Universum least squares twin parametric-margin support vector machine

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Abstract—Universum based algorithms involve universum samples in the classification problem to improve the generalization performance. In order to provide prior information about data, we utilized universum data to propose a novel classification algorithm. In this paper, a novel parametric model for universum based twin support vector machine is presented for classification problems. The proposed model is termed as universum least squares twin parametric-margin support vector machine (ULSTPMSVM). The solution of ULSTPMSVM involves a system of linear equations. This makes the ULSTPMSVM efficient w.r.t. training time. In order to verify the performance of the proposed model, various experiments are carried out on real world benchmark datasets. Statistical tests are performed to verify the significance of the proposed method. The proposed ULSTPMSVM performed better than existing algorithms in terms of classification accuracy and training time for most of the datasets. Moreover, an application of proposed ULSTPMSVM is presented for classification of Alzheimer’s disease data.

Index Terms—Universum, twin parametric model, prior knowledge, magnetic resonance imaging, Alzheimer’s disease.

I. INTRODUCTION

Support vector machine (SVM) [1] is a prominent machine learning algorithm. One of the most efficient classification technique is the twin support vector machine (TWSVM) [2] algorithm. TWSVM constructs twin hyperplanes instead of one, thereby reducing the computation cost of the SVM model. A similar approach is followed to propose the twin support vector regression algorithms, using regularization [3] and weighting based approaches [4]. To improve the generalization performance of TWSVM, various models have been proposed in the past. Linear programming twin support vector machines are proposed by formulating unconstrained optimization problems. Regularization is incorporated with a smoothing technique to propose a smooth linear programming twin support vector machine (SLPT SVM) [5]. Moreover, an iterative algorithm is proposed as Newton method for linear programming twin support vector machine (NLPT SVM) [6].


A noise insensitive loss function is used to propose a general twin support vector machine using pinball loss (Pin-GTSVM) [8]. Further, a sparse model of Pin-GTSVM is proposed as sparse pinball twin support vector machine (SPTWSVM) [9]. To improve the robustness, an improved sparse twin support vector machine (ISPT SVM) is proposed [10] by including the structural risk minimization (SRM) principle. An alternative model termed as twin parametric-margin support vector machine (TPMSVM) [11] is also proposed to improve the generalization performance of TWSVM. Some other twin parametric based SVM models are proposed, such as fuzzy based models [12], [13] to improve the generalization performance.

To reduce the training time of TWSVM, a least squares twin support vector machine (LSTSVM) [14] is proposed in the past. The solution of least squares based algorithms involve a system of linear equations. This makes the LSTSVM algorithm efficient in comparison to TWSVM, which involves a pair of quadratic programming problems (QPPs). Various improvements on LSTSVM are proposed in the literature. For class imbalance learning, a robust fuzzy least squares twin support vector machine (RFLSTSVM-CIL) [15] is proposed to deal with imbalanced datasets. For noisy data, a least squares K-nearest neighbor-based weighted multi-class twin SVM [16] is formulated for multi-class classification. In a comprehensive evaluation [17] of 179 classifiers, the best classification accuracy is obtained by the robust energy based LSTSVM (RELS-T SVM) [18] algorithm.

Weston et al. [19] proposed a universum based support vector machine algorithm. The idea of universum based SVM (USVM) is to introduce prior knowledge of data distribution in the optimization problem of SVM. Due to due to higher generalization performance, universum based algorithms are applied in various applications such as EEG signal classification [20] to facial expression detection [21]. However, the inclusion of additional data points leads to increase in the computation cost of the model [22]–[24]. To remove these drawbacks, a twin support vector machine with universum data (UTSVM) [25] is proposed. Further, a least squares twin
support vector machine with universum data (ULSTPSVM) [26] is also proposed to reduce the computation time of UTSVM. Recently, an efficient angle based universum least squares twin support vector machine (AULSTPSVM) [27] is proposed for classification problems. Some sample screening techniques [28], [29] are proposed to reduce the training time in universe based learning.

Shao et al. [30] proposed a least squares twin parametric-margin support vector machine (LSTPMSVM), where the classifier is determined by parametric-margin based hyperplanes. In the spirit of LSTPMSVM, we propose a novel universum based least squares twin parametric-margin support vector machine (ULSTPMSVM). The key contributions of this paper are as follows:

- The proposed ULSTPMSVM involves an alternative formulation as compared to the existing ULSTPSVM algorithm.
- The solution of ULSTPMSVM involves the solution of system of linear equations. This makes it an efficient and novel universum based classification algorithm.
- The applicability of proposed ULSTPMSVM is shown by experiments on biomedical data.
- Statistical tests are performed to verify the difference between the proposed and existing algorithms.
- Experiments are performed on Alzheimer’s disease (AD) [31] data i.e. structural magnetic resonance imaging (sMRI) images to classify the different types of subjects.

In this work, all vectors are treated as column vectors, and the 2-norm of a vector $x$ is denoted by $\|x\|$. The rest of the paper is organized as follows: Section II presents the related work, while section III discusses the formulation of the proposed algorithm. The results of numerical experiments are presented in section IV. Lastly, section V concludes the paper with future work.

II. RELATED WORK

In this section, we discuss the formulations of LSTPSVM and ULSTPSVM algorithm in brief.

A. Least squares twin support vector machine (LSTPSVM)

The QPPs of LSTPSVM [14] for non-linear case are described as

$$
\min_{w_1, b_1, \eta_1} \frac{1}{2} \|K(A, D^T)w_1 + e_1 b_1\|^2 + \frac{c_1}{2} \eta_1^T \eta_1 \\
\text{s.t. } -(K(B, D^T)w_1 + e_2 b_1) + \eta_1 = e_2,
$$

(1)

$$
\min_{w_2, b_2, \eta_2} \frac{1}{2} \|K(B, D^T)w_2 + e_2 b_2\|^2 + \frac{c_2}{2} \eta_2^T \eta_2 \\
\text{s.t. } K(A, D^T)w_2 + e_1 b_2 + \eta_2 = e_1,
$$

(2)

where $c_i, i = 1, 2$ are positive parameters, $\eta_i, i = 1, 2$ denote the slack variables, $K(., \cdot)$ is the kernel matrix where $D = [A; B]$, and $e_1, e_2$ represent vectors consisting of ones of suitable dimensions.

By using the constraints in their respective objective functions, we get

$$
\min_{w_1, b_1} \frac{1}{2} \|K(A, D^T)w_1 + e_1 b_1\|^2, \\
+ \frac{c_1}{2} \|K(B, D^T)w_1 + e_2 b_1 + e_2\|^2,
$$

(3)

$$
\min_{w_2, b_2} \frac{1}{2} \|K(B, D^T)w_2 + e_2 b_2\|^2 \\
+ \frac{c_2}{2} \| - (K(A, D^T)w_2 + e_1 b_2) + e_1\|^2.
$$

(4)

Taking the gradient of QPP (3) w.r.t. $w_1$ and $b_1$, we get

$$
K(A, D^T)^T (K(A, D^T)w_1 + e_1 b_1) \\
+ c_1 K(B, D^T)^T (K(B, D^T)w_1 + e_2 b_1 + e_2) = 0,
$$

(5)

$$
e_1^T (K(A, D^T)w_1 + e_1 b_1) \\
+ c_1 e_2^T (K(B, D^T)w_1 + e_2 b_1 + e_2) = 0
$$

(6)

Combining Eqs. (5) and (6) and solving, we get

$$
[w_1 \ b_1]^T = -\left(G^T G + \frac{1}{c_1} H^T H\right)^{-1} G^T e_2,
$$

(7)

where $H = [K(A, D^T) \ e_1]$, and $G = [K(B, D^T) \ e_2]$. Similarly, using Eq. (4), we get

$$
[w_2 \ b_2]^T = \left(H^T H + \frac{1}{c_2} G^T G\right)^{-1} H^T e_1.
$$

(8)

For a testing data point $x$, the class is assigned using the following decision function

$$
\text{class}(x) = \arg \min_{i=1,2} \frac{|K(x, D^T)w_i + e_i b_i|}{\|w_i\|}.
$$

(9)

B. Least squares twin support vector machine with universum data (ULSTPSVM)

The QPPs of non-linear ULSTPSVM [25] are written as follows:

$$
\min_{w_1, b_1, \eta_1, \psi_1} \frac{1}{2} \|K(A, D^T)w_1 + e_1 b_1\|^2 + \frac{c_1}{2} \eta_1^T \eta_1 \\
+ \frac{c_3}{2} (\|w_1\|^2 + b_1^2) + \frac{c_4}{2} \psi_1^T \psi_1 \\
\text{s.t. } -(K(B, D^T)w_1 + e_2 b_1) + \eta_1 = e_2, \\
K(U, D^T)w_1 + e_u b_1 + \psi_1 = (-1 + \epsilon)e_u,
$$

(10)

$$
\min_{w_2, b_2, \eta_2, \psi_2} \frac{1}{2} \|K(B, D^T)w_2 + e_2 b_2\|^2 + \frac{c_2}{2} \eta_2^T \eta_2 \\
+ \frac{c_3}{2} (\|w_2\|^2 + b_2^2) + \frac{c_4}{2} \psi_2^T \psi_2 \\
\text{s.t. } K(A, D^T)w_2 + e_1 b_2 + \eta_2 = e_1, \\
-(K(U, D^T)w_2 + e_u b_2 + \psi_2 = (-1 + \epsilon)e_u,
$$

(11)
where \( c_i, i = 1, 2 \) are positive parameters, \( \eta_i, \psi_i, i = 1, 2 \) represent the slack variables, and \( e_1, e_2, e_u \) denote the vectors of ones of suitable dimensions.

Substituting the constraints in their objective functions, we get

\[
\min_{w_1, b_1} \frac{1}{2} \|K(A, D^T)w_1 + e_1b_1\|^2 + c_3 \left( \|w_1\|^2 + b_1^2 \right) \\
+ \frac{c_1}{2} \|K(B, D^T)w_1 + e_2b_1 + e_2\|^2 \\
+ \frac{c_u}{2} \| - (K(U, D^T)w_1 + e_u b_1) + (-1 + \epsilon) e_u \|^2,
\]

\[
(12)
\]

\[
\min_{w_2, b_2} \frac{1}{2} \|K(B, D^T)w_2 + e_2b_2\|^2 + c_4 \left( \|w_2\|^2 + b_2^2 \right) \\
+ \frac{c_2}{2} \| - (K(A, D^T)w_2 + e_1b_2) + e_1\|^2 \\
+ \frac{c_u}{2} \| (K(U, D^T)w_2 + e_u b_2) + (-1 + \epsilon) e_u \|^2.
\]

\[
(13)
\]

Taking the gradient of QPP (12) w.r.t. \( w_1 \) and \( b_1 \) and solving [26], we get

\[
[w_1 \ b_1]^T = - (P^TP + c_1Q^TQ + c_3 I + c_uR^TR)^{-1} (c_1 e_1^T e_2 \\
+ c_u (1 - \epsilon) R^T e_u),
\]

\[
(14)
\]

where \( P = [K(A, D^T) \ c_1], \ Q = [K(B, D^T) \ e_2], \) and \( R = [K(U, D^T) \ e_u] \). Similarly, using Eq. (13), we get

\[
[w_2 \ b_2]^T = (Q^TQ + c_2P^TP + c_4 I + c_u R^TR)^{-1} (c_2 P^T e_1 \\
+ c_u (1 - \epsilon) R^T e_u),
\]

\[
(15)
\]

For a testing data point \( x \), the class is assigned using Eq. (9).

III. PROPOSED UNIVERSUM LEAST SQUARES TWIN PARAMETRIC-MARGIN SUPPORT VECTOR MACHINE (ULSTPMSVM)

In this section, we present the formulation of proposed ULSTPMSVM for the linear and non-linear cases.

A. Linear ULSTPMSVM

The optimization problem of linear ULSTPMSVM is written as

\[
\min_{w_1, b_1, \eta_1, \psi_1} \frac{1}{2} \|w_1\|^2 + b_1^2 + \nu_1 e_2^T (Bw_1 + e_2b_1) + \frac{c_1}{2} \eta_1^T \eta_1 \\
+ \frac{c_u}{2} \psi_1^T \psi_1
\]

\[\text{s.t.} \quad Aw_1 + e_1b_1 = \eta_1, \]

\[Uw_1 + e_ub_1 + (1 - \epsilon) e_u = \psi_1,
\]

\[
(16)
\]

\[
\min_{w_2, b_2, \eta_2, \psi_2} \frac{1}{2} \|w_2\|^2 + b_2^2 - \nu_2 e_1^T (Aw_2 + e_1b_2) + \frac{c_2}{2} \eta_2^T \eta_2 \\
+ \frac{c_u}{2} \psi_2^T \psi_2
\]

\[\text{s.t.} \quad Bw_2 + e_2b_2 = \eta_2, \]

\[Uw_2 + e_ub_2 - (1 - \epsilon) e_u = \psi_2,
\]

\[
(17)
\]

where \( c_i, i = 1, 2 \), \( c_u \) are positive parameters, and \( \eta_i, \psi_i, i = 1, 2 \) represent the slack variables.

Using the constraints of Eqs. (16) and (17) in their respective objective functions, we get

\[
\min_{w_1, b_1} \frac{c_1}{2} (\|Aw_1 + e_1b_1\|^2) + \nu_1 e_2^T (Bw_1 + e_2b_1) \\
+ \frac{1}{2} (\|w_1\|^2 + b_1^2) + \frac{c_u}{2} (\|Uw_1 + e_ub_1 + (1 - \epsilon) e_u\|^2),
\]

\[
(18)
\]

\[
\min_{w_2, b_2} \frac{c_2}{2} (\|Bw_2 + e_2b_2\|^2) - \nu_2 e_1^T (Aw_2 + e_1b_2) \\
+ \frac{1}{2} (\|w_2\|^2 + b_2^2) + \frac{c_u}{2} (\|Uw_2 + e_ub_2 - (1 - \epsilon) e_u\|^2).
\]

\[
(19)
\]

Now, taking the gradient of QPP (18) w.r.t. \( w_1 \) and \( b_1 \) and equating to 0, we get

\[
c_1 A^T (Aw_1 + e_1b_1) + \nu_1 B^T e_2 + w_1 \\
+ c_u U^T (Uw_1 + e_ub_1 + (1 - \epsilon) e_u) = 0,
\]

\[
(20)
\]

\[
c_1 e_1^T (Aw_1 + e_1b_1) + \nu_1 e_2^T e_2 + b_1 \\
+ c_u e_1^T (Uw_1 + e_ub_1 + (1 - \epsilon) e_u) = 0.
\]

\[
(21)
\]

Combining Eqs. (20) and (21) and solving, we get

\[
w_1 \ b_1]^T = - (c_1 H^T H + c_u O^T O + I)^{-1} \nu_1 G^T e_2 \\
+ (1 - \epsilon) c_u O^T e_u),
\]

\[
(22)
\]

where \( H = [A; \ c_1], G = [B; \ e_2], \) and \( O = [U; \ e_u] \). Similarly, using Eq. (19), we get

\[
w_2 \ b_2]^T = (c_2 G^T G + c_u O^T O + I)^{-1} \nu_2 H^T e_1 \\
+ (1 - \epsilon) c_u O^T e_u).
\]

\[
(23)
\]

B. Non-linear ULSTPMSVM

The formulation of non-linear ULSTPMSVM involves kernel generated surfaces. The optimization problem comprises of the following two QPPs.

\[
\min_{w_1, b_1, \eta_1, \psi_1} \frac{1}{2} (\|w_1\|^2 + b_1^2) + \nu_1 e_2^T (K(B, D^T)w_1 + e_2b_1) \\
+ \frac{c_1}{2} \eta_1^T \eta_1 + \frac{c_u}{2} \psi_1^T \psi_1
\]

\[\text{s.t.} \quad K(A, D^T)w_1 + e_1b_1 = \eta_1, \]

\[K(U, D^T)w_1 + e_ub_1 + (1 - \epsilon) e_u = \psi_1,
\]

\[
(24)
\]

\[
\min_{w_2, b_2, \eta_2, \psi_2} \frac{1}{2} (\|w_2\|^2 + b_2^2) - \nu_2 e_1^T (K(A, D^T)w_2 + e_1b_2) \\
+ \frac{c_2}{2} \eta_2^T \eta_2 + \frac{c_u}{2} \psi_2^T \psi_2
\]

\[\text{s.t.} \quad K(B, D^T)w_2 + e_2b_2 = \eta_2, \]

\[K(U, D^T)w_2 + e_ub_2 - (1 - \epsilon) e_u = \psi_2,
\]

\[
(25)
\]

where \( c_i, i = 1, 2 \), \( c_u \) are positive parameters, \( K(\cdot, D^T) \) is the kernel matrix where \( D = [A; B] \), and \( \eta_i, \psi_i, i = 1, 2 \) represent the slack variables.
Substituting the constraints of Eqs. (24) and (25) in their respective objective functions, we get
\[
\min_{w_1, b_1} \frac{1}{2}(\|w_1\|^2 + b_1^2) + \frac{c_1}{2}(\|K(A, D^T)w_1 + e_1b_1\|^2)
+ \nu_1c_2^T(K(B, D^T)w_1 + e_2b_1)
+ \frac{c_2}{2}(\|K(U, D^T)w_1 + e_1b_1 + (1 - \epsilon)e_u\|^2),
\]
(26)
\[
\min_{w_2, b_2} \frac{1}{2}(\|w_2\|^2 + b_2^2) + \frac{c_2}{2}(\|K(B, D^T)w_2 + e_2b_2\|^2)
- \nu_2c_1^T(K(A, D^T)w_2 + e_1b_2)
+ \frac{c_1}{2}(\|K(U, D^T)w_2 + e_1b_2 - (1 - \epsilon)e_u\|^2).
\]
(27)
Solving similar to the linear case, we get
\[
[w_1\ b_1]^T = -(c_1P^TP + c_uR^TR + I)^{-1}(\nu_1Q^Te_2 + (1 - \epsilon)c_uR^Te_u),
\]
(28)
where \(P = [A; e_1], Q = [B; e_2],\) and \(R = [U; e_u].\)
Similarly, using Eq. (27), we get
\[
[w_2\ b_2]^T = (c_2Q^TQ + c_uR^TR + I)^{-1}(\nu_2P^Te_1 + (1 - \epsilon)c_uR^Te_u).
\]
(29)
The decision function of proposed ULSTPMSVM is same as in Eq. (9).

C. Time complexity

The time complexity of proposed ULSTPMSVM is lesser than existing algorithms such as TWSVM and ULSTSV. In comparison to TWSVM where QPPs are solved, proposed ULSTPMSVM solves a system of linear equations, leading to lesser computation cost [30]. Moreover, in comparison to ULSTSV, time complexity of proposed ULSTPMSVM is lesser because ULSTSV involves an additional matrix multiplication term in its solution (Eqs. 14 and 15), as compared to proposed ULSTPMSVM in Eqs. 28 and 29. However, the computation cost of proposed ULSTPMSVM is higher than LSTSV. This is due to the incorporation of universum data points in the proposed ULSTPMSVM algorithm.

IV. EXPERIMENTAL RESULTS

In this section, we show the results of experiments carried out on real world datasets, along with an application on Alzheimer’s disease. The real world datasets are taken from UCI [32], and KEEL repository [33].

All MRI images used in this work were downloaded from the Alzheimer’s Disease Neuroimaging Initiative (ADNI) database (adni.loni.usc.edu). ADNI was started in 2003 as a public-private partnership, led by Principal Investigator Michael W. Weiner, MD. The main objective of ADNI is to find out the effectiveness of neuroimaging techniques like MRI, positron emission tomography (PET), other biological markers, and clinical neuropsychological tests to estimate the onset of Alzheimer’s disease from the state of mild cognitive impairment. For more information, visit www.adni-info.org.

A. Experimental setup

All the experiments are carried out on a PC using Windows 10 OS 64 bit, 3.60 GHz Intel® core™ i7-7700 processor, 16 GB of RAM and MATLAB R2008b environment. To solve the QPPs in case of TWSVM, we utilized MOSEK optimization toolbox (http://www.mosek.com). 5-fold cross-validation is performed for all the methods to obtain the optimal hyperparameters. For non-linear case, Gaussian kernel is employed in all the cases.

For experiments on real world datasets, 50% data is used for training. The parameters \(c\) and \(\nu\) are chosen from the set \(\{10^{-5}, 10^{-4}, ..., 10^0\}\), while \(\mu\) is chosen from \(\{2^{-5}, 2^{-4}, ..., 2^5\}\). The value for \(\epsilon\) is selected from \(\{0.1, 0.4, 0.7\}\). The universum data is generated by performing random averaging of data points in all the cases [20], [21], [25].

In case of Alzheimer’s disease dataset, 40% of the samples are used for training. Freesurfer’s recon-all pipeline (version 6.0.1) [34] is applied for processing the structural MRI (sMRI) images. The total intracranial volume (TIV) is used for normalizing the volumetric features of brain in all the subjects [35]–[37].

B. Real world datasets

In Table I, performance comparison of proposed ULSTPMSVM is shown with existing algorithms on 18 real world benchmark datasets. The existing algorithms used for comparison in this work are TWSVM [2], LSTSV [14], LSTPMSVM [30], and ULSTSV [26]. One can observe that the proposed ULSTPMSVM is showing lowest average rank on the basis of accuracy. In terms of training time, the time taken by proposed ULSTPMSVM is comparable or lesser than the existing algorithms. It is noticeable that the training time of TWSVM is the highest. This is because the solution of TWSVM involves a pair of QPPs, as compared to systems of linear equations in least squares based algorithms.

C. Statistical analysis

To check the statistical difference, the Friedman test [38] is performed with the corresponding posthoc test. First, we assume that there is no difference between the methods. Now, the \(\chi^2\) value is calculated for Friedman test using average ranks \(r_i\) from Table I as,
\[
\chi^2_F = \frac{12 \times 18}{5(5+1)} \left( \sum_{i=1}^{5} r_i^2 - \frac{5(5+1)^2}{4} \right),
\]
\[
\chi^2_F = \frac{12 \times 18}{5(5+1)} \left( (3.8889^2 + 3.4167^2 + 3.0278^2 + 3.0556^2 + 1.6111^2) - \frac{5(5+1)^2}{4} \right) \approx 20.8605.
\]
The \(F_F\) value is calculated as
\[
F_F = \frac{(18 - 1)(20.8605)}{18 \times (5 - 1) - 20.8605} \approx 6.9345.
\]
TABLE I: Comparison of proposed ULSTPMSVM with existing algorithms on classification of real world datasets using Gaussian kernel function.

<table>
<thead>
<tr>
<th>Dataset (Sample size)</th>
<th>TWSVM Accuracy (%)</th>
<th>LSTPMSVM Accuracy (%)</th>
<th>LSTPMSVM Accuracy (%)</th>
<th>ULSTPMSVM Accuracy (%)</th>
<th>Proposed ULSTPMSVM Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(c, µ)</td>
<td>(c, µ)</td>
<td>(c, ν, µ)</td>
<td>(c, ν, µ)</td>
<td>(c, ν, µ)</td>
</tr>
<tr>
<td>Ecoli-0-1_vs_5 (242 x 6)</td>
<td>94.2149</td>
<td>90.9091</td>
<td>95.0413</td>
<td>95.0413</td>
<td>97.5207</td>
</tr>
<tr>
<td>Ecoli-0-1-4-7_vs_5-6 (334 x 6)</td>
<td>97.006</td>
<td>94.6108</td>
<td>97.6048</td>
<td>98.8024</td>
<td>98.8024</td>
</tr>
<tr>
<td>Ecoli-0-2-6-7_vs_3-5 (226 x 7)</td>
<td>93.8053</td>
<td>97.3451</td>
<td>96.4602</td>
<td>92.9204</td>
<td>96.4602</td>
</tr>
<tr>
<td>Ecoli-0-3-4-6_vs_5 (206 x 7)</td>
<td>97.0874</td>
<td>98.0583</td>
<td>95.1546</td>
<td>98.0583</td>
<td>98.0583</td>
</tr>
<tr>
<td>Ecoli-0-6-7_vs_3-5 (224 x 7)</td>
<td>91.0714</td>
<td>94.6429</td>
<td>91.9643</td>
<td>91.0714</td>
<td>95.5357</td>
</tr>
<tr>
<td>Ecoli4 (336 x 7)</td>
<td>97.6331</td>
<td>97.6331</td>
<td>97.6331</td>
<td>98.2249</td>
<td>98.2249</td>
</tr>
<tr>
<td>Glass-0-1-6_vs_2 (194 x 9)</td>
<td>89.6907</td>
<td>91.7526</td>
<td>92.7385</td>
<td>87.6289</td>
<td>92.7385</td>
</tr>
<tr>
<td>Glass-0-4_vs_5 (92 x 9)</td>
<td>91.4894</td>
<td>100</td>
<td>95.7447</td>
<td>100</td>
<td>97.8273</td>
</tr>
<tr>
<td>Heart-stat (270 x 13)</td>
<td>66.9118</td>
<td>62.3</td>
<td>65.4412</td>
<td>63.2353</td>
<td>67.6471</td>
</tr>
<tr>
<td>Led7digit-0-2-4-5-6-7-8-9_vs_1 (444 x 7)</td>
<td>91.8919</td>
<td>93.6937</td>
<td>96.8468</td>
<td>92.3423</td>
<td>92.7928</td>
</tr>
<tr>
<td>Ecoli-0-1-4-6_vs_5 (282 x 6)</td>
<td>98.5816</td>
<td>98.5816</td>
<td>98.5816</td>
<td>98.5816</td>
<td>99.2908</td>
</tr>
<tr>
<td>Ecoli2 (336 x 7)</td>
<td>91.716</td>
<td>86.9822</td>
<td>90.5325</td>
<td>94.0828</td>
<td>92.0777</td>
</tr>
<tr>
<td>Glass4 (282 x 6)</td>
<td>94.4444</td>
<td>96.2963</td>
<td>94.4444</td>
<td>96.2963</td>
<td>97.2222</td>
</tr>
<tr>
<td>Brwisconsin (683 x 9)</td>
<td>98.538</td>
<td>98.2456</td>
<td>98.538</td>
<td>98.2456</td>
<td>98.538</td>
</tr>
<tr>
<td>Ecoli3 (336 x 7)</td>
<td>91.716</td>
<td>86.9822</td>
<td>90.5325</td>
<td>94.0828</td>
<td>92.0777</td>
</tr>
<tr>
<td>Yeast1v7 (460 x 8)</td>
<td>91.7391</td>
<td>93.0435</td>
<td>93.4873</td>
<td>93.4873</td>
<td>93.913</td>
</tr>
<tr>
<td>Ecoli0137vs26 (312 x 7)</td>
<td>95.5128</td>
<td>96.1588</td>
<td>97.4359</td>
<td>96.1588</td>
<td>96.7949</td>
</tr>
<tr>
<td>Votes (436 x 16)</td>
<td>94.4954</td>
<td>94.4954</td>
<td>94.4954</td>
<td>94.0367</td>
<td>94.9541</td>
</tr>
<tr>
<td><strong>Average rank</strong></td>
<td>3.8889</td>
<td>3.4167</td>
<td>3.0278</td>
<td>3.0556</td>
<td>1.6111</td>
</tr>
</tbody>
</table>
This $F$-distribution involves $(5-1, (5-1)(18-1)) = (4, 68)$ degrees of freedom. Thus, for the significance level at $\alpha = 0.05$, the critical value for $F(4, 68)$ is 2.5066. Since, $F_F = 6.9345 > 2.5066$, the null hypothesis is rejected.

To check the pairwise difference between the proposed ULSTPMSVM and existing algorithms, we perform the Nemenyi posthoc test [38]. For significant pairwise difference between the methods at significance level of $\alpha = 0.10$, the average ranks of the algorithms shown in Table I should differ by at least $2.459 \sqrt{\frac{(5+1)(5+2)}{6(5+1)}} \approx 1.296$. The pairwise difference between the methods is shown in Table II.

TABLE II: Pairwise significant difference between proposed ULSTPMSVM and existing algorithms.

<table>
<thead>
<tr>
<th>Statistical difference</th>
<th>TWSVM</th>
<th>LSTSVM</th>
<th>LSTPMSVM</th>
<th>ULSTPMSVM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed ULSTPMSVM</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

It can be stated from Table II that the proposed ULSTPMSVM is significantly better than TWSVM, LSTSVM, LSTPMSVM, and ULSTPMSVM algorithms.

D. Insensitivity performance

The insensitivity analysis of proposed ULSTPMSVM is presented in Fig. 1. The variation of accuracy w.r.t. the penalty parameter $c$ and $\nu$ is shown for 4 datasets viz. Ecoli-0-1-4-7_vs_5-6, Ecoli-0-6-7_vs_3-5, Glass4, and Votes.

One can observe that the accuracy of proposed ULSTPMSVM increases with higher value of $c$, while $\nu$ does not have much effect on the accuracy. However, the accuracy is slightly higher for larger values of $\nu$.

E. Discussion

Experiments show the advantages of proposed ULSTPMSVM over existing algorithms. In comparison to the existing ULSTSV algorithm, the formulation of proposed ULSTPMSVM is novel and efficient. Moreover, in comparison to other algorithms involving QPPs such as TWSVM or UTSVM [25], proposed ULSTPMSVM requires very less computation time. Since, there are not many least squares based twin SVM models using universum, the proposed approach can be beneficial for many applications.

F. Alzheimer disease classification

In Alzheimer disease data, we have considered three classes namely control normal (CN), Alzheimer’s disease (AD), and mild cognitive impairment (MCI) [39], [40]. We include 50 sMRI images of CN and AD each, and 49 sMRI images of MCI, since one MCI image failed to process. The performance of proposed ULSTPMSVM and existing algorithms on classification of Alzheimer data is shown in Table III. One can see that proposed ULSTPMSVM performed better than other algorithms in 2 out of 3 datasets i.e. CN vs MCI, and MCI vs AD.

The highest accuracy of proposed ULSTPMSVM in MCI vs AD indicates that it may be used for the early diagnosis of Alzheimer’s disease. Moreover, the proposed ULSTPMSVM can be used for other diseases such as epilepsy, where the universum data is selected from the dataset itself [20], [23]. This may lead to higher classification accuracy for such problems.

V. Conclusion

A novel universum based algorithm is proposed in this work termed as universum least squares twin parametric-margin support vector machine (ULSTPMSVM). The proposed algorithms show high generalization performance with lesser training time in comparison to existing algorithms. The formulation of ULSTPMSVM is an alternative approach towards universum based learning. The optimization problem of ULSTPMSVM involves a parametric model, solved by a system of linear equations. In terms of statistical difference in the generalization performance, proposed ULSTPMSVM turns out to be significantly better than the existing algorithms. In future, ULSTPMSVM can be applied by selecting universum samples from different classes of the classification problem. Moreover, the proposed ULSTPMSVM performed well for classification of Alzheimer’s disease, showing its applicability on real world biomedical applications. For implementation, the codes of proposed algorithm will be publicly available on the author’s Github page i.e., https://github.com/mtanveer1/.

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Fig. 1: Plots showing insensitivity performance for the penalty parameter $c = c_1 = c_2$ with $\nu = \nu_1 = \nu_2$ for proposed ULSTPMSVM using Gaussian kernel function.

TABLE III: Performance comparison of proposed ULSTPMSVM on classification of Alzheimers data.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>TWSVM Accuracy (%)</th>
<th>LSTSVM Accuracy (%)</th>
<th>LSTPMSVM Accuracy (%)</th>
<th>ULSTSVN Accuracy (%)</th>
<th>Proposed ULSTPMSVM Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CN vs AD</td>
<td>85</td>
<td>80</td>
<td>80</td>
<td>85</td>
<td>76.6667</td>
</tr>
<tr>
<td>CN vs MCI</td>
<td>74.5763</td>
<td>59.322</td>
<td>76.2712</td>
<td>74.5763</td>
<td>76.2712</td>
</tr>
<tr>
<td>MCI vs AD</td>
<td>61.0169</td>
<td>44.0678</td>
<td>61.0169</td>
<td>42.3729</td>
<td>64.4068</td>
</tr>
</tbody>
</table>
