 \leq \mu^U(1-\nu) + \nu^U(1-\mu) \leq \mu^U + \nu^U \leq 1 \\ &\mu^U(1-\nu) + \nu^U(1-\mu) \leq 1. \end{aligned}$$

□

Corollary 3.1: The intervals $I'M'$, $I''N'$ satisfies

$0 \leq I'M' + I''N' \leq 1$. Hence there exists I''' such that
$I''' = [1 - \mu^U(1-\nu) - \nu^U(1-\mu), 1 - \mu^L\mu - \nu^L\nu]$.

Let us add I''' in $I'M' + I''N'$ such that
$I'M' + I''N' + I''' = [1, 1]$
Hence, $I''' = [1 - \mu^U(1-\nu) - \nu^U(1-\mu), 1 - \mu^L\mu - \nu^L\nu]$.

Remark: The Corollary 3.1 constructs IVIFS from IFS with its membership interval, non-membership interval and hesitency interval as $I'M', I''N'$ and I''' respectively.

Theorem 3.2: IVIFS can be converted into IFS.
Proof 3.2:

Let $\mu \in [\mu^L, \mu^U]$, $\nu \in [\nu^L, \nu^U]$ and $\pi \in [\pi^L, \pi^U]$

$$\text{Now, } \mu^L \leq \mu \leq \mu^U \tag{11}$$
$$\text{and } \nu^L \leq \nu \leq \nu^U. \tag{12}$$

Then, by adding Eqn.(11) and Eqn.(12),

$$\begin{aligned} \mu^L + \nu^L &\leq \mu + \nu \leq \mu^U + \nu^U \\ 1 - (\mu^U + \nu^U) &\leq 1 - (\mu + \nu) \\ &\leq 1 - (\mu^L + \nu^L) \end{aligned}$$

$$\begin{aligned} &\text{Since,} \quad \pi^L \leq \pi \leq \pi^U \text{ where } \pi = [1 - (\mu + \nu)] \\ &\text{Now,} \quad \mu + \nu + \pi = \mu + \nu + (1 - (\mu + \nu)) \\ &\qquad = 1. \end{aligned}$$

Hence, (μ, ν, π) is an IFS obtained from IVIFS.

□

Suppose $D_l = (\mu_l, \nu_l, \pi_l)$ be an IFS for grading the l^{th} expert, then the weight λ_l defined for Supplier Selection Problem (SSP) is given by Boran et. al in [42] as

$$\lambda_l = \frac{\left(\mu_l + \pi_l\left(\frac{\mu_l}{\mu_l+\nu_l}\right)\right)}{\sum_{l=1}^{p}\left(\mu_l + \pi_l\left(\frac{\mu_l}{\mu_l+\nu_l}\right)\right)} \quad \text{such that} \quad \sum_{l=1}^{p}\lambda_l = 1 \tag{13}$$

Now we are extending the weights used in [42], [43] for the SSP. The weight of l^{th} expert λ_l can be extended for IVIFS with the help of Theorem 3.3.

Theorem 3.3: Let λ_l be the weight assigned by l^{th} expert when their assessment is given in terms of IFSs. In other words, with universe of discourse containing IFSs.
If the universe of discourse conatins p IVIFSs, then λ_l is an interval of the form $K' = [min(\frac{\lambda_l^L}{\lambda}, \frac{\lambda_l^U}{\lambda'}), max(\frac{\lambda_l^L}{\lambda}, \frac{\lambda_l^U}{\lambda'})]$, where

$$\sum_{l=1}^{p}\lambda_l^L = \lambda \quad \text{and} \quad \sum_{l=1}^{p}\lambda_l^U = \lambda'$$

$$\lambda_l^L = \frac{\left(\mu_l^L + \left(\frac{\pi_l^L\mu_l^L}{\mu_l^U+\nu_l^U}\right)\right)}{\sum_{l=1}^{p}\left(\mu_l^U + \left(\frac{\pi_l^U\mu_l^U}{\mu_l^L+\nu_l^L}\right)\right)} \quad \text{such that} \quad \sum_{l=1}^{p}\lambda_l^L = 1 \tag{14}$$

$$\lambda_l^U = \frac{\left(\mu_l^U + \left(\frac{\pi_l^U\mu_l^U}{\mu_l^L+\nu_l^L}\right)\right)}{\sum_{l=1}^{p}\left(\mu_l^L + \left(\frac{\pi_l^L\mu_l^L}{\mu_l^U+\nu_l^U}\right)\right)} \quad \text{such that} \quad \sum_{l=1}^{p}\lambda_l^U = 1 \tag{15}$$

Proof 3.3: Let $[\mu_l^L, \mu_l^U], [\nu_l^L, \nu_l^U]$ and $[\pi_l^L, \pi_l^U]$ be the membership, non-membership and hesitancy intervals respectively of an IVIFS. As

$$\mu_l^L \leq \mu_l^U, \ \nu_l^L \leq \nu_l^U \text{ and } \pi_l^L \leq \pi_l^U \tag{16}$$
$$\text{So,} \quad \mu_l^L + \nu_l^L \leq \mu_l^U + \nu_l^U \tag{17}$$
$$\pi_l^L\mu_l^L \leq \pi_l^U\mu_l^U \tag{18}$$

Then from Eqn.(17) and Eqn.(18)

$$\frac{\pi_l^L\mu_l^L}{\pi_l^U + \nu_l^U} \leq \frac{\pi_l^U\mu_l^U}{\pi_l^L + \nu_l^L} \tag{19}$$

adding Eqn.(16) and Eqn.(19), we get

$$\mu_l^L + \frac{\pi_l^L\mu_l^L}{\pi_l^U + \nu_l^U} \leq \mu_l^U + \frac{\pi_l^U\mu_l^U}{\pi_l^L + \nu_l^L} \tag{20}$$

Since IVIFS is arbitrary, so Eqn.(20) holds for all $1 \leq l \leq p$, then,

$$\sum_{i=1}^{p}\big(\mu_l^L + \frac{\pi_l^L \mu_l^L}{\pi_l^U + \nu_l^U}\big) \leq \sum_{i=1}^{p}\big(\mu_l^U + \frac{\pi_l^U \mu_l^U}{\pi_l^L + \nu_l^L}\big)$$

i.e. $\lambda \leq \lambda'$.

□

B. Proposed IVIFS-TOPSIS Algorithm for Supplier Selection Problem

Suppose $A = \{A_1, A_2, \dots A_m\}$ is the set of the alternatives, $W = \{W_1, W_2, \dots W_n\}$ is the set of weights and $C = \{C_1, C_2, \dots C_n\}$ be the set of the criteria. Then the steps of Interval-valued Intuitionistic fuzzy TOPSIS method [44] are described underneath as:

$\underline{Step 1.}$ Allocate weights to the experts as per their assement. From the expert panel of p member group, experts find it relatively easier to express their assessment in linguistic terms and IVIFNs are used to explain these linguistic terms So, the weight of l^{th} expert, can be computed using Theorem (3.3).

$\underline{Step 2.}$ Construct aggregated interval-valued intuitionistic fuzzy decision matrix, on the basis of experts assessment. Let $X^{(l)}$=$(x_{ij}^{(l)})_{m\times n}$ be an interval-valued intuitionistic fuzzy decision matrix of each expert. Here, $\lambda = \{\lambda_1, \lambda_2, \dots \lambda_p\}$, denotes the collection of attribute weights given to p experts, where $\sum_{l=1}^{p}\lambda_l = 1, \lambda_l \in [0,1]$. The A_i (set of alternatives) are expressed in linguistic terms by the experts and to provide ratings to A_i, IIFWA [8] operator is used and then ratings are used to construct the interval-valued intutionistic fuzzy decision matrix, X=$(x_{ij})_{m\times n}$, where x_{ij} =$\big([\mu_{ij}^L, \mu_{ij}^U], [\nu_{ij}^L, \nu_{ij}^U)]\big)$ $(i = 1, 2, \dots m; j = 1, 2, \dots n)$.

$$\begin{aligned} x_{ij} &= IIFWA_\lambda(x_{ij}^{(1)}, x_{ij}^{(2)}, \dots x_{ij}^{(p)}) \\ &= \lambda_1 x_{ij}^{(1)} \oplus \lambda_2 x_{ij}^{(2)} \oplus \lambda_3 x_{ij}^{(3)} \oplus \dots\dots \oplus \lambda_p x_{ij}^{(p)} \\ &= \Bigg[\Bigg(1 - \prod_{l=1}^{p}(1-\mu_{ij}^{L(l)})^{\lambda_l}, 1 - \prod_{l=1}^{p}(1-\mu_{ij}^{U(l)})^{\lambda_l}\Bigg), \\ &\qquad \Bigg(\prod_{l=1}^{p}(\nu_{ij}^{L(l)})^{\lambda_l}, \prod_{l=1}^{p}(\nu_{ij}^{U(l)})^{\lambda_l}\Bigg)\Bigg] \end{aligned} \tag{21}$$

The aggregated IVIF decision matrix is determined as:

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1m} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2m} \\ x_{31} & x_{32} & x_{33} & \dots & x_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & x_{n3} & \dots & x_{nm} \end{bmatrix}$$

$\underline{Step 3.}$ Evaluate the weights of each criteria.
It is quite unlikely that all the criteria are equally important, hence a weighted decision matrix need to be obtained in order to blend the usefulness of every criteria, assessed by the experts. Let, l^{th} expert be assigned an IVIFN $w_j^l = \big([\mu_j^{L(l)}, \mu_j^{U(l)}], [\nu_j^{L(l)}, \nu_j^{U(l)})]\big)$ corresponding to each criteria c_j. Now, the weights of each criteria with the help of IIFWA operator [8] are obtained as follows:

$$\begin{aligned} w_j &= IIFWA_\lambda(w_j^{(1)}, w_j^{(2)}, \dots w_j^{(p)}) \\ &= \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \lambda_3 w_j^{(3)} \oplus \dots\dots \oplus \lambda_p w_j^{(p)} \\ &= \Bigg[\Bigg(1 - \prod_{l=1}^{p}(1-\mu_j^{L(l)})^{\lambda_l}, 1 - \prod_{l=1}^{p}(1-\mu_j^{U(l)})^{\lambda_l}\Bigg), \\ &\qquad \Bigg(\prod_{l=1}^{p}(\nu_j^{L(l)})^{\lambda_l}, \prod_{l=1}^{p}(\nu_j^{U(l)})^{\lambda_l}\Bigg)\Bigg] \end{aligned} \tag{22}$$

$W = [w_1, w_2, w_3, \dots w_j], w_j = \big([\mu_j^L, \mu_j^U], [\nu_j^L, \nu_j^U)]\big)$ $(j = 1, 2, \dots, n)$.

$\underline{Step 4.}$ Construct aggregated weighted interval-valued intuitionistic fuzzy decision matrix. Using criteria weights (W) and the aggregated interval-valued intuitionistic fuzzy decision matrix, aggregated weighted interval-valued intuitionistic fuzzy decision matrix is given by formula:

$$\begin{aligned} X \otimes W = \Big\{x, [\mu_{ij}^L \mu_j^L, \mu_{ij}^U \mu_j^U], [\nu_{ij}^L + \nu_j^L - \nu_{ij}^L \nu_j^L, \\ \nu_{ij}^U + \nu_j^U - \nu_{ij}^U \nu_j^U] \mid x \in X\Big\} \end{aligned} \tag{23}$$

$$\begin{aligned} \pi_{X\otimes W} = \big[1 - \mu_{ij}^U \mu_j^U - \nu_{ij}^U - \nu_j^U + \nu_{ij}^U \nu_j^U, \\ 1 - \mu_{ij}^L \mu_j^L - \nu_{ij}^L - \nu_j^L + \nu_{ij}^L \nu_j^L\big] \end{aligned}$$

Then, the aggregated weighted interval-valued intuitionistic fuzzy decision matrix is defined as:

$$X' = \begin{bmatrix} x'_{11} & x'_{12} & x'_{13} & \dots & x'_{1m} \\ x'_{21} & x'_{22} & x'_{23} & \dots & x'_{2m} \\ x'_{31} & x'_{32} & x'_{33} & \dots & x'_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x'_{n1} & x'_{n2} & x'_{n3} & \dots & x'_{nm} \end{bmatrix}$$

where, x'_{ij}=$\big([\mu_{ij}^{L'}, \mu_{ij}^{U'}], [\nu_{ij}^{L'}, \nu_{ij}^{U'})]\big)$ denotes the ij^{th} entry of aggregated weighted intuitionistic fuzzy decision matrix.

$\underline{Step 5.}$ Calculate interval-valued intuitionistic fuzzy positive-ideal solution(R^{p+}) and interval-valued intuitionistic fuzzy negative-ideal solution(R^{p-}). Suppose, J_1 as benefit criteria and J_2 as cost criteria and then R^{p+}, R^{p-} are:

$$R^{p+} = (r_1^{p+}, r_2^{p+}, \cdots, r_n^{p+}) \text{ where,}$$

$$r_j^{p+} = \big([\mu_j^{L(p+)}, \mu_j^{U(p+)}], [\nu_j^{L(p+)}, \nu_j^{U(p+)})]\big), \forall j = 1, 2, \dots n$$

$$\begin{aligned} \big[\mu_j^{L(p+)}, \mu_j^{U(p+)}\big] = [(max(\mu_j^L), max(\mu_j^U) \mid j \in J_1), \\ (min(\mu_j^L), min(\mu_j^U) \mid j \in J_2)] \end{aligned} \tag{24}$$

$$\begin{aligned} \big[\nu_j^{L(p+)}, \nu_j^{U(p+)}\big] = [(min(\nu_j^L), min(\nu_j^U) \mid j \in J_1), \\ (max(\nu_j^L), max(\nu_j^U) \mid j \in J_2)] \end{aligned} \tag{25}$$

$$R^{p-} = (r_1^{p-}, r_2^{p-}, \cdots, r_n^{p-}) \text{ where,}$$

$$r_j^{p-} = \big([\mu_j^{L(p-)}, \mu_j^{U(p-)}], [\nu_j^{L(p-)}, \nu_j^{U(p-)})]\big), \forall j = 1, 2, \dots n$$

$$\left[\mu_j^{L(p-)}, \mu_j^{U(p-)}\right] = [(min(\mu_j^L), min(\mu_j^U) \mid j \in J_1), \\ (max(\mu_j^L), max(\mu_j^U) \mid j \in J_2)] \quad (26)$$

$$\left[\nu_j^{L(p-)}, \nu_j^{U(p-)}\right] = [(max(\nu_j^L), max(\nu_j^U) \mid j \in J_1), \\ (min(\nu_j^L), min(\nu_j^U) \mid j \in J_2)] \quad (27)$$

$Step6$. Evaluation of seperation measures.
In this paper, to calculate separation measure amongst alternatives, normalized Euclidean distance [45] for IVIFS is used, . For every alternative, the separation measures C^+ and C^- of every alternative from (R^{p+}) and (R^{p-}) is calculated as:

$$C^+ = C(X', R^{p+}) = \left(\frac{1}{4n}\sum_{i,j=1}^{n}\left[(\mu_{ij}^{L'} - \mu_j^{L(p+)})^2 + (\mu_{ij}^{U'} - \mu_j^{U(p+)})^2 + (\nu_{ij}^{L'} - \nu_j^{L(p+)})^2 + (\nu_{ij}^{U'} - \nu_j^{U(p+)})^2\right]\right)^{\frac{1}{2}} \quad (28)$$

$$C^- = C(X', R^{p-}) = \left(\frac{1}{4n}\sum_{i,j=1}^{n}\left[(\mu_{ij}^{L'} - \mu_j^{L(p-)})^2 + (\mu_{ij}^{U'} - \mu_j^{U(p-)})^2 + (\nu_{ij}^{L'} - \nu_j^{L(p-)})^2 + (\nu_{ij}^{U'} - \nu_j^{U(p-)})^2\right]\right)^{\frac{1}{2}} \quad (29)$$

$Step7$. Collect the relative closeness coefficient (RCC_i) of every alternative.

$$RCC_i = \frac{C_i^-}{(C_i^+ + C_i^-)}, \quad \text{where } \ 0 \le RCC_i \le 1 \quad (30)$$

$Step8$. Rate the alternatives by obtaining the (RCC_i) in decreasing order. With the maximum value of the relative closeness (RCC_i), is the best alternative.

TABLE I
LINGUISTIC TERMS FOR RATING THE EXPERTS AND THE IMPORTANCE OF CRITERIA

Linguistic terms	IVIFNs
Very important	$([0.75, 0.90], [0.00, 0.10])$
Important	$([0.50, 0.75], [0.10, 0.20])$
Medium	$([0.35, 0.50], [0.20, 0.45])$
Unimportant	$([0.10, 0.35], [0.45, 0.60])$
Very unimportant	$([0.00, 0.10], [0.60, 0.90])$

TABLE II
THE IMPORTANCE OF EXPERTS AND THEIR WEIGHTS

Criteria	$Expert_1$	$Expert_2$	$Expert_3$
C_1	VI	VI	I
C_2	I	I	I
C_3	I	I	M
C_4	M	I	M

TABLE III
LINGUISTIC TERMS FOR RATING THE ALTERNATIVES

Linguistic terms	IVIFN
Extremely good(EG)/ Extremely high(EH)	$([1.00, 1.00], [0.00, 0.00])$
Very very good(VVG)/ Very very high(VVH)	$([0.90, 1.00], [0.00, 0.00])$
Very good(VG)/ Very high(VH)	$([0.80, 0.90], [0.00, 0.10])$
Good(G)/ High(H)	$([0.70, 0.80], [0.10, 0.20])$
Medium Good(MG)/ Medium High(MH)	$([0.60, 0.70], [0.20, 0.30])$
Fair(F)/ Medium (M)	$([0.50, 0.60], [0.30, 0.40])$
Medium Bad(MB)/ Medium Low(ML)	$([0.40, 0.50], [0.40, 0.50])$
Bad(B)/ Low(L)	$([0.25, 0.40], [0.50, 0.60])$
Very Bad(VB)/ Very Low(VL)	$([0.10, 0.25], [0.60, 0.75])$
Very Very Bad(VVB)/ Very Very Low(VVL)	$([0.00, 0.10], [0.75, 0.90])$

TABLE IV
THE RATING OF THE ALTERNATIVES

Criteria	Suppliers	Experts		
		$Expert_1$	$Expert_2$	$Expert_3$
C_1	A_1	G	VG	G
	A_2	MG	G	F
	A_3	VVG	VG	VG
	A_4	MG	G	G
	A_5	F	MG	MG
C_2	A_1	MG	G	MG
	A_2	F	MG	G
	A_3	VG	G	VG
	A_4	F	F	MG
	A_5	MB	F	F
C_3	A_1	VG	G	VG
	A_2	G	MG	MG
	A_3	VG	VG	G
	A_4	VG	G	G
	A_5	G	G	MG
C_4	A_1	H	H	H
	A_2	MH	M	MH
	A_3	VH	VH	H
	A_4	H	MH	MH
	A_5	M	MH	M

IV. NUMERICAL EXAMPLE [42]

An automobile company is desired to select the most suitable supplier for one of the elements in its manufacturing. After initial inspection, four suppliers A_1, A_2, A_3, and A_4 remains for evaluation and fifth supplier A_5 is allowed at the very last moment. A committee of three experts E_1, E_2 and E_3 is formed in order to select the most suitable supplier. Four benefit criteria are considered as:

1) Product quality (C_1)
2) Relationship closeness(C_2)
3) Delivery performance (C_3)
4) Price (C_4)

The hierarchical form of SSP is shown in Fig.(1). To explain SSP, the proposed IVIFS-TOPSIS method is tested and the computing steps are summarized below:

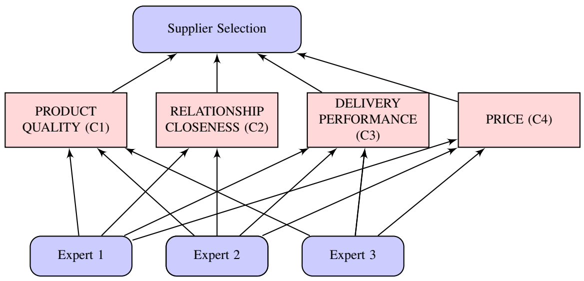

Fig. 1. hierarchical form of supplier selection

TABLE V
THE IMPORTANCE OF EXPERTS AND THEIR WEIGHTS

	$Expert_1$	$Expert_2$	$Expert_3$
Linguistic terms	Very important	Medium	Important
Upper weight	0.4333	0.2416	0.3598
Lower weight	0.3986	0.2384	0.3284

TABLE VI
THE POSITIVE IDEAL SOLUTION (R^{p+}) AND NEGATIVE IDEAL SOLUTION (R^{p-}) FOR EACH EXPERT

Criteria	R^{p+}	R^{p-}
C_1	$([0.585, 0.872], [0.000, 0.118])$	$([0.368, 0.587], [0.249, 0.408])$
C_2	$([0.374, 0.679], [0.108, 0.278])$	$([0.219, 0.437], [0.421, 0.535])$
C_3	$([0.339, 0.618], [0.136, 0.335])$	$([0.279, 0.527], [0.275, 0.433])$
C_4	$([0.195, 0.374], [0.412, 0.591])$	$([0.288, 0.518], [0.179, 0.435])$

TABLE VII
SEPARATION MEASURE AND RELATIVE CLOSENESS COEFFICIENT OF EACH ALTERNATIVE

Alternatives	C^+	C^-	RCC_i
$Supp_1$	0.3376	0.3588	0.5152
$Supp_2$	0.4905	0.262	0.3483
$Supp_3$	0.2380	0.624	0.7238
$Supp_4$	0.4536	0.2419	0.3478
$Supp_5$	0.6205	0.2392	0.2783

$\underline{Step1}$. Calculation of weights are performed according to the assesment of each of the experts (shown in Table V). The assessment of experts are based upon the importance associated to decision criteria (See Table II). To assess the criteria and experts, linguistic terms are used (See Table I). Weights are allocated to experts with the help of Theorem (3.3) and weights are shown in Table V.

$\underline{Step2}$. Formulation of the aggregated intuitionistic fuzzy decision matrix depends upon the ratings given in Table IV. To grade the supplier w.r.t each criteria, each of the experts has used linguistic terms which is given in Table III.
To figure out linguistic terms of Table III, we have used IVIFNs. Later, using Eqn.(21) the aggregated inerval-valued intuitionistic fuzzy decision matrix is constructed and is shown in Table VIII.

$\underline{Step3}$. Calculate weights of criteria according to the assessment of the experts. Corresponding to the linguistic terms, numeric values are given in Table I. Experts evaluation of each of the criteria (See Table II) and assessment of experts were merged with the help of Eqn.(22) to calculate each criteria weight.

$$W_{\{x_1,x_2,x_3,x_4\}} = \begin{bmatrix} (0.671, 0.872) & (0.000, 0.118) \\ (0.488, 0.762) & (0.108, 0.189) \\ (0.441, 0.694) & (0.136, 0.253) \\ (0.381, 0.587) & (0.179, 0.360) \end{bmatrix}^T$$

$\underline{Step4}$. Evaluation of the aggregated weighted intuitionistic fuzzy decision matrix (See Table IX) is carried out with the help of Eqn.(23), after the weights of the criteria and the grading of the supplier has been determined.

$\underline{Step5}$. Evaluate interval-valued intuitionistic fuzzy positive-ideal solution (R^{p+}) and interval-valued intuitionistic fuzzy negative-ideal solution (R^{p-}) with Eqn.(24-27). R^{p-} and R^{p-} are shown in Table VI.

$\underline{Step6}$. Calculate the positive and negative separation measure with the aid of normalized Euclidean distance (See Eqn.(28-29)) for each alternative (See Table VII).

$\underline{Step7}$. Calculate relative closeness coefficients (RCC_i) of each alternative. Table VII shows the RCC_i values.

$\underline{Step8}$. Rank the alternatives, acoording to the value of RCC_i given in Table VII, the $Supp_3$ is selected as the most suited supplier.

In Table X, we can figure out the best and worst supplier for supplier selection problem. Table X, provides the comparison amongst IVIFS-TOPSIS method, distance measure based IFS-TOPSIS [42] and correlation based TOPSIS method [43]. With the introduction of fifth supplier, IVIFS-TOPSIS method offers ranking similar to the ranking obtained in the distance measure based IFS-TOPSIS. The ranking obtained by the proposed method is better than the ranking obtained by correlation based TOPSIS method.

V. CONCLUSION

The ranking obtained for SSP by IVIFS-TOPSIS matches with the ranking given by distance measure based IFS-TOPSIS method. Thus, the equivalence of IVIFS with IFS

TABLE VIII
AGGREGATED INTERVAL-VALUED INTUITIONISTIC FUZZY DECISION MATRIX

Suppliers	C_1	C_2	C_3	C_4
$Supp_1$	$([0.716, 0.840], [0.000, 0.160])$	$([0.614, 0.739], [0.179, 0.261])$	$([0.767, 0.891], [0.000, 0.109])$	$([0.687, 0.811], [0.108, 0.189])$
$Supp_2$	$([0.585, 0.711], [0.205, 0.289])$	$([0.589, 0.718], [0.198, 0.282])$	$([0.632, 0.759], [0.160, 0.241])$	$([0.564, 0.692], [0.179, 0.308])$
$Supp_3$	$([0.872, 1.000], [0.000, 0.000])$	$([0.767, 0.891], [0.000, 0.109])$	$([0.758, 0.881], [0.000, 0.118])$	$([0.632, 0.759], [0.000, 0.118])$
$Supp_4$	$([0.649, 0.774], [0.143, 0.225])$	$([0.524, 0.651], [0.274, 0.349])$	$([0.734, 0.860], [0.000, 0.140])$	$([0.632, 0.759], [0.160, 0.241])$
$Supp_5$	$([0.549, 0.674], [0.249, 0.325])$	$([0.449, 0.573], [0.351, 0.472])$	$([0.656, 0.781], [0.136, 0.219])$	$([0.514, 0.638], [0.284, 0.361])$

TABLE IX
AGGREGATED WEIGHTED INTERVAL-VALUED INTUITIONISTIC FUZZY DECISION MATRIX

Suppliers	C_1	C_2	C_3	C_4
$Supp_1$	$([0.480, 0.732], [0.000, 0.259])$	$([0.300, 0.563], [0.268, 0.401])$	$([0.399, 0.618], [0.136, 0.335])$	$([0.261, 0.476], [0.268, 0.481])$
$Supp_2$	$([0.392, 0.619], [0.205, 0.373])$	$([0.287, 0.547], [0.285, 0.417])$	$([0.279, 0.527], [0.275, 0.433])$	$([0.215, 0.406], [0.326, 0.557])$
$Supp_3$	$([0.585, 0.872], [0.000, 0.118])$	$([0.374, 0.679], [0.108, 0.278])$	$([0.335, 0.612], [0.136, 0.342])$	$([0.288, 0.518], [0.179, 0.436])$
$Supp_4$	$([0.435, 0.675], [0.143, 0.317])$	$([0.256, 0.496], [0.352, 0.472])$	$([0.342, 0.597], [0.136, 0.358])$	$([0.240, 0.445], [0.311, 0.514])$
$Supp_5$	$([0.368, 0.587], [0.249, 0.406])$	$([0.219, 0.437], [0.421, 0.535])$	$([0.290, 0.542], [0.253, 0.417])$	$([0.196, 0.375], [0.412, 0.591])$

TABLE X
COMPARISON TABLE

Method	Ranking		Best supplier
	$Supp_1, Supp_2, Supp_3, Supp_4$	$Supp_1, Supp_2, Supp_3, Supp_4, Supp_5$	
Distance Measure Based IFS-TOPSIS (Boran et al. [42])	$Supp_3 > Supp_1 > Supp_4 > Supp_2$	$Supp_3 > Supp_1 > Supp_2 > Supp_4 > Supp_5$	$Supp_3$
Correlation based TOPSIS (Rinki et al. [43])	$Supp_3 > Supp_1 > Supp_4 > Supp_2$	$Supp_3 > Supp_1 > Supp_4 > Supp_2 > Supp_5$	$Supp_3$
Proposed IVIFS-TOPSIS	$Supp_3 > Supp_1 > Supp_4 > Supp_2$	$Supp_3 > Supp_1 > Supp_2 > Supp_4 > Supp_5$	$Supp_3$

is shown in the application domain too. The weights are assigned unique values, if the problem is dealt in intuitionistic fuzzy domain, but in the interval-valued intuitionistic fuzzy domain weight intervals are obtained. The change of weights by weight intervals do not changes the result of SSP. The distance measure based IVIFS-TOPSIS performs better than correlation coefficient based IFS-TOPSIS.
The construction procedure of the IVIFS using IFS given in this paper, can be easily used to extend many other ranking methods.

In future, we desire to extend IVIFS in other domains too and wish to put in IVIFS to other ranking techniques in MCDM problems.

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