Attitudinal Choquet Integral-Based Stochastic Multicriteria Acceptability Analysis

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***Abstract*—Preference learning is a subfield of machine learning. In the preference learning-fused decision analysis, the utility values of alternatives are inducted from human decision behavior. The commonly used utility model is the weighted summation. The model assumes the independency between criteria and the same attitude towards each performance value of alternatives, which is often unrealistic in practice. To solve this problem, this study presents an ACI-SMAA (Attitudinal Choquet Integral-based Stochastic Multicriteria Acceptability Analysis) model to consider the criteria interaction and attitudinal parameter of human decision behavior in decision analysis. The ACI-SMAA model is helpful to learn human preferences and analyze latent correlations. An application example about household energy selection is used to show the applicability and validity of the proposed model.**

Keywords—Preference learning, Attitudinal Choquet integral, Human decision behavior, Criteria interaction, Stochastic multicriteria acceptability analysis (SMAA).

I. INTRODUCTION

In a decision process with human behavior [1], multi-dimensional information on several criteria is fused in a decision-maker (DM)'s preference. Since it is difficult for DMs to evaluate each alternative on each criterion especially when the number of alternatives or criteria is large, the preferences given by DMs are usually treated as a whole in the way of weighted summation without considering the criteria interaction. In some situations, the utility values deduced by summing the weights of criteria and performances of alternatives on these criteria may produce the paradox with the DM's preference (see *Example 1* in Section II for details). To avoid such paradoxes, the Choquet integral [2] was used as an alternative of the utility model with criteria interaction. If no interaction exists, the Choquet integral is equivalent to the common weighted summation utility model.

Additionally, a DM may own diverse tolerance degrees on alternatives, which can be identified by the "andness" and "orness" measures [3]. In this regard, the attitudinal Choquet integral (ACI) was presented to include the weights of criteria and the attitudes of the DM simultaneously [4]. The ACI is a generalization of the traditional Choquet integral with different attitudes and thus is flexible in decision analysis.

In the preference learning process with the ACI model involving a DM's given preferences, $n+C_n^k$ parameters of the k-additive interaction and one attitudinal value need to be determined where n is the number of criteria. The k-additive interaction means the synergetic or antagonistic effect in the criteria subset whose criteria number is not greater than k [5]. For the simplest condition of the criteria interaction, *i.e.*, $k=2$, there are $n(n+1)/2+1$ parameters to be determined, which requires much cognitive effort from the DM to evolve in the decision analysis process. This is a challenging research issue. To reduce the cognitive burden of DMs, the stochastic multicriteria acceptability analysis (SMAA) [6] was proposed to implement sampling in the feasible solution space without much specific input information. This motivates us to integrate the ACI model with the SMAA model, named the ACI-SMAA model for short to learn the preferences of DMs. An example about household energy selection is provided to validate the model.

II. ATTITUDINAL CHOQUET INTEGRAL

This section introduces the main features of criteria interactions modelled by the ACI [4]. An example about the mobile phone selection starts this section.

Example 1. A DM evaluates three alternative mobile phones based on the performance information on two criteria with a scale [0,10]. The values are listed in Table I. The DM thinks that Meizu is priori to Meitu, whereas there is no difference between Meitu and Gree. That is to say, the DM ranks these three alternatives with the preference order $A_3 \succ A_1 \sim A_2$.

TABLE I. PERFORMANCES OF THREE MOBILE PHONES

Alternatives	**Criteria**	
	Technical feature c_1	***Brand choice*** c_2
Meitu A_1	10	0
Gree A_2	0	10
Meizu A_3	4	5

If we use the common weighted summation to calculate the overall utility values of these alternatives, the three alternatives' ranking can be modelled by

$$\underset{①}{4\times w_1+5\times w_2} > \underset{②}{10w_1} = \underset{③}{10w_2} \tag{1}$$

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where $w_1 + w_2 = 1$, and w_1 and w_2 represent the importance of the two criteria, respectively.

The weights of criteria consist an n-dimensional weight space, $R^n = \left\{ w_i \middle| \sum_{i=1}^{n} w_i = 1, w_i \geq 0 \right\}$. To visualize these inequalities in *Example 1*, Eq. (1) can be transformed into three inequalities: ①② $5w_2 > 6w_1$, ①③ $4w_1 > 5w_2$, and ②③ $w_1 = w_2$, which are shown in Fig. 1.

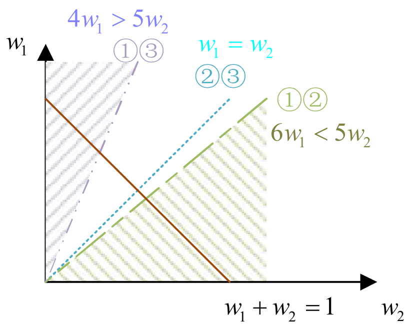

Fig. 1. Solutions of three inequalities in the two-dimensional space

It is not hard to find that Eq. (1) has no solution in the two-dimension weight scape because three lines intersect the normalization constraint line $w_1 + w_2 = 1$ with different points. The infeasibility of Eq. (1) raises the paradox about the DM's preferences. From the perspective of mathematical formulas, we can obtain the infeasible inequality $4w_1 > 5w_2 > 6w_1$ by combining ①② $5w_2 > 6w_1$ and ①③ $4w_1 > 5w_2$. This is contradictory as far as both w_1 and w_2 are non-negative and at least one is positive which is ensured by $w_1 + w_2 = 1$. Such a contradiction was resulted from the weighted summation which has an assumption that criteria are mutually independent. In other words, no interaction exists among criteria.

The Choquet integral [2] is another way to aggregate multiple dimensional information by assigning weights to all possible subsets of the criteria set. In *Example 1*, the subsets of the criteria set, $\{\phi\}$, $\{c_1\}$, $\{c_2\}$, $\{c_1, c_2\}$, should be assigned with the Choquet integral values, $v(\{\phi\}) = 0$, $v(\{c_1\}) = w_1$, $v(\{c_2\}) = w_2$, $v(\{c_1, c_2\}) = 1$, respectively. The mathematical formula of the Choquet integral based on the measured values of subsets is shown as:

$$ChI(x_1, x_2, \cdots, x_n) = \sum_{i=1}^{n} \left[x_{\sigma(i)} - x_{\sigma(i-1)} \right] v(s_i) \quad (2)$$

where $x_{\sigma(i)}$ is the i th largest value in the permutation $\{x_{\sigma(0)}, x_{\sigma(1)}, x_{\sigma(2)}, \cdots, x_{\sigma(n)}\}$ which guarantees $v(x_{\sigma(0)}) = 0$ and $x_{\sigma(0)} \leq x_{\sigma(1)} \leq x_{\sigma(2)} \leq \cdots \leq x_{\sigma(n)}$. s_i which is the subset of the criteria set contains the criteria index $\{i, \cdots, n\}$.

Example 2 (Continued to Example 1). Introducing the Choquet integral in Eq. (2), the DM's preferences can be realized by Eq. (3):

$$v(\{c_2\}) + 4v(\{c_1, c_2\}) > 10v(\{c_1\}) = 10v(\{c_2\}) \quad (3)$$

where $v(\{c_1\}) \geq 0$, $v(\{c_2\}) \geq 0$, and $v(\{c_1, c_2\}) = 1$. In this case, the solution to represent or model the DM's preferences exists when $v(\{c_1\}) = v(\{c_2\}) < 4v(\{c_1, c_2\})/9 = 4/9$. Thus, the importance of technical feature is the same as that of brand choice, and they are both less than or equal to 4/9.

Besides Eq. (2), the Choquet integral can be computed from its Mobius transformation. The Mobius transformation, originated in [7], defines a unique function $m: 2^C \to R$ to mapping the power set of the whole criteria set $\{C\}$ into the real value space R. The Mobius transformation function need to satisfy:

$$v(B) = \sum_{D \subseteq B} m(D) \quad (4)$$

where $m(\phi) = 0$, $\sum_{E \subseteq C} m(E) = 1$ and $\sum_{E \subseteq B} m(E \cup \{c_i\}) \geq 0$. For each criterion c_i, the Mobius value of c_i and another subset $B \subseteq C \setminus \{c_i\}$ should not be less than zero. Based on the Mobius transformation, the Choquet integral can be expressed in a linear form, shown as:

$$ChI_m(x_1, x_2, \cdots, x_n) = \sum_{D \subseteq C} m(D) \times \min_{i \in D} x_i \quad (5)$$

where ChI_m is the weighted summation of the minimal values of all subsets of the criteria set D from C.

The Choquet integral is not able to take into account the attitude of a DM, whereas the criteria interaction and different attitudes for extreme values may exist at the same time [4]. To tackle this issue, the attitudinal Choquet integral (ACI) of the n dimensional data with the attitudinal parameter λ is proposed as:

$$ACI_m(x_1, x_2, \cdots, x_n) = \log_\lambda \left(\sum_{D \subseteq C} m(D) \times \lambda^{\min_{i \in D} x_i} \right) \quad (6)$$

where $\lambda \in (0, +\infty]$ and $\lambda \neq 1$ according to the property of the logarithmic function.

Example 3 (Continued to Example 1). Introducing the solution of Eq. (3) as $v(\{c_1\}) = v(\{c_2\}) = 1/3$ and $v(\{c_1, c_2\}) = 1$ in *Example 1*, the utility values of the three alternatives based on the ACI with different attitudinal parameter λ values range from zero to ten without one can be obtained and visually shown in Fig. 2.

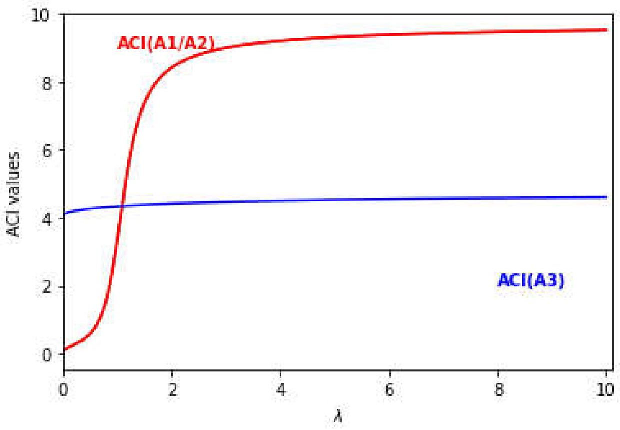

Fig. 2. Attitudinal Choquet integral values of the three alterantives

When λ approaches to zero, the ACI is similar as the minimal operator. The ACI values of alternatives A_1 and A_2 are close to their minimal values, 0, on criterions c_2 and c_1, respectively. The ACI value of alternative A_3 is near to the minimum, 4; when λ is close to one, the ACI degenerates into the Choquet integral; when the value of λ is large, the ACI acts as the maximal operator and ACI values of the three alternatives reach the maximal points. In a nutshell, the ACI is more flexible than the Choquet integral and takes the criteria interaction and DM's attitude into consideration at the same time. However, regarding the application of the ACI, the difficulty of assigning weights to each subset of the criteria set still exists. For n criteria, the importance of 2^n subsets of the criteria set should be determined, in which the importance of the whole criteria set $\{c_1, c_2, \cdots, c_n\}$ is one.

It is not easy to assign weights for all possible criteria subsets with interactions. That is to say, the importance degrees of some criteria subsets may not be available. The original Choquet integral or ACI does not have the ability to tackle the imperfect input information. To reduce human cognitive effort and consider criteria interaction simultaneously, the stochastic multicriteria acceptability analysis (SMAA) was introduced to generate recommended solutions with imperfect data (see [6] for details regarding the SMAA). The literature about the useful technique for stochastic analysis fused with the criteria interaction modelled by the Choquet integral is tabulated in Table II.

TABLE II. LITERATURE RELATED TO THE CHOQUET INEGRAL AND STOCHASTIC ANALYSIS

Preference models	Stochastic analysis techniques	References
Choquet integral	SMAA	[8] [9] [10]
Bipolar Choquet integral	SMAA	[11]
Level dependent Choquet integral	SMAA	[12]
ACI	SMAA with an attitudianl parameter	This study

The first column of Table II describes four preference models in human decision behavior. The Choquet integral [8-10] is the preliminary model, a special case of the ACI. The bipolar Choquet integral [11] describes the positive and negative interactions by the integral ranging from minus one to one. The level dependent Choquet integral [12] sets a threshold of alternative performances for generating interaction values. These three models failed to take DM's attitudes into consideration. This is the motivation of this study and Section III tries to fill in this research gap.

III. ACI-SMAA: ATTITUDINAL CHOQUET INTEGRAL-BASED STOCHASTIC MULTICRITERIA ACCEPTABILITY ANALYSIS

This section presents the ACI-SMAA model, a technique for preference learning in decision analysis. The criteria interaction and DM's attitudes are considered at the same time in the ACI-SMAA.

A. ACI-based human decision behavior modelling

DMs usually have two preference types, "preferential" and "indifferent", which can be summarized into a weak preference order, $\succeq$, on the alternative set in the multi-attribute utility theory [13, 14]. The weak preference relation between any two alternatives A_1 and A_2 is $A_1 \succeq A_2$, denoting that A_1 is at least as good as A_2. Such a weak preference can be divided into preferential relation, meaning $A_1 \succ A_2$ and not $A_2 \succeq A_1$, and indifferent relation, i.e., $A_1 \succ A_2$ and $A_2 \succeq A_1$. The preferential and indifferent relations can be represented as:

$$\begin{aligned} A_1 \succ A_2 &\Rightarrow U(A_1) > U(A_2) \\ A_1 = A_2 &\Rightarrow U(A_1) = U(A_2) \end{aligned} \tag{7}$$

where "$\succ$" and "$=$" denote "preferential" and "indifferent", respectively.

The overall utility value of an alternative can be calculated in two ways under different assumptions. In the first case, the relation among criteria is assumed independent and the utility function $U(A_i) = \sum_{j=1}^{n} w_j x_{ij}$ is additive, where w_j denotes the importance of criterion c_j. In the other case, criteria are assumed dependent as interactions among criteria exist and the utility function $U(A_i) = CI(A_i) = \sum_{D \subseteq C} m(D) \times \min_{i \in D} x_i$ is non-additive, where D is the possible subsets of all criteria and $m(D)$ represents the Mobius transformation value of the importance of criteria subset D. The Mobius value based on Eq. (4) can be obtained by

$$m(B) = \sum_{D \subseteq B} (-1)^{|B-D|} v(D) \tag{8}$$

where $v(D)$ is the importance of the criteria set D.

Considering the criteria interaction and the attitude of a DM, the preferential and indifferent relations of alternatives can be converted as:

$$\begin{aligned} A_1 \succ A_2 &\Rightarrow ACI(A_1) > ACI(A_2) \\ A_1 = A_2 &\Rightarrow ACI(A_1) = ACI(A_2) \end{aligned} \tag{9}$$

where the function ACI aggregates the performance values of alternatives on all criteria based on the interaction and diverse views on extreme performance values.

Example 4 (Continued to Example 1). The preference order $A_3 \succ A_1 \sim A_2$ in *Example 1* aggregated by the ACI can be measured as:

$$
\begin{aligned}
& ACI_m(A_1) = ACI_m(A_2) \\
& \Rightarrow \log_\lambda\left(m(\{c_1\})\lambda^{10} + m(\{c_2\})\lambda^0 + m(\{c_1,c_2\})\lambda^{\min\{0,10\}}\right) \\
& = \log_\lambda\left(m(\{c_1\})\lambda^{0} + m(\{c_2\})\lambda^{10} + m(\{c_1,c_2\})\lambda^{\min\{0,10\}}\right)
\end{aligned}
\quad (10)
$$

$$
\begin{aligned}
& ACI_m(A_3) > ACI_m(A_1) \\
& \Rightarrow \log_\lambda\left(m(\{c_1\})\lambda^{4} + m(\{c_2\})\lambda^5 + m(\{c_1,c_2\})\lambda^{\min\{4,5\}}\right) \\
& > \log_\lambda\left(m(\{c_1\})\lambda^{10} + m(\{c_2\})\lambda^{0} + m(\{c_1,c_2\})\lambda^{\min\{0,10\}}\right)
\end{aligned}
\quad (11)
$$

Equation (10) deduces $m(\{c_1\}) = m(\{c_2\})$. As $\lambda \geq 1$ and $m(\{c_1,c_2\}) + 2m(\{c_1\}) = 1$, in Eq. (11), the logarithmic function can be offset by the increasing property and thus Eq. (11) can be transformed into

$$
\begin{aligned}
& m(\{c_1\})\lambda^4 + m(\{c_2\})\lambda^5 + m(\{c_1,c_2\})\lambda^4 > \\
& m(\{c_1\})\lambda^{10} + m(\{c_2\})\lambda^0 + m(\{c_1,c_2\})\lambda^0
\end{aligned}
$$

$$
\Rightarrow \begin{aligned}
& m(\{c_1\})\lambda^4 + m(\{c_2\})\lambda^5 + m(\{c_1,c_2\})\lambda^4 \\
& > m(\{c_1\})\lambda^{10} + m(\{c_2\}) + m(\{c_1,c_2\})
\end{aligned}
$$

$$
\Rightarrow \begin{aligned}
& m(\{c_1\})(\lambda^4 + \lambda^5) + m(\{c_1,c_2\})\lambda^4 \\
& > m(\{c_1\})(\lambda^{10} + 1) + m(\{c_1,c_2\})
\end{aligned}
$$

$$
\Rightarrow m(\{c_1,c_2\})(\lambda^4 - 1) > m(\{c_1\})(\lambda^{10} + 1 - \lambda^4 - \lambda^5)
$$

$$
\Rightarrow (1 - 2m(\{c_1\}))(\lambda^4 - 1) > m(\{c_1\})(\lambda^{10} + 1 - \lambda^4 - \lambda^5)
$$

$$
\Rightarrow \lambda^4 - 1 > m(\{c_1\})(\lambda^{10} - \lambda^5 + \lambda^4 - 1)
$$

In addition, the constraint of Eq. (3), $v(\{c_1\}) = v(\{c_2\}) < 4v(\{c_1,c_2\})/9 = 4/9$, denotes that $m(\{c_1,c_2\}) \geq 1/9$. Then, Eq. (11) can also be solved from the perspective of the value range of $m(\{c_1,c_2\})$, shown as follows:

$$
\Rightarrow m(\{c_1,c_2\})(\lambda^4 - 1) > m(\{c_1\})(\lambda^{10} + 1 - \lambda^4 - \lambda^5)
$$

$$
\Rightarrow 2m(\{c_1,c_2\})(\lambda^4 - 1) > [1 - m(\{c_1,c_2\})](\lambda^{10} + 1 - \lambda^4 - \lambda^5)
$$

$$
\Rightarrow m(\{c_1,c_2\})(\lambda^{10} - \lambda^5 + \lambda^4 - 1) > \lambda^{10} - \lambda^5 - \lambda^4 + 1
$$

Similarly, the constraint that the attitudinal parameter belongs to (0,1) can be obtained as well.

Solving Eqs. (10) and (11), $\lambda^4 - 1 > m(\{c_1\})(\lambda^{10} - \lambda^5 + \lambda^4 - 1)$, $m(\{c_1\}) = m(\{c_2\}) \geq 0$, $m(\{c_1,c_2\})(\lambda^{10} - \lambda^5 + \lambda^4 - 1) > \lambda^{10} - \lambda^5 - \lambda^4 + 1$ and $m(\{c_1\}) + m(\{c_2\}) + m(\{c_1,c_2\}) = 1$ are obtained as the constraints of the DM's preference order $A_3 \succ A_1 \sim A_2$ with $\lambda > 1$. As $\lambda \in (0, +\infty]$ and $\lambda \neq 1$, the Mobius transformed values of all possible subsets, $m(\{c_1\})$, $m(\{c_2\})$, $m(\{c_1,c_2\})$, cannot be restricted as specific values. Compared with the derived constraint, $v(\{c_1\}) = v(\{c_2\}) < 4v(\{c_1,c_2\})/9 = 4/9$ by the Choquet integral, the constraints deduced by Eqs. (10) and (11) limit the original cubic space for the Mobius values in a complex way with respect to λ. In other words, the input information of human decision behavior restricts the solution space by the ACI, which is difficult to obtain the analytical solutions of Mobius values. The limited solution space based on the ACI preference orders may be not convex, and the nonconvex space is not appropriate for the Hit-and-Run sampling [15]. Therefore, the rejection sampling [16] is adopted in this study instead of the hit and run algorithm [17-19].

B. *Descriptive measures in the ACI-SMAA for learning human preferences*

This section adopts the idea of the descriptive measures in the SMAA [6] to depict the output results and help human understand their internal preferences.

The original SMAA [20] used three descriptive measures, namely, the acceptability index, confidence factor and central weight vectors. Afterwards, SMAA-2 [21] utilized rank acceptability index, confidence factor and central weight vector. As for the SMAA-Choquet model [8], it considered the three descriptive measures in SMAA-2 as well but with the difference that the central weight vector contains the interactions between criteria. This study adopts the rank acceptability index, central attitudinal Choquet integral value and central Mobius vector with the focus on criteria interactions and attitudes at the same time.

1) *Rank acceptabiltiy index*

DMs' preferences restrict the feasible space of Mobius values and the attitudinal parameter. Let $M_i^r(x)$ denote the decision space which supports alternative A_i to be ranked at the r-th position with the criteria's Mobius values m and the attitudinal parameter λ. The rank acceptability index related to the support statement is calculated by

$$
q_i^r = \int_x f_X(x) \int_{M_i^r(x)} f_M(m) f_\lambda(\lambda) \, d\lambda \, dm \, dx \quad (12)
$$

where q_i^r represents the acceptability ratio of alternative A_i ranked at the r th position. f_X, f_M and f_λ are three probability density functions for the performance value x, the Mobius value m and the attitudinal parameter λ, respectively.

2) *Central attitudinal value*

Based on the rank acceptability of the first position, the central attitudinal value can be calculated by

$$\lambda_i^c = \frac{1}{q_i^1}\int_X f_X(x)\int_{M_i^r(x)} f_M(m) f_\lambda(\lambda)\lambda d\lambda dm dx \quad (13)$$

where q_i^1 means alternative A_i is the best option. Other symbols own the same meaning in Eq. (12).

3) Central Mobius vector

Compared with other SMAA models [20, 21], the interaction-considered central Mobius vector can be computed by

$$m_i^c = \frac{1}{q_i^1}\int_X f_X(x)\int_{M_i^r(x)} f_M(m) f_\lambda(\lambda) m d\lambda dm dx \quad (14)$$

where m_i^c is devised for the best alternative.

The computational complexity could be high if functions f_X, f_M and f_λ are determined specifically to implement straightforward integration on each dimension in the multi-dimensional integrals of Eqs. (12)-(14). For example, for a decision problem with four alternatives and four criteria, total dimensions for rank acceptability index is 20, because the outer integration in Eq. (12) goes through four-dimensional criteria space, and the inner one needs to integrate the space of all alternatives' performances on each criterion (4*4=16). To avoid the big computational effort, the Monte Carlo simulation technique [22] is adopted to sample in the feasible space and to obtain the approximate values of the integrals given in Eqs. (12)-(14).

IV. An Illustrative Exmaple: Household Energy Selection

This section gives an application example about how to solve household energy selection problem by using the proposed ACI-SMAA, and then compare the result with those obtained by the original utility model without criteria interactions and human attitudes.

A. Case description

Selecting the right household energy plays a vital role in the energy consumption management and the decision process is usually involving multiple dimensional information. The research report on China's household energy consumption [23] shows that China's household energy consumption level is at a relatively low level in the world, with biomass energy and electricity being the main sources of energy. The survey found that the average energy consumption of a standard Chinese household was 1087 kg (without transportation), 1208 kg(including transportation) of standard coal. To improve the energy utilization efficiency, the household could choose different types of energy according to diverse objectives. Four energy alternatives are obtained with respect to four criteria as shown in Table III.

TABLE III. Literature Related To The Choquet Inegral And Stochastic Analysis

Alternatives/criteria	Cooking	Heating	Hot water	Household appliances
Biomass energy A_1	10	0	4	2
Central heating A_2	0	10	1	5
Electricity A_3	4	5	3	3
Pipeline gas A_4	2	7	2	5

In this case, the k -order additive Choquet integral [5] is set as the 2-order additive Choquet. Then, $n(n+1)/2$ numbers of Mobius values and the attitudinal parameter λ should be determined by the DM, which is not a small cognitive effort-required work. The SMAA technique was introduced to analyze the preferences of the DM and help the DM to make decision. Suppose that the DM gives a preference as the alternative A_3 is better than A_2. The ACI-based preference of the DM can be modelled as the following constraints:

$$\begin{aligned} & ACI(A_2) < ACI(A_3) \\ & m(\{c_{i1}\}) \geq 0, \forall c_{i1} \in C \\ & m(\{c_{i1}\}) + \sum_{c_{i2}\in C\setminus\{c_{i1}\}} m(\{c_{i1}, c_{i2}\}) \geq 0 \\ & \sum_{c_{i1}\in C} m(\{c_{i1}\}) + \sum_{c_{i1},c_{i2}\in C} m(\{c_{i1}, c_{i2}\}) = 1 \end{aligned} \quad (15)$$

The first constraint denotes the DM's preference. The second constraint represents the non-negativity of each criterion. The third constraint indicates that the summation of the Mobius value of criterion c_{i1}, $m(\{c_{i1}\})$, and other interactions between the criterion c_{i1} and other criteria, $\sum_{c_{i2}\in C\setminus\{c_{i1}\}} m(\{c_{i1}, c_{i2}\})$, should be not less than zero. The last constraint shows the boundary of the importance of all criteria. There are more unknown variables than the number of constraints in Eq. (15). So, there might be multiple solutions of Eq. (15). In the following, the ACI-SMAA is introduced to sample possible solutions of Eq. (15) and analyze the unknown preferences on the remaining alternatives.

B. Solving the case by the ACI-SMAA

The preference $ACI(A_2) < ACI(A_3)$ is derived at first.

$$ACI(A_2) = \log_\lambda \begin{pmatrix} m(\{c_1\}) + \lambda^{10} m(\{c_2\}) + \lambda m(\{c_3\}) + \lambda^5 m(\{c_4\}) \\ + m(\{c_1, c_2\}) + m(\{c_1, c_3\}) + m(\{c_1, c_4\}) + \\ \lambda m(\{c_2, c_3\}) + \lambda^5 m(\{c_2, c_4\}) + \lambda m(\{c_3, c_4\}) \end{pmatrix}$$

$$ACI(A_3) = \log_\lambda \begin{pmatrix} \lambda^4 m(\{c_1\}) + \lambda^5 m(\{c_2\}) + \lambda^3 m(\{c_3\}) + \lambda^3 m(\{c_4\}) \\ + \lambda^4 m(\{c_1, c_2\}) + \lambda^3 m(\{c_1, c_3\}) + \lambda^3 m(\{c_1, c_4\}) + \\ \lambda^3 m(\{c_2, c_3\}) + \lambda^3 m(\{c_2, c_4\}) + \lambda^3 m(\{c_3, c_4\}) \end{pmatrix}$$

Then, the rejection sampling is implemented to analyze the DM's preferences in Eq. (15). It is observed that the DM may not tolerant some bad values because $A_3 \succ A_2$ and these utility values of the two alternatives are the same in the original utility model. Owing to this idea, the attitudinal parameter is restricted from zero to one. The rejection sampling process would stop when the number of acceptable samples reach one thousand. Using these samples, the ranks acceptability indices and central ACI values are listed in Tables IV-VI.

TABLE IV. RANKS ACCEPTABILITY INDICES

Alternatives	q_i^1	q_i^2	q_i^3	q_i^4
Biomass energy	7.3%	15.7%	**44.6%**	32.4%
Central heating	0.0%	0.8%	31.7%	**67.5%**
Electricity	**78.9%**	17.4%	3.7%	0.0%
Pipeline gas	13.8%	**66.1%**	20.0%	0.1%

TABLE V. CENTRAL ATTITUDINAL VECTOR AND CENTRAL MOBIUS VECTOR OF SIGNLE CRITERION

Alternatives	λ	$m(c_1)$	$m(c_2)$	$m(c_3)$	$m(c_4)$
A_1	0.7549	0.4611	0.1306	0.3400	0.1582
A_3	0.4722	0.2331	0.2303	0.2578	0.2312
A_4	0.5062	0.1989	0.2137	0.2373	0.2703

TABLE VI. CENTRAL MOBIUS VALUES OF INTERACTIONS

	$m(c_1,c_2)$	$m(c_1,c_3)$	$m(c_1,c_4)$	$m(c_2,c_3)$	$m(c_2,c_4)$	$m(c_3,c_4)$
A_1	-0.1718	-0.0440	0.0778	0.0864	0.0598	-0.0981
A_3	0.1975	-0.2526	0.0972	0.0916	-0.2656	0.1795
A_4	-0.3214	0.3405	0.0914	0.0724	0.3145	-0.4176

In Table IV, alternative A_2 cannot be ranked at the first position with the 0.0% probability because the DM thinks that A_3 is better than A_2. Alternative A_3 is the most likely to be the best with 81% probability. The data in Table IV can be visually shown in Fig. 3, where the height in the histogram represents the probability.

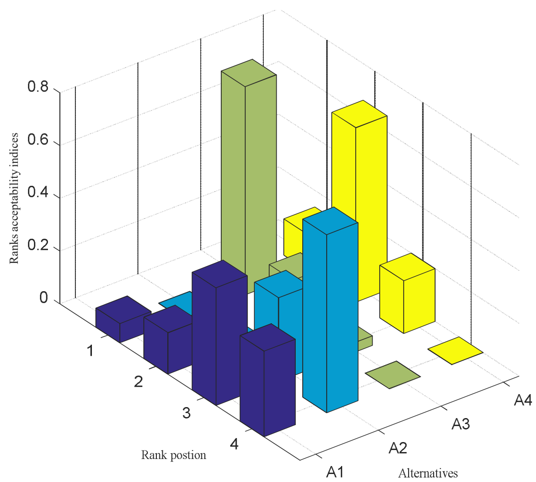

Fig. 3. Ranks acceptability indicies of four alterantives

Tables V and VI show that criteria c_1 and c_2 have the antagonistic effect. Alternative A_1 becomes the better alternative when criterion c_1 is more important. To sum up, the recommended ranking of alternatives is $A_3 \succ A_4 \succ A_1 \succ A_2$ with the maximal likelihood.

C. Discussions

The original utility model without criteria interactions and the attitudinal parameter would regard four alternatives as the same utility value. This violates the DM's given preference $A_3 \succ A_2$. In this example, the original utility model is not appropriate.

Some managerial implications can be summarized from the example. The input about the performances of alternatives and a single preference on two alternatives $A_3 \succ A_2$ are used in the ACI-SMAA model, and it outputs the most possible ranking of alternative energies. Compared with the original utility model, the ACI-SMAA requires less input information on criteria or alternatives and learn the DM's preference as much as possible. For policy-makers of energy management, the ACI-SMAA is an efficient tool to analyze other alternatives of household energies from the perspective of the DM's preferences.

V. CONCLUSIONS

This paper presented an ACI-SMAA model to analyze human decision behaviors on the basis of criteria interactions and DMs' attitudes in decision analysis. The interaction effect of the DM's criteria and attitudes were depicted by the ACI. The human decision behaviors on alternatives were transformed into the inequality or equality in the preference space. The Monte Carlo simulation based on rejection sampling was used to obtain the approximate values of three descriptive measures, namely, the rank acceptability index, central attitudinal value and central Mobius vector. Based on the sampling results, potential preferences of human decision behaviors and ranking results of alternatives could be acquired for decision analysis.

For the future research direction, the multiple criteria hierarchy process [24] for the attitudinal Choquet integral with SMAA is an interesting topic with challenges.

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