Adaptive Neural Consensus Control for Nonlinear Strict-Feedback Multiagent Systems With Switching Directed Topology*

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***Abstract*—This paper focuses on solving the adaptive consensus tracking problem of uncertain nonlinear strict-feedback multiagent systems with switching directed topology. From a viewpoint of switched system, the neighborhood synchronization error can be regarded as a nonlinear switching system and the switching signal is caused by the change of the topology, then an appropriate common Lyapunov function is constructed for the whole multiagent system with switching topology. By using the radial basis function neural networks (RBFNNs), the unknown nonlinear functions are compensated during the backstepping design procedure. Especially, instead of the common first-order filter in the conventional dynamic surface control (CDSC) technique, a novel nonlinear observer is presented to improve the control performance. A new common adaptive neural consensus control protocol is proposed for such systems based on the structure property of RBFNNs and the common Lyapunov function method. The developed control scheme guarantees that the consensus tracking errors between all followers' outputs and the output of leader can converge to a small neighborhood of the origin in the presence of switching directed communication topology. Finally, two illustrative examples are provided to show the effectiveness of the proposed consensus control methodology.**

***Index Terms*—nonlinear multiagent system, switching topology, adaptive consensus control, neural network, dynamic surface control**

I. INTRODUCTION

In the past decade, the nonlinear multiagent system has been one of the major concerns for researchers due to its wide applications in various fields such as unmanned air vehicles, mobile robots, and biological systems [1]- [2]. As a fundamental issue for the coordination of multiagent systems, consensus control thus has become a hot topic of research. Most of the available work focused on the first- or second-order multiagent systems [3]- [4], but an increasing attention has been paid to nonlinear multiagent systems with high-order dynamics since they are more suitable to describe the real-life systems. In particular, the well-known backstepping technique has been extended to the distributed control design for nonlinear multiagent systems [5]- [6]. Nevertheless, the repeated differentiation of virtual control functions easily gives rise to the problem of "explosion of complexity" during the process of backstepping design [7]. Thus, by including a first-order filtering of the virtual control function at each step, the CDSC method was presented to solve this problem and applied to many kinds of nonlinear systems [8]- [9]. Furthermore, the dynamic surface design was extended and applied to solving the distributed control problems of nonlinear multiagent systems [10]- [13]. Whereas it is worth noting that the derivatives of virtual control functions bounded by some unknown constants in the compact sets are not compensated during the CDSC design procedure, thus the control performance degrades inevitably to some extent.

On the other hand, it is requisite to point out that most of the previous results focus on multiagent systems with fixed communication topologies [3]- [6], [10]- [13]. Nevertheless, for practical multiagent systems, their interaction topologies are often unreliable because of the limited sensing region of sensors or the effect of obstacles. So far, many efforts have been made in solving consensus problems under switching topologies [14]- [22]. As mentioned in [14], the consensus of first-order multiagent systems with undirected topology can be achieved by "jointly connectivity" over a time interval. The results in [14] were extended to the directed information topology. By constraining the time interval between the consecutive switching, i.e., dwell time, a number of multiagent systems with switching topology were studied in [15]- [19] and some effective control protocols were presented for such systems. Besides, the cases of Markovian switching topologies have been addressed for several classes of multiagent systems in [20] and [21]. However, in the real world, the switching mechanism of communication topology may be usually unknown or too complicate to be used in the consensus analysis and control design, such that it is significant and challenging to investigate the case of switching topology under arbitrary switching. In [22], a common Lyapunov function is proposed to deal with the consensus problem of linear multiagent system under directed networks with switching topology. Unfortunately, no

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enough attention has been paid to the problem of adaptive distributed control for uncertain nonlinear multiagent systems with arbitrary switching communication topology.

With the preceding motivation, we are devoted to solving the adaptive consensus tracking problem of uncertain nonlinear strict-feedback multiagent systems with switching directed topology. Compared with the existing results, the main contributions of this paper are threefold: (1) It is the first time to effectively solve the consensus control problem of high-order strict-feedback nonlinear multiagent systems in the presence of switching topology. (2)A viewpoint of switched nonlinear system is exploited to investigate the consensus control problem of nonlinear multiagent systems with switching communication topology, and a common Lyapunov function is constructed to overcome the difficulties arising from the change of directed communication topology, such that the restriction on the information of switching topology can be greatly relaxed. (3)A novel nonlinear filter is presented to replace the common first-order filter, and to improve the performance of the CDSC design.

II. PROBLEM FORMULATION

A. Graph theory

A high-order nonlinear multi-agent system which is composed of $N+1$ agents including leader labeled as 0 and followers labeled as $1, 2, \dots, N$ is considered in this paper. Actually, with the change of interactions among the agents in the graph, there must exist the switching communication topologies. In order to depict the switching topologies, a finite family of all possible topologies is defined as $\bar{\mathcal{G}} = \{\bar{\mathcal{G}}_1, \bar{\mathcal{G}}_2, \dots, \bar{\mathcal{G}}_q\}$ and the set of index is described as $Q = \{1, 2, \dots, q\}$. For any $k \in Q$, $\sigma(t) = k$ means the topology $\bar{\mathcal{G}}_{\sigma(t)}$ (or $\bar{\mathcal{G}}_k$) is activated at time t for the multiagent system, where the switching signal $\sigma(t) : [0, +\infty) \to Q$ is a right continuous piecewise constant function. $\bar{\mathcal{G}}_k = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}_k\}$ with $\bar{\mathcal{V}} = \{0, 1, \dots, N\}$ and $\mathcal{E}_k$ denoting the communication among N followers and one leader. For a directed graph $\bar{\mathcal{G}}_k$, the direct edge $e_{ij}^k = (j, i) \in \mathcal{E}_k$ means that agent i can get information from agent j, not vice versa. Denote $\mathcal{N}_i^k = \{j | e_{ij}^k \in \mathcal{E}_k\}$ as the set of nodes with edges coming to node i. Under the directed graph $\bar{\mathcal{G}}_k$, the leader's adjacency matrix is defined as $B_k = \mathrm{diag}\{b_1^k, b_2^k, \dots, b_N^k\}$ with $b_i^k > 0$ if the leader $0 \in \mathcal{N}_i^k$ and $b_i^k = 0$ otherwise. In order to describe the communication of the followers separately, a subgraph of $\bar{\mathcal{G}}_k$ is defined as $\mathcal{G}_k = \{\mathcal{V}, \mathcal{E}_k\}$, where $\mathcal{V} = \{1, \dots, N\}$ denotes the set of N follower agents and $\mathcal{E}_k$ is a set of edges which stands for the communication between follower agents.Accordingly, the Laplacian matrix L_k is defined as $L_k = D_k - A_k$, where $D_k = \mathrm{diag}\{d_1^k, \dots, d_N^k\}$ with $d_i^k = \sum\limits_{j \in \mathcal{N}_i^k} a_{ij}^k$ and $A_k = [a_{ij}^k] \in R^{N \times N}$ defined as: when $e_{ij}^k \in \mathcal{E}_k$, $a_{ij}^k > 0$; when $e_{ij}^k \notin \mathcal{E}_k$, $a_{ij}^k = 0$.

B. Problem statement

Consider a nonlinear multiagent system consisting of a leader labeled as 0 and N followers labeled as $1, 2, \dots, N$, in which the ith follower's dynamics can be modeled as follows:

$$\begin{cases} \dot{x}_{i,m} = g_{i,m}(\bar{x}_{i,m}) x_{i,m+1} + f_{i,m}(\bar{x}_{i,m}), \\ \qquad m = 1, \dots, n_i - 1 \\ \dot{x}_{i,n_i} = g_{i,n_i}(\bar{x}_{i,n_i}) u_i + f_{i,n_i}(\bar{x}_{i,n_i}) \\ y_i = x_{i,1} \end{cases} \tag{1}$$

where $x_{i,m} \in R, m = 1, 2, \dots, n_i$ and $y_i \in R$ are the states and the output of the ith agent, respectively; denote $\bar{x}_{i,m} = [x_{i,1}, x_{i,2}, \dots, x_{i,m}]^T \in R^m$ for $m = 1, 2, \dots, n_i$ and let $x_{i,n_i+1} = u_i$. $f_{i,m}(\bar{x}_{i,m}) : R^m \to R$ and $g_{i,j} : R^m \to R$ are unknown smooth functions. $u_i \in R$ is the control input signal of the ith follower.

For the nonlinear multiagent systems (1) under switching directed communication graphs, the control objective is to design a distributed adaptive control protocol u_i such that all the outputs of followers y_i can synchronize with the output signal $y_r(t)$ of the leader within a certain error range. In other words, the consensus tracking of the entire multiagent systems can be achieved.

To attain the above control objective, some useful assumptions, definition and lemmas are introduced as follows.

Assumption 1. For $\forall\, m = 1, 2, \dots, n_i$, the signs of unknown nonlinear functions $g_{i,m}(\cdot), \forall i \in \mathcal{V}$ are known, and there exist unknown positive constants g_i^* such that $g_i^* \leq |g_{i,m}(\cdot)| < \infty$. Without loss of generality, it is further assumed that $g_i^* \leq g_{i,m}(\cdot) < \infty$.

Assumption 2. For the directed diagraph $\bar{\mathcal{G}}_k, \forall k \in Q$, the ith follower can only receive its neighboring agents' state messages, and the directed graph $\bar{\mathcal{G}}_k$ contains a spanning tree with the leader designated as the root node.

Assumption 3. The leader's output $y_r(t)$ and its derivatives $\dot{y}_r(t)$ and $\ddot{y}_r$ are continuous and bounded. Moreover, the output of leader can be acquired by its neighboring agents only.

Definition 1 [23]. For the nonlinear multiagent systems (1) under the directed switching topologies, the distributed consensus tracking errors between the followers and the leader are called to be cooperatively semiglobally uniformly ultimately bounded (CSUUB) if for any $y_i(t_0) - y_r(t_0) \in \Omega_{i,0}$ with $\Omega_{i,0}$ being a given compact set, there exist constant $\epsilon > 0$ and time $T(y_i(t_0) - y_r(t_0), \epsilon)$, such that $\|\mathrm{y} - \mathrm{y}_r\|^2 \leq \epsilon$ holds for $\forall t \geq t_0 + T$, where $\mathrm{y} = [y_1, \dots, y_N]^T$ and $\mathrm{y}_r = [y_r, \dots, y_r]^T$.

Lemma 1 [23]. For the directed diagraph $\bar{\mathcal{G}}_k, \forall k \in Q$ and the leader's adjacency matrix B_k, if there exists any $b_i^k > 0$, then $L_k + B_k$ is nonsingular.

Lemma 2 [23]. For multiagent systems with a directed diagraph $\bar{\mathcal{G}}_k, \forall k \in Q$, let $z_1 = [z_{1,1}, z_{2,1}, \dots, z_{N,1}]^T$, $\mathrm{y} = [y_1, y_2, \dots, y_N]^T$, and $\mathrm{y}_r = [y_r, y_r, \dots, y_r]^T$, then $\|\mathrm{y} - \mathrm{y}_r\| \leq \frac{\|z_1\|}{\sigma_{\min}(L_k + B_k)}$ holds, where $z_{i,1}, i = 1, 2, \dots, N$ will be defined later, and $\sigma_{\min}(L_k + B_k)$ is the minimum singular value of $L_k + B_k$.

Lemma 3 [24]. For any $\kappa > 0$ and any $z \in R$, the following inequality $0 \leq |z| - z \tanh(\frac{z}{\kappa}) \leq 0.2785\kappa$ holds.

In this paper, an unknown nonlinear function $f(Z) : R^m \to R$ on a compact set Ω_Z can be estimated by an RBFNN $\hat{f}(Z) = W^{*T}\Phi(Z)$, where $Z \in \Omega_Z \in R^m$ is the input vector, $W = [w_1, \dots, w_l]^T \in R^l$ stands for the weight vector with $l > 1$ being the number of NN nodes, and $\Phi(Z) = [\Phi_1(Z), \ldots, \Phi_l(Z)]^T$ is the basis function vector with $\Phi_i(Z) = \exp[\frac{-(Z-\mu_i)^T(Z-\mu_i)}{\eta_i^2}], i = 1, 2, \ldots, l$, in which $\mu_i \in R^m$ stands for the center of the basis function, and η_i is called the width of the basis function. Obviously, there exists an ideal constant weight vector W, such that $f(Z) = W^T\Phi(Z) + \varepsilon(Z), \forall Z \in \Omega_Z$, where $W := \arg\min_{W \in R^l}\{\sup_{z\in\Omega_Z} \left|f(Z) - W^{*T}\Phi(Z)\right|\}$, and $\varepsilon(Z)$ is the approximation error satisfying $\|\varepsilon(Z)\| \le \varepsilon^*$ with $\varepsilon^* > 0$ being a constant.

In order to facilitate the control design, an interesting characteristic of RBFNN is given by the following lemma.

Lemma 4 [5]. $\Phi(\bar{Z}_q) = [\Phi_1(\bar{Z}_q), \Phi_2(\bar{Z}_q), \ldots, \Phi_l(\bar{Z}_q)]^T$ is the basis function vector of RBFNN. The input vector is $\bar{Z}_q = [Z_1, Z_2, \ldots, Z_q]^T$, and for any positive integer $p \le q$, the inequality $\left\|\Phi(\bar{Z}_q)\right\|^2 \le \left\|\Phi(\bar{Z}_p)\right\|^2$ is established.

III. MAIN RESULT

A. Adaptive control design

In this section, the common Lyapunov function approach, the RBFNN approximation technique and the dynamic surface control method will be combined to develop a distributed adaptive neural control protocol for nonlinear multiagent system with arbitrary switching communication topology. The overall design procedure will include n_i steps for ith follower (1). At step $m, m = 1, 2, \ldots, n_i - 1$, an intermediate control functions α_m will be constructed. In order to alleviate the computing complexity caused by the repeated differentiations of α_{ij} during the regular backstepping design, a novel nonlinear filter will be proposed to obtain the filtered intermediate control function $s_{i,j}$. Finally, the actual control protocol u_i is obtained at step n_i.

Firstly, based on the communication topology $\bar{\mathcal{G}}_k, \forall k \in Q$, the neighborhood synchronization error $z_{i,1}$ is defined as

$$z_{i,1} = \sum_{j=1}^{N} a_{ij}^k(y_i - y_j) + b_i^k(y_i - y_r), \tag{2}$$

where a_{ij}^k is the element of the graph adjacency matrix A_k, and b_i^k represents the communication transfer weight between ith agent and the leader.

For $i = 1, \ldots, N$ and $j = 1, \ldots, n_i - 1$, the adaptive DSC design is begun with the following coordinate transformations

$$z_{i,j+1} = x_{i,j+1} - s_{i,j}, e_{i,j} = s_{i,j} - \alpha_{i,j}, \tag{3}$$

where $\alpha_{i,j}$ is the intermediate control function to be developed later, and $s_{i,j}$ is the filtered intermediate control function by passing $\alpha_{i,j}$ through the following nonlinear filter

$$\tau_{i,j}\dot{s}_{i,j} = -e_{i,j} - \tau_{i,j}\hat{M}_{i,j}\tanh(\tfrac{\hat{M}_{i,j}e_{i,j}}{\kappa_{i,j}}), \tag{4}$$

where $\tau_{i,j} > 0$ is a time constant, $\kappa_{i,j} > 0$ is a design constant, and $\hat{M}_{i,j}$ is the estimate of unknown constant $M_{i,j}$ to be given later. The estimation error $\tilde{M}_{i,j}$ is defined as $\tilde{M}_{i,j} = \hat{M}_{i,j} - M_{i,j}$, and the adaptive law for $\hat{M}_{i,j}$ is designed by

$$\dot{\hat{M}}_{i,j} = -\gamma_{i,j}\hat{M}_{i,j} + \chi_{i,j}\left|e_{i,j}\right|, \tag{5}$$

where $\gamma_{i,j}$ and $\chi_{i,j}$ are positive adjusting parameters.

Step 1: Combining (1) - (3), the time derivative of $z_{i,1}$ is easily derived by

$$\begin{aligned}\dot{z}_{i,1} &= (\sum_{j=1}^{N} a_{ij}^k + b_i^k)[g_{i,1}(z_{i,2} + e_{i,1} + \alpha_{i,1}) + f_{i,1}] \\ &\quad - \sum_{j=1}^{N} a_{ij}^k(g_{j,1}x_{j,2} + f_{j,1}) - b_i^k\dot{y}_r.\end{aligned} \tag{6}$$

Motivated by the viewpoint in [25], the dynamics of $z_{i,1}$ can be seen as a switched nonlinear system which the switching signal is caused by the change of communication topology. Then the common Lyapunov function method can be extended to explore it [26].

For any communication topology $\bar{\mathcal{G}}_k, \forall k \in Q$, construct the Lyapunov function candidate $V_{i,1}$ as $V_{i,1} = \frac{1}{2}z_{i,1}^2 + \frac{c_i^* g_i^*}{2\lambda_{i,1}}\tilde{\theta}_{i,1}^2$, where $\lambda_{i,1}$ is a positive constant, and $\tilde{\theta}_{i,1} := \hat{\theta}_{i,1} - \theta_{i,1}$ with $\hat{\theta}_{i,1}$ being the estimated value of $\theta_{i,1} = \max_{k\in\Omega}\{\frac{\|W_{i,1,k}\|^2}{c_i^* g_i^*}\}$ and $c_i^* := \min_{k\in Q}\{\sum_{j=1}^{N} a_{ij}^k + b_i^k\}$. Then the time derivative of $V_{i,1}$ is obtained as $\dot{V}_{i,1} = z_{i,1}\dot{z}_{i,1} + \frac{c_i^* g_i^*}{\lambda_{i,1}}\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}$.

An unknown nonlinear function is defined as $\bar{f}_{i,1}$ and can be approximated by using RBFNNs $W_{i,1,k}^T\Phi_{i,1}(Z_{i,1})$, i.e.

$$\begin{aligned}\bar{f}_{i,1} &= (\sum_{j=1}^{N} a_{ij}^k + b_i^k)f_{i,1} - \sum_{j=1}^{N} a_{ij}^k(g_{j,1}x_{j,2} + f_{j,1}) - b_i^k\dot{y}_r \\ &= W_{i,1,k}^T\Phi_{i,1}(Z_{i,1}) + \varepsilon_{i,1,k}(Z_{i,1}),\end{aligned} \tag{7}$$

where $Z_{i,1} = \left[\bar{x}_{i,2}, \bar{x}_{j,2}, b_i^k\dot{y}_r\right]^T$ $(j \in \mathcal{N}_i^k)$ is the input vector, and the approximation error $\varepsilon_{i,1,k}$ satisfies $|\varepsilon_{i,1,k}(Z_{i,1})| \le \varepsilon_{i,1}^*$, $\forall\, k \in Q$ with $\varepsilon_{i,1}^* > 0$ being a constant.

With the help of (7) and Lemma 4, it is easy to obtain that

$$\begin{aligned}z_{i,1}\bar{f}_{i,1} &\le |z_{i,1}|\left(\|W_{i,1,k}\|\,\|\Phi_{i,1}(Z_{i,1})\| + |\varepsilon_{i,1,k}(Z_{i,1})|\right) \\ &\le |z_{i,1}|\left(\|W_{i,1,k}\|\,\|\Phi_{i,1}(\xi_{i,1})\| + |\varepsilon_{i,1,k}(Z_{i,1})|\right) \\ &\le \tfrac{1}{2h_{i,1}^2}c_i^* g_i^* z_{i,1}^2\theta_{i,1}\Phi_{i,1}^T(\xi_{i,1})\Phi_{i,1}(\xi_{i,1}) + \tfrac{1}{2}h_{i,1}^2 \\ &\quad + \tfrac{c_i^* g_i^*}{2}z_{i,1}^2 + \tfrac{\varepsilon_{i,1}^{*2}}{2c_i^* g_i^*}.\end{aligned} \tag{8}$$

where $\xi_{i,1} = x_{i,1}$, and $h_{i,1}$ is a positive constant.

Choose the intermediate control function $\alpha_{i,1}$ as

$$\alpha_{i,1} = -\tfrac{1}{2h_{i,1}^2}z_{i,1}\hat{\theta}_{i,1}\Phi_{i,1}^T(\xi_{i,1})\Phi_{i,1}(\xi_{i,1}) - (p_{i,1} + \tfrac{1}{2})z_{i,1}, \tag{9}$$

where $p_{i,1}$ is a positive design parameter.

The adaptive law of $\hat{\theta}_{i,1}$ is given by

$$\dot{\hat{\theta}}_{i,1} = -\beta_{i,1}\hat{\theta}_{i,1} + \tfrac{\lambda_{i,1}}{2h_{i,1}^2}z_{i,1}^2\Phi_{i,1}^T(\xi_{i,1})\Phi_{i,1}(\xi_{i,1}), \tag{10}$$

where $\beta_{i,1} > 0$ is a design parameter. It is worth pointing out that the initial value of $\hat{\theta}_{i,1}$ is set to meet $\hat{\theta}_{i,1}(0) > 0$ in order that $\hat{\theta}_{i,1}$ can be seen as a positive variable for all $t \ge 0$.

Then, it follows (6)-(10) that

$$\dot{V}_{i,1} \le -c_i^* g_i^* p_{i,1} z_{i,1}^2 + (\sum_{j=1}^{N} a_{ij}^k + b_i^k) g_{i,1} z_{i,1}(z_{i,2} + e_{i,1}) + \varpi_{i,1} - \frac{\beta_{i,1} c_i^* g_i^*}{\lambda_{i,1}} \tilde{\theta}_{i,1} \hat{\theta}_{i,1}, \quad (11)$$

where $\varpi_{i,1} = \frac{1}{2} h_{i,1}^2 + \frac{1}{2c_i^* g_i^*} \varepsilon_{i,1}^{*2}$.

Step m ($2 \le m \le n_i$): According to (1), (3) and (4), it is easy to obtain the dynamic system of $z_{i,m}$ as

$$\dot{z}_{i,m} = g_{i,m}(z_{i,m+1} + e_{i,m} + \alpha_{i,m}) + f_{i,m} + \hat{M}_{i,m-1} \tanh(\frac{\hat{M}_{i,m-1} e_{i,m-1}}{\kappa_{i,m-1}}) + \frac{e_{i,m-1}}{\tau_{i,m-1}}. \quad (12)$$

For any communication topology $\bar{\mathcal{G}}_k, \forall k \in Q$, define the Lyapunov function candidate $V_{i,m}$ as $V_{i,m} = V_{i,m-1} + \frac{1}{2} z_{i,m}^2$.

Define the unknown nonlinear function $\bar{f}_{i,m}$ as $\bar{f}_{i,2,k} = f_{i,2} + (\sum_{j=1}^{N} a_{ij}^k + b_i^k) g_{i,1} z_{i,1} + \hat{M}_{i,1} \tanh(\frac{\hat{M}_{i,1} e_{i,1}}{\kappa_{i,1}}) + \frac{e_{i,1}}{\tau_{i,1}}$ and $\bar{f}_{i,m,k} = f_{i,m} + g_{i,m-1} z_{i,m-1} + \hat{M}_{i,m-1} \tanh(\frac{\hat{M}_{i,m-1} e_{i,m-1}}{\kappa_{i,m-1}}) + \frac{e_{i,m-1}}{\tau_{i,m-1}} - \Delta_{i,m}(Z_{i,m})$ for $m = 3, \ldots, n_i$, where $\Delta_{i,m}(Z_{i,m})$ is a continuous function to be specified later. Reconstruct $\bar{f}_{i,m,k}$ by using RBFNNs $W_{i,m,k}^T \Phi_{i,m}(Z_{i,m})$, then we have

$$\bar{f}_{i,m,k} = W_{i,m,k}^T \Phi_{i,m}(Z_{i,m}) + \varepsilon_{i,m,k}(Z_{i,m}), \quad (13)$$

where $Z_{i,2} = [x_{i,1}, x_{i,2}, z_{i,1}, e_{i,1}, \hat{M}_{i,1}]^T$ and $Z_{i,m} = [\bar{x}_{i,m}, z_{i,m-1}, e_{i,1}, \ldots, e_{i,m-1}, \hat{\theta}_{i,2}, \hat{M}_{i,m-1}]^T$ for $m = 3, \ldots, n_i$. Moreover, the reconstruction error is subject to $|\varepsilon_{i,m,k}(Z_{i,m})| \le \varepsilon_{i,m}^*$ with $\varepsilon_{i,m}^*$ being a positive constant.

Choose the intermediated control function $\alpha_{i,m}$ as

$$\alpha_{i,m} = -\frac{\hat{\theta}_{i,2} z_{i,m}}{2h_{i,m}^2} \Phi_{i,m}^T(Z_{i,m}) \Phi_{i,m}(Z_{i,m}) - (p_{i,m} + \frac{1}{2}) z_{i,m}, \quad (14)$$

where $h_{i,m}$ and $p_{i,m}$ are positive design parameters.

By using (11)-(14), taking the derivation of $V_{i,m}$ yields

$$\begin{aligned}\dot{V}_{i,m} \le& -c_i^* g_i^* p_{i,1} z_{i,1}^2 - \sum_{j=2}^{m} g_i^* p_{i,j} z_{i,j}^2 + (\sum_{j=1}^{N} a_{ij}^k + b_i^k) g_{i,1} e_{i,1} z_{i,1} \\ &+ \sum_{j=2}^{m} g_{i,j} e_{i,j} z_{i,j} + g_{i,m} z_{i,m} z_{i,m+1} - \frac{\beta_{i,1} c_i^* g_i^*}{\lambda_{i,1}} \tilde{\theta}_{i,1} \hat{\theta}_{i,1} \\ &+ \frac{g_i^*}{\lambda_{i,2}} \tilde{\theta}_{i,2} (\dot{\hat{\theta}}_{i,2} - \sum_{j=2}^{m} \frac{\lambda_{i,2}}{2h_{i,j}^2} z_{i,j}^2 \Phi_{i,j}^T(Z_{i,j}) \Phi_{i,j}(Z_{i,j})) \\ &+ \sum_{m=3}^{n_i} z_{i,m} \Delta_{i,m}(Z_{i,m}) + \varpi_{i,m}, \end{aligned} \quad (15)$$

where $\varpi_{i,m} = \frac{1}{2} \sum_{j=1}^{m} h_{i,j}^2 + \frac{1}{2c_i^* g_i^*} \varepsilon_{i,1}^{*2} + \frac{1}{2g_i^*} \sum_{j=2}^{m} \varepsilon_{i,j}^{*2}$.

So far, the inductive design steps are completed. Especially, when $m = n_i$, the actual control input for ith agent system appears, i.e., $u_i = \alpha_{i,n_i}$, and $z_{i,n_i+1} = 0$. Moreover, the adaptive parameter $\hat{\theta}_{i,2}$ is updated by

$$\dot{\hat{\theta}}_{i,2} = -\beta_{i,2} \hat{\theta}_{i,2} + \sum_{j=2}^{n_i} \frac{\lambda_{i,2}}{2h_{i,j}^2} z_{i,j}^2 \Phi_{i,j}^T(Z_{i,j}) \Phi_{i,j}(Z_{i,j}), \quad (16)$$

where $\beta_{i,2}$ is a positive design constant. Similar to the adaptive parameter $\hat{\theta}_{i,1}$, the initial value of $\hat{\theta}_{i,2}$ is also set by $\hat{\theta}_{i,2}(0) > 0$ such that $\hat{\theta}_{i,2}(t) > 0$ holds for all $t > 0$.

B. Consensus analysis

From equations (3)-(4) and for $j = 1, \ldots, n_i - 1$, the dynamics of boundary layer errors are given by

$$\dot{e}_{i,j} = -\frac{e_{i,j}}{\tau_{i,j}} - \hat{M}_{i,j} \tanh(\frac{\hat{M}_{i,j} e_{i,j}}{\kappa_{i,j}}) - \dot{\alpha}_{i,j}, \quad (17)$$

where $\dot{\alpha}_{i,1} = \frac{\partial \alpha_{i,1}}{\partial x_{i,1}} \dot{x}_{i,1} + \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_{i,1}} \dot{\hat{\theta}}_{i,1} + \frac{\partial \alpha_{i,1}}{\partial z_{i,1}} \dot{z}_{i,1} + \frac{\partial \alpha_{i,1}}{\partial \dot{y}_r} \ddot{y}_r$, and $\dot{\alpha}_{i,j} = \sum_{m=1}^{j} \frac{\partial \alpha_{i,j}}{\partial x_{i,m}} \dot{x}_{i,m} + \frac{\partial \alpha_{i,j}}{\partial z_{i,j}} \dot{z}_{i,j} + \frac{\partial \alpha_{i,j}}{\partial z_{i,j-1}} \dot{z}_{i,j-1} + \sum_{m=2}^{j} \frac{\partial \alpha_{i,j}}{\partial e_{i,m-1}} \dot{e}_{i,m-1} + \frac{\partial \alpha_{i,j}}{\partial \hat{\theta}_{i,2}} \dot{\hat{\theta}}_{i,2} + \frac{\partial \alpha_{i,j}}{\partial \hat{M}_{i,j-1}} \dot{\hat{M}}_{i,j-1}$.

For the ith multiagent subsystem in any topology $\bar{\mathcal{G}}_k$, consider the common Lyapunov function candidate V_i as $V_i = V_{i,n_i} + \frac{1}{2} \sum_{m=1}^{n_i-1} e_{i,m}^2 + \frac{1}{2} \sum_{m=1}^{n_i-1} \frac{1}{\chi_{i,m}} \tilde{M}_{i,m}^2$.

By using the inequalities $-\tilde{\theta}_{i,q} \hat{\theta}_{i,q} \le -\frac{1}{2} \tilde{\theta}_{i,q}^2 + \frac{1}{2} \theta_{i,q}^2$ with $q = 1, 2$, we have

$$-\frac{\beta_{i,1} c_i^* g_i^*}{\lambda_{i,1}} \tilde{\theta}_{i,1} \hat{\theta}_{i,1} \le -\frac{\beta_{i,1} c_i^* g_i^*}{2\lambda_{i,1}} \tilde{\theta}_{i,1}^2 + \frac{\beta_{i,1} c_i^* g_i^*}{2\lambda_{i,1}} \theta_{i,1}^2, \quad (18)$$

$$-\frac{\beta_{i,2} g_i^*}{\lambda_{i,2}} \tilde{\theta}_{i,2} \hat{\theta}_{i,2} \le -\frac{\beta_{i,2} g_i^*}{2\lambda_{i,2}} \tilde{\theta}_{i,2}^2 + \frac{\beta_{i,2} g_i^*}{2\lambda_{i,2}} \theta_{i,2}^2. \quad (19)$$

Note that $\dot{\hat{\theta}}_{i,2}$ is relevant to the error variables $z_{i,2}, z_{i,3}, \ldots, z_{i,n_i}$, and is also related to the state variables $x_{i,2}, x_{i,3}, \ldots, x_{i,n_i}$, thus the term $e_{i,m-1} \frac{\partial \alpha_{i,m-1}}{\partial \hat{\theta}_{i,2}} \dot{\hat{\theta}}_{i,2}$ should be handled in a different way from the common DSC design [10]- [13]. By using the fact that $0 < \Phi_{i,m}^T \Phi_{i,m} \le l_{i,m}$ with $l_{i,m}$ being the number of NN's nodes, it is easy to obtain that

$$\begin{aligned} & -\sum_{m=3}^{n_i} e_{i,m-1} \frac{\partial \alpha_{i,m-1}}{\partial \hat{\theta}_{i,2}} \dot{\hat{\theta}}_{i,2} \\ \le & -\sum_{m=3}^{n_i} e_{i,m-1} \frac{\partial \alpha_{i,m-1}}{\partial \hat{\theta}_{i,2}} [-\beta_{i,2} \hat{\theta}_{i,2} \\ & + \sum_{j=2}^{m-1} \frac{\lambda_{i,2}}{2h_{i,j}^2} z_{i,j}^2 \Phi_{i,j}^T(Z_{i,j}) \Phi_{i,j}(Z_{i,j})] \\ & + \sum_{m=3}^{n_i} \frac{\lambda_{i,2}}{2h_{i,m}^2} l_{i,m} z_{i,m}^2 \sum_{j=3}^{m} |e_{i,j-1} \frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_{i,2}}|. \end{aligned} \quad (20)$$

Thus, for $m = 3, \ldots, n_i$, define the function $\Delta_{i,m}(Z_{i,m})$ as $\Delta_{i,m}(Z_{i,m}) = -\frac{\lambda_{i,2}}{2h_{i,m}^2} l_{i,m} z_{i,m} \sum_{j=3}^{m} |e_{i,j-1} \frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_{i,2}}|$ such that $\sum_{m=3}^{n_i} [z_{i,m} \Delta_{i,m}(Z_{i,m}) - e_{i,m-1} \frac{\partial \alpha_{i,m-1}}{\partial \hat{\theta}_{i,2}} \dot{\hat{\theta}}_{i,2}] \le 0$.

Moreover, designate some continuous functions as $\mathcal{B}_{i,1} = -\dot{\alpha}_{i,1} + (\sum_{j=1}^{N} a_{ij}^k + b_i^k) g_{i,1} z_{i,1}$ and $\mathcal{B}_{i,m} = -\sum_{j=1}^{m} \frac{\partial \alpha_{i,m}}{\partial x_{i,j}} \dot{x}_{i,j} - \frac{\partial \alpha_{i,m}}{\partial z_{i,m}} \dot{z}_{i,m} - \frac{\partial \alpha_{i,m}}{\partial z_{i,m-1}} \dot{z}_{i,m-1} - \sum_{j=1}^{m} \frac{\partial \alpha_{i,m}}{\partial e_{i,j}} \dot{e}_{i,j} - \frac{\partial \alpha_{i,m}}{\partial \hat{M}_{i,j-1}} \dot{\hat{M}}_{i,j-1} + g_{i,m} z_{i,m} + \frac{\partial \alpha_{i,m}}{\partial \hat{\theta}_{i,2}} [\beta_{i,2} \hat{\theta}_{i,2} - \sum_{j=2}^{m} \frac{\lambda_{i,2}}{2h_{i,j}^2} z_{i,j}^2 \Phi_{i,j}^T(Z_{i,j}) \Phi_{i,j}(Z_{i,j})]$.

Obviously, for $m = 1, 2, \ldots, n_i - 1$ and $\forall k \in Q$, the continuous functions $|\mathcal{B}_{i,m}|$ acquire their maximums $M_{i,m}$ on the compact sets $\Omega_{i,m,k} := \{\frac{1}{2}\sum_{j=1}^{m+1} z_{i,j}^2 + \frac{1}{2}\sum_{j=1}^{m} e_{i,j}^2 + \frac{c_i^* g_i^*}{2\lambda_{i,1}}\tilde{\theta}_{i,1}^2 + \frac{g_i^*}{2\lambda_{i,2}}\tilde{\theta}_{i,2}^2 + \frac{1}{2}\sum_{j=1}^{m}\frac{1}{\chi_{i,j}}\tilde{M}_{i,j}^2 + \sum_{j\in N_i^k}(\frac{1}{2}z_{j,2}^2 + \frac{1}{2}z_{j,3}^2 + \frac{1}{2}e_{j,1}^2 + \frac{1}{2}e_{j,2}^2 + \frac{1}{2\chi_{j,1}}\tilde{M}_{j,1}^2 + \frac{1}{2\chi_{j,2}}\tilde{M}_{j,2}^2 + \frac{c_j^* g_j^*}{2\lambda_{j,1}}\tilde{\theta}_{j,1}^2 + \frac{g_j^*}{2\lambda_{j,2}}\tilde{\theta}_{j,1}^2) \le r_0\}$ and $\Omega_0 = \{y_r^2 + \dot{y}_r^2 + \ddot{y}_r^2 \le B_r\}$ with B_r being a positive constant, i.e., it is obtained that $|\mathcal{B}_{i,m}| \le M_{i,m}$ with $M_{i,m}$ being estimated by $\hat{M}_{i,m}$ for $m = 1, 2, \ldots, n_i - 1$. Then, the following inequality is easily established

$$e_{i,m}\mathcal{B}_{i,m} \le |e_{i,m}|\, M_{i,m} = |e_{i,m}|\,(\hat{M}_{i,m} - \tilde{M}_{i,m}). \qquad (21)$$

From Lemma 3, we have

$$\hat{M}_{i,m}\,|e_{i,m}| \le \hat{M}_{i,m}e_{i,m}\tanh(\tfrac{\hat{M}_{i,m}e_{i,m}}{\kappa_{i,m}}) + 0.2785\kappa_{i,m}. \qquad (22)$$

Combining (16)-(22) easily gives

$$\dot{V}_i \quad \le \quad -\psi_i V_i + \zeta_i, \qquad (23)$$

where $\psi_i = \min\limits_{j=1,\ldots,n_i-1}\{2c_i^* g_i^* p_{i,1}, 2g_i^* p_{i,j+1}, \beta_{i,1}, \beta_{i,2}, \frac{2}{\tau_{i,j}}, \gamma_{i,j}\}$ and $\zeta_i = \frac{1}{2}\sum_{j=1}^{n_i} h_{i,j}^2 + \frac{1}{2c_i^* g_i^*}\varepsilon_{i,1}^{*2} + \frac{1}{2g_i^*}\sum_{j=2}^{n_i}\varepsilon_{i,j}^{*2} + \frac{\beta_{i,1}c_i^* g_i^*}{2\lambda_{i,1}}\theta_{i,1}^2 + \frac{\beta_{i,2}g_i^*}{2\lambda_{i,2}}\theta_{i,2}^2 + \frac{1}{2}\sum_{m=1}^{n_i-1}\frac{\gamma_{i,m}}{\chi_{i,m}}M_{i,m}^2 + 0.2785\sum_{m=1}^{n_i-1}\kappa_{i,m}$.

For all the whole multiagent systems with any communication topology $\bar{\mathcal{G}}_k, k \in Q$, designate the common Lyapunov function candidate V as $V = \sum_{i=1}^{N} V_i$. Then, from (23), taking the derivative of V yields

$$\dot{V} \le -\psi V + \zeta, \qquad (24)$$

where $\psi = \min\limits_{i=1,2,\ldots,N}\psi_i$ and $\zeta = \sum_{i=1}^{N}\zeta_i$.

Using the similar arguments in [11] - [13], for a given constant $r_0 > 0$, if $V = r_0$ and $\psi > \frac{\zeta}{r_0}$, then $\dot{V} \le 0$. Accordingly, if $V(0) \le r_0$, then $V(t) \le r_0$, $\forall\, t \ge 0$, which also means that (24) holds for all $t \ge 0$ and $V(0) \le r_0$. Multiplying both sides of (24) by $e^{\psi t}$ and integrating them over $[0, t]$ gives

$$0 \le V(t) \le \frac{\zeta}{\psi} + [V(0) - \frac{\zeta}{\psi}]e^{-\psi t}, \qquad (25)$$

which also implies that $V(t) \le \frac{\zeta}{\psi}, t \to \infty$.

In view of the definition of V and (25), we have

$$\|z_1\|^2 \le 2V(t) \le \frac{2\zeta}{\psi}, t \to \infty. \qquad (26)$$

where $z_1 = [z_{1,1}, z_{2,1}, \ldots, z_{N,1}]^T$.

The values of the parameters $p_{i,1}, p_{i,j}, \beta_{i,1}, \beta_{i,2}, \chi_{i,j}$ can be chosen large, and the values of the parameters $\lambda_{i,1}, \lambda_{i,2}, \gamma_{i,j}, h_{i,j}, \tau_{i,j}$ can be adjusted small, such that for a given small constant $\epsilon > 0$, the following inequality is satisfied

$$\frac{\zeta}{\psi} \le \frac{\epsilon}{2}\min_{k\in Q}\{\sigma_{\min}(L_k + B_k)\}. \qquad (27)$$

By applying (26)-(27) and Lemma 2, the tracking error vector satisfies $\|\mathrm{y} - \mathrm{y_r}\|^2 \le \epsilon,\ t \to \infty$, which also indicates that the tracking errors $y_i - y_r (i = 1, \ldots, N)$ are CSUUB in any communication topology $\bar{\mathcal{G}}_k, k \in Q$.

At this stage, the following theorem is presented to summarize the main result.

Theorem 1. Consider the uncertain strict-feedback nonlinear multiagent systems (1) with switching communication topology satisfying Assumptions 1-3, if the distributed adaptive neural control protocols (9),(14) together with the nonlinear filters (4), (5) and the parameter adaptive laws (10), (16) are utilized, then, for any initial condition satisfying $V(0) \le r_0$ and $\hat{\theta}_{i,j}(0) > 0 (j = 1, 2)$, all signals in the closed-loop system are uniformly ultimately bounded and the consensus tracking errors can remain in a small neighborhood of the origin.

Remark 1. From a practical viewpoint, a restriction that the initial values should meet $V(0) \le r_0$ is imposed on all the variables. Nevertheless, it is requisite to note that r_0 can be chosen enough large, thus this restriction is quite relaxed and the forgoing condition $\psi > \frac{\zeta}{r_0}$ could be easily fulfilled.

Remark 2. Note that the repeated differentiations of intermediate control functions must result in the problem of "explosion of complexity" during the traditional backstepping design, the conventional DSC (CDSC) method often introduce the common first-order filter to solve the problem [10]- [13]. However, the effects of unknown nonlinear functions $\mathcal{B}_{i,j}(\cdot)$ arising from the dynamics of boundary layer errors are not compensated during the CDSC technique, which usually cause the degradation of consensus tracking performance. During the modified DSC (MDSC) method in this paper, some novel nonlinear filters (4) and the adjusting laws (5) are included to compensate for the unknown bounds $M_{i,j}$ of nonlinear functions $\mathcal{B}_{i,j}(\cdot)$.

IV. SIMULATION STUDIES

In order to verify the effectiveness of the preceding theoretical results, two simulation examples are presented in this section. The first example is a numerical example, and the second one is a practical example which can further show the applicability of the proposed control protocol.

Example 1. The uncertain nonlinear multiagent systems consisting of five followers and one leader, where the dynamics of the ith$(i = 1, 2, \ldots, 5)$ follower is given by

$$\begin{cases} \dot{x}_{i,1} = g_{i,1}(x_{i,1})x_{i,2} + f_{i,1}(x_{i,1}) \\ \dot{x}_{i,2} = g_{i,2}(x_{i,1}, x_{i,2})u_i + f_{i,2}(x_{i,1}, x_{i,2}) \\ y_i = x_{i,1} \end{cases} \qquad (28)$$

where $g_{1,1} = 3 + 0.5\sin(x_{1,1})$, $f_{1,1} = 0.1x_{1,1}\sin(x_{1,1})$, $g_{1,2} = 2 + 0.5\sin(x_{1,1}) + \sin(x_{1,2})$, $f_{1,2} = 0.5x_{1,1}\sin(x_{1,1}) + 0.5\cos(x_{1,2})$; $g_{2,1} = 2 + \cos(x_{2,1})$, $f_{2,1} = 1.5x_{2,1}$, $g_{2,2} = 2.5 + 0.5\cos(x_{2,1}) - \sin(x_{2,2})$, $f_{2,2} = x_{2,1} + 0.7x_{2,2}\cos(x_{2,2})$; $g_{3,1} = 1.7 + 0.4\sin(x_{3,1})$, $f_{3,1} = 0.5x_{3,1}\cos(x_{3,1})$, $g_{3,2} = 3 + \frac{1}{x_{3,1}^2+1} + \cos(x_{3,2})$, $f_{3,2} = x_{3,1} + 0.5x_{3,2}\cos(x_{3,2})$; $g_{4,1} = 2 + \cos(x_{4,1})$, $f_{4,1} = 0.5x_{4,1}\cos(x_{4,1})$, $g_{4,2} = 1.5 + \sin(x_{4,1} + x_{4,2})$, $f_{4,2} = x_{4,1}\sin(x_{4,1}) + \cos(x_{4,2})$; $g_{5,1} = 3 + \cos(x_{5,1})$,

$f_{5,1} = 0.2x_{5,1}$, $g_{5,2} = 2.5 + \cos(x_{5,1}) + 0.3\cos(x_{5,2})$, and $f_{5,2} = 0.7x_{5,2}\sin(x_{5,1})$.

The communication graphs of all the agents including five followers and one leader are shown as Fig.1, and Fig. 2 describes the change among four communication topologies.

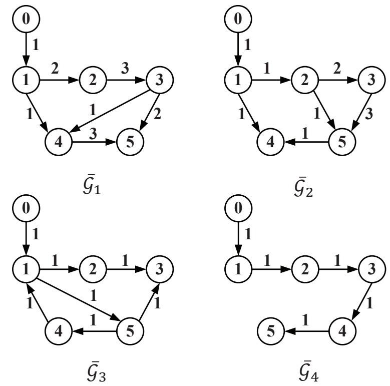

Fig. 1. Three communication diagraphs $\bar{\mathcal{G}}_1 - \bar{\mathcal{G}}_4$ for Example 1.

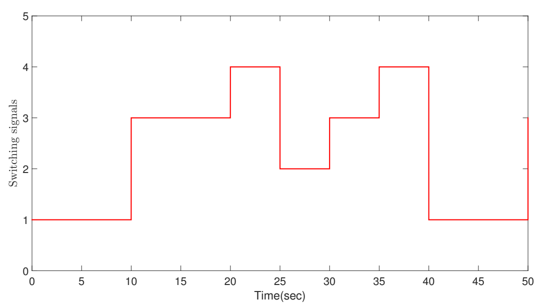

Fig. 2. The switching mechanism for Example 1.

In this simulation, the output signals of five followers are forced to follow the leader's output signal $y_r = \sin(t) + \sin(0.5t)$. For RBFNNs and $i = 1, 2, \ldots, 5$, $\Phi_{i,1}(\xi_{i,1})$ contain 3^2 neurons with center evenly located in $[-3, 3] \times [-3, 3]$,and the width set by 1. $\Phi_{i,2}(Z_{i,2})$ contain 3^5 neurons with center evenly located in $[-3, 3] \times [-3, 3] \times [-3, 3] \times [-3, 3] \times [-3, 3]$ and the widths set by 1.

Moreover, the design parameters are chosen as follows: $\beta_{1,1} = 17$, $\lambda_{1,1} = 0.1$, $h_{1,1} = 0.05$, $p_{1,1} = 7$, $\tau_{1,1} = 0.006$, $\kappa_{1,1} = 0.04$, $\gamma_{1,1} = 0.01$, $\chi_{1,1} = 50$, $\beta_{1,2} = 13$, $\lambda_{1,2} = 0.05$, $h_{1,2} = 0.05$, $p_{1,2} = 10$; $\beta_{2,1} = 13$, $\lambda_{2,1} = 0.6$, $h_{2,1} = 0.6$, $p_{2,1} = 10$, $\tau_{2,1} = 0.002$, $\kappa_{2,1} = 0.05$, $\gamma_{2,1} = 0.1$, $\chi_{2,1} = 60$, $\beta_{2,2} = 15$, $\lambda_{2,2} = 0.6$, $h_{2,2} = 0.05$, $p_{2,2} = 5$; $\beta_{3,1} = 17$, $\lambda_{3,1} = 0.6$, $h_{3,1} = 0.5$, $p_{3,1} = 3$, $\tau_{3,1} = 0.005$, $\kappa_{3,1} = 0.04$, $\gamma_{3,1} = 0.1$, $\chi_{3,1} = 60$, $\beta_{3,2} = 16$, $\lambda_{3,2} = 0.7$, $h_{3,2} = 0.07$, $p_{3,2} = 20$; $\beta_{4,1} = 13$, $\lambda_{4,1} = 0.8$, $h_{4,1} = 0.05$, $p_{4,1} = 10$, $\tau_{4,1} = 0.005$, $\kappa_{4,1} = 0.04$, $\gamma_{4,1} = 0.15$, $\chi_{4,1} = 50$, $\beta_{4,2} = 6$, $\lambda_{4,2} = 0.8$, $h_{4,2} = 0.05$, $p_{4,2} = 3$; $\beta_{5,1} = 10$, $\lambda_{5,1} = 1.7$, $h_{5,1} = 0.5$, $p_{5,1} = 5$, $\tau_{5,1} = 0.005$, $\kappa_{5,1} = 0.04$, $\gamma_{5,1} = 0.2$, $\chi_{5,1} = 55$, $\beta_{5,2} = 13$, $\lambda_{5,2} = 0.8$, $h_{5,2} = 0.04$, $p_{5,2} = 6$.

The initial states are chosen as $x_{1,1}(0) = 0.1$, $x_{1,2}(0) = 0.2$;$x_{2,1}(0) = 0.15$, $x_{2,2}(0) = 0.1$; $x_{3,1}(0) = 0.1$,$x_{3,2}(0) = 0.05$; $x_{4,1}(0) = 0.2$, $x_{4,2}(0) = 0.15$;$x_{5,1}(0) = 0.05$, $x_{5,2}(0) = 0.1$. For the adaptive parameters,the initial values are set by $\hat{\theta}_{1,1}(0) = 1.5$, $\hat{M}_{1,1}(0) = 2$, $\hat{\theta}_{1,2}(0) = 2$; $\hat{\theta}_{2,1}(0) = 2.2$, $\hat{M}_{2,1}(0) = 1.7$, $\hat{\theta}_{2,2}(0) = 2.5$; $\hat{\theta}_{3,1}(0) = 1.3$, $\hat{M}_{3,1}(0) = 1.5$, $\hat{\theta}_{3,2}(0) = 1.5$; $\hat{\theta}_{4,1}(0) = 1.7$, $\hat{M}_{4,1}(0) = 1.5$, $\hat{\theta}_{4,2}(0) = 1$; $\hat{\theta}_{5,1}(0) = 1$, $\hat{M}_{5,1}(0) = 1.3$, $\hat{\theta}_{5,2}(0) = 0.8$.

The simulation results are shown in Figs.3-6. The outputs of followers and leader are depicted in Fig. 3. Fig. 4 gives the control input signals of five followers. The boundedness of adaptive parameters $\hat{\theta}_{i,1}$ and $\hat{\theta}_{i,2}$ are demonstrated in Fig. 5 and Fig. 6, respectively.

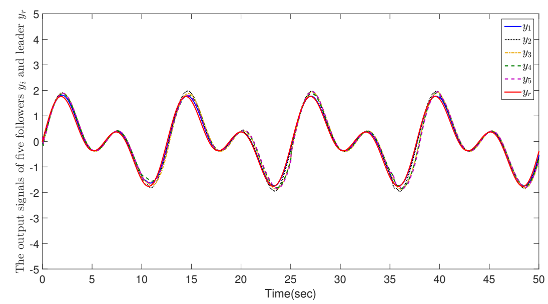

Fig. 3. Consensus tracking outputs for Example 1.

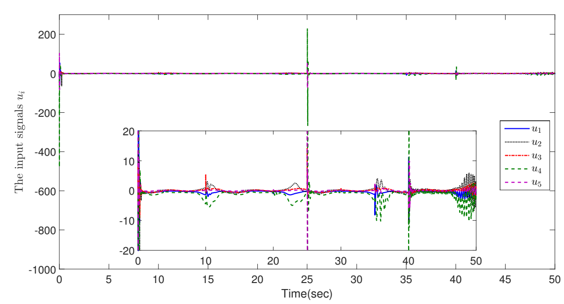

Fig. 4. Control inputs of five followers for Example 1.

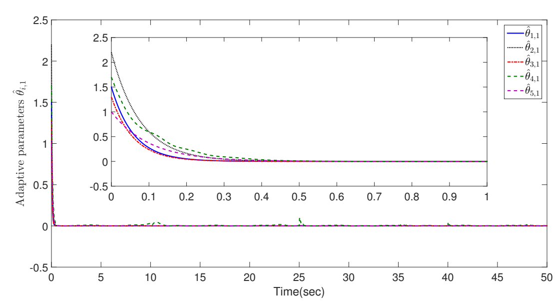

Fig. 5. Adaptive parameters $\hat{\theta}_{i,1}$ of five followers for Example 1.

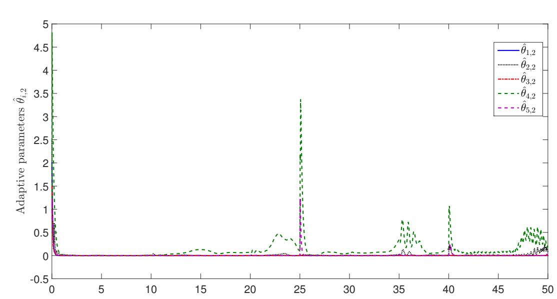

Fig. 6. Adaptive parameters $\hat{\theta}_{i,2}$ of five followers for Example 1.

Example 2. In this section, a group of one-link manipulators is provided to verify the applicability of the proposed control protocol. The dynamics of the system can be described by [13]

$$\begin{cases} D_i\ddot{q}_i + B_i\dot{q}_i + N_i\sin(q_i) = I_i \\ M_i\dot{I}_i = -H_iI_i - K_{m,i}\dot{q}_i + V_i \end{cases} \tag{29}$$

where q_i, $\dot{q}_i$, $\ddot{q}_i$, I_i and V_i denote the angular position, velocity, acceleration, motor current and input voltage, respectively. Let $x_{i,1} = q_i$, $x_{i,2} = \dot{q}_i$, $x_{i,3} = \ddot{q}_i$ and control input $u_i = V_i$, then the equation (29) can be expressed as follows

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \dot{x}_{i,2} = g_{i,2}x_{i,3} + f_{i,2}(\bar{x}_{i,2}) \\ \dot{x}_{i,3} = g_{i,3}u_i + f_{i,3}(\bar{x}_{i,3}) \\ y_i = x_{i,1} \end{cases} \tag{30}$$

where $g_{i,2} = 1/D_i$, $f_{i,2} = [-N_i\sin(x_{i,1}) - B_ix_{i,2}]/D_i$, $g_{i,3} = 1/M_i$, and $f_{i,3} = (-K_{m,i}x_{i,2} - H_ix_{i,3})/M_i, i = 1,2,3$. The parameters are chosen as $D_i = 1$, $B_i = 1$, $N_i = 10$, $M_i = 0.05$, $H_i = 0.5$ and $K_{m,i} = 10$.

Consider the multiagent system with three followers described as (30) and one leader, the distributed adaptive neural control protocols proposed in section III are adopted for each follower to synchronize with the leader's output $y_r = \frac{\pi}{2}\sin(t) + \cos(0.5t)$. In the simulation, there exist three different communication topologies which change as the time. The topology graphs are shown in Fig.7, and the switching mechanism among them is exhibited in Fig.8.

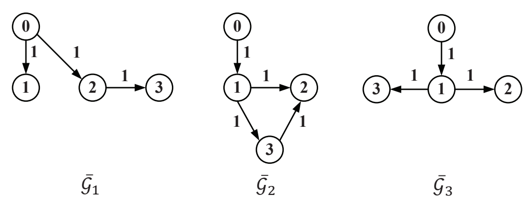

Fig. 7. Three communication diagraphs $\bar{\mathcal{G}}_1 - \bar{\mathcal{G}}_3$ for Example 2.

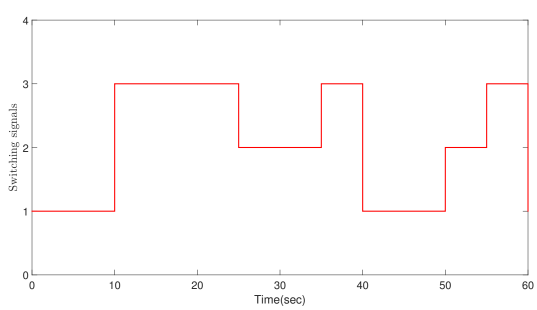

Fig. 8. The switching mechanism for Example 2.

For RBFNNs and $i = 1,2,3$, $\Phi_{i,1}(\xi_{i,1})$ contain 3 neurons with center evenly located in $[-3,3]$. $\Phi_{i,2}(Z_{i,2})$ contain 3^5 neurons with center located in $[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]$ evenly.$\Phi_{i,3}(Z_{i,3})$ contain 3^8 neurons with center located in $[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]$ evenly. All the width set by 1.

The design parameters are chosen as follows: $\beta_{1,1} = 1.7$, $\lambda_{1,1} = 0.3$, $h_{1,1} = 0.5$, $p_{1,1} = 15$, $\tau_{1,1} = 0.036$, $\kappa_{1,1} = 0.04$, $\gamma_{1,1} = 0.2$, $\chi_{1,1} = 50$, $h_{1,2} = 0.5$, $p_{1,2} = 9$, $\tau_{1,2} = 0.036$, $\kappa_{1,2} = 0.04$, $\gamma_{1,2} = 0.1$, $\chi_{1,2} = 55$, $\beta_{1,2} = 1.3$, $\lambda_{1,2} = 0.5$, $h_{1,3} = 0.7$, $p_{1,3} = 7$; $\beta_{2,1} = 1.3$, $\lambda_{2,1} = 0.6$, $h_{2,1} = 0.6$, $p_{2,1} = 12$, $\tau_{2,1} = 0.036$, $\kappa_{2,1} = 0.04$, $\gamma_{2,1} = 0.15$, $\chi_{2,1} = 60$, $h_{2,2} = 0.5$, $p_{2,2} = 5$, $\tau_{2,2} = 0.036$, $\kappa_{2,2} = 0.04$, $\gamma_{2,2} = 0.1$, $\chi_{2,2} = 54$, $\beta_{2,2} = 1.6$, $\lambda_{2,2} = 0.1$, $h_{2,3} = 0.7$, $p_{2,3} = 5$; $\beta_{3,1} = 2$, $\lambda_{3,1} = 0.6$, $h_{3,1} = 0.4$, $p_{3,1} = 13$, $\tau_{3,1} = 0.036$, $\kappa_{3,1} = 0.04$, $\gamma_{3,1} = 0.1$, $\chi_{3,1} = 65$, $h_{3,2} = 0.08$, $p_{3,2} = 10$, $\tau_{3,2} = 0.036$, $\kappa_{3,2} = 0.04$, $\gamma_{3,2} = 0.2$, $\chi_{3,2} = 60$, $\beta_{3,2} = 1.7$, $\lambda_{3,2} = 0.4$, $h_{3,3} = 0.7$, and $p_{3,3} = 10$. All the initial values are given by $x_{1,1}(0) = 0.1$, $x_{1,2}(0) = 0.2$, $x_{1,3}(0) = 0.1$; $x_{2,1}(0) = 0.1$, $x_{2,2}(0) = 0.05$, $x_{2,3}(0) = 0.14$; $x_{3,1}(0) = 0.1$, $x_{3,2}(0) = 0.07$, $x_{3,3}(0) = 0.13$; $\hat{\theta}_{1,1}(0) = 0.1$, $\hat{M}_{1,1}(0) = 1$, $\hat{M}_{1,2}(0) = 2.1$, $\hat{\theta}_{1,2}(0) = 0.3$; $\hat{\theta}_{2,1}(0) = 0.2$, $\hat{M}_{2,1}(0) = 1.5$, $\hat{M}_{2,2}(0) = 1.6$, $\hat{\theta}_{2,2}(0) = 0.2$; $\hat{\theta}_{3,1}(0) = 0.15$, $\hat{M}_{3,1}(0) = 2.3$, $\hat{M}_{3,2}(0) = 2$, and $\hat{\theta}_{3,2}(0) = 0.15$.

Figs.9-13 give the simulation results. Fig.9 illustrates the outputs $y_i(i = 1,2,3)$ of followers and the output y_r of leader,which implies that the satisfactory consensus tracking performance can be achieved by the proposed control scheme. Fig.10 depicts the distributed control input signals for three followers. Finally, the trajectories of adaptive parameters $\hat{\theta}_{i,j}, i = 1,2,3, j = 1,2$ are kept bounded as seen in Figs.11 and 12. Let $E(t) = \sqrt{\sum_{i=1}^{3}(y_i - y_r)^2}$ denote the consensus tracking error of system (30) with switching communication topology. Fig.13 shows the comparison with the CDSC scheme, which indicates that the consensus tracking performance can be improved by the modified DSC (MDSC) method.

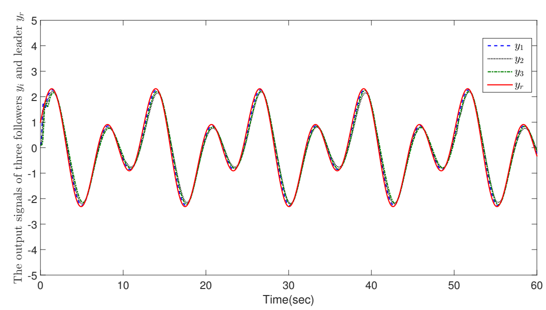

Fig. 9. Consensus tracking outputs for Example 2.

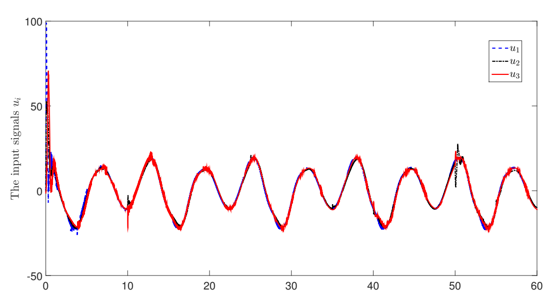

Fig. 10. Control inputs of three followers for Example 2.

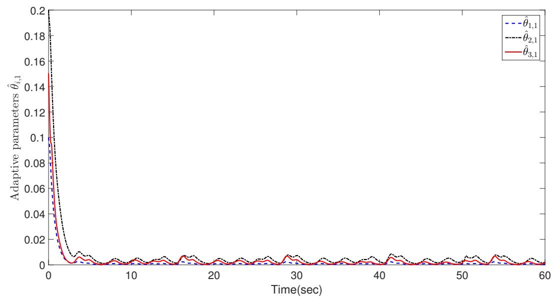

Fig. 11. Adaptive parameters $\hat{\theta}_{i,1}$ of three followers for Example 2.

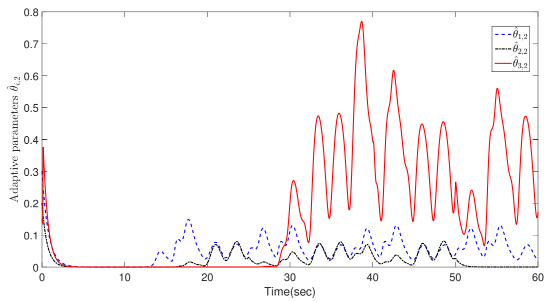

Fig. 12. Adaptive parameters $\hat{\theta}_{i,2}$ of three followers for Example 2.

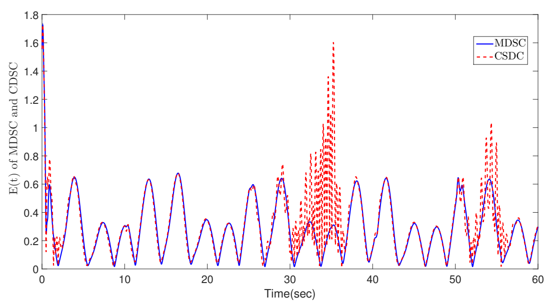

Fig. 13. Tracking errors $E(t)$ of MDSC and CDSC for Example 2.

V. Conclusions

This paper solved the problem of adaptive neural consensus control for uncertain strict-feedback nonlinear multiagent systems with the switching directed topology. The given design methodology has the following characteristics: (1) The switching mechanism of communication topology need not be known in prior; (2) The common Lyapunov function is constructed to develop a novel distributed adaptive neural control protocol for the arbitrary switching communication topology; (3) A new nonlinear observer is proposed to improve the convention DSC design method. Our future work will explore the adaptive output feedback control for nonlinear multiagent systems with the switching communication topology.

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